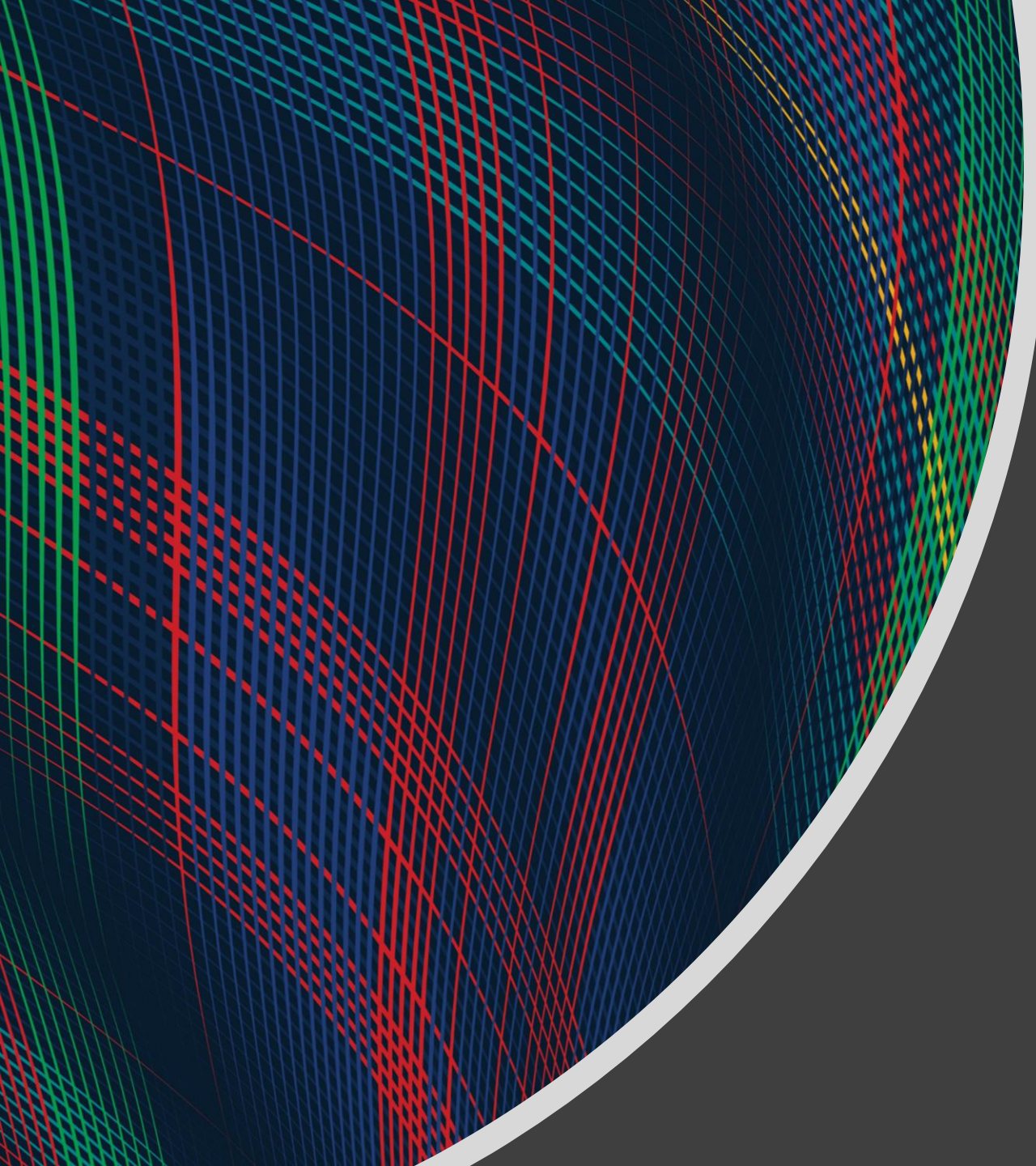


# Plan

Wrap-up Overview → Lecture 1 slides

## Proof Techniques

- Proof by cases
- Disprove by counterexample
- Proof by contrapositive



10-607  
Computational  
Foundations for  
Machine Learning

Proof Techniques

Instructor: Pat Virtue

# Proof Techniques

## Proof by Cases

# Proof by Cases

Goal is to prove  $X$

1  $\phi \vee \psi$  Assume

2 Case 1:  $\phi$

a)

b)

c)  $\phi \Rightarrow X$

\_\_\_\_\_  
\_\_\_\_\_  
\_\_\_\_\_

3 Case 2:  $\psi$

a)

b)

c)

d)  $\psi \Rightarrow X$

\_\_\_\_\_  
\_\_\_\_\_  
\_\_\_\_\_  
\_\_\_\_\_

4  $X$

by  $\vee$  elimination

# Proof by Cases

Proposition: Let  $a, b \in \mathbb{Z}$ . If  $ab$  is even, then either  $a$  is even or  $b$  is even (or both).

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→ whiteboard

# Proof Techniques

Proof by cases

Disproof by counterexample

- One example is not sufficient to prove
- One counterexample is sufficient to disprove

# Disprove by counterexample

Proposition: Let  $a, b \in \mathbb{Z}$ . If  $a$  is odd and  $b$  is odd, then  $a+b$  is odd



# Poll 1

Given model  $m$ : {A: True, B: False}

Does  $m$  satisfy the following sentence:

$$(A \Rightarrow B) \Leftrightarrow (\neg B \Rightarrow \neg A)$$

i. Yes

ii. No

iii. Not enough information

iv. Syntax error in sentence

## Poll 2

Truth table for  $(A \Rightarrow B) \Leftrightarrow (\neg B \Rightarrow \neg A)$

How many rows do we need (excluding a header)?

- i. 2
- ii. 4
- iii. 6
- iv. 8
- v. 16

# Poll 3

Truth table for  $(A \Rightarrow B) \Leftrightarrow (\neg B \Rightarrow \neg A)$

How many columns should we have?

- i. 2
- ii. 3
- iii. 4
- iv. 5
- v. 6