Automated Bilateral Multiple-issue Negotiation with No Information About Opponent

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***Abstract*—In this paper, we investigate offer generation methods for automated negotiation on multiple issues with no information about the opponent's utility function. In existing negotiation literature, it is usually assumed that an agent has full information or probabilistic beliefs about the other agent's utility function. However, it is usually not possible for agents to have complete information about the other agent's preference or accurate probability distributions. We prove that using an *alternating projection strategy*, it is possible to reach an agreement in general *automated multi-attribute negotiation*, where the agents have *nonlinear utility functions and no information* about the utility function of the other agent. We also prove that rational agents do not have any incentive to deviate from the proposed strategy. We further present simulation results to demonstrate that the solution obtained from our protocol is quite close to the Nash bargaining solution.**

I. Introduction

In bilateral multi-attribute negotiation, two parties (or agents) with limited common knowledge about each other's preferences want to arrive at an agreement over a set of issues when they have (possibly) conflicting preferences over the issues. In the extant literature on theoretical analysis of negotiation, it is usually assumed that an agent either has (a) a complete knowledge of the preference structure of the opponents (i.e., the utility of the agents are assumed to be known, e.g., [1]) or has (b) a probability distribution over the preferences of the agents is known (e.g., [2], [3], [4]). Furthermore, much of the literature on negotiation with incomplete information has focused on developing equilibrium strategies for single issue negotiation. Computational modelers, whose goal is to build protocols and strategies for software agents to negotiate, consider multi-attribute negotiations and usually provide heuristic strategies for the negotiating agent. Most of the literature also assumes that the agents have linear additive utility functions. In general, the negotiation may involve multiple issues, the utility functions of the agents may be nonlinear, and an agent may not have any knowledge about the utility of the other player. A fundamental open question in bilateral negotiation in such a general setting, that we study in this paper, is the following: *Is it possible to design negotiation strategies for agents so that they come to an agreement given that they have no prior knowledge about their opponent's utility function*?

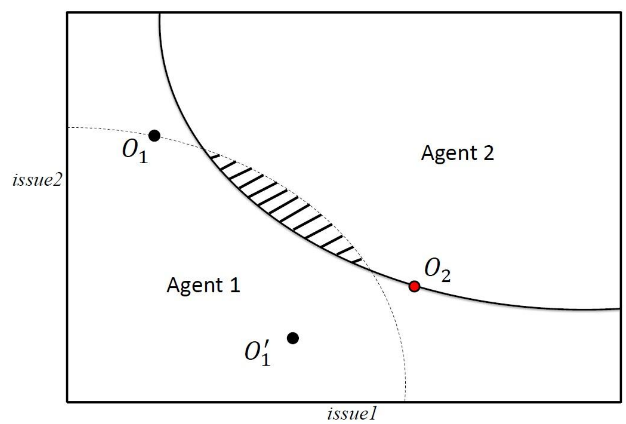

Fig. 1. Illustrative sketch of the offer space of 2 agents negotiating on 2 issues. The 2 curves (called reservation curves) denote the set of all offers with value equal to the reservation utility of the agents. The convex sets bounded by the two curves are the feasible offer sets of the agents (e.g., O_1' is a feasible offer for agent 1). The zone of agreement is the common intersection of the two sets (hatched region). Each agent knows its own reservation curve (e.g., agent 2 knows only the solid curve). The agents know neither the other agent's reservation curve, nor the zone of agreement.

Formally, there is a set of n (≥ 1) issues and 2 negotiators, with each negotiator having a private utility function (known only to her). The utility functions can be nonlinear but they are assumed to be (strictly) concave. Each agent also has a private reservation utility, and any offer that gives a utility less than her reservation utility is not acceptable to an agent. The agents use an alternating-offer protocol to negotiate [5]. Figure 1 gives a geometric view of the *offer space* for two agents negotiating on two issues. The *zone of agreement* (hatched region in Figure 1) is the set of offers that is acceptable to both agents. Any point within the zone of agreement is called a *satisficing agreement* and the goal of the agents is to find an agreement that satisfies both agents. However, either agent does not know the other agent's utility function, and therefore, the zone of agreement is unknown. *Thus, geometrically speaking, in negotiation, the goal of the agents is to find a point in the zone of agreement, where none of the agents have any explicit knowledge of the zone of agreement*. Note that if the zone of agreement is empty, there is no agreement that can be achieved for the negotiation problem.

