15-462 Computer Graphics I
Lecture 13

Clipping

Line Clipping
Polygon Clipping
Clipping in Three Dimensions
[Angel 8.3-8.7]

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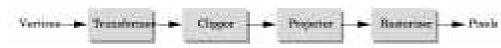
http://www.cs.cmu.edu/~fp/courses/graphics/

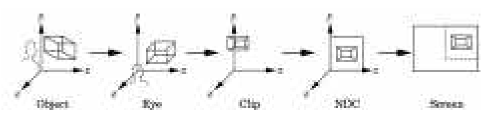
The Graphics Pipeline, Revisited



- Must eliminate objects outside viewing frustum
- Tied in with projections
  - Clipping: object space (eye coordinates)
  - Scissoring: image space (pixels in frame buffer)
- Introduce clipping in stages
  - 2D (for simplicity)
  - 3D (as in OpenGL)
- In a later lecture: scissoring

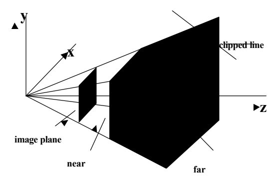
Transformations and Projections

- Sequence applied in many implementations
  1. Object coordinates to
  2. Eye coordinates to
  3. Clip coordinates to
  4. Normalized device coordinates to
  5. Screen coordinates



Clipping Against a Frustum

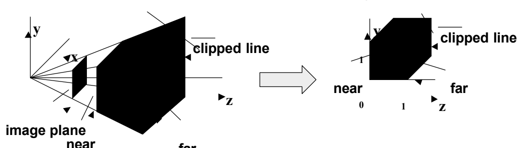
- General case of frustum (truncated pyramid)



- Clipping is tricky because of frustum shape

Perspective Normalization

- Solution:
  - Implement perspective projection by perspective normalization and orthographic projection
  - Perspective normalization is a homogeneous tfm.



See [Angel Ch. 5.8]

The Normalized Frustum

- OpenGL uses -1 $\leq$ x,y,z $\leq$ 1 (others possible)
- Clip against resulting cube
- Clipping against programmer-specified planes is different and more expensive
- Often a useful programming device

The Viewport Transformation

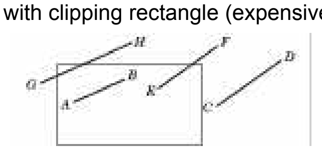
- Transformation sequence again:
  1. Camera: From object coordinates to eye coords
  2. Perspective normalization: to clip coordinates
  3. Clipping
  4. Perspective division: to normalized device coords.
  5. Orthographic projection (setting $z_p = 0$)
  6. Viewport transformation: to screen coordinates
- Viewport transformation can distort
- Often in OpenGL: resize callback

Line-Segment Clipping

- General: 3D object against cube
- Simpler case:
  - In 2D: line against square or rectangle
  - Before scan conversion (rasterization)
  - Later: polygon clipping
- Several practical algorithms
  - Avoid expensive line-rectangle intersections
  - Cohen-Sutherland Clipping
  - Liang-Barsky Clipping
  - Many more [see Foley et al.]

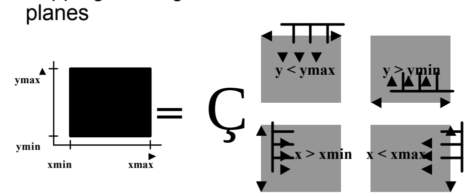
Clipping Against Rectangle

- Line-segment clipping: modify endpoints of lines to lie within clipping rectangle
- Could calculate intersections of line (segments) with clipping rectangle (expensive)



Cohen-Sutherland Clipping

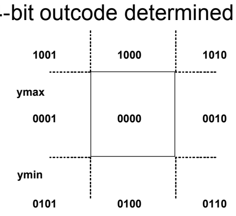
- Clipping rectangle as intersection of 4 half-planes



- Encode results of four half-plane tests
- Generalizes to 3 dimensions (6 half-planes)

