Computational Foundations for ML

10-607

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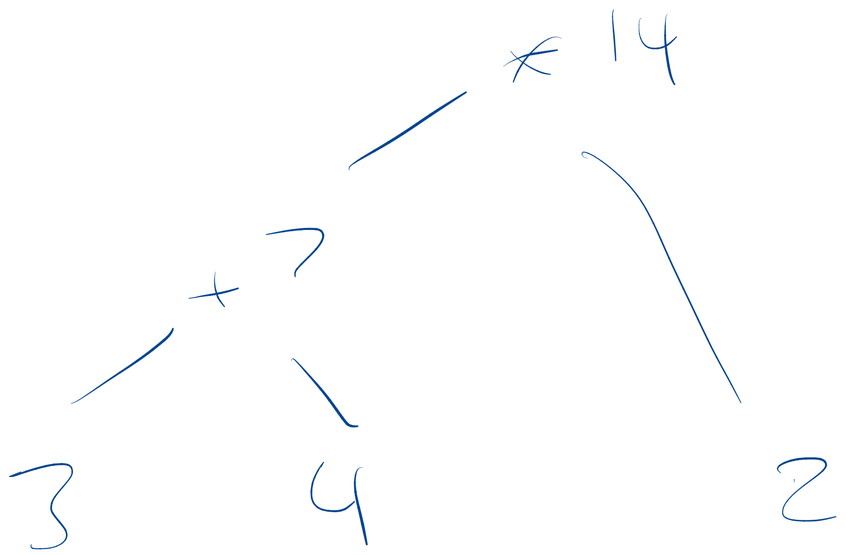



```
LOAD    $4    R1
LOAD    $3    R2
ADD     R1, R2, R3
LOAD    $2    R4
MUL     R3, R4, R5
```

