Determining CMM Motion on Free-Form Objects Using Implicit Polynomials[1]

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Abstract

This paper addresses the question of determining the motion tracked by a coordinate measuring machine (CMM) on an entire free-form curve or surface defined by an implicit polynomial (IP) equation. We also develop procedures for quantifying the errors in manufactured parts during test and measurement. In particular, given an IP equation of a model object, we describe how the stylus of a CMM can be programmed to move along the surface of a newly manufactured object in order to make tests and measurements along the path. The points along the path of the stylus are determined from the model IP equation.

1 Introduction

One often must compare several (ideally) identical objects to a model object, such as the turbine blades in a jet engine, the propeller blades of a ship, or the contoured grooves in a gear. If the profile and tooth trace errors of a gear exceed tolerance errors, the running qualities of the gear decrease significantly, causing increased tip wear that can seriously degrade overall gear performance [13]. More serious consequences can occur if manufactured objects, such as the turbine blades in a jet engine are not precisely fabricated

Coordinate measuring machines (CMM) are designed to measure complex shapes. They are particularly important when used to quickly and accurately inspect a large variety of manufactured parts, especially those with sculptured, or free-form shapes, such as gears, propeller blades etc., as specified in some quantitative way by an engineer or designer. Master measurements generally are entered into the database of the CMM computer so that it can trace along appropriate profiles and compare its measurement data to the stored data.

Here we assume that the objects being traced by the stylus of the CMM machine are modeled by implicit polynomial (IP) equations, which are beginning to play a more important role in manufacturing. We also assume that the stylus of the CMM machine begins its movement very close to some initial point on an IP path to be traced. Subsequent points along the path are then determined using a new perpendicular distance approximation algorithm together with Newton's root finding procedure. Certain geometrical features of the implicit curve or surface, such as its tangent and curvature are used as well.

Unfortunately, there is very little in the literature on implicit polynomial models for large or entire free-form shapes because of the lack of tractable computational procedures for obtaining and modifying such models[3]. Recently, however, some useful new procedures and computer algorithms have been developed for describing and analyzing them [4, 5, 6, 7, 8, 9, 10], as well as fitting IP equations to 2-D and 3-D boundary data sets [11, 12]. These new results have proven to be very useful in modeling and measuring free-form manufactured parts.

This paper is organized as follows: A new algorithm for approximating the perpendicular distance between a point and an implicit polynomial curve is outlined in Section 2. Section 3 serves to present a fast new procedure for defining a sequence of points for a CMM to follow and measure along a curve or surface defined by an IP equation. Experimental results which illustrate these procedures are then given in Section 4, and we conclude with some final observations in Section 5.

2 A Point to IP Curve Distance Approximation

In order to find the exact point on an IP curve that has the minimum distance to an external point, one must determine the actual Euclidean distance from any point on an IP curve to the external point. As noted by Taubin in [1], this distance cannot be computed directly. Therefore, one must use either an iterative procedure or an approximation. In [1], Taubin employs an iterative procedure that utilizes a "scaled" algebraic distance defined by $|F_n(x, y)| / \|\nabla F_n(x, y)\|$.

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A new perpendicular distance approximation procedure was recently presented in [14] to rapidly and accurately determine the minimum distance from any well-defined external point to a curve or surface defined by the zero set of an implicit polynomial equation. We will employ this procedure here to approximate the point on an implicit polynomial curve that has the minimum distance to a defined external point. This new approximation procedure has been shown to be faster than the iterative approach first presented by Taubin, while producing perpendicular distance errors very close to those of Taubin.

2.1 The 2-D Case:

To illustrate this procedure, consider Figure 1, where $f_n(x,y)=0$ represents an implicit polynomial curve and (x_m, y_m) denotes some arbitrary external point. The points (x_0, y_m) and (x_m, y_0) are the perpendicular projections of the external point onto the IP curve. To determine these points, we simply substitute x_m for x in $f_n(x,y)=0$ to obtain a polynomial function of y alone. We then solve for the corresponding y_0 using Newton's successive approximation root-finding procedure, which converges very fast.

