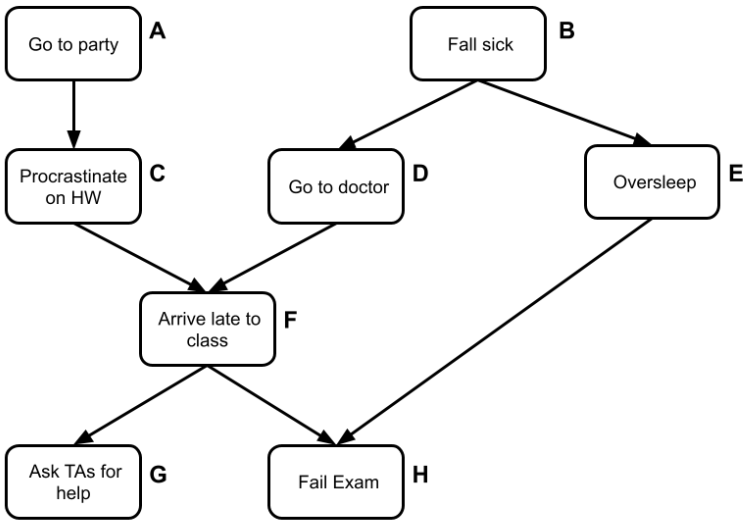
Learning Objectives

- To practice computing samples and probabilities for a Bayes Net

Suppose you are concerned about your social life, physical health, and procrastination and how it impacts your performance in class. You draw the following Bayes Net. The letters are provided for ease of reference to the variables.



You are given the following CPT tables. Hint: you can infer missing values if you know which distributions must sum to 1.

A	P(A)
+a	0.25

B	P(B)
+b	0.4

A	C	$P(C\|A)$
+a	+c	0.8
-a	+c	0.3

B	D	$P(D\|B)$
+b	+d	0.35
-b	+d	0.40

B	E	$P(E\|B)$
+b	+e	0.15
-b	+e	0.55

C	D	F	$P(F\|C,D)$
+c	+d	+f	0.8
+c	-d	+f	0.6
-c	+d	+f	0.5
-c	-d	+f	0.1

F	G	$P(G\|F)$
+f	+g	0.45
-f	+g	0.50

E	F	H	$P(H\|E,F)$
+e	+f	+h	0.9
+e	-f	+h	0.15
-e	+f	+h	0.3
-e	-f	+h	0.4

Q1. Bayes Net Sampling

From the lecture on Bayes Net Inference, we learned that:

$$P(A, D, E, H) = P(A) \sum_c P(c|A) \sum_f P(f|c, D) P(H|E, f) \sum_b P(b) P(D|b) P(E|b) \sum_g P(g|f)$$

This is complicated to compute. Instead, let's try to sample this distribution.

(a) Use the random numbers below (divide by 100 and check the relevant probability distributions) to create 10 samples, and then estimate the distribution $P(A, D, E, H)$ from those samples. Use the random numbers for sampling variables in alphabetical order of the variables you need to sample (left to right).

S1: 47 67 65 41 51 44 50 40
S2: 90 45 56 95 55 97 80 94
S3: 19 92 77 37 75 02 34 78
S4: 15 33 32 57 63 42 46 28
S5: 73 60 00 62 89 24 66 53
S6: 64 69 48 30 21 59 79 01
S7: 61 18 88 09 49 99 91 74
S8: 11 23 31 82 12 86 68 04
S9: 13 05 17 03 72 14 35 83
S10: 25 43 87 71 39 70 93 85

P(A,D,E,H) =

A	D	E	H	Count	Probability
+a	+d	+e	+h		
+a	+d	+e	-h		
+a	+d	-e	+h		
+a	+d	-e	-h	2	.2
+a	-d	+e	+h	1	.1
+a	-d	+e	-h		
+a	-d	-e	+h	1	.1
+a	-d	-e	-h		
-a	+d	+e	+h	1	.1
-a	+d	+e	-h		
-a	+d	-e	+h		
-a	+d	-e	-h	1	.1
-a	-d	+e	+h		
-a	-d	+e	-h	2	.2
-a	-d	-e	+h		
-a	-d	-e	-h	2	.2

Space for writing your samples:

sample 1: -a, -b, -c, -d, +e, -f, -g, -h
sample 2: -a, -b, -c, -d, -e, -f, -g, -h
sample 3: +a, -b, -c, +d, -e, +f, +g, -h
sample 4: +a, +b, +c, -d, -e, +f, -g, +h
sample 5: -a, -b, +c, -d, -e, +f, -g, -h
sample 6: -a, -b, -c, +d, +e, -f, -g, +h
sample 7: -a, +b, -c, +d, -e, -f, -g, -h
sample 8: +a, +b, +c, -d, +e, -f, -g, +h
sample 9: +a, +b, +c, +d, -e, +f, +g, -h
sample 10: -a, -b, -c, -d, +e, -f, -g, -h

P(+a,+d,-e,-h) =
.2

Rejection sampling. Now we're given the evidence +b, +c, +g.

