

Spatial Data Structures

Hierarchical Bounding Volumes
Regular Grids
Octrees
BSP Trees
Constructive Solid Geometry (CSG)
[Angel 9.10]

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<http://www.cs.cmu.edu/~fp/courses/graphics/>

Ray Tracing Acceleration

- Faster intersections
 - Faster ray-object intersections
 - Object bounding volume
 - Efficient intersectors
 - Fewer ray-object intersections
 - Hierarchical bounding volumes (boxes, spheres)
 - Spatial data structures
 - Directional techniques
- Fewer rays
 - Adaptive tree-depth control
 - Stochastic sampling
- Generalized rays (beams, cones)

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2

Spatial Data Structures

- Data structures to store geometric information
- Sample applications
 - Collision detection
 - Location queries
 - Chemical simulations
 - Rendering
- Spatial data structures for ray tracing
 - Object-centric data structures (bounding volumes)
 - Space subdivision (grids, octrees, BSP trees)
 - Speed-up of 10x, 100x, or more

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Bounding Volumes

- Wrap complex objects in simple ones
- Does ray intersect bounding box?
 - No: does not intersect enclosed objects
 - Yes: calculate intersection with enclosed objects
- Common types
 - Boxes, axis-aligned
 - Boxes, oriented
 - Spheres
 - Finite intersections or unions of above



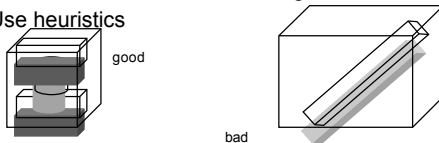
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Selection of Bounding Volumes

- Effectiveness depends on:
 - Probability that ray hits bounding volume, but not enclosed objects (tight fit is better)
 - Expense to calculate intersections with bounding volume and enclosed objects
- Amortize calculation of bounding volumes
- Use heuristics



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5

Hierarchical Bounding Volumes

- With simple bounding volumes, ray casting still has requires $O(n)$ intersection tests
- Idea use tree data structure
 - Larger bounding volumes contain smaller ones etc.
 - Sometimes naturally available (e.g. human figure)
 - Sometimes difficult to compute
- Often reduces complexity to $O(\log(n))$

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Ray Intersection Algorithm

- Recursively descend tree
- If ray misses bounding volume, no intersection
- If ray intersects bounding volume, recurse with enclosed volumes and objects
- Maintain near and far bounds to prune further
- Overall effectiveness depends on model and constructed hierarchy

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Spatial Subdivision

- Bounding volumes enclose objects, recursively
- Alternatively, divide space
- For each segment of space keep list of intersecting surfaces or objects
- Basic techniques
 - Regular grids
 - Octrees (axis-aligned, non-uniform partition)
 - BSP trees (recursive Binary Space Partition, planes)

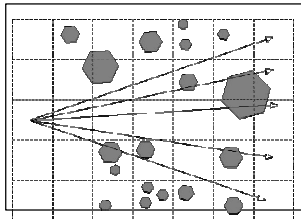
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Grids

- 3D array of cells (voxels) that tile space
- Each cell points to all intersecting surfaces
- Intersection alg steps from cell to cell



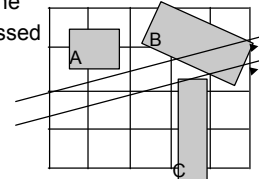
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Caching Intersection points

- Objects can span multiple cells
- For A need to test intersection only once
- For B need to cache intersection and check next cell for closer one
- If not, C could be missed (yellow ray)



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Assessment of Grids

- Poor choice when world is non-homogeneous
- Size of grid
 - Too small: too many surfaces per cell
 - Too large: too many empty cells to traverse
 - Can use alg like Bresenham's for efficient traversal
- Non-uniform spatial subdivision more flexible
 - Can adjust to objects that are present

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11

Outline

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- Constructive Solid Geometry (CSG)

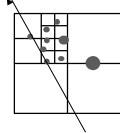
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Quadtrees

- Generalization of binary trees in 2D
 - Node (cell) is a square
 - Recursively split into 4 equal sub-squares
 - Stop subdivision based on number of objects
- Ray intersection has to traverse quadtree
- More difficult to step to next cell



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Octrees

- Generalization of quadtree in 3D
- Each cell may be split into 8 equal sub-cells
- Internal nodes store pointers to children
- Leaf nodes store list of surfaces
- Adapts well to non-homogeneous scenes

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14

Assessment for Ray Tracing

- Grids
 - Easy to implement
 - Require a lot of memory
 - Poor results for non-homogeneous scene
- Octrees
 - Better on most scenes (more adaptive)
- Alternative: nested grids
- Spatial subdivision expensive for animations
- Hierarchical bounding volumes
 - Natural for hierarchical objects
 - Better for dynamic scenes

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Other Spatial Subdivision Techniques

- Relax rules for quadtrees and octrees
- k-dimensional tree (k-d tree)
 - Split at arbitrary interior point
 - Split one dimension at a time
- Binary space partitioning tree (BSP tree)
 - In 2 dimensions, split with any line
 - In k dims. split with k-1 dimensional hyperplane
 - Particularly useful for painter's algorithm
 - Can also be used for ray tracing [see handout]



