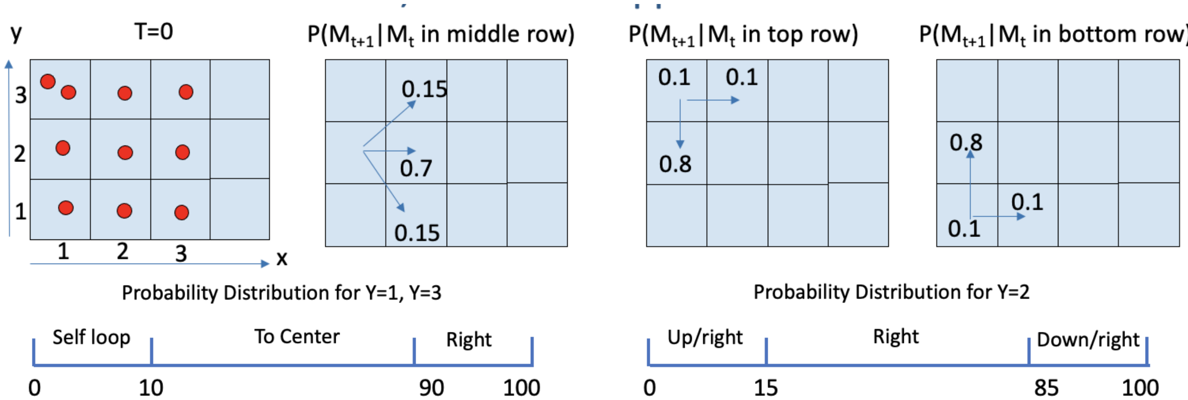
15-281 Spring 2023 — Lecture Activity

Learning Objectives

- To practice particle filtering
- To run Predict (Propagate Forward) and Update (Weight and Resample) steps

Q1. Particle Filtering

Suppose we have a robot on a 4x3 grid as shown below. Locations on the map (denoted $m = (x, y), m \in M$) can be referenced by their x, y locations. The robot knows it is not in the right column but otherwise, it has no clue where it is. We initialize 10 particles across the nine squares as shown in the grid to the left. The transition function from each row are given below. We're going to run particle filtering for two time steps to predict $P(M_2|e_1, e_2)$. Notice that this process will be faster than performing exact inference using the Forward algorithm.

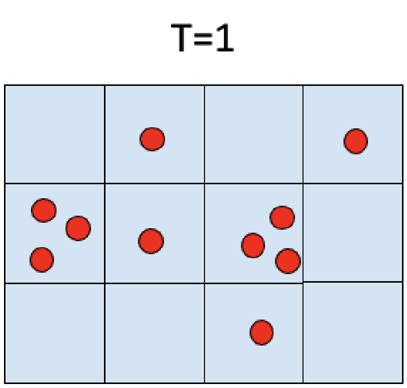


(a) PREDICT! We are given the initial distribution $P(M_0)$. Now we need to Propagate each of the particles forward to time $T = 1$. Use the random number sequence below to sample from $P(M_t|M_{t-1})$ and draw your new particles on the blank map below.

Sample particles in order left to right, bottom to top (i.e., (1,1), (2,1), (3,1), (1,2), (2,2), ...).

Use the following rules for selecting the next state. From the center (Y=2), choose up/right in range [0,15), right in range [15, 85), and down/right in range [85,100). From the top and bottom (Y=1, Y=3), choose self loop in range [0,10), center [10,90), and right [90,100).

66 94 38 30 25 02 18 88 09 31



particle	new location
(1,1)	(1,2)
(2,1)	(3,1)
(3,1)	(3,2)
(1,2)	(2,2)
(2,2)	(3,2)
(3,2)	(4,3)
(1,3)	(1,2)
(1,3)	(1,2)
(2,3)	(2,3)
(3,3)	(3,2)

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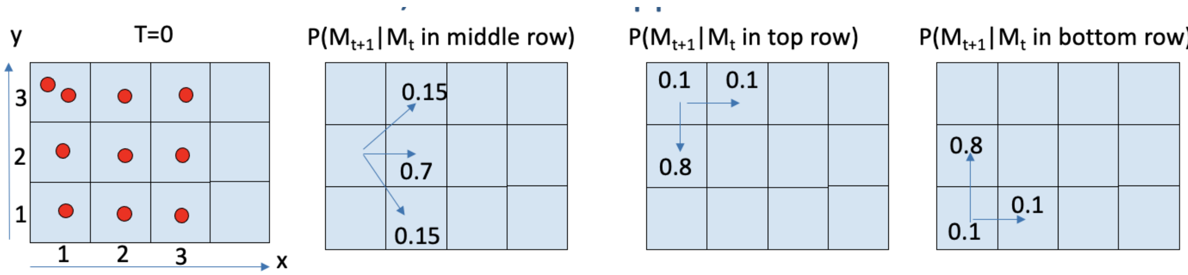


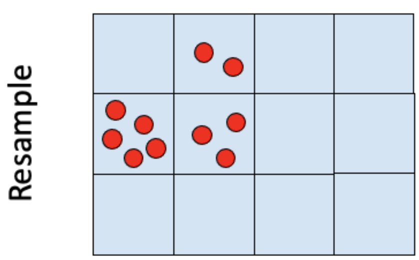
(b) WEIGHT! Now you observe some set e_1. Weigh each sample by it's likelihood $P(e_1|m)$ for each m which is given below. Blank grids mean the probability is 0. Notice that the grid does not sum to 1; instead $\sum_e P(e|m) = 1$ for each m. You should list to the right the weight of each location m as the count of the particles in m multiplied by the probability $P(e_1|m)$. Normalize so that the sum of the weights equals 1.

