# Modules II

15-150

Lecture 17: 🎃👻🎃, 2024

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# Announcement: midterm II

Be on time; next lecture starts at 12:30pm!

## When and where:

- Thursday, **November 7**, **11:00am—12:20pm**.
- **MM 103** (Sections A—D), **PH 100** (Sections E—L).

## Scope:

- Lectures: 1—15.
- Labs: 1—8 and midterm review section of Lab 10.
- Assignments: up to including Exceptions/Regex.

## What you may have on your desk:

- Writing utensils, we provide paper, something to drink/eat, tissues.
- 8.5” x 11” cheatsheet (back and front), handwritten or typeset.
- No cell phones, laptops, or any other smart devices.

# Recap

Abstraction through separating specification from implementation:

- Specification: externally visible promise deliver.
- Implementation: internal choice of how to deliver promise.
- Allows us to hide implementation details from the client.
  - Representation independence: the client becomes independent of the choice of internal representation.
  - Any two implementations that satisfy specifications are indistinguishable to the client and thus equal.
- Facilitates modular reasoning (component-wise reasoning).

# Recap

SML modules facilitate abstraction:

- Specification: **signature**.
- Implementation: **structure**.

SML modules allow us to control the "flow of information":

- Structures can **hide** auxiliary, implementation-specific components, not specified by signature.
- **Transparent ascription**: for undefined type specified in signature, **representation type** chosen by structure is **revealed**.
- **Opaque ascription**: for undefined type specified in signature, **representation type** chosen by structure is **hidden**.

# Today

- Functors (aka functions on structures).
- Type classes (aka descriptive signatures).
- Deeper exploration of transparent and opaque ascription.

Correspondence:

| | | |
|---|---|---|
| specification | signature | type |
| implementation | structure | value |
| mapping | functor | function |

loosely

- Let’s resume our dictionary example!

# Example: dictionary

A dictionary is a collection of pairs of the form

```
(key, value)
```

where all keys must be unique within a dictionary.

```
signature DICT =
sig
  type key = string               (* concrete type *)
  type 'a entry = key * 'a        (* concrete type *)
  type 'a dict                    (* abstract type *)
end
```

Let's use strings as keys for now

Permit value type to be polymorphic

# Example: dictionary

A dictionary is a collection of pairs of the form

```
(key, value)
```

where all keys must be unique within a dictionary.

```
signature DICT =
sig
  type key = string                       (* concrete type *)
  type 'a entry = key * 'a                (* concrete type *)
  type 'a dict                            (* abstract type *)



end
```

# Example: dictionary

A dictionary is a collection of pairs of the form

```
(key, value)
```

where all keys must be unique within a dictionary.

```
signature DICT =
sig
  type key = string                       (* concrete type *)
  type 'a entry = key * 'a                (* concrete type *)
  type 'a dict
  val empty : 'a dict
  val lookup : 'a dict -> key -> 'a option
  val insert : 'a dict * 'a entry -> 'a dict
end
```

Replace entry, if key already exists

# Search tree representation of dictionary

Implementation: **represent** dictionary as a binary search tree, where

`(key, value)`

are stored in nodes.

- Representation **invariant**:
  - Tree must be sorted.
  - Keys must be unique.
- All functions declared by structure
  - may **assume** that received tree is sorted,
  - and must **assert** that returned tree is sorted.
  - (Similarly for key uniqueness.)

# Search tree representation of dictionary

```
signature DICT =
sig
  type key = string           (* concrete type *)
  type 'a entry = key * ‘a (* concrete type *)
  type 'a dict                (* abstract type *)
  val empty : 'a dict
  val lookup : 'a dict -> key -> 'a option
  val insert : 'a dict * 'a entry -> 'a dict
end
```

Explore :>

```
structure BST : DICT =
struct
  type key = string
  type 'a entry = key * 'a
  datatype 'a tree =
    Empty | Node of 'a tree * 'a entry * 'a tree
  type 'a dict = 'a tree
```