(b) Suppose we use the same random numbers and resulting samples as above. Reject the ones that don't match the evidence +b, +c, +g. Then, estimate the distribution $P(A, D, E, H)$ from those remaining samples. Use the random numbers for sampling variables in alphabetical order of the variables you need to sample (left to right).

S1: 47 67 65 41 51 44 50 40
S2: 90 45 56 95 55 97 80 94
S3: 19 92 77 37 75 02 34 78
S4: 15 33 32 57 63 42 46 28
S5: 73 60 00 62 89 24 66 53
S6: 64 69 48 30 21 59 79 01
S7: 61 18 88 09 49 99 91 74
S8: 11 23 31 82 12 86 68 04
S9: 13 05 17 03 72 14 35 83
S10: 25 43 87 71 39 70 93 85

How many samples are accepted? How many more samples do you need to collect if we want 10 samples?

Accepted Samples:	**Samples Remaining to Collect:**
1: Sample S9	9

How many rows does your distribution table have? What is $P(+a, +d, -e, -h| + b, +c, +g)$ using only the samples accepted above?

P(A,D,E,H| $+b, +c, +g$**) =**

A	D	E	H	Count	Probability
+a	+d	+e	+h		
+a	+d	+e	-h		
+a	+d	-e	+h		
+a	+d	-e	-h	1	1.0
+a	-d	+e	+h		
+a	-d	+e	-h		
+a	-d	-e	+h		
+a	-d	-e	-h		
-a	+d	+e	+h		
-a	+d	+e	-h		
-a	+d	-e	+h		
-a	+d	-e	-h		
-a	-d	+e	+h		
-a	-d	+e	-h		
-a	-d	-e	+h		
-a	-d	-e	-h		

$P(+a, +d, -e, -h| + b, +c, +g) =$
1.0

For Likelihood Weighted Sampling, answer the following questions.

(c) What is the sample weight for +a, +b, +c, -d, +e, -f, +g, +h (evidence is +b,+c,+g)?

Weight:
The weight of a sample is the product of the evidence variables given their parents. In this case, we multiply $w = P(+b)P(+c|+a)P(+g|-f) = .4 * .8 * .5 = .16$

(d) What is the sample weight for +a, +b, +c, +d, +e, +f, +g, +h (evidence is +b,+c,+g)?

Weight:
The weight of a sample is the product of the evidence variables given their parents. In this case, we multiply $w = P(+b)P(+c|+a)P(+g|+f) = .4 * .8 * .45 = .144$

(e) What is the sample weight for -a, +b, +c, -d, +e, -f, +g, +h (evidence is +b,+c,+g)?

Weight:
The weight of a sample is the product of the evidence variables given their parents. In this case, we multiply $w = P(+b)P(+c|-a)P(+g|-f) = .4 * .3 * .5 = .06$

(f) Suppose we create 10 samples using likelihood weighted sampling. Note that +b, +c, +g are held constant. Fill in the table below.

sample 1: -a, +b, +c, -d, +e, -f, +g, -h
sample 2: -a, +b, +c, -d, -e, -f, +g, -h
sample 3: +a, +b, +c, +d, -e, +f, +g, -h
sample 4: +a, +b, +c, -d, -e, +f, +g, +h
sample 5: -a, +b, +c, -d, -e, +f, +g, -h
sample 6: -a, +b, +c, +d, +e, -f, +g, +h
sample 7: -a, +b, +c, +d, -e, -f, +g, -h
sample 8: +a, +b, +c, -d, +e, -f, +g, +h
sample 9: +a, +b, +c, +d, -e, +f, +g, -h
sample 10: -a, +b, +c, -d, +e, -f, +g, -h

What is the probability P(+a, +b, +c, -d, +e, +g, +h) (evidence is +b,+c,+g)? Notice no f is given.