The line through (x_0, y_m) and (x_m, y_0) is then defined by

$$\det\begin{bmatrix} x-x_0 & y-y_m \\ x_m-x_0 & y_0-y_m \end{bmatrix} = (x-x_0)(y_0-y_m)-(y-y_m)(x_m-x_0)=0\,,$$

which implies that the line through (x_m, y_m) perpendicular to this line is defined by

$$\frac{x-x_m}{y_m-y_0}=\frac{y-y_m}{x_m-x_0} \quad \Rightarrow \quad y=\left(\frac{x_m-x_0}{y_m-y_0}\right)(x-x_m)+y_m$$

Substituting this value for y into $f_n(x,y)=0$ implies a polynomial function of x alone which then is solved for x_f, again using Newton's procedure. The corresponding y_f point is then given by $y_f=(x_m-x_0)(x_f-x_0)/(y_m-y_0)+y_m$.

2.2 The 3-D Case:

A 3-D "extension" of our minimum distance strategy is depicted in Figure 2. In particular, the equation of the plane through (x_0, y_m, z_m), (x_m, y_0, z_m) and (x_m, y_m, z_0) is defined by

$$\det\begin{bmatrix} x-x_0 & y-y_m & z-z_m \\ x_m-x_0 & y_0-y_m & 0 \\ x_m-x_0 & 0 & z_0-z_m \end{bmatrix}=0$$

or

$$\underbrace{(y_m-y_0)(z_m-z_0)}_{a}(x-x_0)+\underbrace{(x_m-x_0)(z_m-z_0)}_{b}(y-y_m)+\underbrace{(x_m-x_0)(y_m-y_0)}_{c}(x_m-x_0)=0,$$

where *a*, *b* and *c* define the normal direction to the plane [2]. The line through the measured point (x_m, y_m, z_m) perpendicular to this plane is then defined by

$$\frac{x-x_m}{a}=\frac{y-y_m}{b}=\frac{z-z_m}{c}$$

Substituting $(x-x_m)b/a+y_m$ for *y* and $(x-x_m)c/a+z_m$ for *z* into $f_n(x,y,z)=0$ implies a polynomial equation in *x* alone that can readily be solved for x_f using Newton's root finding procedure, as in the 2-D case. The corresponding y_f

and z_f are subsequently given by $(x_f - x_m)b/a + y_m$ and $(x_f - x_m)c/a + z_m$, respectively. The line from (x_f, y_f, z_f) to (x_m, y_m, z_m) is the gradient vector from P_1 to P_1', analogous to that depicted in Figure 1 in the 2-D case.

3 Determining CMM Motion Along an IP Curve

Once an external point is projected onto a defined IP curve as an initial CMM measurement point, the next point in the CMM measurement sequence is determined by using the geometric features of an IP curve, such as its gradient and tangent vectors. In particular, consider the implicit polynomial equation of a curve and some initial point P_1', which is either on the curve or very close to it, as shown in Figure 3. The horizontal and vertical projections of this point onto the implicit polynomial curves are also depicted in Figure 3. Next, the point P_1 on the implicit polynomial curve that has minimum perpendicular distance to the external point P_1' is found using the approximation procedure of Section 2. The gradient vector at point P_1 is then directly computed from knowledge of the IP equation as

$$\nabla f(x, y)\Big|_{P_1} = \left\langle \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y} \right\rangle\Bigg|_{P_1}$$

By rotating the gradient vector 90^o, we obtain the tangent vector

$$\vec{T} = \left\langle \frac{\partial f}{\partial y}, -\frac{\partial f}{\partial x} \right\rangle\Bigg|_{P_1},$$

as shown in Figure 3.

We now use the tangent vector at point P_1 to estimate the next point in the CMM sequence. Moving along one step-size (*h*) of the tangent vector defines the next external point $P_2' = P_1 + h\vec{T}$ as shown in Figure 3. We then apply the minimum perpendicular distance approximation procedure described in Section 2 to find the closest point P_2 to P_2', on the implicit polynomial curve as shown in Figure 3. P_3 and the subsequent sequential points are found in an analogous manner, as shown in Figure 3.