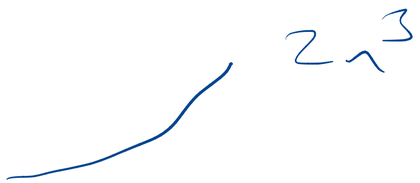
(3+4)*2

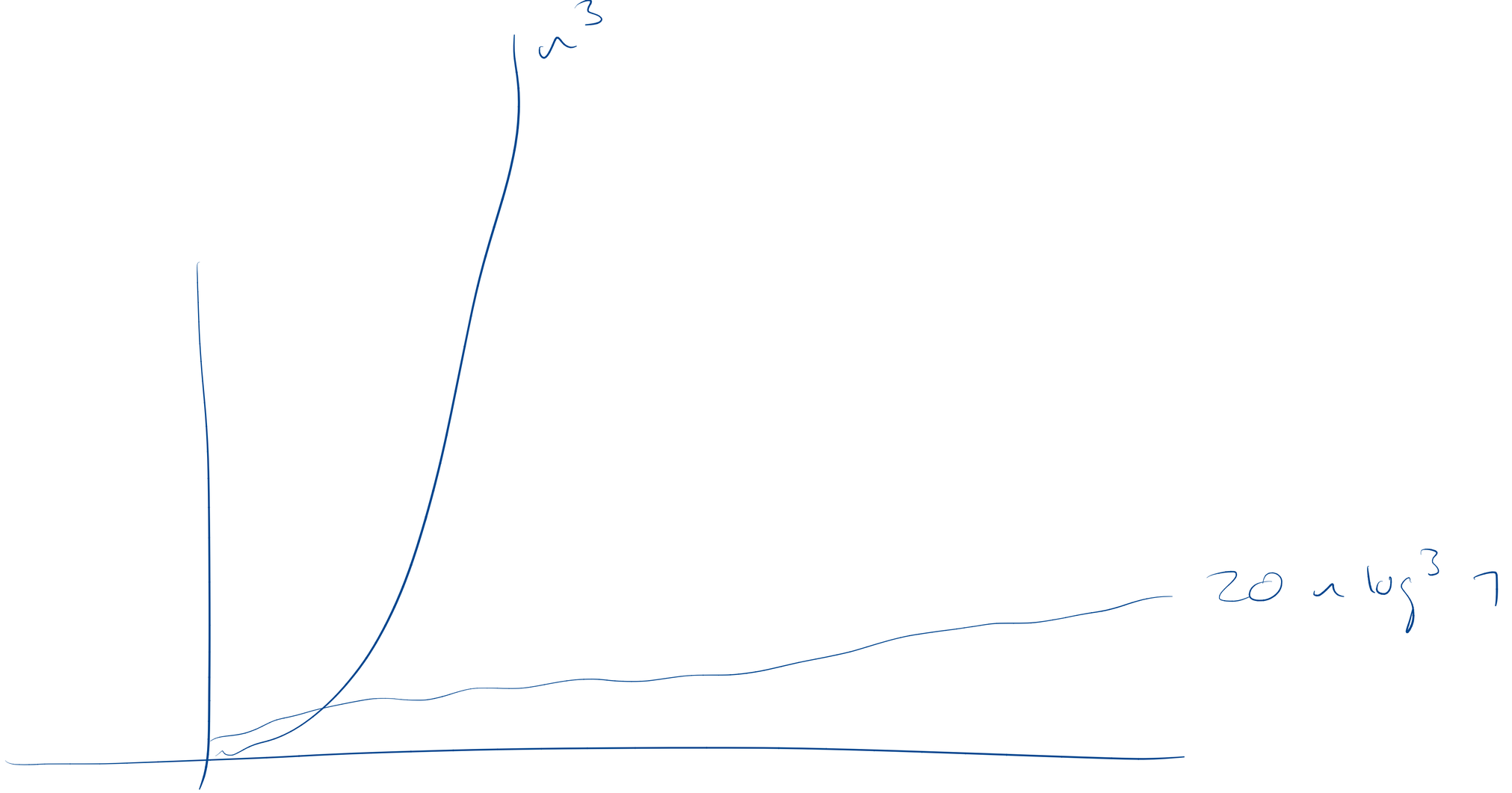
$2n^3$

$4m \log^3 n$ ← # nonzero entries

$m = 5n$

$20n \log^3 n$




n^3
$20 \cdot n \log^3 7$

$f(n) \geq 0 \qquad n \geq 1$

$g(n) \geq 0$

$$O(g(n)) = \{ f \mid \exists\, c > 0.\ \exists\, n_0 > 1.\ \forall\, n > n_0.\ f(n) \leq c\, g(n) \}$$