Let us consider two agents negotiating on a single issue (e.g., a buyer and a real estate agent negotiating on the price of a house). If the zone of agreement is non-empty (i.e., the lowest price at which the seller is willing to sell the house is less than the highest price the buyer is willing to pay), and the agents are willing to concede enough, there will always be an agreement reached in the negotiation (since an offer of an agent with utility equal to her reservation utility is acceptable to the other agent). However, for two agents negotiating on two issues, even if an agent makes an offer with utility equal to her reservation utility the offer may not be acceptable to her opponent. For example, Figure 1 shows that although the offers O_1 and O_2 give agents 1 and 2 their least possible utilities (i.e., they concede as much as they can), O_1 and O_2 do not lie in the (unknown) zone of agreement and hence are not acceptable offers. Therefore, developing methods to find offers that provably lie in the zone of agreement with agents not having any information about opponents utility function is a challenging problem, that, hitherto has not been addressed. Provable convergence to an agreement is a desirable property to have for designing automated negotiation agents.

In this paper, we prove that an alternating projection strategy for generating offers that has been proposed in the literature [6], [7] guarantees that the agents reach an agreement. The alternating projection strategy consists of two steps: (a) A concession step in which the agents reduce the utility of offers that are acceptable to them (unless they have reached their reservation utility) (b) An offer generation step in which they use the previous offer of their opponent to generate a new offer with utility equal to their current acceptable utility. We show that the convergence holds for general concave utility functions irrespective of the specific concession strategy the agents adopt (as long as the agents concede up to their individual reservation utilities). In previous work, the concession strategies used by the agents were not reactive to whether the opponent was conceding or not. Hence it was not clear whether the agents had any incentive to concede. We prove that if the agents use reactive concession strategies, i.e., each agent concedes by an amount proportional to her evaluation of the amount of concession of her opponent, then the agents have no incentive to deviate from the concession strategy. To the best of our knowledge, *this is the first paper that gives negotiation strategies with guaranteed convergence to a satisficing solution for general multi-attribute, bilateral negotiation with agents having nonlinear utility functions and no knowledge about other agents preferences*. We also demonstrate the performance of the alternating projection strategy through simulations.

The remainder of the paper is organized as follows: In Section II, we give a review of related literature. In Section III, we outline the framework of automated negotiation. Thereafter, in Section IV, we present our convergence proof of the alternating projection protocol for offer generation. In Section VII we discuss the simulation results and in Section VIII we conclude the paper with a discussion of future work.

II. Related Literature

Theoretical study of negotiation has been done in the economics literature as well as in artificial intelligence (AI) literature. For the literature using non-cooperative game theoretic models of negotiation, the alternating-offer game, which was first proposed by Rubinstein [5], is one of the most popular negotiation protocols for bilateral single-issue setting. Work in economics using the framework of the alternating-offer game often focus on single issue problem. In the original alternating-offer game in [5], as well as subsequent literature (e.g., [8]), the two players (or agents) with complete information have incentive to concede because it is assumed that the utility of the negotiation outcome decreases with time. Transaction cost of bargaining is another reason for the players to concede (see [9]). Some studies (e.g., [10], [11]) consider outside options as an alternative incentive for the players (or agents) to concede over time. These studies also extend the alternating-offer game to the setting where the two players have incomplete and asymmetric information, i.e., they are uncertain about the opponent's type. In our setting, the agents have no information about the opponent's utility structure or type, and no time discounting effect, transaction cost, or outside options are considered. In our setting, the players concede as a part of the search process to achieve a possible agreement in the absence of any information about the opponent's utility.

The alternating-offer game has also been extended to multi-agent or multi-issue negotiation (e.g., [12] , [13], [14]). They usually assume that there are two issues in the negotiation and that the agents utility functions are linear and additive on the values of the two issues (e.g. [15], [16]). However, the agents in real-world are likely to have much more complex utility functions, and the information might be incomplete. Thus, the optimal negotiation strategies and the equilibrium under those negotiation settings with simplified assumptions are difficult to apply in practice. In our setting, there is no knowledge about the other agent's utility function. To the best of our knowledge, no notion of equilibrium solutions has been developed in such no-knowledge settings. Hence, we use the notion of satisficing solution.

The literature using AI methods focus on developing tractable heuristics for negotiating agents to generate offers. Although there is a large body of automated negotiation literature [17], most prior work assumes either full information or commonly known random distributions. In the presence of incomplete information, Bayesian learning has been proposed in agents' negotiation strategy [18]. A classification method for learning an opponent's preferences during a bilateral multi-issue negotiation using Bayesian techniques is developed in [19]. However, the Bayesian updating rule is only applicable when the agents are of certain set of types. There are also works that utilize a non-biased mediator in the negotiation strategy (e.g., [20], [6]). In [6], the authors propose a Pareto-optimal mediating protocol where, in each period, the mediator provides a negotiation baseline and the agents propose base offers on this line. In [20], negotiations mediated by automated mediator are concluded significantly faster than non-mediated ones by conducting an experiment. However, the existence of a non-biased mediator cannot always be assumed.

