

15-462 Computer Graphics I

Lecture 17

# Spatial Data Structures

Hierarchical Bounding Volumes

Regular Grids

Octrees

BSP Trees

Constructive Solid Geometry (CSG)

[Angel 9.10]

April 1, 2003

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<http://www.cs.cmu.edu/~fp/courses/graphics/>

# Ray Tracing Acceleration

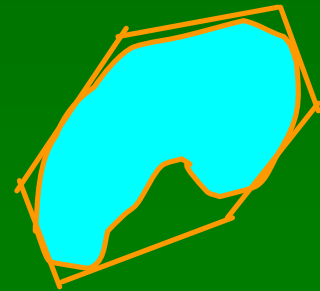
- Faster intersections
  - Faster ray-object intersections
    - Object bounding volume
    - Efficient intersectors
  - Fewer ray-object intersections
    - Hierarchical bounding volumes (boxes, spheres)
    - Spatial data structures
    - Directional techniques
- Fewer rays
  - Adaptive tree-depth control
  - Stochastic sampling
- Generalized rays (beams, cones)

# Spatial Data Structures

- Data structures to store geometric information
- Sample applications
  - Collision detection
  - Location queries
  - Chemical simulations
  - Rendering
- Spatial data structures for ray tracing
  - Object-centric data structures (bounding volumes)
  - Space subdivision (grids, octrees, BSP trees)
  - Speed-up of 10x, 100x, or more

# Bounding Volumes

- Wrap complex objects in simple ones
- Does ray intersect bounding box?
  - No: does not intersect enclosed objects
  - Yes: calculate intersection with enclosed objects
- Common types
  - Boxes, axis-aligned
  - Boxes, oriented
  - Spheres
  - Finite intersections or unions of above

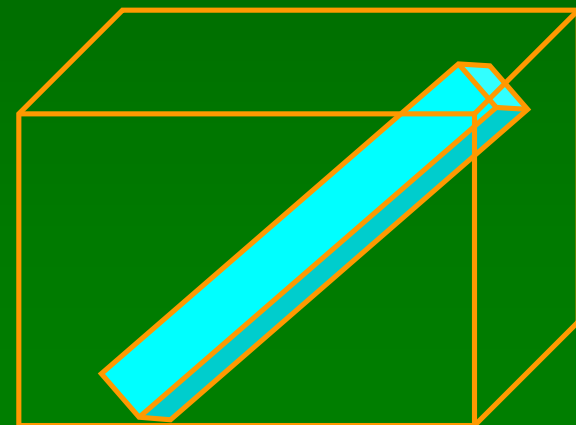


# Selection of Bounding Volumes

- Effectiveness depends on:
  - Probability that ray hits bounding volume, but not enclosed objects (tight fit is better)
  - Expense to calculate intersections with bounding volume and enclosed objects
- Amortize calculation of bounding volumes
- Use heuristics



bad



# Hierarchical Bounding Volumes

- With simple bounding volumes, ray casting still has requires  $O(n)$  intersection tests
- Idea use tree data structure
  - Larger bounding volumes contain smaller ones etc.
  - Sometimes naturally available (e.g. human figure)
  - Sometimes difficult to compute
- Often reduces complexity to  $O(\log(n))$

# Ray Intersection Algorithm

- Recursively descend tree
- If ray misses bounding volume, no intersection
- If ray intersects bounding volume, recurse with enclosed volumes and objects
- Maintain near and far bounds to prune further
- Overall effectiveness depends on model and constructed hierarchy