First half of table:

$P(A, +b, +c, D, E, F, +g, H) =$

A	D	E	F	H	Count	S_{WS}	Weight	Probability
+a	+d	+e	+f	+h		0	1	0
+a	+d	+e	-f	+h		0	1	0
+a	+d	+e	+f	-h		0	1	0
+a	+d	+e	-f	-h		0	1	0
+a	+d	-e	+f	+h		0	1	0
+a	+d	-e	-f	+h		0	1	0
+a	+d	-e	+f	-h	2	.2	.4*.8*.45	.0288
+a	+d	-e	-f	-h		0	1	0
+a	-d	+e	+f	+h		0	1	0
+a	-d	+e	-f	+h	1	.1	.4*.8*.5	.016
+a	-d	+e	+f	-h		0	1	0
+a	-d	+e	-f	-h		0	1	0
+a	-d	-e	+f	+h	1	.1	.4*.8*.45	.0144
+a	-d	-e	-f	+h		0	1	0
+a	-d	-e	+f	-h		0	1	0
+a	-d	-e	-f	-h		0	1	0

Second half of table:

$P(A, +b, +c, D, E, F, +g, H) =$

A	D	E	F	H	Count	S_{WS}	Weight	Probability
-a	+d	+e	+f	+h		0	1	0
-a	+d	+e	-f	+h	1	.1	.4*.3*.5	.006
-a	+d	+e	+f	-h		0	1	0
-a	+d	+e	-f	-h		0	1	0
-a	+d	-e	+f	+h		0	1	0
-a	+d	-e	-f	+h		0	1	0
-a	+d	-e	+f	-h		0	1	0
-a	+d	-e	-f	-h	1	.1	.4*.3*.5	.006
-a	-d	+e	+f	+h		0	1	0
-a	-d	+e	-f	+h		0	1	0
-a	-d	+e	+f	-h		0	1	0
-a	-d	+e	-f	-h	2	.2	.4*.3*.5	0.012
-a	-d	-e	+f	+h		0	1	0
-a	-d	-e	-f	+h		0	1	0
-a	-d	-e	+f	-h	1	.1	.4*.3*.45	.0054
-a	-d	-e	-f	-h	1	.1	.4*.3*.5	.006

Note that this table looks the same as part a only because I changed the samples.
You should create the table for the remaining non-evidence nodes, then multiply the sample by the weight. $w = P(+b)P(+c|+a)P(+g|-f) = .4 * .8 * .5 = .16$.

What is the probability P(+a, +b, +c, -d, +e, +g, +h) (evidence is +b,+c,+g)?
P(+a, +b, +c, -d, +e, +g, +h) = P(+a, +b, +c, -d, +e, +f, +g, +h)+P(+a, +b, +c, -d, +e, -f, +g, +h) = .016+0 = .016

(g) Work in a group to each create 2 samples using Gibbs sampling for P(A,B,E,H|+c,+d,+f,+h). For this example, use 8 iterations per sample (2 times per sampled variable). Start with the random assignment {+a, +b, +c, +d, +e, +f, +g, +h} for each sample, and sample the variables in alphabetical order. Use the random values below for your sampling.

S1: `48 67 65 41 51 44 50 40` S2: `90 45 56 95 55 97 80 94`

How many rows does your distribution table have?

Sample 1: start with {+a, +b, +c, +d, +e, +f, +g, +h}

Resample A, choose -a because .48 (above) > :

$$P(A|+b,+c,+d,+e,+f,+g,+h) = \frac{P(A,+b,+c,+d,+e,+f,+g,+h)}{P(+b,+c,+d,+e,+f,+g,+h)}$$

$$= \frac{P(A)P(+b)P(+c|A)P(+d|+b)P(+e|+b)P(+f|+c,+d)P(+g|+f)P(+h|+e,+f)}{\sum_a P(a)P(+b)P(+c|a)P(+d|+b)P(+e|+b)P(+f|+c,+d)P(+g|+f)P(+h|+e,+f)}$$

$$= \frac{P(A)P(+c|A)*[P(+b)P(+d|+b)P(+e|+b)P(+f|+c,+d)P(+g|+f)P(+h|+e,+f)]}{[\sum_a P(a)P(+c|a)]*[P(+b)P(+d|+b)P(+e|+b)P(+f|+c,+d)P(+g|+f)P(+h|+e,+f)]}$$

$$= \frac{P(A)P(+c|A)}{\sum_a P(a)P(+c|a)}.P(+a|...) = \frac{.25*.8}{.25*.8+.75*.3} = .2/.425 = .47$$

Resample B, choose -b because .67 (above) > :

$$P(B|-a,+c,+d,+e,+f,+g,+h) = \frac{P(-a,B,+c,+d,+e,+f,+g,+h)}{P(-a,+c,+d,+e,+f,+g,+h)}$$

$$= \frac{P(-a)P(B)P(+c|-a)P(+d|B)P(+e|B)P(+f|+c,+d)P(+g|+f)P(+h|+e,+f)}{\sum_b P(-a)P(b)P(+c|-a)P(+d|b)P(+e|b)P(+f|+c,+d)P(+g|+f)P(+h|+e,+f)}$$

$$= \frac{P(B)P(+d|B)P(+e|B)*[P(+a)P(+c|-a)P(+f|+c,+d)P(+g|+f)P(+h|+e,+f)]}{[\sum_b P(b)P(+d|b)P(+e|b)]*[P(-a)P(+c|-a)P(+f|+c,+d)P(+g|+f)P(+h|+e,+f)]}$$

$$= \frac{P(B)P(+d|B)P(+e|B)}{\sum_b P(b)P(+d|b)P(+e|b)}.P(+b|...) = \frac{.4*.35*.15}{.4*.35*.15+.6*.4*.55} = .021/.153 = .14$$