3.1 Determining h

To determine an appropriate value for the scalar *h*, we first define an upper and lower limit for the point-to-point displacement that our particular CMM can make in any one move, namely ULD (Upper Limit for Displacement) and LLD (Lower Limit for Displacement), respectively. Clearly, the actual displacement of the CMM machine, $h\vec{T}$, must satisfy the relations:

$$h\vec{T} \geq LLD \qquad\qquad h\vec{T} \leq ULD$$

We next employ the vector length $\left|\vec{T}\right| = \sqrt{(df/dx)^2 + (df/dy)^2}$ to determine the following two limiting values for *h*:

$$h_l = \frac{LLD}{\left|\vec{T}\right|_{max}} \qquad\qquad h_u = \frac{ULD}{\left|\vec{T}\right|_{min}}$$

An initial value for the scalar *h* is then defined by $h = (h_l + h_u)/2$.

3.2 The Stylus Configuration

We next observe that a CMM with a rigid stylus might not be able to physically contact the sequential IP points defined by our procedure. However, this is not the case if a spring-loaded stylus were employed whose zero mid-point was programmed to contact our defined IP points. Such a stylus would not only track an IP curve using our sequential IP points, but it also would provide an immediate error measurement proportional to its displacement on either side of its mid-point.

It is important to note that because the gradient and the tangent vectors are inversely related to curvature, in regions of high curvature the length of the tangent vector decreases. This implies that our CMM sequence point determination procedure automatically decreases tracking speed in areas of high curvature vice-versa. Furthermore, the gradient vector from P_i' to P_i clearly specifies the orientation of the stylus during motion.

3.3 A Simple Example

To demonstrate how different values of *h* affect the locations of sequence points along a curved path, consider the ellipse shown in Figure 4 (a), which is defined by the equation

$$x^2 + 8y^2 = 1$$

Figure 4 (b) and (c) shows resultant sequence points and gradient vectors at those points defined by h values of 0.01 and 0.05, respectively. Note that the lengths of the gradient vectors in regions of high curvature are shorter, and vice-versa, which implies an automatic slowing down of the CMM tracing procedure in high curvature regions and speeding it up in regions of low curvature. It is also important to note that a smaller value for *h* will slow down the entire tracing speed, while choosing a high value of *h* speeds up the entire tracing procedure.

4 An Illustration

To further illustrate our sequential CMM tracking point procedure, consider the IP curve defined by

$$\begin{aligned} f(x, y) &= x^4 - 0.0666x^3 y + 0.8462x^2 y^2 - 0.9050xy^3 + 0.4656y^4 + 0.2146x^3 - 0.3950x^2 y \\ &+ 0.5634xy^2 - 0.0918y^3 - 0.1542x^2 + 0.0260xy - 0.0149y^2 - 0.0537x + 0.0330y - 0.0099 = 0 \end{aligned}$$

whose gradient vectors are shown are shown in Figure 5.

5 The 3-D Case

In the 3-D case, we assume that measurements are to be made about an implicit surface defined by some $f_n(x,y,z) = 0$ in an intersecting plane defined by $ax + by + cz + d = 0$. A configuration (translation and rotation) coordinate transformation can then be applied which transforms the intersecting plane to the $\bar{z} = 0$ plane, thus transforming $f_n(x,y,z) = 0$ to an equivalent surface defined by some $\bar{f}_n(\bar{x},\bar{y},\bar{z}) = 0$, and a resulting 2-D curve defined by $\bar{f}_n(\bar{x},\bar{y},0) = 0$, as depicted in Figure 6. The 2-D results outlined earlier can then be employed to determine a sequence of CMM measurement points, together with tangential directions, around the 2-D curve in the $\bar{z} = 0$ plane. Furthermore, the gradient vector from any external 3-D point $\bar{P}_i'$ (in the $\bar{z} = 0$ plane) to the corresponding minimum distance point $\bar{P}_i$, as depicted in Figure 2, would specify the orientation of the stylus during the CMM motion. Note that in this 3-D case, the perpendicular distance point $\bar{P}_i$ from $\bar{P}_i' = (\bar{x}_{mi}, \bar{y}_{mi}, 0)$ to the surface would not necessarily lie in the $\bar{z} = 0$ plane.

6 Conclusions

Implicit polynomials represent a useful new procedure for modeling and measuring free-form curves and surfaces. In this paper, we described a method of determining CMM motion for measurement on a variety of manufactured free-form planar surfaces that are implicitly defined. Curves on more general 3-D implicit surfaces can be traced using same procedure. The algorithm to find approximate perpendicular distance to a surface is outlined in Section 2.2. However, since a 3-D surface has a tangent plane, unlike a curve that has a tangent vector, some additional constraint must be employed to determine the CMM surface direction. To summarize, we have once again shown, as in an earlier paper [14], that much can be gained in ease of fabrication and measurement if implicit polynomials are used to define, measure and model free-form manufactured objects.