Outcodes

- Divide space into 9 regions
- 4-bit outcode determined by comparisons

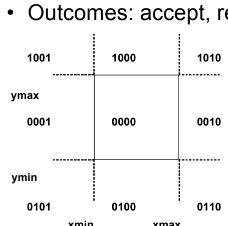


b_o: $y > y_{max}$
b_1: $y < y_{min}$
b_2: $x > x_{max}$
b_3: $x < x_{min}$

- o_1 = outcode(x_1,y_1) and o_2 = outcode(x_2,y_2)

Cases for Outcodes

- Outcomes: accept, reject, subdivide



$o_1 = o_2 = 0000$: accept
o_1 & $o_2 \neq 0000$: reject
$o_1 = 0000$, $o_2 \neq 0000$: subdiv
$o_1 \neq 0000$, $o_2 = 0000$: subdiv
o_1 & $o_2 = 0000$: subdiv

Cohen-Sutherland Subdivision

- Pick outside endpoint ($o \neq 0000$)
- Pick a crossed edge ($o = b_0b_1b_2b_3$ and $b_k \neq 0$)
- Compute intersection of this line and this edge
- Replace endpoint with intersection point
- Restart with new line segment
  - Outcodes of second point are unchanged
- Must converge (roundoff errors?)

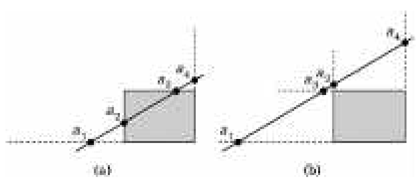
Liang-Barsky Clipping

- Starting point is parametric form

$$\begin{aligned} \mathbf{p}(\alpha) &= (1-\alpha)\mathbf{p}_1 + \alpha\mathbf{p}_2, \quad 0 \leq \alpha \leq 1 \\ x(\alpha) &= (1-\alpha)x_1 + \alpha x_2 \\ y(\alpha) &= (1-\alpha)y_1 + \alpha y_2 \end{aligned}$$

- Compute four intersections with extended clipping rectangle
- Will see that this can be avoided

Ordering of intersection points



- Order the intersection points
- Figure (a): $1 > \alpha_4 > \alpha_3 > \alpha_2 > \alpha_1 > 0$
- Figure (b): $1 > \alpha_4 > \alpha_2 > \alpha_3 > \alpha_1 > 0$

Liang-Barsky Efficiency Improvements

- Efficiency improvement 1:
  - Compute intersections one by one
  - Often can reject before all four are computed
- Efficiency improvement 2:
  - Equations for α_3, α_2

$$\begin{aligned} y_{max} &= (1-\alpha_3)y_1 + \alpha_3 y_2 \\ x_{min} &= (1-\alpha_2)x_1 + \alpha_2 x_2 \end{aligned}$$

$$\alpha_3 = \frac{y_{max} - y_1}{y_2 - y_1} \quad \alpha_2 = \frac{x_{min} - x_1}{x_2 - x_1}$$

  - Compare α_3, α_2 without floating-point division

Line-Segment Clipping Assessment

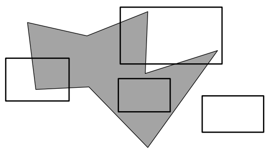
- Cohen-Sutherland
  - Works well if many lines can be rejected early
  - Recursive structure (multiple subdiv) a drawback
- Liang-Barsky
  - Avoids recursive calls (multiple subdiv)
  - Many cases to consider (tedious, but not expensive)
  - Used more often in practice (?)

Outline

- Line-Segment Clipping
  - Cohen-Sutherland
  - Liang-Barsky
- Polygon Clipping
  - Sutherland-Hodgeman
- Clipping in Three Dimensions

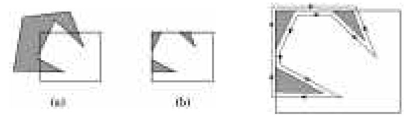
Polygon Clipping

- Convert a polygon into one ore more polygons
- Their union is intersection with clip window
- Alternatively, we can first tesselate concave polygons (OpenGL supported)