Resample E, choose +e because .65 (above) < :

$$P(E|-a,-b,+c,+d,+f,+g,+h) = \frac{P(-a,-b,+c,+d,E,+f,+g,+h)}{P(-a,-b,+c,+d,+f,+g,+h)}$$

$$= \frac{P(E|-b)P(+h|E,+f)}{\sum_e P(e|-b)P(+h|e,+f)}.P(+e|...) = \frac{.55*.9}{.55*.9+.45*.3} = .495/.63 = .79$$

Resample H, choose +h because .41 (above) < :

$$P(H|-a,-b,+c,+d,+e,+f,+g) = \frac{P(-a,-b,+c,+d,+e,+f,+g,H)}{P(-a,-b,+c,+d,+e,+f,+g)} = \frac{P(H|+e,+f)}{\sum_h P(h|+e,+f)}.\ P(+h|...) = .9$$

Resample A, choose -a because .51 (above) > :

$$P(+a|...) = \frac{P(+a)P(+c|+a)}{\sum_a P(a)P(+c|a)} = .47$$

Resample B, choose -b because .44 (above) > :

$$P(+b|...) = \frac{P(+b)P(+d|+b)P(+e|+b)}{\sum_b P(+b)P(+d|+b)P(+e|+b)} = .14$$

Resample E, choose +e because .50 (above) < :

$$P(+e|...) = \frac{P(+e|-b)P(+h|+e,+f)}{\sum_e P(e|-b)P(+h|e,+f)} = .79$$

Resample H, choose +h because .40 (above) < :

$$P(+h|...) = \frac{P(+h|+e,+f)}{\sum_h P(h|+e,+f)} = .9$$

Sample 1 = -a, -b, +c, +d, +e, +f, +g, +h

Sample 2: start with {+a, +b, +c, +d, +e, +f, +g, +h}

Resample A, choose -a because .90 (above) > :

$$P(+a|...) = \frac{P(+a)P(+c|+a)}{\sum_a P(a)P(+c|a)} = .47$$

Resample B, choose -b because .45 (above) > :

$$P(+b|...) = \frac{P(+b)P(+d|+b)P(+e|+b)}{\sum_b P(+b)P(+d|+b)P(+e|+b)} = .14$$

Resample E, choose +e because .56 (above) < :

$$P(+e|...) = \frac{P(+e|-b)P(+h|+e,+f)}{\sum_e P(e|-b)P(+h|e,+f)} = .79$$

Resample H, choose -h because .95 (above) < :

$$P(+h|...) = \frac{P(+h|+e,+f)}{\sum_h P(h|+e,+f)} = .9$$

Resample A, choose -a because .55 (above) > :

$$P(+a|...) = \frac{P(+a)P(+c|+a)}{\sum_a P(a)P(+c|a)} = .47$$

Resample B, choose -b because .97 (above) $>$:

$$P(+b|...) = \frac{P(+b)P(+d|+b)P(+e|+b)}{\sum_b P(+b)P(+d|+b)P(+e|+b)} = .14$$

Resample E, choose -e because .80 (above) $>$:

$$P(+e|...) = \frac{P(+e|-b)P(+h|+e,+f)}{\sum_e P(e|-b)P(+h|e,+f)} = .79$$

Resample H, choose -h because .94 (above) $>$:

$$P(+h|...) = \frac{P(+h|-e,+f)}{\sum_h P(h|-e,+f)} = .3$$

Sample 2 = -a, -b, +c, +d, -e, +f, +g, -h

Q2. Theory Questions

(a) In part (a) there were two ways to sample a marginal distribution – using prior sampling and likelihood weighted sampling. Why isn't likelihood weighted sampling appropriate?

Why?
Likelihood weighted sampling finds the distribution of Q (all values of query variables) **and** e (given evidence) variables. We don't have any given evidence so each sample would have equal weight.

(b) Which sampling algorithms will help us estimate $P(A, +b, +c, D, E, +g, H)$?

Algorithms:
Likelihood weighted sampling

(c) Which will help us estimate $P(A, D, E, H| + b, +c, +g)$?

Algorithms:
rejection sampling and Gibbs sampling