Math Foundations for ML

10-606

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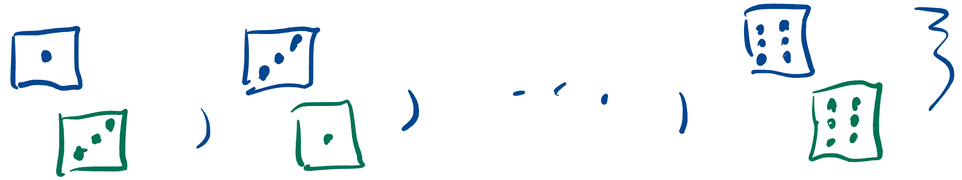
Notes and reminders

- Hand in Lab 3 today
- Upcoming: Lab 4 (Monday)
- Exam schedule should post soon (for time and place of Quiz 2)

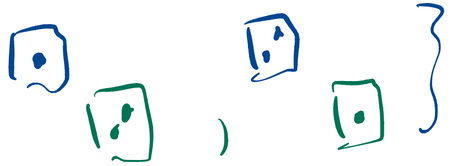
$U = \{$  $\}$

$\sum_{a \in U} P(a) = 1$

$P(1,3)$

$1/36$

$\hookrightarrow \geq 0$

$E_3 = \{$  $\}$

$P(E_3) = \frac{1}{36} + \frac{1}{36} = \frac{1}{18}$

$A \cup B$ = either A or B happens

$A \vee B$

$A \cap B$ = both A AND B

$A \wedge B$

$A \setminus B$

$A \wedge \neg B$

$A^c = U \setminus A = \neg A$

suit = ♤ ∧ pips % 2 = 0

A	B	A → B
F	F	T
F	T	T
T	F	F
T	T	T

A	B	A ⟷ B
F	F	T
F	T	F
T	F	F
T	T	T

$\neg(A \to B)$

$\neg(\neg A \vee B)$

$\underbrace{\neg\neg A}_{A} \wedge \neg B$

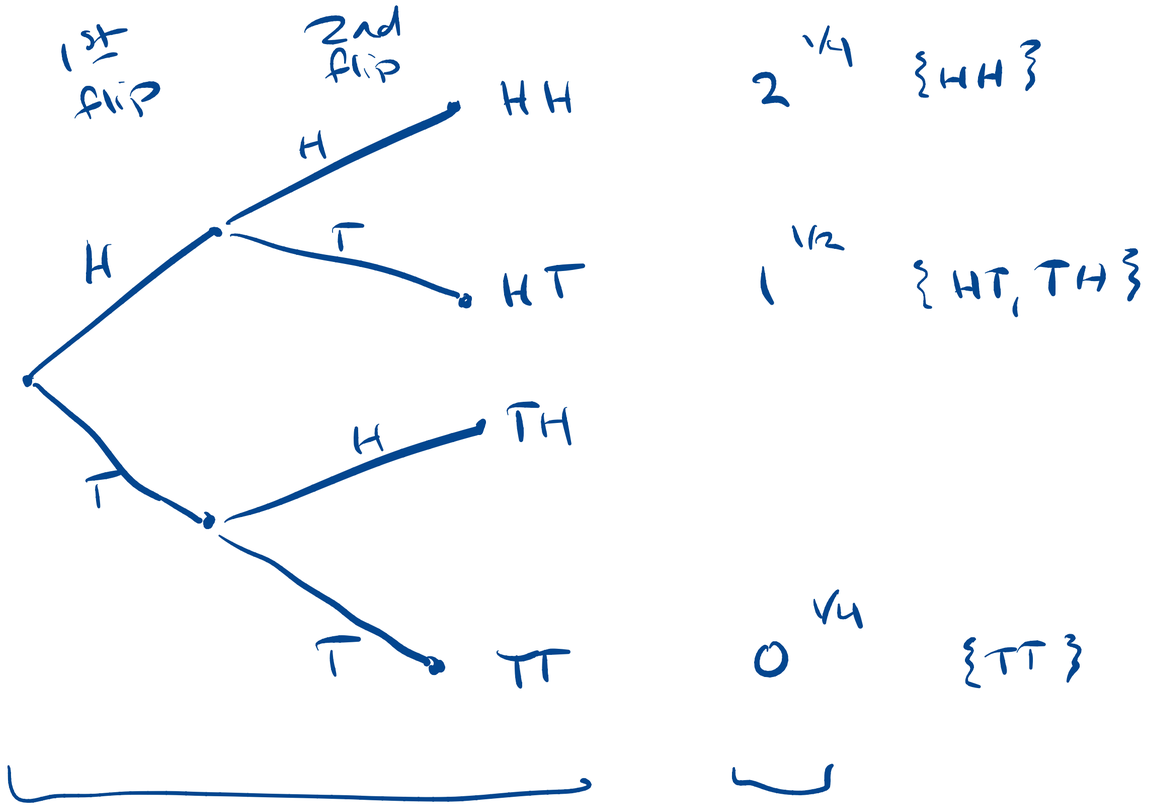
$A \wedge \neg B$

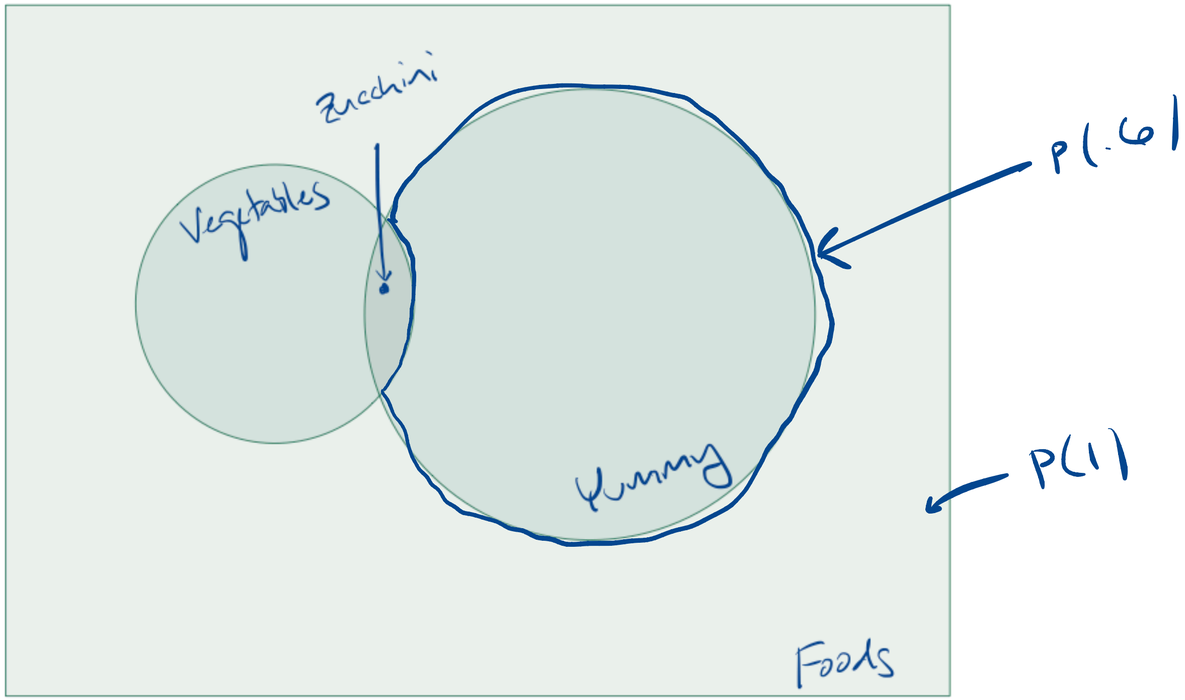
$U = \mathbb{R}$ A e.g. $[1.7, 1.9]$

$f(x) \geq 0$ density $\int_U f(x)\,dx = 1$

$P(A) = \int_A f(x)\,dx$

$P(X = [t - \frac{\epsilon}{2}, t + \frac{\epsilon}{2}]) \approx \epsilon f(t)$


1st flip
2nd flip
H
T
H
T
H
T
HH
HT
TH
TT