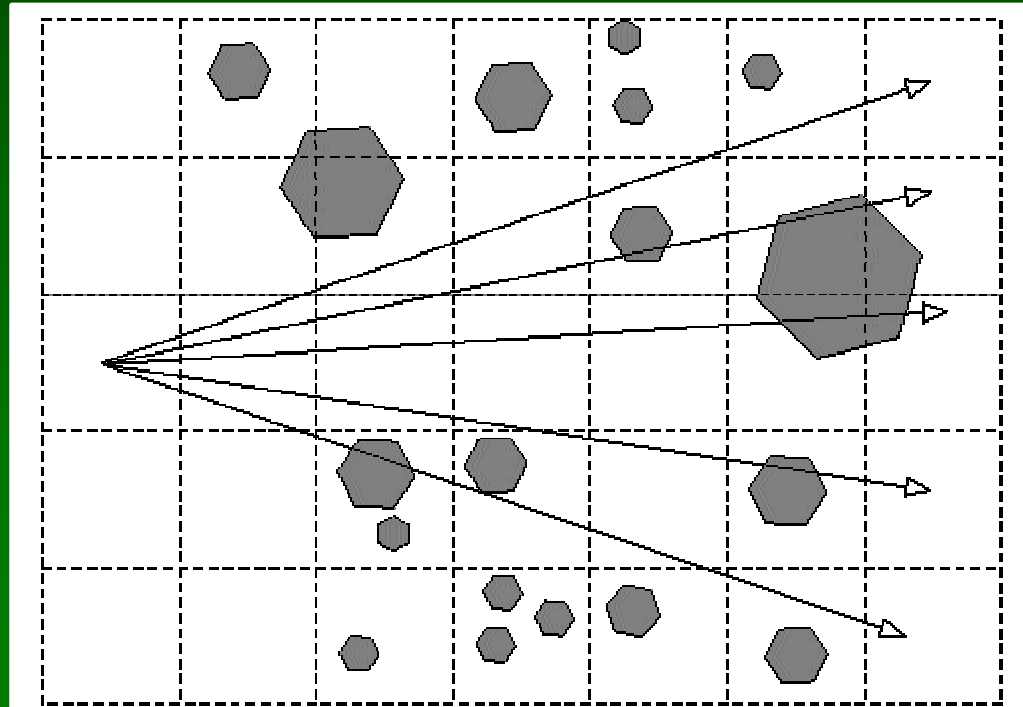
# Spatial Subdivision

- Bounding volumes enclose objects, recursively
- Alternatively, divide space
- For each segment of space keep list of intersecting surfaces or objects
- Basic techniques
  - Regular grids
  - Octrees (axis-aligned, non-uniform partition)
  - BSP trees (recursive Binary Space Partition, planes)



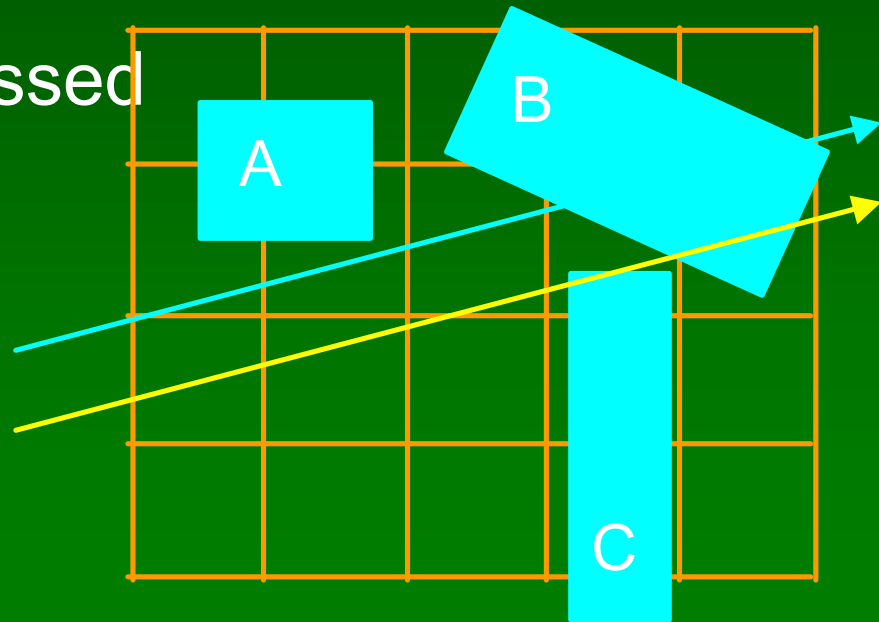
# Grids

- 3D array of cells (voxels) that tile space
- Each cell points to all intersecting surfaces
- Intersection alg steps from cell to cell



# Caching Intersection points

- Objects can span multiple cells
- For A need to test intersection only once
- For B need to cache intersection and check next cell for closer one
- If not, C could be missed (yellow ray)



