

# Computational Foundations for ML

10-607

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# Notes and reminders

- Lab 3 due today
- HW2 out (due date extended to 5/2)
- Room for Quiz 2: GHC 4215

meanings	$X_i$	prob $P(X_i)$	code $C_i$	# bits in $C_i$ $l_i$
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$l_i \approx -\log_2 P(X_i)$  (exact for long sequences of  $X_i$ 's and optimal code)

Use on average  $-\sum_i P(X_i) \log_2 P(X_i) \equiv \text{entropy } P(\cdot)$   
 $H(P) = \uparrow$

$0 \leq H(P) \leq \log_2 n$   
 $\uparrow$  # of meanings



True dist  $P_i = P(X_i)$

Use  $q_i$

Spent  $-\sum_i P_i \log_2 q_i$

vs. opt  $-\sum_i P_i \log_2 P_i$

Waste =  $-\sum_i P_i \log \frac{q_i}{P_i} = KL(P \parallel q)$

$= \sum_i P_i \log \frac{P_i}{q_i}$

$$P = \left( \frac{1}{4} \quad \frac{1}{4} \quad \frac{1}{4} \quad \frac{1}{4} \right)$$

$$Q = \left( \frac{1}{2} \quad \frac{1}{4} \quad \frac{1}{8} \quad \frac{1}{8} \right)$$

design for P:

true P: 2 bits / meaning

true Q: 2 bits / meaning

design for Q

true P: 2.25 bits/m

true Q: 1.75 bits/m

$$D(\underbrace{P}_{\text{true}} \mid \underbrace{Q}_{\text{designed}}) = 2.25 - 2 = 0.25 \text{ bits}$$

$$D(q|p) = 2 - 1.75 = 0.25 \text{ bits}$$

	$X_1$	$X_2$	$X_3$
q	$\frac{3}{7}$	$\frac{3}{7}$	$\frac{1}{7}$
p	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$

X	Y	P
pancakes	L	$\frac{1}{2}$
cereal	L	$\frac{1}{6}$
pancakes	R	$\frac{1}{6}$
cereal	R	$\frac{1}{6}$

$$\frac{2}{3} D\left(\left(\frac{3}{4}, \frac{1}{4}\right) \middle| \left(\frac{2}{3}, \frac{1}{3}\right)\right)$$

$$+ \frac{1}{3} D\left(\left(\frac{1}{2}, \frac{1}{2}\right) \middle| \left(\frac{2}{3}, \frac{1}{3}\right)\right)$$

$\approx 0.44$  bits

$$\frac{1}{2} \log_2 \frac{1/2}{2/3} + \frac{1}{2} \log_2 \frac{1/2}{1/3}$$

$$E_Y \left( D(P(X|Y=y) | P(X)) \right) = I(X, Y) = I(Y, X)$$



# Exercise

## Information gain

which is better to observe,  $X_1$  or  $X_2$ ?  
(If we want to know  $Y$ )

$$H(Y) = - \left[ \frac{3}{5} \log_2 \frac{3}{5} + \frac{2}{5} \log_2 \frac{2}{5} \right] \approx 0.971 \text{ bits}$$

$X_1$	$X_2$	$Y$
1	1	0
0	1	0
1	0	0
0	1	1
0	0	1

$\frac{2}{3}, \frac{1}{3}$   
0.92 bits

1 bit