Midterm Exam 1A

15110 Principles of Computing Spring 2015

February 18, 2015

Instructions

- Answer each question neatly in the space provided.
- There are 7 questions totaling 34 subproblems on 12 pages. Not all problems are the same size or difficulty.
- Please read each question carefully. You have 50 minutes for this exam. No electronic devices allowed. Good luck!

	Max	Score
1	6	
2	16	
3	20	
4	21	
5	22	
6	10	
7	5	
Total:	100	

Questions

- 1. This question deals with history of computing devices.
 - (a) (2 points) What is the significance of the invention of integrated circuits in the history of computing?
 - (b) (2 points) Moore's Law says that the number of integrated circuit chips in a computer doubles every 2 years, which implies that computers become twice as powerful every 2 years. According to Moore's Law, how many years from now will computers be 32 times more powerful as they are now?

(c) (2 points) A Gigabyte (GB) is 2³⁰ Bytes and a Kilobyte (KB) is 2¹⁰ Bytes. If you have a storage device with a capacity of 16 GB, how many 32KB files can you fit in that device? Express the result as a power of 2.

- 2. This question focuses on expressions, data types, and variable assignments.
 - (a) (4 points) For each of the following Python expressions, write down the value that is output when the expression is evaluated using a Python interpreter. Write **error** if you think the expression will raise an error.
 - 15 % 3
 - 12 + 3 * 5 == 75
 - "CMU" + "15110"
 - "1000" + 500
 - (b) (4 points) Suppose that we type the following assignments and expressions in a Python shell **in the given order**. For each of the expressions below write down the value that will be output by Python.

>>> a = 10
>>> b = 20
>>> c = a + b
>>> a + 5
----->>> a + b
----->>> b = b + 1
>>> c
----->>> a + b

(d) (2 points) Show how to create a list of every integer between 0 and 100, inclusive, named nums1 using Python, sorted in increasing order.

nums1 = list(_____)

(e) (2 points) Let nums2 and nums3 be two non-empty lists. Write a Python command that will append the last element of nums3 to the end of nums2.
 ______append(______).

- 3. This question focuses on the basics of Python functions and tracing.
 - (a) (3 points) In economics, the percentage rate of inflation for a period of time is calculated based on the final value F of a commodity and the initial value I of the commodity, using the formula $((F-I)/I) \times 100$. Write a Python function inflation_rate(initial, final) to compute and return the inflation rate given the initial and final values of a commodity.

(b) (4 points) Using the function from part (a), write a Python function average_inflation_rate(), that computes and returns the average rate of inflation for the 3-year period represented in the table below:

Year	Initial value	Final value
1	23.50	24.00
2	24.00	24.25
3	24.25	24.38

The function is **required** to call the function from part (a).

(c) (3 points) Consider the simple function given below.

```
def twice(n):
```

```
print(2*n)
```

When we compare twice(5) with 10 the Python interpreter would return False after printing 10 as shown below. Explain why it gives False as a result of the comparison. >>> twice(5) == 10 10

False

(d) (6 points) Consider the following Python function where **m** and **n** are assumed to be a positive integers:

```
def mystery(n, m):
    p = 0
    e = 0
    while e < m:
        p = p + n
        e = e + 1
    return p</pre>
```

Trace this function for n = 4, m = 3, showing the value of e and p in the table above at the end of each iteration of the loop. The initial values of p and e are given for you in the table. Use as many spaces as you need.

р	е
	=======
0	0

- (e) (2 points) Which of the following functions is being computed by mystery above? Circle your answer.
 - 1. *nm*

- $2. \ n+m$
- 3. n^m
- $4. \ m^n$
- 5. None of these
- (f) (2 points) Suppose that the return statement was indented as below. What would mystery (4, 3) return in this case?

```
def mystery(n, m):
    p = 0
    e = 0
    while e < m:
        p = p + n
        e = e + 1
        return p</pre>
```

- 4. This question focuses on searching.
 - (a) (5 points) Below is a Python function that takes a list of integers nums and an integer divisor as inputs, and searches for a number in the list that is evenly divisible by divisor. It returns the index of the first occurrence of such a number, or None if there is no such number. For example, when the function is called with [101, 4, 12, 24] for nums and 2 for divisor it should return 1. This is because 4 is the first item in the list that is divisible by 2. Complete the missing parts of the function.

def first_divisible(nums, divisor):

for _____:
 if _____:

return _____

return _____

Assuming that the list vals is [100, 66, 55, 64, 41, 35, 18, 64], write the output from each of the following calls to first_divisible.

- (b) (2 points) first_divisible(vals, 9)
- (c) (2 points) first_divisible(vals, 101)
- (d) (2 points) How many times would the for-loop iterate if we evaluated first_divisible(list(range(10, 0, -1)), 12)?