2 1/4 {HH}
1 1/2 {HT, TH}
0 1/4 {TT}


Zucchini
Vegetables
Yummy
Foods
P(.|.)
P(.)

$$\frac{52!}{47!\,5!} = \#\text{ of 5-card hands (unordered)}$$

$$\binom{52}{5}$$

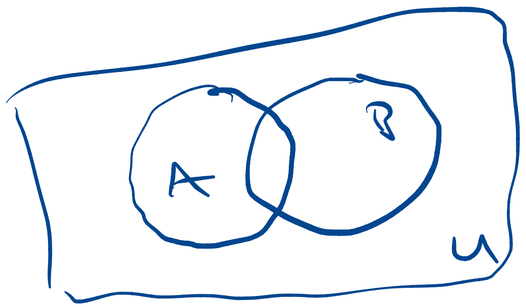
$$13 \cdot 4 \cdot \frac{48 \cdot 47}{2}$$

$$\binom{48}{2}$$

$$\frac{13 \cdot 4 \cdot \binom{48}{2}}{\binom{52}{5}} \approx 2.26\%$$

$A \cap B = \phi$

A	$P(A)$
B	$P(B)$



$$P(A \cup B) \xrightarrow{\text{disjoint}} P(A) + P(B)$$

$$\hookrightarrow \text{o/w} \quad \leq P(A) + P(B)$$

$$= P(A) + P(B) - P(A \cap B)$$

What is P(5 right, in test on 7 examples)

if classifier is useless

$$\frac{\binom{7}{5}}{2^7} = \frac{21}{128} = 16.4\%$$

At least 5: $\frac{21+7+1}{128} = \frac{29}{128} = 22.7\%$

$\binom{7}{5}$ $\binom{7}{6}$ $\binom{7}{7}$

RV: $U \to \mathbb{R}$

$X(\square\ \square) = 4$

$X = 4$

$\exp(3X) \cdot \cos(Y/2\pi)$

X^2

p	eyes	height
0.01	blue	150 cm
0.03	brown	155 cm
0.02	⋮	⋮
⋮		

$Z = f(X, Y)$

$Z(\omega) = f(X(\omega), Y(\omega))$ $\quad \hookrightarrow \in U$

eyes	150	151	152	...
blue	0.01			
brown				
green				
⋮				

A	B	C	P
F	F	F	0.30
F	F	T	
F	T	F	
F	T	T	
T	F	F	
T	F	T	
T	T	F	
T	T	T	0.10