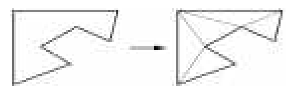


Concave Polygons

- Approach 1: clip and join to a single polygon

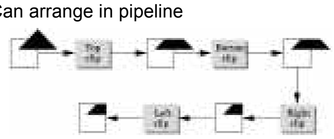


- Approach 2: tesselate and clip triangles



Sutherland-Hodgeman I

- Subproblem:
  - Input: polygon (vertex list) and single clip plane
  - Output: new (clipped) polygon (vertex list)
- Apply once for each clip plane
  - 4 in two dimensions
  - 6 in three dimensions
  - Can arrange in pipeline



Sutherland-Hodgeman II

- To clip vertex list (polygon) against half-plane:
  - Test first vertex. Output if inside, otherwise skip.
  - Then loop through list, testing transitions
    - In-to-in: output vertex
    - In-to-out: output intersection
    - out-to-in: output intersection and vertex
    - out-to-out: no output
  - Will output clipped polygon as vertex list
- May need some cleanup in concave case
- Can combine with Liang-Barsky idea

Other Cases and Optimizations

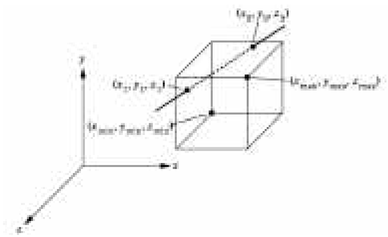
- Curves and surfaces
  - Analytically if possible
  - Through approximating lines and polygons otherwise
- Bounding boxes
  - Easy to calculate and maintain
  - Sometimes big savings

Outline

- Line-Segment Clipping
  - Cohen-Sutherland
  - Liang-Barsky
- Polygon Clipping
  - Sutherland-Hodgeman
- Clipping in Three Dimensions

Clipping Against Cube

- Derived from earlier algorithms
- Can allow right parallelepiped



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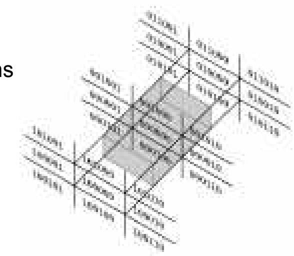
Cohen-Sutherland in 3D

- Use 6 bits in outcode
  - b_4: $z > z_{max}$
  - b_5: $z < z_{min}$
- Other calculations as before



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Liang-Barsky in 3D

- Add equation $z(\alpha) = (1- \alpha)\, z_1 + \alpha\, z_2$
- Solve, for $\mathbf{p}_0$ in plane and normal $\mathbf{n}$:

$$\mathbf{p}(\alpha) = (1-\alpha)\mathbf{p}_1 + \alpha\mathbf{p}_2$$
$$\mathbf{n} \cdot (\mathbf{p}(\alpha) - \mathbf{p}_0) = 0$$

- Yields

$$\alpha = \frac{\mathbf{n} \cdot (\mathbf{p}_0 - \mathbf{p}_1)}{\mathbf{n} \cdot (\mathbf{p}_2 - \mathbf{p}_1)}$$

- Optimizations as for Liang-Barsky in 2D

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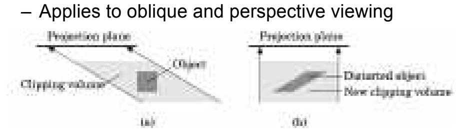
Perspective Normalization

- Intersection simplifies for orthographic viewing
  - One division only (no multiplication)
  - Other Liang-Barsky optimizations also apply
- Otherwise, use perspective normalization
  - Reduces to orthographic case
  - Applies to oblique and perspective viewing



Normalization of oblique projections

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Summary: Clipping

- Clipping line segments to rectangle or cube
  - Avoid expensive multiplications and divisions
  - Cohen-Sutherland or Liang-Barsky
- Clipping to viewing frustum
  - Perspective normalization to orthographic projection
  - Apply clipping to cube from above
- Client-specific clipping
  - Use more general, more expensive form
- Polygon clipping
  - Sutherland-Hodgeman pipeline

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Preview and Announcements

- Scan conversion
- Anti-aliasing
- Other pixel-level operations
- Assignment 5 due a week from Thursday!
- Start early!
- Sriram’s office hours now Mon 4:30-6:30
- Movie
- Returning Midterm

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