weight of (1,2) = 3*.5 = 1.5. Normalized: 1.5/2.5 = .6
weight of (2,2) = 1*.5 = .5. Normalized: .5/2.5 = .2
weight of (2,3) = 1*.5 = .5. Normalized: .5/2.5 = .2
all others are 0

(c) RESAMPLE! You should find that 3 locations have non-zero weight – (1,2), (2,2), and (2,3). The weights from (b) form a probability distribution. Use the probability distribution (in **the order P((1,2)), P((2,2)), P((2,3))**) and the random numbers below to create 10 new particles based on this distribution.

Draw the particles on the grid below or write them in the space to the right.

74 33 01 63 92 91 53 67 27 04



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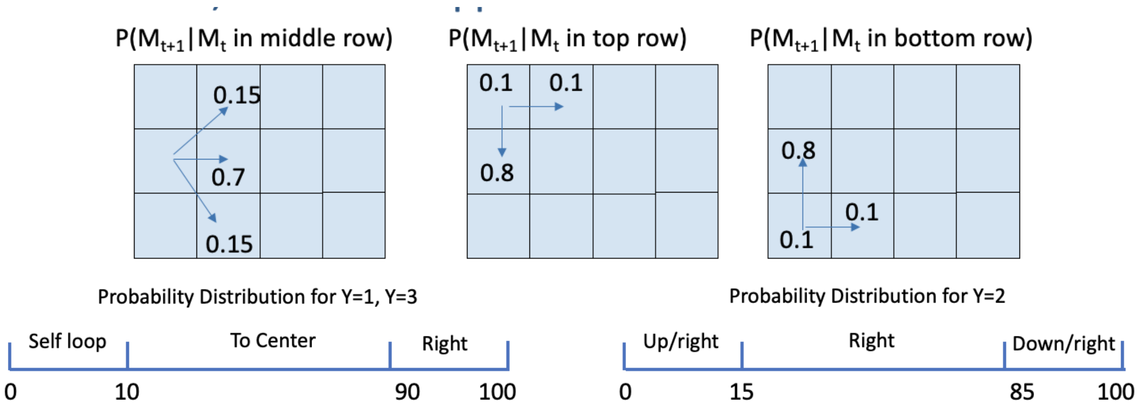
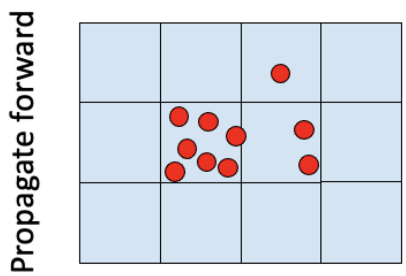


Now use your particles from part (c) and repeat the process for time 2.

(d) PREDICT! Propagate your new samples forward using the transition probabilities above by first moving all the samples at (1,2) then (2,2), then (2,3). Use the same rules for the distribution as part (a).

17 71 83 26 55 09 58 73 61 15

Draw your new sample locations below or write them in the space to the right.



(e) WEIGHT! Now you observe some set e_2. Weigh each sample by it's likelihood $P(e_2|m)$ for each m which is given below. Blank grids mean the probability is 0. Notice that the grid does not sum to 1; instead $\sum_e P(e|m) = 1$ for each m. You should list to the right the weight of each location m as the count of the particles in m multiplied by the probability $P(e_2|m)$. Normalize so that the sum of the weights equals 1.

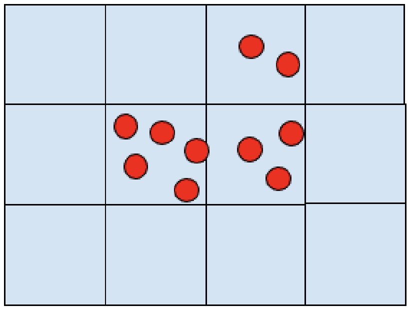
$P(e_2|m)$

	.05	.4	
	.3	.5	
	.05	.2	

weight of (2,2) = 7*.3 = 2.1. Normalize 2.1/3.5 = .6
weight of (3,2) = 2*.5 = 1.0. Normalize 1.0/3.5 = .29
weight of (3,3) = 1*0.4 = 0.4. Normalize 0.4/3.5 = .11
all others are 0

(f) RESAMPLE! You should have 3 particle locations – (2,2), (3,2), and (3,3). Use the weights from (e) as a probability distribution in the same way as (c) and sample to create 10 new particles. Draw them on the grid below or write them in the space to the right.

51 77 41 96 29 37 66 70 92 23



(g) What is the probability that the robot is in (3,2) after 2 time steps $P(M_2 = (3,2)|e_1, e_2)$?

Based on our sample distribution, $P(M_2 = (3,2)|e_1, e_2) = .3$

(h) Suppose we want to propogate forward again for time step 3. What is the joint probability that two particles in (2,2) will both move to (3,3)?

Based on the transition probabilities and that each particle is independent of all others, the probability is 0.15*0.15 = .0225