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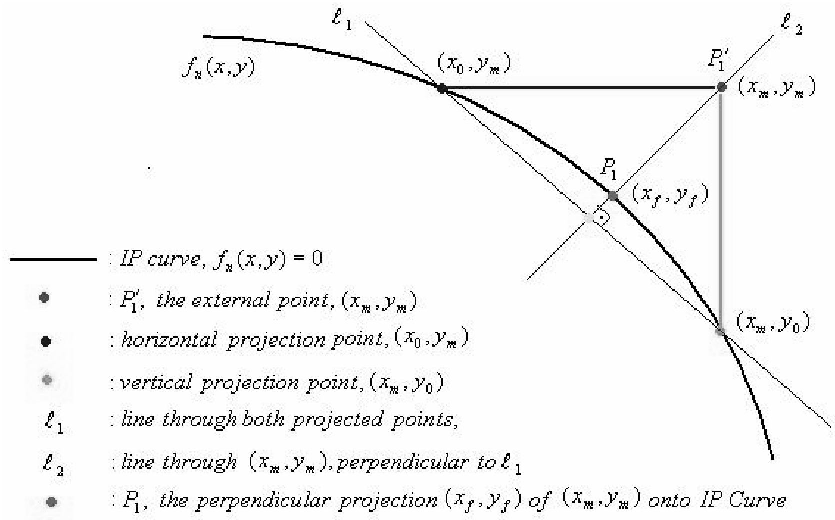

Figure 1 Approximating a point (x_f, y_f) on an IP curve that has minimum distance to (x_m, y_m).

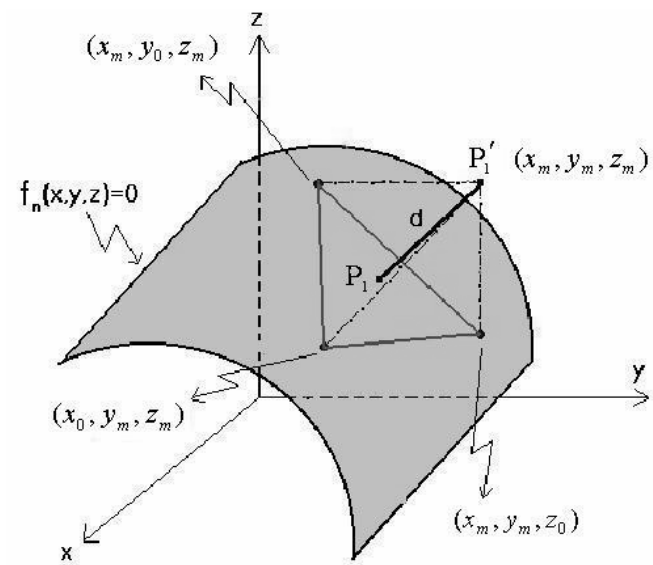

Figure 2 Approximating a point (x_f, y_f, z_f) on an IP surface that has minimum distance to (x_m, y_m, z_m).

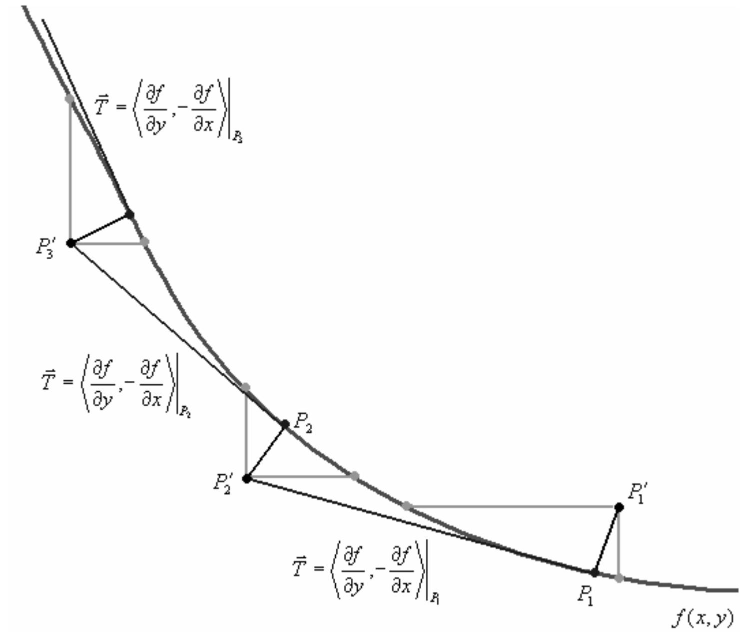

Figure 3 The determination of CMM sequential points .