When the agents' utility functions have general forms [6] propose a shortest-distance proposing method that reaches negotiation outcome with no knowledge of opponent utility function. The agents operate in a offer counter-offer paradigm [5]. Each agent starts with her highest utility offer and use a concession strategy by which they determine their current utility. The agent then proposes the closest offer (among all the offers that corresponds to her current utility) to the opponents offer. Similar methods have been proposed in more specific settings [21], [22], [23]. Although, it was shown through simulations in [6] (and also in an extension of the protocol to three agents negotiating on two issues in [7]), that the agents using the protocol always reach an agreement, there was no formal proof of convergence. Our paper is the first to mathematically prove the convergence of a negotiation protocol by applying the alternating projection theory [24].

III. The Negotiation Framework

We consider two self-interested agents negotiating on a set of issues $j \in \{1, 2, ..., n\}$. Let $i \in \{1, 2\}$ denote the two agents. We assume that the issues take on continuous values and the negotiation domain for each issue is $\Omega_j = [0, 1]$, with 0 and 1 corresponding to the extreme values of the issues. We assume that the utility function of each agent is strictly convex, a widely applied assumption in economics (see [25]). The utility function of agent i, $u_i(x), i = 1, 2$ is continuous and concave $\forall x \in [0, 1]^n$. Without loss of generality, we can normalize the range of agent i's utility function to $[0, 1]$. The properties of the utility function ensure the monotonicity of preferences, i.e., an agent's utility is increasing or decreasing in one issue if the other issues are held constant. Each agent, i, has a *reservation utility*, ru_i. Any offer with utility less than its reservation utility is not acceptable to an agent. The set of all feasible offers that an agent i can accept is $A^i = \{x \in [0, 1]^n \,|u_i(x) \geq ru_i\}$. The set A^i is strictly convex for each i. The *zone of agreement*, $\mathcal{Z}$, is defined as the common intersection of the feasible offer sets of both agents, i.e., $\mathcal{Z} = A^1 \cap A^2$. Since the zone of agreement is the intersection of two convex sets, it is a convex set. For a solution to exist to any negotiation problem, the zone of agreement has to be non-empty. Any point within the zone of agreement is acceptable to both agents and we call such a solution a *satisficing solution* to the negotiation.

A. The Negotiation Model

The alternating-offer game, which was first proposed by [5], is one of the most popular negotiation models. In an alternating-offer game, an agent proposes her offer and the other agent responds to the offer by either proposing a new offer or accepting the offer. If the offer made by the opponent is within her acceptable offer set, an agent accepts the offer, otherwise she proposes a counter-offer. This process continues until an agreement or the negotiation deadline is reached.

Solution Concept for Negotiation: There have been different definitions proposed for a *proper* negotiation solution. Axiomatic solution concepts has been proposed for bargaining games (e.g., Nash bargaining solution [1], Kalai-Smorodinsky solution [26], egalitarian solution [27], pareto-optimal solution). The set of points that satisfy these different solution requirements are all subsets of the zone of agreement. However, computing them requires that all the agents know each other's utility functions. Since an agent does not know the utility function of her opponent, we use a satisficing solution as our solution concept. A satisficing solution is any agreement that gives the negotiators a utility greater than or equal to their reservation utility. The use of a satisficing solution in this very general setting where the agents have no information about their opponents is in the spirit of Herbert Simon [28].

Informally speaking, a negotiating agent not only wants to reach an agreement with the other agent but also may want to obtain as much utility as possible. Thus, when agents start out in a negotiation, they want to propose offers that have the highest utility for them and gradually move towards offers with lower utility. However, they will neither propose nor accept any offer with utility lower than their reservation utility. This intuition implies that, during negotiation, agents gradually reduce the utility of offers acceptable to them (which is very often seen in practice). Consequently, we assume that during the negotiation, agents use a *concession strategy* (e.g., the time-dependent strategy in [29]) to determine their current utility at time t (denoted by $s_i(t)$). This concession continues until an agent reaches her reservation utility. In other words, $s_i(t)$ is a *monotonically decreasing* function of t and $s_i(t) \geq ru_i, \forall t$. We do not make any assumptions on the manner in which the agents reduce their utilities. For agent i, let A_t^i be the set of all offers that have utilities higher than $s_i(t)$ at time t. The set, $A_t^i = \{x \in [0, 1]^n \,|u_i(x) \geq s_i(t)\}$, is called the *current feasible offer set* of agent i. For all t, A_t^i is a convex set and $A_1^i \subseteq A_2^i \subseteq \cdots \subset A^i$. The boundary of the set A_t^i is called the *indifference surface (or curve)* of agent i at time t.

Problem Statement The problem that we are studying in this paper can be formally stated as follows: *Given* 2 *agents negotiating on* n *issues where (a) each agent,* i*, has a strictly concave private utility function,* u_i*, and a strategy for concession that is monotonically decreasing with time (up to the reservation utility,* ru_i*), and (b) the zone of agreement has a nonempty interior, find a method for computing the offer an agent should propose such that it is guaranteed that the* *agents will eventually reach an agreement*.