Transparent ascription can be useful for debugging purposes.

```
end
```

# Search tree representation of dictionary

```
signature DICT =
sig
  type key = string              (* concrete type *)
  type 'a entry = key * ‘a (* concrete type *)
  type 'a dict                   (* abstract type *)
  val empty : 'a dict
  val lookup : 'a dict -> key -> 'a option
  val insert : 'a dict * 'a entry -> 'a dict
end
```

```
structure BST : DICT =
struct
  type key = string
  type 'a entry = key * 'a
  datatype 'a tree =
    Empty | Node of 'a tree * 'a entry * 'a tree
  type 'a dict = 'a tree



end
```

forced by signature

# Search tree representation of dictionary

```
signature DICT =
sig
  type key = string              (* concrete type *)
  type 'a entry = key * ‘a (* concrete type *)
  type 'a dict                (* abstract type *)
  val empty : 'a dict
  val lookup : 'a dict -> key -> 'a option
  val insert : 'a dict * 'a entry -> 'a dict
end
```

```
structure BST : DICT =
struct
  type key = string
  type 'a entry = key * 'a
  datatype 'a tree =
    Empty | Node of 'a tree * 'a entry * 'a tree
  type 'a dict = 'a tree
```

Because datatype is not declared in signature, constructors (and thus pattern matching) are not available outside signature.

```
end
```

But bindings externally visible due to transparent ascription.

# Search tree representation of dictionary

```
signature DICT =
sig
  type key = string               (* concrete type *)
  type 'a entry = key * ‘a (* concrete type *)
  type 'a dict                    (* abstract type *)
  val empty : 'a dict
  val lookup : 'a dict -> key -> 'a option
  val insert : 'a dict * 'a entry -> 'a dict
end
```

```
structure BST : DICT =
struct
  type key = string
  type 'a entry = key * 'a

  datatype 'a tree =
    Empty | Node of 'a tree * 'a entry * 'a tree

  type 'a dict = 'a tree



end
```

# Search tree representation of dictionary

```
signature DICT =
sig
  type key = string           (* concrete type *)
  type 'a entry = key * ‘a (* concrete type *)
  type 'a dict                (* abstract type *)
  val empty : 'a dict
  val lookup : 'a dict -> key -> 'a option
  val insert : 'a dict * 'a entry -> 'a dict
end
```

```
structure BST : DICT =
struct
  type key = string
  type 'a entry = key * 'a
  datatype 'a tree =
    Empty | Node of 'a tree * 'a entry * 'a tree
  type 'a dict = 'a tree
  val empty = Empty
  fun insert ...
  fun lookup ...
end
```

explore next!

# Search tree representation of dictionary

```
(* ins : 'a dict * 'a entry -> 'a dict *)
fun insert (Empty, e) = Node(Empty, e, Empty)
  | insert (Node(lt, e’ as (k’,_), rt),
            e as (k,_)) =
```

Layered pattern matching

# Search tree representation of dictionary

```
(* ins : 'a dict * 'a entry -> 'a dict *)
fun insert (Empty, e) = Node(Empty, e, Empty)
  | insert (Node(lt, e’ as (k’,_), rt),
            e as (k,_)) =
```

# Search tree representation of dictionary

```
(* ins : 'a dict * 'a entry -> 'a dict *)
fun insert (Empty, e) = Node(Empty, e, Empty)
  | insert (Node(lt, e’ as (k’,_), rt),
            e as (k,_)) =
      (case String.compare(k,k’) of
```

# Search tree representation of dictionary

```
(* ins : 'a dict * 'a entry -> 'a dict *)
fun insert (Empty, e) = Node(Empty, e, Empty)
  | insert (Node(lt, e' as (k',_), rt),
            e as (k,_)) =
      (case String.compare(k,k') of
        EQUAL => Node(lt, e, rt)
```

Replace existing entry with new one

# Search tree representation of dictionary

```
(* ins : 'a dict * 'a entry -> 'a dict *)
fun insert (Empty, e) = Node(Empty, e, Empty)
  | insert (Node(lt, e’ as (k’,_), rt),
            e as (k,_)) =
      (case String.compare(k,k’) of
        EQUAL => Node(lt, e, rt)
```

# Search tree representation of dictionary

```
(* ins : 'a dict * 'a entry -> 'a dict *)
fun insert (Empty, e) = Node(Empty, e, Empty)
  | insert (Node(lt, e’ as (k’,_), rt),
            e as (k,_)) =
      (case String.compare(k,k’) of
        EQUAL => Node(lt, e, rt)
        | LESS => Node(insert(lt,e), e’, rt)
```