(e) (5 points) Suppose we want to return, not the first, but the last element in the list that is divisible by the parameter divisor. Complete the function definition below to accomplish this task. If you're not able to make your answer conform to the partial definition, write your answer in the space provided.

def last_divisible(nums, divisor):

for _____:

if _____:

return _____

return _____

- (f) (2 points) Supposing the length of a list vals is fixed at n elements, what kind of list contents would result in the longest running time when evaluating first_divisible(vals, 2)?
- (g) (2 points) Supposing the length of a list vals is fixed at n elements. What kind of list contents would result in the shortest running time when evaluating first_divisible(vals, 2)?

- (h) (1 point) Assuming the input list has length n, give the big-O notation for the **worst-case** time complexity for first_divisible.
- 5. This question focuses on searching and sorting.
 - (a) (6 points) Recall that the recursive binary search algorithm we studied in class works with a range to search defined by two indexes: lower and upper. The value of lower is one less than the index of the first element in the range, and the value of upper is one greater than the index of the last element in the range. Finally, the algorithm also computes the midpoint mid = (lower + upper) // 2.

Suppose we do a binary search in the list

[0, 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28, 30, 32]

for the value 23. Complete the table below to show how the values of lower, upper, and mid change during the search, until and including the point where the base case is reached. (You should not need all the space provided.)

lower	upper	mid
-1	17	8
Ĩ	11	0

(b) (2 points) Give the big-O notation for the **worst-case** time complexity of binary search of a list of n elements.

- (c) (5 points) Below is a list of functions you have seen that describe the running times of algorithms for inputs of size n. Order them by asymptotic rate of growth (smallest rate to largest rate) by writing the corresponding numbers in order in the space provided.
 - 1. $n \log n$
 - 2. $\log n$
 - 3. n
 - 4. 1
 - 5. n^2
- (d) (1 point) What kind of input causes the binary search algorithm to exhibit its *best-case* time complexity?
- (e) (4 points) In order to use binary search, we must have a sorted list. Suppose we begin with an unsorted list of length n, sort it using mergesort, and then search it once using binary search. Give the big-O notation for the worst-case time complexity of the whole process. For full credit, make sure you give the smallest and simplest function possible, and show your work. (Remember, if there are multiple terms, you should throw out the ones with lower growth rates)
- (f) (2 points) If an algorithm has worst-case time complexity $O(\log n)$ for an input of size n, doubling the input size increases the running time by how much? Circle your answer.
 - 1. a constant amount
 - 2. a factor of two
 - 3. a factor of four
 - 4. zero
 - 5. the running time is squared

(g) (2 points) Suppose we sort a list of 8 elements using mergesort. There will be eight singleelement lists to be pairwise merged.

List the other merge operations needed to complete the mergesort, giving the list sizes and number of lists of each size. For instance, you might say "five 3-element lists and two 5-element lists" (except that would be wrong!)

- 6. This question deals with correctness of functions and testing.
 - (a) (2 points) Complete the following function definition so that it finds the first pair of adjacent items in the input list that are in descending order, and returns the index of the first element of the pair. If there is no such pair, it returns None. Example:

```
>>> search_pair([1,3,4,6,4,7,6])
```

3

Notice that 3 is the index of 6, which is the first element of the first pair in descending order. (It may be helpful to remember that range(n) is empty, if n is 0 or less, and a for-loop on an empty range skips the loop body.)

```
def search_pair(items):
```

```
for i in range(_____):
    assert i+1 < len(items)
    if items[i] > items[i + 1]:
        return i
return None
```

- (b) (2 points) Explain in one sentence the purpose of the assert statement.
- (c) (6 points) Below is a test function for search_pair. It only tests two kinds of input. Add three test cases: one for an empty list, one for a list containing only one element, and one for a list where the only two adjacent elements in descending order are the last two.

```
def test_search_pair():
```

```
assert search_pair(list(range(10))) == None
assert search_pair([4, 5, 4, 5]) == 1
```

```
print("Test complete")
```

- 7. This question is based on your readings from the book Blown to Bits.
 - (a) (3 points) Here is a quote from the book:

Imagine one sick person infecting two healthy people, and the next day each of those two infects two others, and the next day after that each of those four infects two others, and so on. The number of newly infected each day grows from two to four to eight. In a week, 128 people come down with the disease in a single day, and twice that number are now sick, but in a population of ten million, no one notices. Even after two weeks, barely three people in a thousand are sick. But after another week, 40% of the population is sick, and society collapses.

The hypothetical epidemic overwhelms society at the end of three weeks, with 2^{22} people infected. At what point in time was the epidemic only half as severe, with 2^{21} people infected?

(b) (2 points) Event Data Recorders (EDRs) are common in modern cars. What kind of data do they record, and why do police sometimes download the data from cars involved in an accident?