# Assessment of Grids

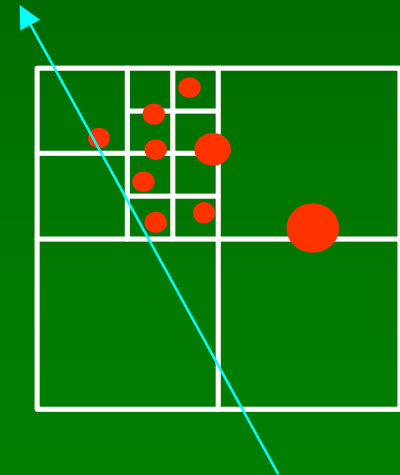
- Poor choice when world is non-homogeneous
- Size of grid
  - Too small: too many surfaces per cell
  - Too large: too many empty cells to traverse
  - Can use alg like Bresenham's for efficient traversal
- Non-uniform spatial subdivision more flexible
  - Can adjust to objects that are present

# Outline

- Hierarchical Bounding Volumes
- Regular Grids
- **Octrees**
- BSP Trees
- Constructive Solid Geometry (CSG)

# Quadtrees

- Generalization of binary trees in 2D
  - Node (cell) is a square
  - Recursively split into 4 equal sub-squares
  - Stop subdivision based on number of objects
- Ray intersection has to traverse quadtree
- More difficult to step to next cell



# Octrees

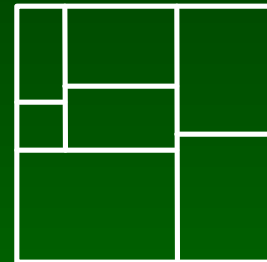
- Generalization of quadtree in 3D
- Each cell may be split into 8 equal sub-cells
- Internal nodes store pointers to children
- Leaf nodes store list of surfaces
- Adapts well to non-homogeneous scenes

# Assessment for Ray Tracing

- Grids
  - Easy to implement
  - Require a lot of memory
  - Poor results for non-homogeneous scene
- Octrees
  - Better on most scenes (more adaptive)
- Alternative: **nested grids**
- Spatial subdivision expensive for animations
- Hierarchical bounding volumes
  - Natural for hierarchical objects
  - Better for dynamic scenes

# Other Spatial Subdivision Techniques

- Relax rules for quadtrees and octrees
- k-dimensional tree (k-d tree)
  - Split at arbitrary interior point
  - Split one dimension at a time
- Binary space partitioning tree (BSP tree)
  - In 2 dimensions, split with any line
  - In k dims. split with k-1 dimensional hyperplane
  - Particularly useful for painter's algorithm
  - Can also be used for ray tracing [\[see handout\]](#)





# Outline

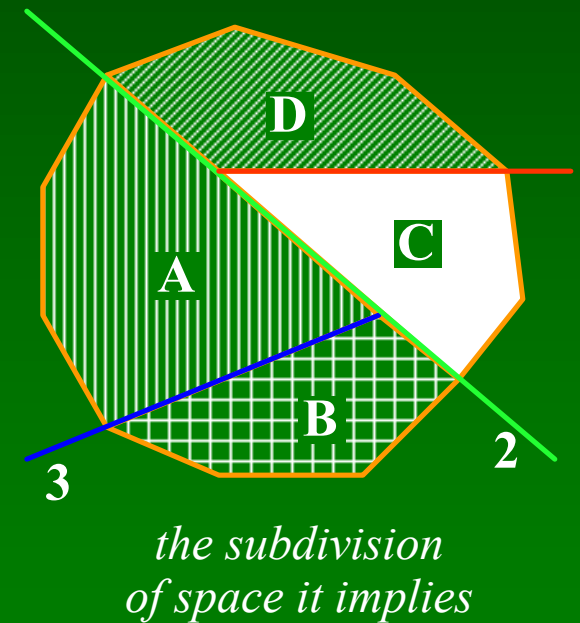
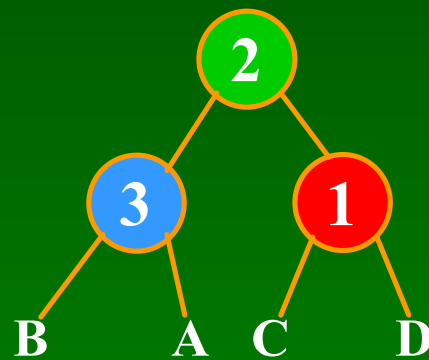
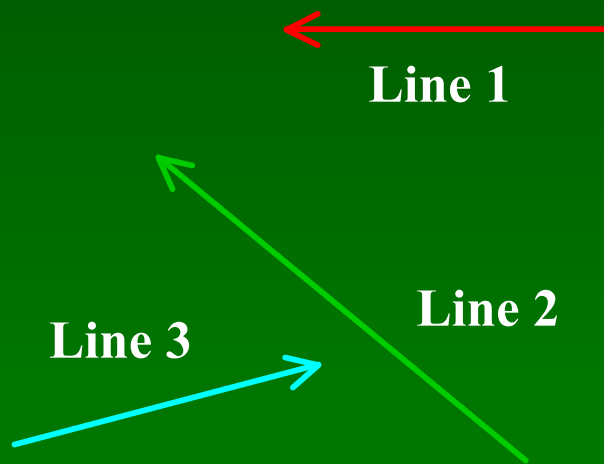
- Hierarchical Bounding Volumes
- Regular Grids
- Octrees
- **BSP Trees**
- Constructive Solid Geometry (CSG)

