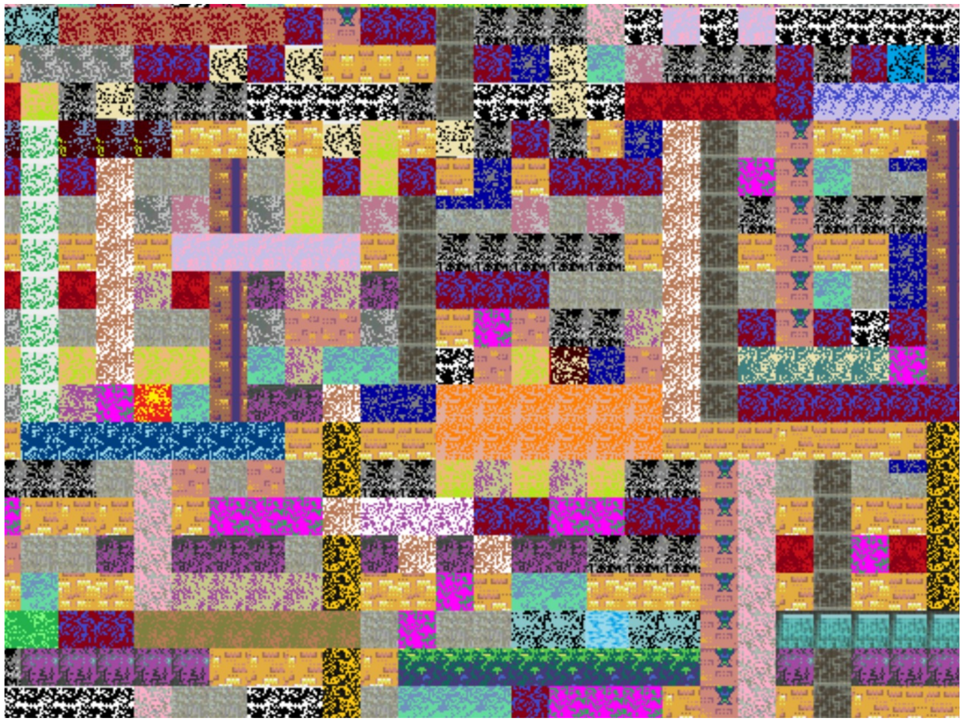
Try if you'd like…

https://high-level-4.herokuapp.com/experiment



https://rach0012.github.io/humanRL_website/

Announcements

Assignments

- P3: Logic Plan and Classical Planning due Tomorrow 3/17 10pm
- HW7 (written) due Tuesday 3/21 10pm
- HW8 (online) due Tuesday 3/28 10pm

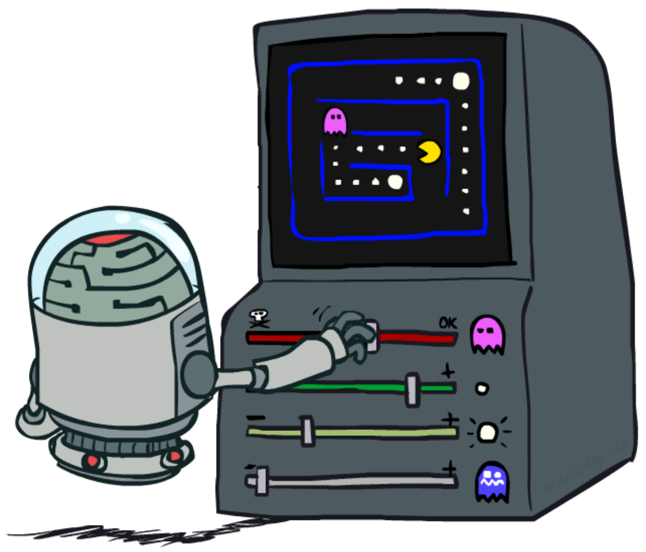
Coming up

- Midterm 2 3/30
- Review Session – TBA
- Topics: Logic, Classical Planning, MDPs, RL, Probability, Bayes Nets up to 3/28
- More info next week

- Midsemester and TA Feedback form! See Piazza – 1 participation point

AI: Representation and Problem Solving

Reinforcement Learning II



Instructor: Stephanie Rosenthal

Slide credits: CMU AI and http://ai.berkeley.edu

Reinforcement Learning

We still assume an MDP:

- A set of states $s \in S$
- A set of actions (per state) A
- A model T(s,a,s')
- A reward function R(s,a,s')

Still looking for a policy $\pi(s)$

New twist: don't know T or R, so must try out actions

Big idea: Compute all averages over transition probabilities using sample outcomes

Model-Free Learning

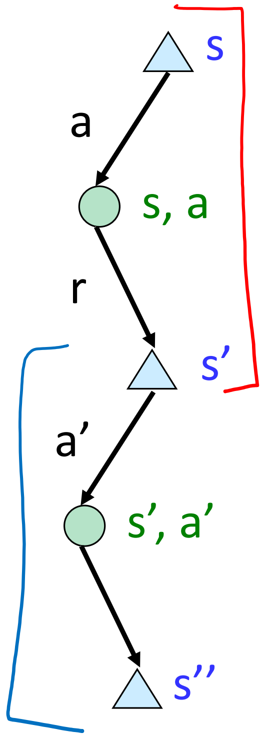
Model-free (temporal difference) learning

- Experience world through episodes

$$(s, a, r, s', a', r', s'', a'', r'', s'''' \ldots)$$

- Update estimates each transition (s, a, r, s')

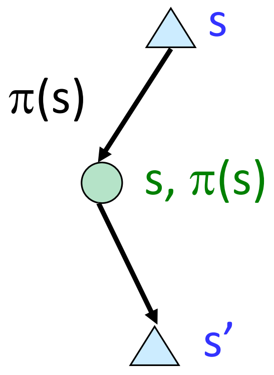
- Over time, updates will mimic Bellman updates



Temporal Difference Learning

Big idea: learn from every experience!

- Update V(s) each time we experience a transition (s, a, s', r)
- Likely outcomes s' will contribute updates more often



Temporal difference learning of values

- Policy still fixed, still doing evaluation!
- Move values toward value of whatever successor occurs: running average

passive

Sample of V(s): $sample = r + \gamma V^{\pi}(s')$

learning rate

Update to V(s): $V^{\pi}(s) \leftarrow (1-\alpha)\, V^{\pi}(s) + (\alpha)\, sample$

∇Error

Same update: $V^{\pi}(s) \leftarrow V^{\pi}(s) + \alpha\, \boxed{sample - V^{\pi}(s)}$

new old

Same update: $V^{\pi}(s) \leftarrow V^{\pi}(s) - \alpha \nabla Error$ $\quad Error = \frac{1}{2}\left(sample - V^{\pi}(s)\right)^2$

Quick Calculus Quiz

$$f(x) = \frac{1}{2}(\underline{y} - x)^2$$

What is $\frac{df}{\underline{dx}}$?

$$\cancel{2}\left(\tfrac{1}{2}\cancel{2}(y-x)'\right)\boxed{(-1)}$$

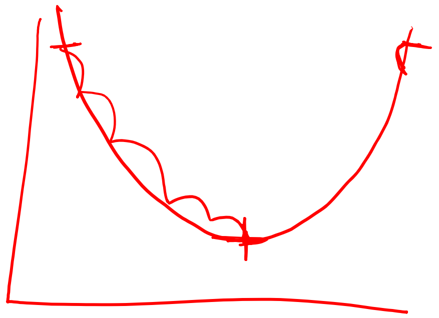
$$2 - x$$

$$\boxed{-1}$$

$$-(y-x)$$

$$x-y$$

Gradient Descent



$$f(x) = \frac{1}{2}(y - x)^2$$

$$\frac{df}{dx} = -(y - x)$$

Goal: find x that minimizes $f(x)$

1. Start with initial guess, x_0
2. Update x by taking a step in the direction that $f(x)$ is changing fastest (in the negative direction) with respect to x:

   $x \leftarrow x - \alpha \nabla_x f$, where α is the step size or learning rate
3. Repeat until convergence

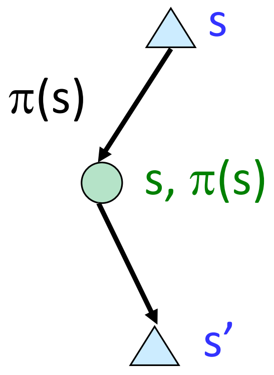
TD goal: find value(s), V, that minimizes difference between sample(s) and V

$$V \leftarrow V - \alpha \nabla_V Error \quad V^{\pi}(s) \qquad Error(V) = \frac{1}{2}(sample - V)^2$$

Temporal Difference Learning

Big idea: learn from every experience!