Agent Strategy: The negotiation strategy that we will use consists of two steps [6]. When it is an agent's turn to make an offer, the agent first checks if the current offer lies within her acceptable offer set. If the current offer is satisficing the agents reach an agreement and the negotiation ends. Otherwise, the agent reduces her current utility and thus increases the set of offers acceptable to her. She then generates an offer on the indifference surface corresponding to her current utility by projecting her opponent's offer to her current indifference surface. Note that this method generates an offer that is satisficing to the agent and closest (in terms of Euclidean distance) to the offer made by her opponent.

In the next section, we will first present the alternating projection method for computing offers for an agent and then give a convergence proof for the method. The convergence proof is asymptotic in nature, i.e., it is proven that the agents will converge to a common point in the zone of agreement as time tends to infinity. This is useful when there is no deadline for negotiation as is the case in many application domains. The infinite time convergence is unavoidable, since the only requirement we have on an agent's concession strategy is that the strategy is monotonically decreasing. Thus, it is possible that an agent may concede so slowly so as to reach its reservation utility as time tends to infinity. Now, if there are negotiation deadlines then questions of finite time convergence are relevant. We formulate this question and study it in a later section.

IV. Offer Generation Method

In the alternating projection offer generation method, if an agent rejects her opponent's offer, she proposes her own offer by choosing the projection of the opponent's offer to her current indifference surface. Figure 2 presents an example of the alternating projection proposing protocol negotiating on two issues. In this example, the solid indifference curves belong to agent 1 and the dashed indifference curves belong to agent 2. In period $t-4$, agent 1 proposes an offer x^1_{t-4}. In period $t-3$, agent 2 rejects this offer and identifies x^2_{t-3} on her indifference curve such that x^2_{t-3} is the projection of x^1_{t-4} to her indifference curve. In period $t-2$, agent 1 rejects this offer and identifies x^1_{t-2} by projection of x^2_{t-3} to her current indifference curve. The process continues until an offer is accepted or the deadline is reached.

A. Convergence of Alternating Projection Method

The convergence of a negotiation strategy implies that the negotiating agents are guaranteed to reach an agreement if the zone of agreement is not empty. We first introduce some notation. Let agent i at period $t+1$ propose an offer $x^i_{t+1} = P_{A^i_{t+1}}(x^j_t)$, if $x^j_t \notin A^i_{t+1}$, where $i,j \in \{1,2\}$, $i \neq j$ and $P_{A^i_{t+1}}(x^j_t)$ is the projection of x^j_t onto the convex face of A^i_{t+1}.

We state the following theorem without proof, since it will be useful in our proof of convergence (please see [24] for proof).

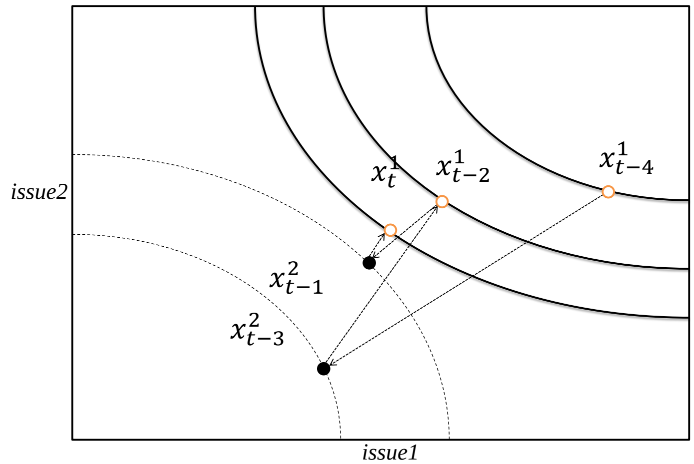

Fig. 2. The alternating projection protocol for two issues and two agents.

Theorem IV.1. *The projection map P_K for a closed convex set K in a real space satisfies $\|P_K(x)-P_K(y)\| \leq \|x-y\|$,for all x,y with equality holding only if $\|x-P_K(x)\| = \|y-P_K(y)\|$.*

The key idea in the proof is to first show that the distance between the offers of the two agents strictly decreases over time under the alternating projection proposing method (see Lemma IV.2).