# Search tree representation of dictionary

```
(* ins : 'a dict * 'a entry -> 'a dict *)
fun insert (Empty, e) = Node(Empty, e, Empty)
  | insert (Node(lt, e’ as (k’,_), rt),
            e as (k,_)) =
      (case String.compare(k,k’) of
        EQUAL => Node(lt, e, rt)
        | LESS => Node(insert(lt,e), e’, rt)
        | GREATER => Node(lt, e’, insert(rt,e)))
```

# Search tree representation of dictionary

```
(* lookup : 'a dict -> key -> 'a option *)
fun lookup tree key =
 let
    fun lk (Empty) = NONE
    | lk (Node(left, (k,v), right)) =
      (case String.compare(key,k) of
        EQUAL => SOME(v)
        | LESS => lk left
        | GREATER => lk right)
  in
    lk tree
  end
```

# Search tree representation of dictionary

Let's interact with **BST**:

```
val d = BST.insert(BST.insert(BST.insert(
          BST.empty,("a",1)),("b",2)),("c",3))
```

What is the type of **d**?

```
int BST.dict
```

The binding for **d** will be revealed because of opaque ascription. However, because the tree datatype is not declared in the signature, a client cannot pattern match on its constructors.

Now consider: `val look = BST.lookup d`

What is the type of **look**?

```
BST.key -> int option
```

# Search tree representation of dictionary

Let's interact with **BST**:

```
val d = BST.insert(BST.insert(BST.insert(
          BST.empty,(“a",1)),("b",2)),("c",3))
```

What is the type of **d**?

```
int BST.dict
```

Now consider: `val look = BST.lookup d`

What is the type of **look**?

```
BST.key -> int option
```

Now consider:

```
val x = look “e”
val y = look “a”
```

Bindings: `[NONE/x, (SOME 1)/y]`

# Let's reconsider our DICT signature

```
signature DICT =
sig
  type key = string                    (* concrete type *)
  type 'a entry = key * 'a             (* concrete type *)
  type 'a dict                         (* abstract type *)
  val empty : 'a dict
  val lookup :
  val insert :
end
```

What if we needed keys other than strings?

We could try to make key polymorphic too.

# Let's reconsider our DICT signature

```
signature DICT =
sig
  type 'a key = 'a                    (* concrete type *)
  type ('a,'b) entry = 'a key * 'b (* concrete type *)
  type ('a,'b) dict                   (* abstract type *)
  val empty : ('a,'b) dict
  val lookup :
  val insert :
end
```

- What if we needed keys other than strings?
- We could try to make key polymorphic too.

# Let's reconsider our DICT signature

```
signature DICT =
sig
  type 'a key = 'a                     (* concrete type *)
  type ('a,'b) entry = 'a key * 'b (* concrete type *)
  type ('a,'b) dict                           ype *)
  val empty : ('a,'b) dict
  val lookup :
  val insert :
end
```

How to implement now?

Used `String.compare`

- What if we needed keys other than strings?
- We could try to make key polymorphic too.
- Keys should become comparable!

# Let’s reconsider our DICT signature

```
lookup:

insert:
```

→ **Keys should become comparable!**

# Let’s reconsider our DICT signature

```
lookup: ('a *'a -> order) -> ('a,'b) dict -> ‘a ->'b option

insert: ('a *'a -> order) -> (('a,'b) dict * ('a,'b) entry)
                          -> ('a,'b) dict
```

➡ Keys should become comparable!

➡ Require a comparison function as an argument.

➡ Restricts polymorphism of keys!

# Let's update our BST structure accordingly

```
structure BST : DICT =
struct
  type 'a key = ‘a

  type ('a,'b) entry = 'a key * 'b

  datatype ('a,'b) dict = Empty | Node of
             ('a,'b) dict * ('a,'b) entry * ('a,'b) dict

  val empty = Empty

  fun insert cmp d k =

  fun lookup cmp (d, k) =
end
```

As specified by signature

# Let’s update our BST structure accordingly

```
structure BST : DICT =
struct
  type 'a key = ‘a

  type ('a,'b) entry = 'a key * 'b

  datatype ('a,'b) dict = Empty | Node of
           ('a,'b) dict * ('a,'b) entry * ('a,'b) dict

  val empty = Empty

  fun insert cmp d k =

  fun lookup cmp (d, k) =
end
```

Again, binary search tree as representation type.