# BSP Trees

- Split space with any line (2D) or plane (3D)
- Applications
  - Painters algorithm for hidden surface removal
  - Ray casting
- Inherent spatial ordering given viewpoint
  - Left subtree: in front, right subtree: behind
- Problem: finding good space partitions
  - Proper ordering for any viewpoint
  - Balance tree
- For details, see <http://reality.sgi.com/bspfaq/>

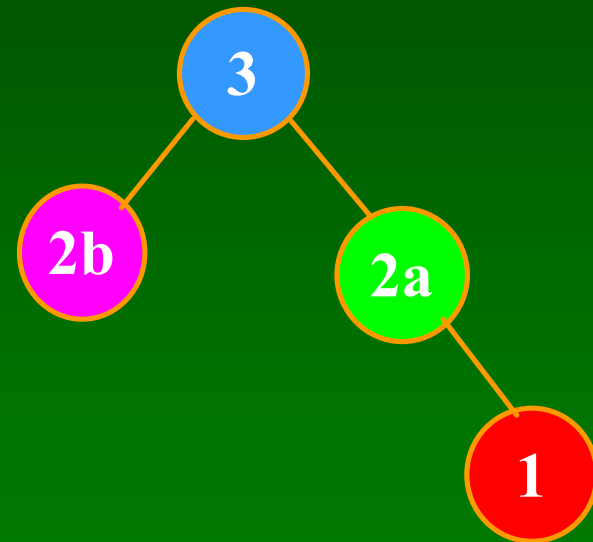
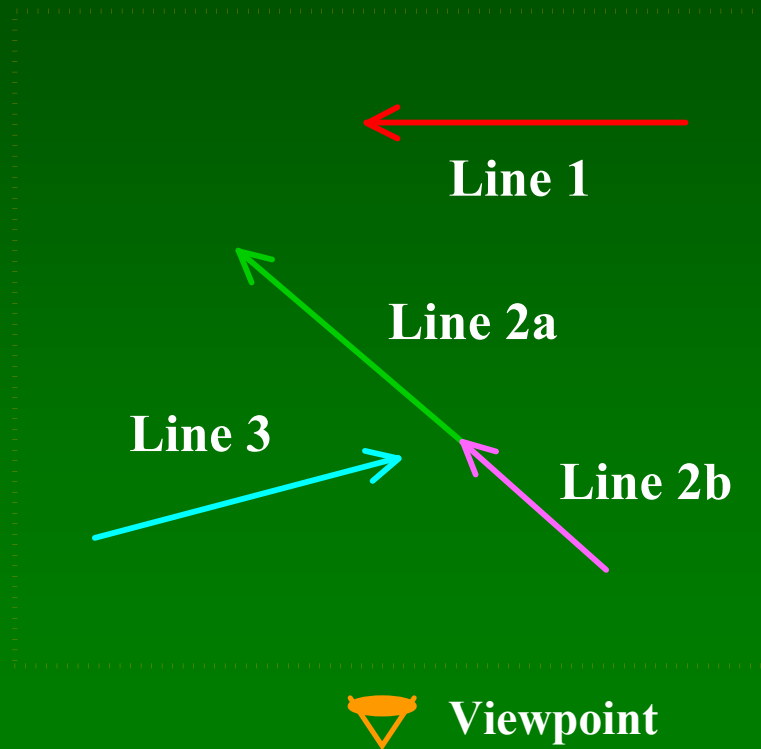
# Building a BSP Tree

- Use hidden surface removal as intuition
- Using line 1 or line 2 as root is easy



# Splitting of surfaces

- Using line 3 as root requires splitting



# Building a Good Tree

- Naive partitioning of  $n$  polygons yields  $O(n^3)$  polygons (in 3D)
- Algorithms with  $O(n^2)$  increase exist
  - Try all, use polygon with fewest splits
  - Do not need to split exactly along polygon planes
- Should balance tree
  - More splits allow easier balancing
  - Rebalancing?