Lemma IV.2. *For any 3 sequential offers proposed by the two agents, x^i_{t-1}, x^j_t and x^i_{t+1}, we have $\|x^i_{t+1}-x^j_t\| < \|x^j_t - x^i_{t-1}\|$.*

Proof: Define $\tilde{x}^i_t$ as follows: $\tilde{x}^i_{t+1} = P_{A^i_{t-1}}(x^j_t)$, if $x^j_t \notin A^i_t$, where $i,j \in \{1,2\}$, $i \neq j$ and $P_{A^i_{t-1}}(x^j_t)$ is the projection of x^j_t onto the convex face of A^i_{t-1}. By Theorem IV.1, we obtain $\|x^j_t - x^i_{t-1}\| \geq \|P_{A^i_{t-1}}(x^j_t) - P_{A^j_t}(x^i_{t-1})\| = \|\tilde{x}^i_{t+1} - x^j_t\|$. Since $A^i_t \subset A^i_{t+1}$ for $i \in \{1,2\}$, it is easy to show that $\|\tilde{x}^i_{t+1} - x^j_t\| > \|x^i_{t+1} - x^j_t\|$. We therefore have $\|x^i_{t+1} - x^j_t\| < \|\tilde{x}^i_{t+1} - x^j_t\| \leq \|x^j_t - x^i_{t-1}\|$. ■

Using Lemma IV.2 we can obtain the following result.

Lemma IV.3. *For a given negotiation problem, if the zone of agreement has a nonempty interior and there are infinite periods to negotiate, i.e. $T \to \infty$, the agents are guaranteed to reach an agreement using a monotonic concession strategy and an alternating projection proposing protocol .*

Proof: Suppose $\exists\, s$ such that $A^1_s \cap A^2_s \neq \varnothing$, (otherwise for $\forall t$, $A^1_t \cap A^2_t = \varnothing$, which implies no interior point in the zone of agreement) then as $A^i_t \subset A^i_{t+1}$ for $i \in \{1,2\}$, we have for all $t_1 \geq s$ and $t_2 \geq s$, $A^1_{t_1} \cap A^2_{t_2} \neq \varnothing$. Since the distance between the offers decrease with time (c.f. Lemma IV.2) and there is a non-empty intersection of the acceptable offer sets of the two agents, the offers will converge to a point in the intersection of the two sets. This can be shown using standard arguments for proving the convergence of the alternating projection method to a point within the set $C \cap D$ when $C \cap D \neq \varnothing$, where C

and D are two fixed convex sets [24]. ■

V. Finite Time Convergence of Offer Generation Method

In the previous section, we proved that as long as the zone of agreement has a non-empty interior and the agents concede so that they reach their reservation utilities, the agents can reach an agreement. We did not make any assumption about the time an agent takes to concede to the reservation utility. In this section, we study the finite time convergence properties of the alternating projection method for offer generation. This is relevant for negotiation with deadlines. For negotiation with deadlines, one needs to be careful with the question of whether the agents will reach an agreement or not. Even if the zone of agreement is non-empty, whether the agents reach an agreement or not depends on the relative volume of the zone agreement with respect to the whole offer space as well as the rate of concession of the agents. For example, for a negotiation deadline of T, consider the case where two agents use the concession strategy that they will propose offers with their highest utility up to time $T-1$ and then propose an offer with utility equal to their reservation utility at time T. Technically, the agents use a monotonically decreasing concession strategy and reaches the reservation utility at time T. However, the intersection of the current feasible offer sets up to time $T-1$ may be empty, and although the zone of agreement is non-empty, the proposal at time T may not be within the zone of agreement and therefore, there will be no agreement.

To understand finite time convergence properties of a method for computing offers, we study the following question: *Given that the concession strategies of the agents are such that all the agents reach their reservation utilities in finite time, say T_0, do the agents converge to an agreement in finite time (provided the zone of agreement has a non-empty interior)*? Note that we do not make any assumptions about the specific concession strategy used by the agents.

The answer to this question is yes in general. We first give a sketch of the formal proof of this claim for two agents negotiating on two issues to show the basic intuition.

Theorem V.1. *For two agents negotiating on two issues, if the agents use concession strategies such that they reach their reservation utilities in finite time, then they can reach an agreement in finite time (assuming that the zone of agreement has a non-empty interior).*

Proof: Without loss of generality, we can assume that each agent starts with an offer on her indifference curve corresponding to her reservation utility (since by assumption she reaches her reservation utility in finite time). Figure 3 shows the indifference curves corresponding to the reservation utilities for two agents, say 1 and 2, negotiating on two issues. Since the indifference curves are strictly concave, they intersect at exactly two points. Let A_1 and A_2 be the feasible offer sets for the two agents. Let θ be the minimum of the two angles made by the tangents at the intersection points. In Figure 3, $\angle AOB = \theta$, and OA and OB are the tangents to the two curves at O. Let P be the initial offer of agent 1. The projection of P on A_2 is Q. The projection of Q on A_1 is R and the projection of R on A_2 is S, and so on. Thus the offer sequence is $\{P, Q, R, S, \dots\}$. Let A be the intersection of the tangent to A_1 at O with PQ. Construct the sequence $\{A, B, C, D, E, \dots\}$, where $AB \perp OB$, $BC \perp OA$, $CD \perp OB$ and so on. Since the sets A_1 and A_2 are strictly convex, the sequence $\{P, R, \dots\}$ approaches O faster than the sequence $\mathcal{S} = \{A, C, E, \dots\}$. Thus, if the sequence $\{A, C, E, \dots\}$ reaches O in finite time then the sequence $\{P, R, \dots\}$ reaches O in finite time.