This time, with polymorphic key.

# Let’s update our BST structure accordingly

```
structure BST : DICT =
struct
  type 'a key = ‘a

  type ('a,'b) entry = 'a key * 'b

  datatype ('a,'b) dict = Empty | Node of
                ('a,'b) dict * ('a,'b) entry * ('a,'b) dict

  val empty = Empty

  fun insert cmp d k =

  fun lookup cmp (d, k) =
end
```

As before.

# Let’s update our BST structure accordingly

```
structure BST : DICT =
struct
  type 'a key = 'a

  type ('a,'b) entry = 'a key * 'b

  datatype ('a,'b) dict = Empty | Node of
            ('a,'b) dict * ('a,'b) entry * ('a,'b) dict

  val empty = Empty

  fun insert cmp d k =

  fun lookup cmp (d, k) =
end
```

Bodies of `insert` and `lookup` now use `cmp` instead of `String.compare`.

# Let’s update our BST structure accordingly

```
structure BST : DICT =
struct
  type 'a key = 'a

  type ('a,'b) entry = 'a key * 'b

  datatype ('a,'b) dict = Empty | Node of
              ('a,'b) dict * ('a,'b) entry * ('a,'b) dict

  val empty = Empty

  fun insert cmp d k =

  fun lookup cmp (d, k) =
end
```

Does this do the trick?

# Let's update our BST structure accordingly

```
fun insert cmp d k =
fun lookup cmp (d, k) =
```

Does this do the trick?

# Let’s update our BST structure accordingly

```
fun insert cmp d k =
fun lookup cmp (d, k) =
```

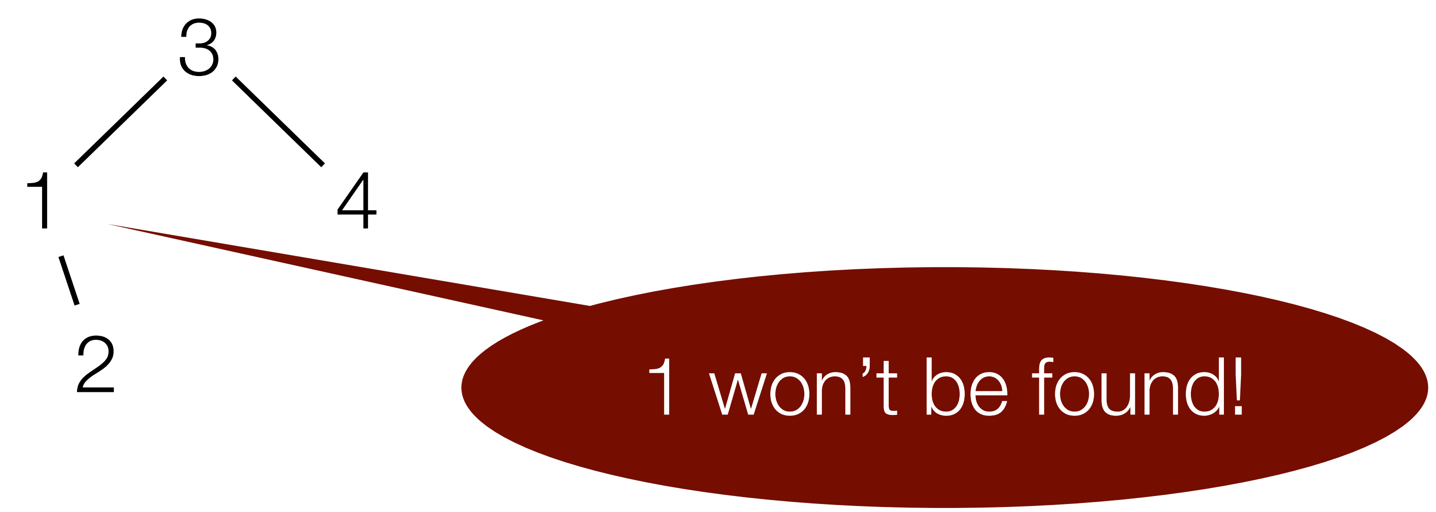
Does this do the trick?  Well, not quite.

What if a client provides different `cmp` functions to `insert` than to `lookup`, for example?