# Painter's Algorithm with BSP Trees

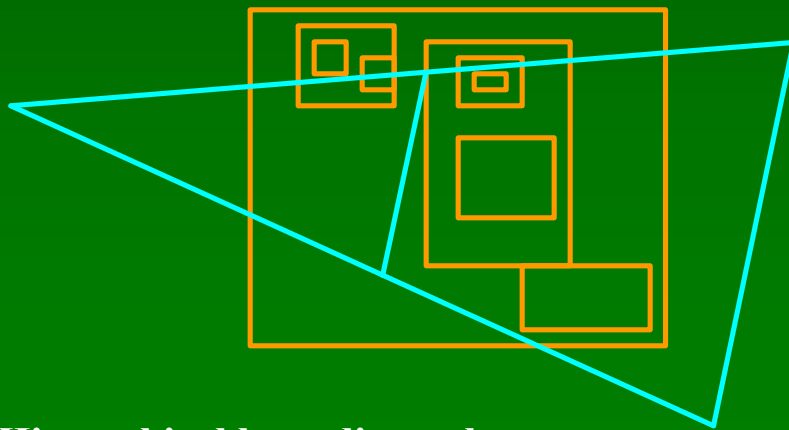
- Building the tree
  - May need to split some polygons
  - Slow, but done only once
- Traverse back-to-front or front-to-back
  - Order is viewer-direction dependent
  - What is front and what is back of each line changes
  - Determine order on the fly

# Details of Painter's Algorithm

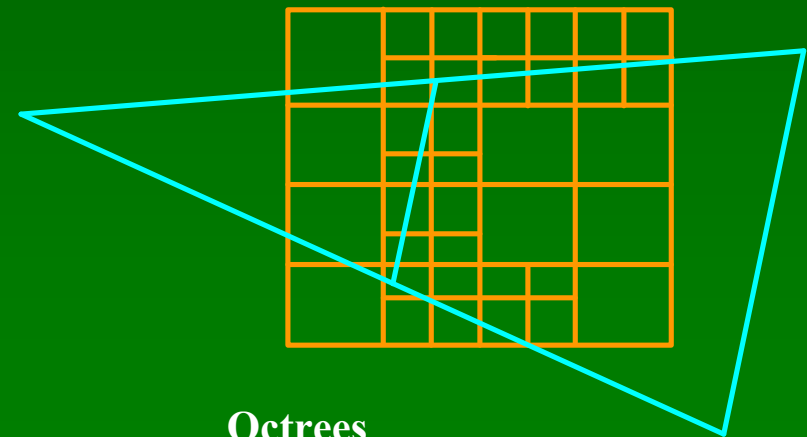
- Each face has form  $Ax + By + Cz + D$
- Plug in coordinates and determine
  - Positive: front side
  - Zero: on plane
  - Negative: back side
- Back-to-front: inorder traversal, farther child first
- Front-to-back: inorder traversal, near child first
- Do backface culling with same sign test
- Clip against visible portion of space (portals)

# Clipping With Spatial Data Structures

- Accelerate clipping
  - Goal: accept or rejects whole sets of objects
  - Can use an spatial data structures
- Scene should be mostly fixed
  - Terrain fly-through
  - Gaming