Now, $AB = OA\sin(\theta), \angle ABC = \angle AOB = \theta$. Therefore $AC = AB\sin(\angle ABC) = OA\sin^2(\theta)$. Similarly $CE = OC\sin^2(\theta)$, and so on. Since the angle θ is a constant for any given negotiation problem, at each step, the distance of a point in the sequence $\mathcal{S}$ reduces by a constant ratio. Hence, the sequence $\mathcal{S}$ converges to O in finite time, which implies that the sequence $\{P, R, \dots\}$ reaches O in finite time. Thus, the agents reach the same point in the zone of agreement in finite time. ■

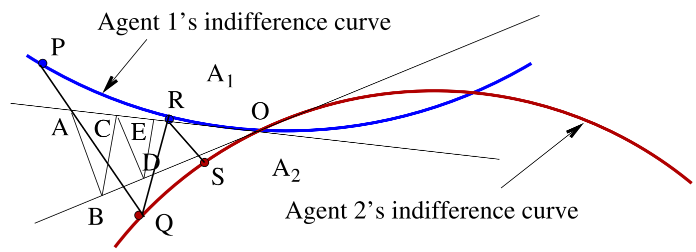

Fig. 3. Two agents negotiating on two issues.

VI. Incentive of Agents to Concede

In the previous sections, we proved that the alternating projection strategy proposed in the literature guarantees that the agents converge to an agreement provided the zone of agreement is non-empty and the agents concede. We first note that if an agent 1 concedes, there is no incentive for her to propose an offer on the indifference surface that is not the projection of her opponent 2's offer. This is because all points on the indifference surface of agent 1 have the same utility and by proposing another point she may decrease the chance of reaching an agreement. However, one can argue that an agent may want to deviate from the proposal strategy by not conceding since that may give her more utility. For example, since agent 2's concession strategy is independent of agent 1's strategy, agent 2 will continue to concede till she reaches her reservation utility. Thus, agent 1 may gain more utility by not conceding.

In this section, we show that there is a *reactive concession strategy*, namely, concede by an amount proportional to the perceived change in utility of the opponent's offer that is rational (i.e., if agents follow this strategy, they do not have any incentive to deviate). More precisely, we prove that if any of the agents do not concede, it is possible for the opponent to determine this within a finite number of rounds and hence stop conceding. This, combined with the fact that an agent does not know her opponents utility provides the threat of the negotiation coming to a stall, even if the zone of agreement is non-empty. Since the utility of a negotiated agreement is not worse than the utility for breakdown (which can be thought of as the reservation utility), it is rational for the agent to concede.

We now prove that if agent 1 stops conceding and agent 2 uses a reactive strategy, the negotiation can stall. As shown in Figure 4, let agent 1 propose x_t^1 at time t. If agent 1 stops to concede from time t, all offers proposed by agent 1 after time t are on the indifference surface $u_1(x) = u_1(x_t^1) = s_1(t)$. Let x^* be the point on the indifference surface $u_1(x) = s_1(t)$ such that $u_2(x) = u_2(x^*)$ is the highest possible perceived utility by agent 2. Therefore, $\Delta = u_2(x^*) - u_2(x_t^1)$ would be the largest possible perceived utility improvement for agent 2. Hence, using the reactive strategy, agent 2 would concede by at most Δ. If $\Delta < (s_2(t) - u_2(x^*))$, where $s_2(t)$ is the current utility level of agent 2 at time t, the negotiation will stall (see Figure 4 for the two-issue case). Since agent 1 has no knowledge about the utility function of agent 2, she is uncertain about whether the largest possible perceived utility improvement for agent 2, Δ, is larger than $s_2(t) - u_2(x^*)$. Thus, agent 1 is not sure about whether there will be an agreement or not if she stops to concede from time t. Since an agreement would provide higher utility than her reserved utility for no agreement, agent 1 would not stop conceding. By a similar argument, agent 2 would keep conceding through the negotiation process.

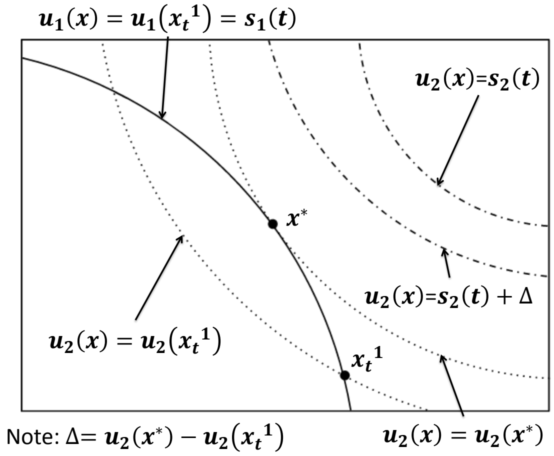

Fig. 4. Figure for proving that there is an incentive to concede.