# Let's update our BST structure accordingly

- Does this do the trick? Well, not quite.
- What if a client provides different `cmp` functions to `insert` than to `lookup`, for example?

For example, a client creates the following tree using `insert` and `Int.compare`:




For `lookup` of `1`, the client now uses:

```
fun cmp (x,y) = Int.compare (y,x)
```

# Let's update our BST structure accordingly

- Does this do the trick? Well, not quite.
- What if a client provides different `cmp` functions to `insert` than to `lookup`, for example?
- Can we enforce the invariant, that all operations use the same comparison function by typing?
- Yes, but we need type classes for this!

# Type classes

Type class

A signature specifying a type and associated operations.

No expectation that specification is exhaustive.

Example:

```
signature ORDERED =
sig
  type t      (* parameter *)
  val compare : t * t -> order
end
```

Signature **ORDERED** specifies an “ordered type class” to consist of a type **t** along with a comparison function **compare** for **t**.

# Type classes

Example:

```
signature ORDERED =
sig
  type t        (* parameter *)
  val compare : t * t -> order
end
```

- Even though `t` is not concrete, it is not abstract.
- We expect `t` to be some already existing type, hence use the comment **parameter**.
- Signature `ORDERED` is said to be **descriptive**.
- Signature `DICT` is in contrast **prescriptive**, defining an abstract type with all its operations, exhaustively.

# Type classes

- Even though `t` is not concrete, it does not have to be abstract.
- We expect `t` to be some already existing type, hence use the comment **parameter**.
- Signature `ORDERED` is said to be **descriptive**.
- Signature `DICT` is in contrast **prescriptive**, defining an abstract type with all its operations, exhaustively.
- We tend to use transparent ascription for descriptive signatures (aka type classes), and opaque ascription for prescriptive signatures.

# Perspective of types in signatures

Concrete:

Signature dictates representation type, which is thus visible to client.

Abstract:

Signature hides representation type.  Client code must work regardless of the representation type chosen by structure.

Parameter:

Client supplies the type, implementation must work with whatever the clients supplies.

# Different ways of implementing ORDERED

```
structure IntLt : ORDERED =
struct
  type t = int
  val compare = Int.compare
end
```

```
signature ORDERED =
sig
  type t     (* parameter *)
  val compare : t * t -> order
end
```

# Different ways of implementing ORDERED

```
structure IntLt : ORDERED =
struct
  type t = int
  val compare = Int.compare
end

structure IntGt : ORDERED =
struct
  type t = int
  fun compare(x,y) = Int.compare(y,x)
end
```

```
signature ORDERED =
sig
  type t      (* parameter *)
  val compare : t * t -> order
end
```

# Different ways of implementing ORDERED

```
signature ORDERED =
sig
  type t      (* parameter *)
  val compare : t * t -> order
end
```

```
structure IntLt : ORDERED =
struct
  type t = int
  val compare = Int.compare
end

structure IntGt : ORDERED =
struct
  type t = int
  fun compare(x,y) = Int.compare(y,x)
end

structure StringLt : ORDERED =
struct
  type t = string
  val compare = String.compare
end
```

# Redefine DICT using type class ORDERED

```
signature DICT =
sig
  type key = string                        (* concrete  *)
  type 'a entry = key * 'a                 (* concrete  *)
  type 'a dict                             (* abstract  *)
  val empty : 'a dict
  val lookup : 'a dict -> key -> 'a option
  val insert : 'a dict * 'a entry -> 'a dict
end
```

Use type class as a parameter

# Redefine DICT using type class ORDERED

```
signature DICT =
sig
  structure Key = ORDERED                        (* parameter *)
  type 'a entry = key * 'a                       (* concrete  *)
  type 'a dict                                   (* abstract  *)
  val empty : 'a dict
  val lookup : 'a dict -> key -> 'a option
  val insert : 'a dict * 'a entry -> 'a dict
end
```

→ Any structure implementing `DICT` will comprise a sub-structure implementing `ORDERED`.

# Redefine DICT using type class ORDERED

```
signature DICT =
sig
  structure Key = ORDERED                     (* parameter *)
  type 'a entry = key * 'a                    (* concrete  *)
  type 'a dict                                (* abstract  *)
  val empty : 'a dict
  val lookup : 'a dict -> key -> 'a option
  val insert : 'a dict * 'a entry -> 'a dict
end
```

Use type class' type `t`

Any structure implementing `DICT` will comprise a sub-structure implementing `ORDERED`.