**Hierarchical bounding volumes**



**Octrees**



# Data Structure Demos

- BSP Tree construction

<http://symbolcraft.com/graphics/bsp/index.html>

- KD Tree construction

<http://www.rolemaker.dk/nonRoleMaker/uni/algogem/kdtree.htm>

# Real-Time and Interactive Ray Tracing

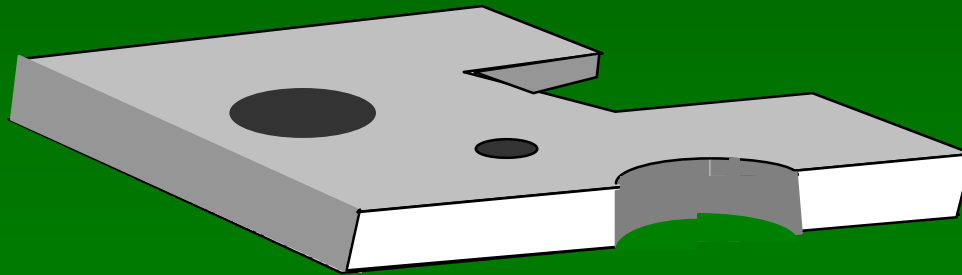
- Interactive ray tracing via space subdivision  
<http://www.cs.utah.edu/~reinhard/egwr/>
- Interactive ray tracing with good hardware  
<http://www.cs.utah.edu/vissim/projects/raytracing/>

# Outline

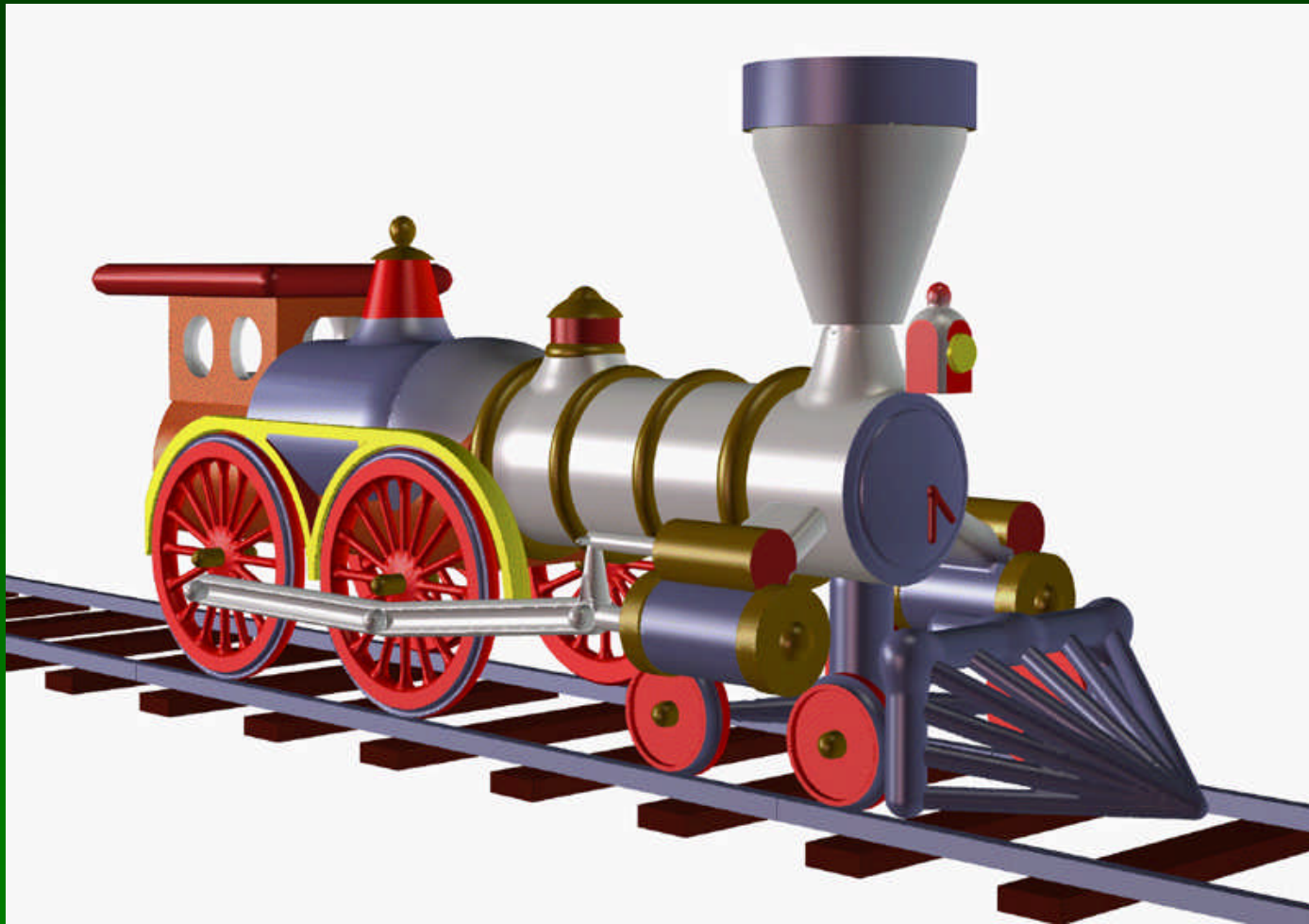
- Hierarchical Bounding Volumes
- Regular Grids
- Octrees
- BSP Trees
- **Constructive Solid Geometry (CSG)**