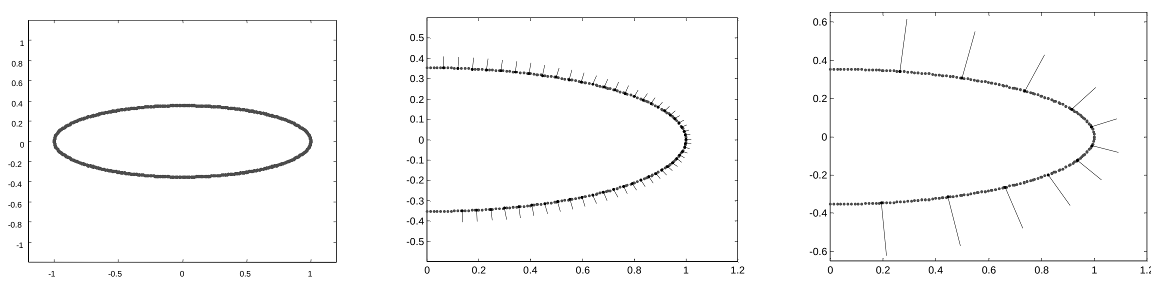

Figure 4 (a) Ellipse, $x^2 + 8y^2 = 1$**. The gradients vectors at some external points when (b)** ***h*****=0.01 and (c) h=0.05.**

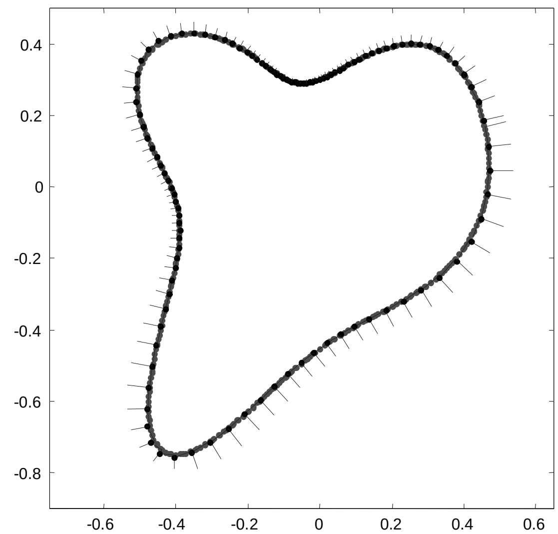

Figure 5 An implicit curve with its gradient vectors. Note that as the curvature increases, the magnitudes of the gradient vectors decrease, which implies slower CMM tracking at points of high curvature and vice-versa.

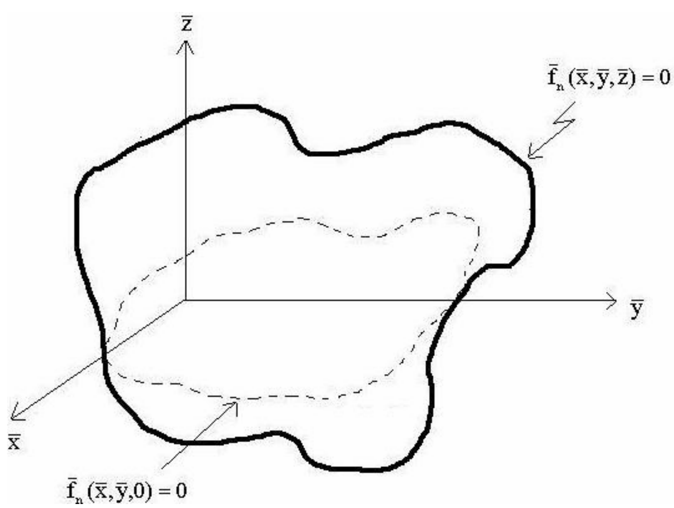

Figure 6 A 3-D surface (solid) intersected by a $\bar{z}=0$ plane (dashed).