$f \in O(g)$

$f(n) \in O(g(n))$

$$f(n) = 3n^2 + 5n + 99$$

~~$g(n) \stackrel{?}{=} 1000000\,n$~~

$3n^2 \leq 3n^2$ $\quad 5n \leq n^2 \quad \forall n \geq 5$

$5 \leq n \quad 5 \leq n$

$n \quad n \quad n \leq n$

$99 \leq n^2 \quad n \geq 10$

$3n^2 + 5n + 99 \leq \underbrace{5}_{c} n^2 \quad n \geq \underbrace{10}_{n_0}$

$f \in O(n^2)$

$f = O(n^2)$

$f \in \Theta(n^2)$

$f \in O(n^3)$

$f \in O(527n^2 + 3)$

$f \in O(g) \qquad g \in O(h) \rightarrow f \in O(h)$

$\hookrightarrow g \in \Omega(f)$

$g \in \Theta(f)$
$f \in \Theta(g)$
} both O and Ω

$f \in O(g)$

$s \in O(t)$

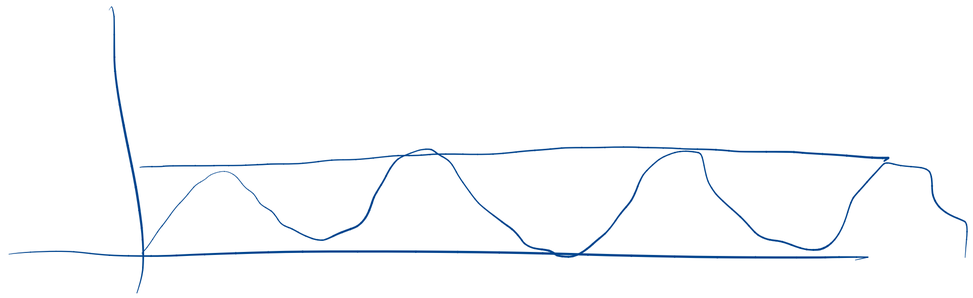
$f \cdot s \in O(g \cdot s)$

$f \cdot s \in O(g \cdot t)$

$f + s \in O(g + t)$

$n^p \in O(n^q) \quad p \leq q$

$f(n) = 1 \notin O(\overbrace{\sin^2 \frac{n}{2\pi}}^{g(n)})$



$\sin^2\left(\frac{1}{2}\pi\right) \in O(1)$ ✓

$\binom{n}{2} \in O(n^2)$

$\binom{n}{k} \in O(?)$

$O(n!)$ $O(n^n)$

$=$

$\frac{n!}{k!\,(n-k)!}$

$n\,(n-1)\,(n-2)\,\cdots\,(n-k+1)\cdot\frac{1}{k!} \in O(n^k)$

$\uparrow\ \uparrow\ \uparrow\ \qquad\uparrow\ \qquad\uparrow$

$\le n\ \le n\ \le n\ \qquad \le n\ \qquad \le 1$

$f(n) \in \tilde{O}(g(n))$

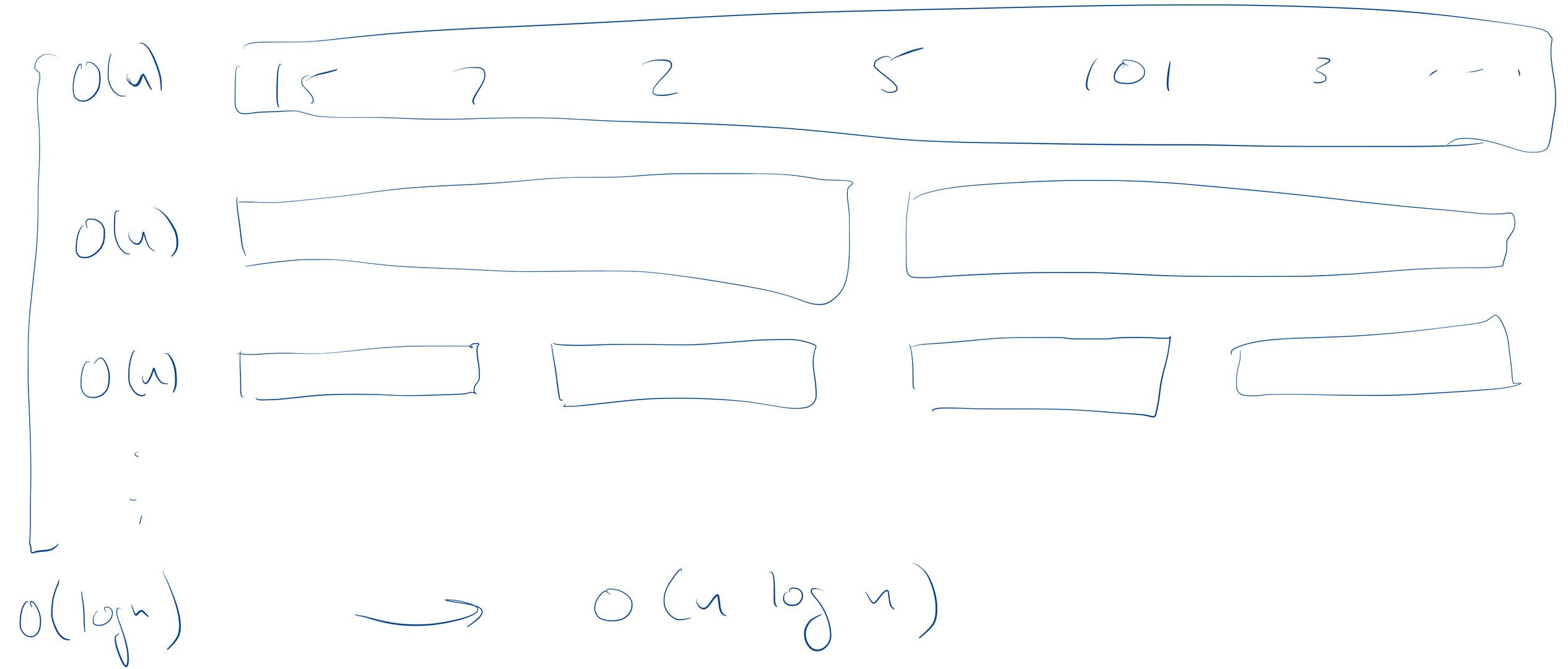
"like $O(g(n))$ but ignoring log factors"

quick sorting $\in O(n \log n)$

$\in \tilde{O}(n)$

$f(n) \in 2^{O(n)}$

$3^n \notin O(2^n)$


O(n)
15 7 2 5 101 3 ...
O(n)
O(n)
O(log n)
O(n log n)

$O(\log n)$

n