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16

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BSP Trees

- Split space with any line (2D) or plane (3D)
- Applications
 - Painters algorithm for hidden surface removal
 - Ray casting
- Inherent spatial ordering given viewpoint
 - Left subtree: in front, right subtree: behind
- Problem: finding good space partitions
 - Proper ordering for any viewpoint
 - Balance tree
- For details, see <http://reality.sgi.com/bspfaq/>

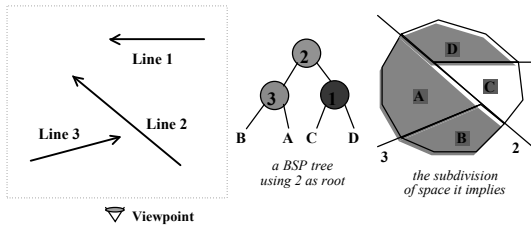
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Building a BSP Tree

- Use hidden surface removal as intuition
- Using line 1 or line 2 as root is easy



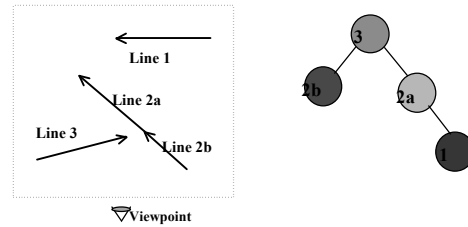
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Splitting of surfaces

- Using line 3 as root requires splitting



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Building a Good Tree

- Naive partitioning of n polygons yields $O(n^3)$ polygons (in 3D)
- Algorithms with $O(n^2)$ increase exist
 - Try all, use polygon with fewest splits
 - Do not need to split exactly along polygon planes
- Should balance tree
 - More splits allow easier balancing
 - Rebalancing?

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Painter's Algorithm with BSP Trees

- Building the tree
 - May need to split some polygons
 - Slow, but done only once
- Traverse back-to-front or front-to-back
 - Order is viewer-direction dependent
 - What is front and what is back of each line changes
 - Determine order on the fly

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Details of Painter's Algorithm

- Each face has form $Ax + By + Cz + D$
- Plug in coordinates and determine
 - Positive: front side
 - Zero: on plane
 - Negative: back side
- Back-to-front: inorder traversal, farther child first
- Front-to-back: inorder traversal, near child first
- Do backface culling with same sign test
- Clip against visible portion of space (portals)

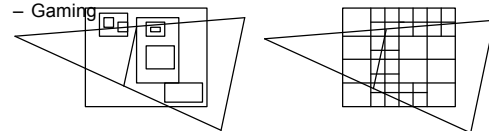
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Clipping With Spatial Data Structures

- Accelerate clipping
 - Goal: accept or rejects whole sets of objects
 - Can use an spatial data structures
- Scene should be mostly fixed
 - Terrain fly-through
 - Gaming



Hierarchical bounding volumes

Octrees

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Data Structure Demos

- BSP Tree construction
<http://symbolcraft.com/graphics/bsp/index.html>
- KD Tree construction
<http://www.rolemaker.dk/nonRoleMaker/uni/algogem/kdtree.htm>

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Real-Time and Interactive Ray Tracing

- Interactive ray tracing via space subdivision
<http://www.cs.utah.edu/~reinhard/eqwr/>
- Interactive ray tracing with good hardware
<http://www.cs.utah.edu/vissim/projects/raytracing/>

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26

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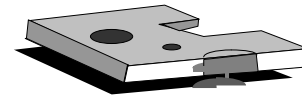
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Constructive Solid Geometry (CSG)

- Generate complex shapes with simple building blocks (boxes, spheres, cylinders, cones, ...)
- Particularly applicable for machined objects
- Efficient with ray tracing



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Example: A CSG Train



Brian Wyvill et al., U. of Calgary

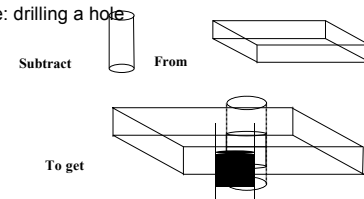
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Boolean Operations

- Intersection and union
- Subtraction
 - Example: drilling a hole



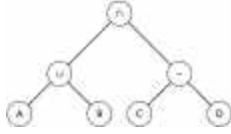
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CSG Trees

- Set operations yield tree-based representation



- Use these trees for ray/objects intersections
- Think about how!

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Implicit Functions for Booleans

- Solid as implicit function, $F(x,y,z)$
 - $F(x, y, z) < 0$ interior
 - $F(x, y, z) = 0$ surface
 - $F(x, y, z) > 0$ exterior
- For CSG, use $F(x, y, z) \in \{-1, 0, 1\}$
- $F_{A \cap B}(\mathbf{p}) = \max(F_A(\mathbf{p}), F_B(\mathbf{p}))$
- $F_{A \cup B}(\mathbf{p}) = \min(F_A(\mathbf{p}), F_B(\mathbf{p}))$
- $F_{A-B}(\mathbf{p}) = \max(F_A(\mathbf{p}), -F_B(\mathbf{p}))$

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32

Summary

- Hierarchical Bounding Volumes
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33

Preview

- Radiosity
- Assignment 6 due Thursday
- Assignment 7 (ray tracing) out Thursday

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34