On a Logical Foundation for Explicit Substitutions

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Traditionally, calculi of explicit substitution [1] have been conceived as an implementation technique for β-reduction and studied with the tools of rewriting theory. This computational view has been extremely fruitful (see [2] for a recent survey) and raises the question if there may also be a more abstract underlying logical foundation.

Some forms of explicit substitution have been related to cut in the intuitionistic sequent calculus [3]. While making a connection to logic, the interpretation of explicit substitutions remains primarily computational since they do not have a reflection at the level of propositions, only at the level of proofs.

In recent joint work [4], we have shown how explicit substitutions naturally arise in the study of intuitionistic modal logic. Their logical meaning is embodied by a contextual modality which captures all assumptions a proof of a proposition may rely on. Explicit substitutions mediate between such contexts and therefore, intuitively, between worlds in a Kripke-style interpretation of modal logic.

In this talk we review this basic observation about the logical origin of explicit substitutions and generalize it to a multi-level modal logic. Returning to the computational meaning, we see that explicit substitutions are the key to a λ-calculus where variables, meta-variables, meta-meta-variables, etc. can be unified without the usual paradoxes such as lack of α-conversion. We conclude with some speculation on potential applications of this calculus in logical frameworks or proof assistants.

References

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