Midterm Review Questions

15-462 Graphics I
Spring 2003

March 3, 2003

WARNING: These questions are only intended to help you think about the material. They likely bear little or no resemblance to the questions on the exam (which we – the TAs – have not seen). Consult the sample midterm for a better idea of the kinds of problems to expect.

1 Affine transformations

1. Suppose we have some transformation matrix $\mathbf{T}$ that we want to apply to some shape. What matrix do we need to multiply the shape *normals* by to preserve the correct shading?
2. Consider the following matrix,

$$\mathbf{Q} = \begin{bmatrix} 7 & 0 & 0 & 4 \\ 0 & \frac{1}{4} & 0 & -3 \\ 0 & 0 & -4 & 0 \\ 0 & 0 & 0 & 2 \end{bmatrix} \tag{1}$$

   Describe the behaviour of this transformation matrix on the unit cube centered on the origin.
3. Harry Bovik has come up with a transformation that maps a square to a trapezoid in the plane. Is this affine? Can this be expressed as a linear transformation?

2 Projective transformations

Consider the projective transformation

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1/d & 0 \end{pmatrix}$$

1. If we apply a uniform scale to all points before performing the projection, how will the projected points be different from the case where no scale is applied? Show your reasoning mathematically and explain the result geometrically.
2. In general, any two parallel lines transformed by a projection will converge and intersect at a vanishing point. In fact, some parallel lines will *not* intersect; i.e., they will remain parallel, even after undergoing this perspective projection. Which lines are these? Geometrically speaking, what do all sets of these lines have in common?
3. (bonus) Hobbits were made to appear around 4'6" tall in comparison to Gandalf's 6'2" in the Lord of the Rings movies even though the real actors for the hobbits were much taller. This technique, used in Hollywood for years, is known as forced perspective. Describe a method you would use in OpenGL to implement this effect. Also, how would you contend with instances where you had to move the camera - i.e. avoiding parallax?

3 Shading

The Phong shading equation can be described by the following equation,

$$I_{\text{phong}} = k_d + k_a I_a + \sum_i \frac{1}{a + bd_i + cd_i^2} I_{\ell_i} \left[k_d(\mathbf{N} \cdot \mathbf{L}_i)_+ + k_s(\mathbf{V} \cdot \mathbf{R}_i)_+^{n_i} \right]$$

You may assume that all light sources are point sources.

1. The following variables are listed in the Phong equation:

$$I_{\text{phong}} \quad a \quad b \quad c \quad d_i \quad k_e \quad k_a \quad k_d \quad k_s \quad I_a \quad I_{\ell_i} \quad \mathbf{L}_i \quad \mathbf{R}_i \quad \mathbf{N} \quad \mathbf{V}$$

   In general, which of these quantities are affected if ...

   (a) ... the viewing direction changes?

   (b) ... the orientation of the surface changes? Assume that the change in orientation is about the point being viewed.

   (c) ... the position (but not the orientation) of the point being viewed changes?

2. Given a single light source, describe the relationships between $\mathbf{N}$, $\mathbf{L}$, and $\mathbf{V}$ that would result in a point shaded with the Phong model appearing maximally bright.

3. Describe the difference between Goreaud shading and Phong shading. Describe the difference between Phong *shading* and Phong *illumination*.

4 Curves and surfaces

1. Suppose that we join two Bezier curves of degree 2, using the control point sequences $(\mathbf{p}_0, \mathbf{p}_1, \mathbf{p}_2)$ and $(\mathbf{p}_2, \mathbf{p}_3, \mathbf{p}_4)$, respectively. What conditions must be satisfied by these five points to have C^1 continuity at the join point?

2. Consider a quadratic Bezier curve determined by three control points: $\mathbf{p}_0$, $\mathbf{p}_1$, and $\mathbf{p}_2$. Given any value of t in the interval $[0, 1]$, we have two points $\mathbf{a}$, $\mathbf{b}$ on $\overline{\mathbf{p}_0\mathbf{p}_1}$ and $\overline{\mathbf{p}_1\mathbf{p}_2}$, respectively, such that the line segment $\overline{\mathbf{ab}}$ is tangent to the Bezier curve at t.

   (a) For what value of t does the triangle $\Delta\mathbf{ap}_1\mathbf{b}$ have the maximum area? Prove your answer.

   (b) For what value of t is the line segment $\overline{\mathbf{ab}}$ parallel to $\overline{\mathbf{p}_0\mathbf{p}_2}$? Prove your answer.

3. Describe the effect on the cubic Bezier curve when you make control points $\mathbf{p}_0$ and $\mathbf{p}_1$ exactly the same.

4. Of the 16 control points for a Bezier surface $(\mathbf{p}_{00}, \mathbf{p}_{01}, \ldots, \mathbf{p}_{33})$, which ones are always interpolated by a Bezier surface? A B-spline surface?

5. Consider the various curves we went over in class (interpolating, Hermite, Bezier, B-Splines). Which of these have

   - C^1 continuity? C^2 continuity? Across multiple segments?
   - Local control?
   - The convex hull property?
   - Interpolation?