VII. Simulation Results

The convergence proofs of the alternating projection protocol presented above is either asymptotic in nature, or gives finite time convergence if the agents concede to their reservation utilities in finite time. It is imperative to also understand the practical convergence performance of our algorithm with increasing number of issues and the quality of the solution obtained. In this section, we present simulation results that show that the algorithm always converges (to a finite numerical precision) in finite number of rounds (that depends on the desired numerical precision). We evaluate our solution with respect to the Nash bargaining solution [1].

For performing simulations, we choose a time-dependent concession strategy function that has been suggested in the literature [6], [13], [7]. The current utility of agent i in round t is given by

$$s_i(t) = 1 - (1 - ru_i)\left(\frac{t}{T}\right)^{1/\beta_i}, \quad (1)$$

where ru_i is the reservation utility and T is the negotiation deadline. In Equation (1), the reservation utility smoothly decays from a maximum value of 1 at $t = 0$ to a value of ru_i at $t = T$, with the decay rate controlled by the parameter β_i. For all the presented simulation results, $T = 200$ rounds.

We have assumed a very general *hyperquadric* utility function [30] for the agents.

$$u_i(x) = 1 - \sum_{k=1}^{Q} |H_k(x)|^{n_k},$$

where x is the n-dimensional proposal vector, $H_k(x) = \sum_{j=1}^{N} a_{kj}x_j, n_k = l_k/m_k, l_k, m_k \in \mathbb{Z}^+; f(x)$ is strictly concave if $1 < n_k < \infty$. Hyperquadrics are a very general class of functions used in computer graphics [30] and can model a wide range of convex functions. The feasible set of offers for an agent i at time t is the intersection of the unit n-dimensional hypercube $[0,1]^N$ with $u_i(x) \geq s_i(t)$. Popular convex functions for modeling utilities in economics like the Cobb-Douglas functions can be shown to be special cases of the hyperquadric function. The sole reason for using this function is that it is possible to generate a wide variety of preference structures for the agents with these functions.

In order to evaluate the quality of our negotiation solutions, we compare it against the Nash bargaining solution which is also Pareto optimal. The Nash bargaining solution maximizes the joint utility (i.e., the product of the utilities) of the agents. For the class of (strictly) concave utility functions that we consider, we can obtain the Nash bargaining solution by solving a convex optimization problem and hence we can easily find this solution irrespective of the number of negotiation issues. In the paper, we have used the solver CVX [31], [32] implemented in MATLAB for obtaining the Nash bargaining solution.

Figure 5 shows a typical sequence of offers generated by the two agents negotiating over three issues (for simplicity of presentation). Agents' utility functions are randomly created hyperquadrics (whose domain is the unit square). The boundary of the acceptable set of offers for the reservation utility is shown for each agent by the solid and dashed lines. The decay parameter β_i in Equation 1 is 1.2 and the reservation utility is 0.2 for each agent. The ratio of the joint utility obtained by our algorithm to the Nash bargaining solution is 0.9928. The utilities obtained by agents 1 and 2 with the corresponding Nash solution utilities in parenthesis are: 0.2408(0.2219) and 0.2672 (0.2921).

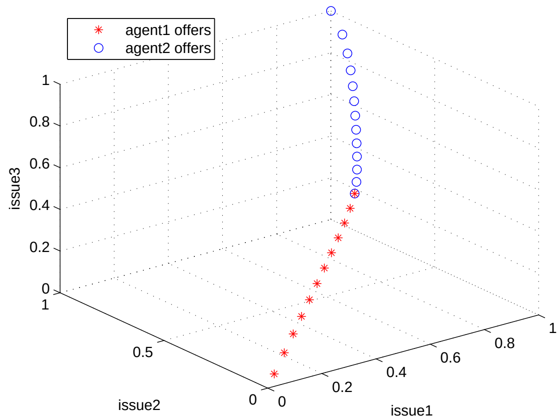

Fig. 5. Sequence of offers made by the two agents in a three-issue negotiation scenario using the alternating projection algorithm.

TABLE I
PERFORMANCE OF THE ALTERNATING PROJECTION ALGORITHM FOR RANDOMLY GENERATED PROBLEMS WITH DIFFERENT NUMBER OF ISSUES.

