PAC Learning: Theorem 1

A $h \in \mathcal{H}$ is consistent with the training data if $\hat{R}(h) = 0$

Union bound

$P(A \cup B) \le P(A) + P(B)$

Theorem 1:

$$N \ge \frac{1}{\epsilon}\left[\log|H| + \log\frac{1}{\delta}\right]$$

N examples is sufficient to ensure that with prob $1-\delta$ all $h \in H$ with $\hat{R}(h) = 0$ have $R(h) \le \epsilon$

$P(\hat{R}(h_1)=0)$
$+ P(\hat{R}(h_2)=0)$
$+ P(\hat{R}(h_3)=0)$
$\vdots$

(1) Assume k "bad" hypotheses $h_1, h_2, \dots, h_k$ with $R(h_i) > \epsilon$

(2) Pick bad h_i: Prob h_i is consistent w/ first train point $\le 1-\epsilon$

Prob $\underbrace{h_i \text{ is consistent w/ first } N \text{ train points}}_{\hat{R}(h_i)=0} \le (1-\epsilon)^N$

(3) Prob at least one bad h_i is consistent w/ first N train points $\le k(1-\epsilon)^N$

$\le |H|(1-\epsilon)^N$

(4) $1 - x \le e^{-x} \implies |H|(1-\epsilon)^N \le |H| e^{-\epsilon N}$

(5) Calc N and δ s.t. $\boxed{|H| e^{-\epsilon N} \le \delta}$

(6) Solve N $\quad |H| \frac{1}{\delta} \le \frac{1}{e^{-\epsilon N}}$

$\boxed{\log|H| + \log 1/\delta \le \epsilon N}$

Assume $N \ge \frac{1}{\epsilon}[\log|H| + \log 1/\delta]$

with prob δ

$\exists\, h$ st $\underbrace{R(h) > \epsilon}_{A}$ and $\underbrace{\hat{R}(h)=0}_{B}$ "bad"

with prob $1-\delta$

all $h \in H$ with $R(h) > \epsilon$ have $\hat{R}(h) > 0$ "good"

$\implies$ all $h \in H$ with $\underbrace{\hat{R}(h)=0}_{\neg B}$ have $\underbrace{R(h) \le \epsilon}_{\neg A}$

Contrapositive

$A \Rightarrow B$

$\neg B \Rightarrow \neg A$