# Redefine DICT using type class ORDERED

```
signature DICT =
sig
  structure Key = ORDERED              (* parameter *)
  type 'a entry = Key.t * 'a          (* concrete  *)
  type 'a dict                        (* abstract  *)
  val empty : 'a dict
  val lookup : 'a dict -> key -> 'a option
  val insert : 'a dict * 'a entry -> 'a dict
end
```

Any structure implementing `DICT` will comprise a sub-structure implementing `ORDERED`.

# Redefine DICT using type class ORDERED

```
signature DICT =
sig
  structure Key = ORDERED                (* parameter *)
  type 'a entry = Key.t * 'a             (* concrete  *)
  type 'a dict                           (* abstract  *)
  val empty : 'a dict
  val lookup : 'a dict -> key -> 'a option
  val insert : 'a dict * 'a entry -> 'a dict
end
```

Use type class' type t

Any structure implementing DICT will comprise a sub-structure implementing ORDERED.

# Redefine DICT using type class ORDERED

```
signature DICT =
sig
  structure Key = ORDERED                    (* parameter *)
  type 'a entry = Key.t * 'a                 (* concrete  *)
  type 'a dict                               (* abstract  *)
  val empty : 'a dict
  val lookup : 'a dict -> Key.t -> 'a option
  val insert : 'a dict * 'a entry -> 'a dict
end
```

→ Any structure implementing `DICT` will comprise a sub-structure implementing `ORDERED`.

# Let's define dictionaries with different keys!

Using our structures defined earlier implementing type class ORDERED, we can define dictionary structures with different keys:

```
structure IntLtDict : DICT =
struct



end
```

# Let’s define dictionaries with different keys!

Using our structures defined earlier implementing type class ORDERED, we can define dictionary structures with different keys:

```
structure IntLtDict : DICT =
struct
  structure Key = IntLt


end
```

# Let’s define dictionaries with different keys!

Using our structures defined earlier implementing type class ORDERED, we can define dictionary structures with different keys:

```
structure IntLtDict : DICT =
struct
  structure Key = IntLt
  (* code as before but now using Key.t instead of key
     and Key.compare instead of String.compare *)
end

structure IntGtDict : DICT =
struct




end
```

# Let’s define dictionaries with different keys!

Using our structures defined earlier implementing type class ORDERED, we can define dictionary structures with different keys:

```
structure IntLtDict : DICT =
struct
  structure Key = IntLt
  (* code as before but now using Key.t instead of key
     and Key.compare instead of String.compare *)
end

structure IntGtDict : DICT =
struct
  structure Key = IntGt


end
```

# Let’s define dictionaries with different keys!

Using our structures defined earlier implementing type class ORDERED, we can define dictionary structures with different keys:

```
structure IntLtDict : DICT =
struct
  structure Key = IntLt
  (* code as before but now using Key.t instead of key
     and Key.compare instead of String.compare *)
end

structure IntGtDict : DICT =
struct
  structure Key = IntGt
  (* code as before but now using Key.t instead of key
     and Key.compare instead of String.compare *)
end
```

# Let’s define dictionaries with different keys!

```
structure IntLtDict : DICT =
struct
  structure Key = IntLt
  (* code as before but now using Key.t instead of key
     and Key.compare instead of String.compare *)
end

structure IntGtDict : DICT =
struct
  structure Key = IntGt
  (* code as before but now using Key.t instead of key
     and Key.compare instead of String.compare *)
end

structure StringLtDict : DICT =
struct
  structure Key = StringLt
  (* code as before but now using Key.t instead of key
     and Key.compare instead of String.compare *)
end
```

Only differ in **Key**!

# Is that it?

- Have we solved the problem of inserting with one comparison function but looking up elements with a different one?
- Can we avoid rewriting (copying & pasting) the same code over and over when implementing dictionaries with different keys?

# Is that it?

**Have we solved the problem of inserting with one comparison function but looking up elements with a different one?**

For example, could we accidentally insert into a dictionary using `IntLtDict.insert` but then lookup using `IntGtDict.lookup`?

After all, `IntLtDict.Key.t` and `IntGtDict.Key.t` are both `int`.

No, this is not possible! `IntGtDict.dict` and `IntLtDict.dict` are different types.