# Constructive Solid Geometry (CSG)

- Generate complex shapes with simple building blocks (boxes, spheres, cylinders, cones, ...)
- Particularly applicable for machined objects
- Efficient with ray tracing



# Example: A CSG Train



Brian Wyvill et al., U. of Calgary

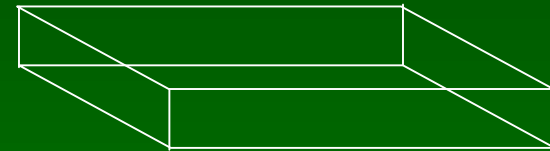
# Boolean Operations

- Intersection and union
- Subtraction
  - Example: drilling a hole

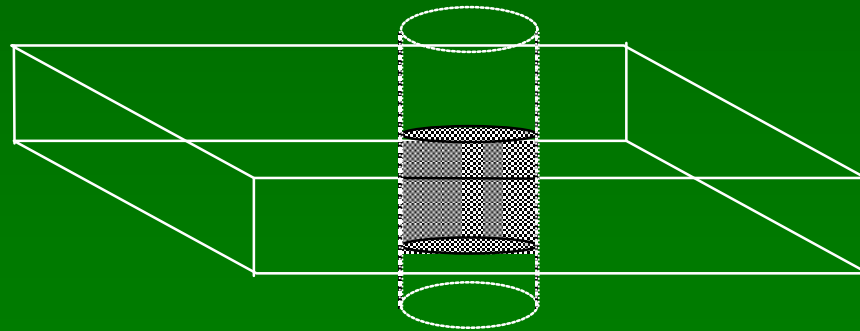
**Subtract**



**From**

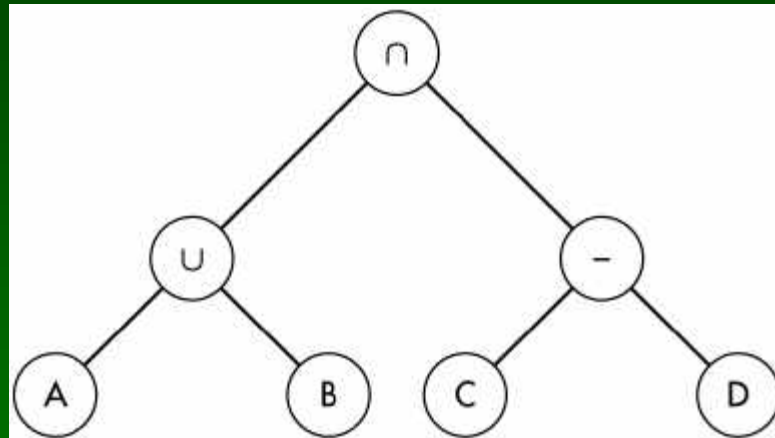


**To get**



# CSG Trees

- Set operations yield tree-based representation



- Use these trees for ray/objects intersections
- Think about how!

# Implicit Functions for Booleans

- Solid as implicit function,  $F(x,y,z)$ 
  - $F(x, y, z) < 0$  interior
  - $F(x, y, z) = 0$  surface
  - $F(x, y, z) > 0$  exterior
- For CSG, use  $F(x, y, z) \in \{-1, 0, 1\}$
- $F_{A \cap B}(\mathbf{p}) = \max(F_A(\mathbf{p}), F_B(\mathbf{p}))$
- $F_{A \cup B}(\mathbf{p}) = \min(F_A(\mathbf{p}), F_B(\mathbf{p}))$
- $F_{A - B}(\mathbf{p}) = \max(F_A(\mathbf{p}), -F_B(\mathbf{p}))$



# Summary

- Hierarchical Bounding Volumes
- Regular Grids
- Octrees
- BSP Trees
- Constructive Solid Geometry (CSG)

# Preview

- Radiosity
- Assignment 6 due Thursday
- Assignment 7 (ray tracing) out Thursday