- Update V(s) each time we experience a transition (s, a, s', r)
- Likely outcomes s' will contribute updates more often



Temporal difference learning of values

- Policy still fixed, still doing evaluation!
- Move values toward value of whatever successor occurs: running average

Sample of V(s): $sample = r + \gamma V^{\pi}(s')$

Update to V(s): $V^{\pi}(s) \leftarrow (1-\alpha) V^{\pi}(s) + (\alpha)\, sample$

Same update: $V^{\pi}(s) \leftarrow V^{\pi}(s) + \alpha\, [sample - V^{\pi}(s)]$

Same update: $V^{\pi}(s) \leftarrow V^{\pi}(s) - \alpha \nabla Error$ $\qquad Error = \frac{1}{2}\left(sample - V^{\pi}(s)\right)^2$

Poll 1

TD ~~update~~ temporal difference:

$$V^\pi(s) = V^\pi(s) + \alpha\, [\overbrace{\overset{\text{sample}}{r + \gamma V^\pi(s')} - V^\pi(s)}^{\text{diff}}]$$

Which converts TD values into a policy?

$$\pi(s) = \arg\max_a Q(s,a)$$

A) Value iteration: $V_{k+1}(s) = \max_a \sum_{s'} P(s'|s,a)[R(s,a,s') + \gamma V_k(s')], \quad \forall\, s$

B) Q-iteration: $Q_{k+1}(s,a) = \sum_{s'} P(s'|s,a)[R(s,a,s') + \gamma \max_{a'} Q_k(s',a')], \quad \forall\, s,a$

C) Policy extraction: $\pi_V(s) = \operatorname{argmax}_a \sum_{s'} P(s'|s,a)[R(s,a,s') + \gamma V(s')], \quad \forall\, s$

D) Policy evaluation: $V^\pi_{k+1}(s) = \sum_{s'} P(s'|s,\pi(s))[R(s,\pi(s),s') + \gamma V^\pi_k(s')], \quad \forall\, s$

E) Policy improvement: $\pi_{new}(s) = \operatorname{argmax}_a \sum_{s'} P(s'|s,a)[R(s,a,s') + \gamma V^{\pi_{old}}(s')], \quad \forall\, s$

[✓ F) None of the above]

MDP/RL Notation

Standard expectimax:	$V(s) = \max_a \sum_{s'} P(s'\|s,a) V(s')$
Bellman equations:	$V^*(s) = \max_a \sum_{s'} P(s'\|s,a)[R(s,a,s') + \gamma V^*(s')]$
Value iteration:	$V_{k+1}(s) = \max_a \sum_{s'} P(s'\|s,a)[R(s,a,s') + \gamma V_k(s')], \quad \forall\, s$
Q-iteration:	$Q_{k+1}(s,a) = \sum_{s'} P(s'\|s,a)[R(s,a,s') + \gamma \max_{a'} Q_k(s',a')], \quad \forall\, s,a$
Policy extraction:	$\pi_V(s) = \operatorname{argmax}_a \sum_{s'} P(s'\|s,a)[R(s,a,s') + \gamma V(s')], \quad \forall\, s$
Policy evaluation:	$V^{\pi}_{k+1}(s) = \sum_{s'} P(s'\|s,\pi(s))[R(s,\pi(s),s') + \gamma V^{\pi}_k(s')], \quad \forall\, s$
Policy improvement:	$\pi_{new}(s) = \operatorname{argmax}_a \sum_{s'} P(s'\|s,a)[R(s,a,s') + \gamma V^{\pi_{old}}(s')], \quad \forall\, s$
Value (TD) learning:	$V^{\pi}(s) = V^{\pi}(s) + \alpha\,[r + \gamma\, V^{\pi}(s') - V^{\pi}(s)]$
Q-learning:	$Q(s,a) = Q(s,a) + \alpha[r + \gamma \max_{a'} Q(s',a') - Q(s,a)]$

Q-Learning

We'd like to do Q-value updates to each Q-state:

$$\underline{\underline{Q_{k+1}(s,a)}} \leftarrow \sum_{s'} T(s,a,s') \left[R(s,a,s') + \gamma \max_{a'} Q_k(s',a')\right]$$