ML type checker will thus prevent intermingling of dictionaries.

Remark: Had we implemented `dict` in terms of a representation type available in the client's scope, we should have used opaque ascription!

# Is that it?

- Have we solved the problem of inserting with one comparison function but looking up elements with a different one?
  - YES!
- Can we avoid rewriting (copying & pasting) the same code over and over when implementing dictionaries with different keys?
  - YES, but we need to use a functor for this!
- A **functor** creates a structure, given a structure as an argument.

Let’s write a functor that creates a structure ascribing to `DICT`, given a structure ascribing to `ORDERED` as an argument.

# Avoid the bloat with a functor!

```
functor TreeDict (K : ORDERED) : DICT =
struct



end
```

Argument structure, of type `ORDERED`

Note: ":" denotes typing, not ascription mode.

# Avoid the bloat with a functor!

```
functor TreeDict (K : ORDERED) : DICT =
struct



end
```

Structured returned, transparently ascribing to `DICT`

Denotes ascription mode, as usual.

# Avoid the bloat with a functor!

```
functor TreeDict (K : ORDERED) : DICT =
struct





end
```

# Avoid the bloat with a functor!

```
functor TreeDict (K : ORDERED) : DICT =
struct
  structure Key = K




end
```

# Avoid the bloat with a functor!

```
functor TreeDict (K : ORDERED) : DICT =
struct
  structure Key = K
  type 'a entry = Key.t * 'a


end
```

# Avoid the bloat with a functor!

```
functor TreeDict (K : ORDERED) : DICT =
struct
  structure Key = K
  type 'a entry = Key.t * 'a
  datatype 'a dict = ...

end
```

# Avoid the bloat with a functor!

```
functor TreeDict (K : ORDERED) : DICT =
struct
  structure Key = K
  type 'a entry = Key.t * 'a
  datatype 'a dict = ...
  (* code as before, but using Key.t and Key.compare *)
end
```

# Avoid the bloat with a functor!

```
functor TreeDict (K : ORDERED) : DICT =
struct
  structure Key = K
  type 'a entry = Key.t * 'a
  datatype 'a dict = ...
  (* code as before, but using Key.t and Key.compare *)
end
```

Now, we can define our earlier dictionaries as:

```
structure IntLtDict = TreeDict(IntLt)
structure IntGtDict = TreeDict(IntGt)
structure StringLtDict = TreeDict(StringLt)
```

# Avoid the bloat with a functor!

```
functor TreeDict (K : ORDERED) : DICT =
struct
  structure Key = K
  type 'a entry = Key.t * 'a
  datatype 'a dict = ...
  (* code as before, but using Key.t and Key.compare *)
end
```

Let's use opaque ascription instead!

# Avoid the bloat with a functor!

```
functor TreeDict (K : ORDERED) :> DICT =
struct
  structure Key = K
  type 'a entry = Key.t * 'a
  datatype 'a dict = ...
  (* code as before, but using Key.t and Key.compare *)
end
```

- But now we hide the representation type for `Key.t`.
- But we want it to be known that `Key.t` is the same as the input key `K.t`!
- To rectify this, we need to add a `where` clause.

# Avoid the bloat with a functor!

```
functor TreeDict (K : ORDERED) :> DICT
                          where type Key.t = K.t =
struct
  structure Key = K
  type 'a entry = Key.t * 'a
  datatype 'a dict = ...
  (* code as before, but using Key.t and Key.compare *)
end
```

- But now we hide the representation type for `Key.t`.
- But we want it to be known that `Key.t` is the same as the input key `K.t`!
- To rectify this, we need to add a `where` clause.

# Summary

- **Prescriptive** signatures exhaustively specify a type's operations, typically using **opaque** ascription.
- **Descriptive** signatures (aka type classes) expose a type parameter's operations, typically using **transparent** ascription.
- A **functor** creates a structure, given a structure as an argument.
  - Functor arguments are typically type classes to prevent code redundancy.

A word on syntax:

# Summary

A word on syntax:

- Functors only take a single structure as an argument.
- Multiple argument structures can be passed using nested structures or using specialized syntax.
  - More on this in labs and homework.
- Similarly, multiple where clauses are supported, using different syntactic forms.
  - More on this in labs and homework.

That's all for today. Happy 🎃👻🎃!