No. of issues	No. of rounds		Ratio of Joint Utility	
	Mean	SD	Mean	SD
2	31.2	5.2	0.9676	0.0339
3	36.9	5.0	0.9586	0.0440
5	41.8	4.8	0.9432	0.0522
7	44.9	3.6	0.9202	0.0714
9	45.5	2.8	0.9583	0.0265

In order to test the performance of our algorithm with increasing number of issues, we ran simulations with randomly generated utility functions with different number of issues for various values of β_i. Table I presents typical results for two agents negotiating on m issues for a random choice of β_i, namely, $\beta_i \in [0.8, 1.2]$, for $i = 1, 2$. The reservation utility of the two agents are selected to ensure that the zone of agreement is non-empty. The number of issues are varied between 2 and 9. The results are averaged over 100 random runs for each row of the table. The numerical tolerance used for convergence is 0.001. As can be seen from Table I (second and third columns), the number of rounds required to arrive at an agreement do not change much with the increase in number of issues. The solution obtained is very near to the Nash bargaining solution (fourth and fifth columns).

In the previous simulations, an agent's concession strategy was not reactive to her opponent's concession strategy. We now consider the case where each agent adapts her concession strategy according to her perception of her opponent's amount of concession. The perceived utility of agent i for agent j's offer, x_j, is $u_i(x_j)$. Thus, the change in perceived utility of agent i for agent j's offer at round t is

$$\Delta u_i(t) = (u_i(x_j(t)) - u_i(x_j(t-2)))^+$$

where $y^+ = \max\{0, y\}$. The current utility of agent i in round t is given by

$$s_i^{\text{adjust}}(t) = \min\{s_i(t), s_i(t-2) + \Delta u_i(t)\}, \qquad (2)$$

TABLE II
PERFORMANCE OF THE ALTERNATING PROJECTION ALGORITHM WITH REACTIVE CONCESSION STRATEGY.

No. of issues	No. of rounds		Ratio of Joint Utility	
	Mean	SD	Mean	SD
2	30.7	4.5	0.9752	0.0395
3	34.5	5.6	0.9755	0.0283
5	41.9	4.6	0.9149	0.1450
7	43.1	2.7	0.9498	0.0767
9	45.1	3.4	0.9462	0.0532

where $s_i(t)$ is the current utility of agent i in the original time-dependent concession strategy.

Figure 6 shows a simulation where the agent 1 stops conceding after reaching half of its reservation utility. Since agent 2 is reactive, it realizes within a few steps that agent 1 is not conceding and it stops conceding. Hence, although the zone of agreement of the two agents is nonempty, the two agents do not reach an agreement, as the agents stop conceding.

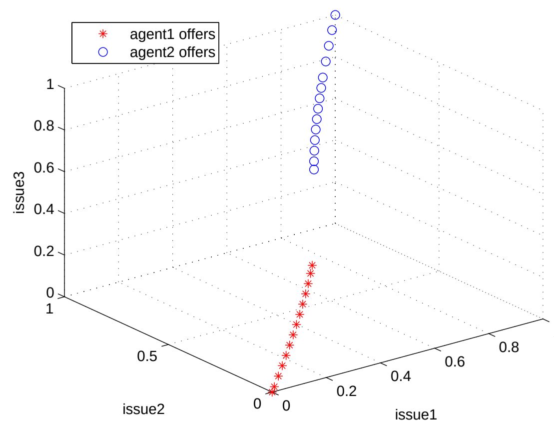

Fig. 6. Sequence of offers made by the two agents without a final agreement in a three-issue negotiation scenario using the reactive concession strategy when agent 1 stops conceding.

Table II shows the performance of the algorithm when the adjusted time-dependent concession strategy is used. Using similar setting as Table I, two agents negotiate on m issues for a random choice of $\beta_i \in [0.8, 1.2]$, $i = 1, 2$. Comparing the results in Table II with Table I, we find that both the number of rounds and the ratio of joint utility are close. It suggests that the adjusted time-dependent concession strategy does not affect the performance of the algorithm adversely.

VIII. CONCLUDING REMARKS

In this paper, we prove that the alternating projection method is guaranteed to enable negotiating agents to arrive at an agreement for general automated multi-attribute negotiation. We show that agents can arrive at an agreement, even if they have no knowledge about each other's utility function. This is the first paper that formally establishes the convergence of a proposal method by investigating the geometric properties of the negotiation process. The convergence guarantees hold for any nonlinear concave utility function. We also show that in the absence of any information about the opponent this offer generation strategy along with a reactive concession strategy is a rational strategy. Using simulations we demonstrated that the solution obtained by our algorithm is quite close to the Nash bargaining solution (that maximizes the joint utility of the agents). The negotiation converges in a reasonable number of iterations and scales well as the number of issues are increased.

Several broader issues need to be further addressed. One possibility is to generalize this alternating projection method to negotiation between multiple (more than two) agents, and investigate whether the convergence of the method still holds. At present our agents are myopic in nature and do not try to learn the other agents utility function from the sequence of offers. It would also be interesting to investigate whether the agents can be incorporated with some learning capability so that they converge to a better agreement.

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