Exercise 7.11 For each of the following entailments, give a derivation in Elf (if it holds in intuitionistic logic) or indicate that it does not hold.

1. $\forall x.\ A(x) \wedge B(x) \vdash (\forall x.\ A(x)) \wedge (\forall x.\ B(x))$.
2. $(\forall x.\ A(x)) \wedge (\forall x.\ B(x)) \vdash \forall x.\ A(x) \wedge B(x)$.
3. $\forall x.\ \bot \vdash \bot$.
4. $\bot \vdash \forall x.\ \bot$.
5. $\forall x.\ \forall y.\ A(x, y) \vdash \forall y.\ \forall x.\ A(x, y)$.
6. $\forall x.\ \neg A(x) \vdash \neg \exists x.\ A(x)$.
7. $\neg \exists x.\ A(x) \vdash \forall x.\ \neg A(x)$.
8. $\exists x.\ A(x) \vee B(x) \vdash (\exists x.\ A(x)) \vee (\exists x.\ B(x))$.
9. $(\exists x.\ A(x)) \supset B \vdash \forall x.\ (A(x) \supset B)$.
10. $\forall x.\ (A(x) \supset B) \vdash (\exists x.\ A(x)) \supset B$.
11. $(\forall x.\ A(x)) \supset B \vdash \exists x.\ (A(x) \supset B)$.
12. $\exists x.\ (A(x) \supset B) \vdash (\forall x.\ A(x)) \supset B$.

Exercise 7.12 Show that the system of natural deduction extended with the rule of excluded middle XM is equivalent to the system of natural deduction extended with proof by contradiction $\bot_C$ (see page 242). Implement this proof in Elf.