- But can't compute this update without knowing T, R

Instead, compute average as we go

- Receive a sample transition (s,a,r,s')
- This sample suggests

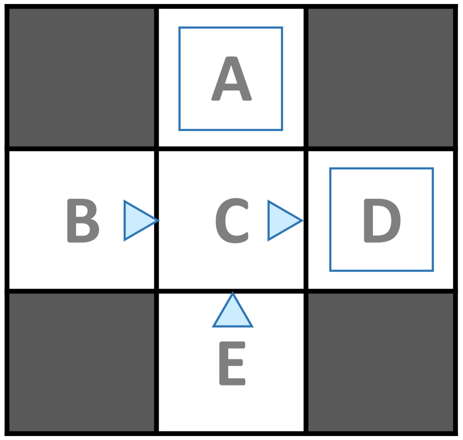
$$Q(s,a) \approx r + \gamma \overbrace{\left[\max_{a'} Q(s',a')\right]}^{V(s')}$$

- But we want to average over results from (s,a) (Why?)
- So keep a running average

$$Q(s,a) \leftarrow (1-\alpha)Q(s,a) + (\alpha)\left[r + \gamma \max_{a'} Q(s',a')\right]$$

Q-Learning + Lecture 14 Poll 2

Input S,A



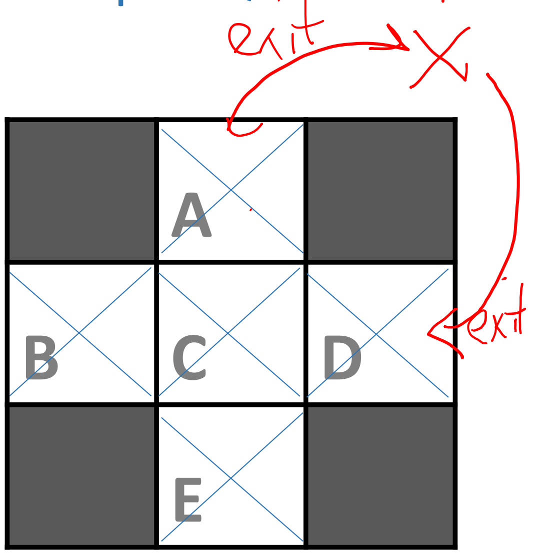
Assume: γ = 1
α = 0.5

Observed Episodes (Training)

Episode 1	Episode 2
B, east, C, -1	B, east, C, -1
C, east, D, -1	C, east, D, -1
D, exit, x, +10	D, exit, x, +10

Episode 3	Episode 4
E, north, C, -1	E, north, C, -1
C, east, D, -1	C, east, A, -1
D, exit, x, +10	A, exit, x, -10

Output Q-Values



$$Q(s,a) \leftarrow (1-\alpha)Q(s,a) + (\alpha)\left[r + \gamma \max_{a'} Q(s',a')\right]$$

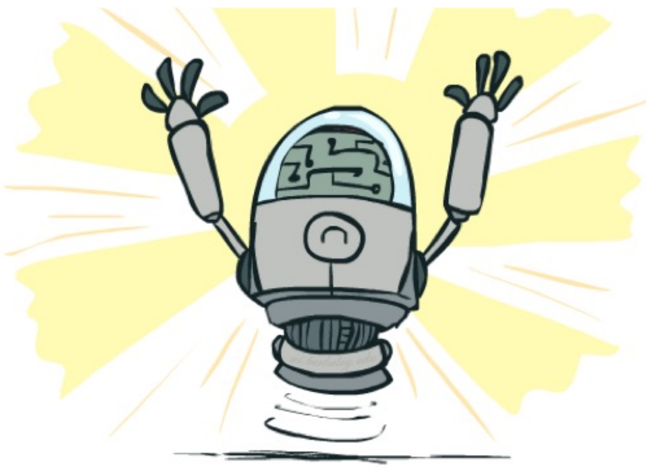
Q-Learning Properties

Amazing result: Q-learning converges to optimal policy -- even if you're acting suboptimally!

This is called off-policy learning

Caveats:

- You have to explore enough ☆
- You have to eventually make the learning rate small enough $\alpha \downarrow$
- … but not decrease it too quickly
- Basically, in the limit, it doesn't matter how you select actions (!)



The Story So Far: MDPs and RL

Known MDP: Offline Solution

Goal	Technique
Compute V*, Q*, π*	Value / policy iteration
Evaluate a fixed policy π	Policy evaluation

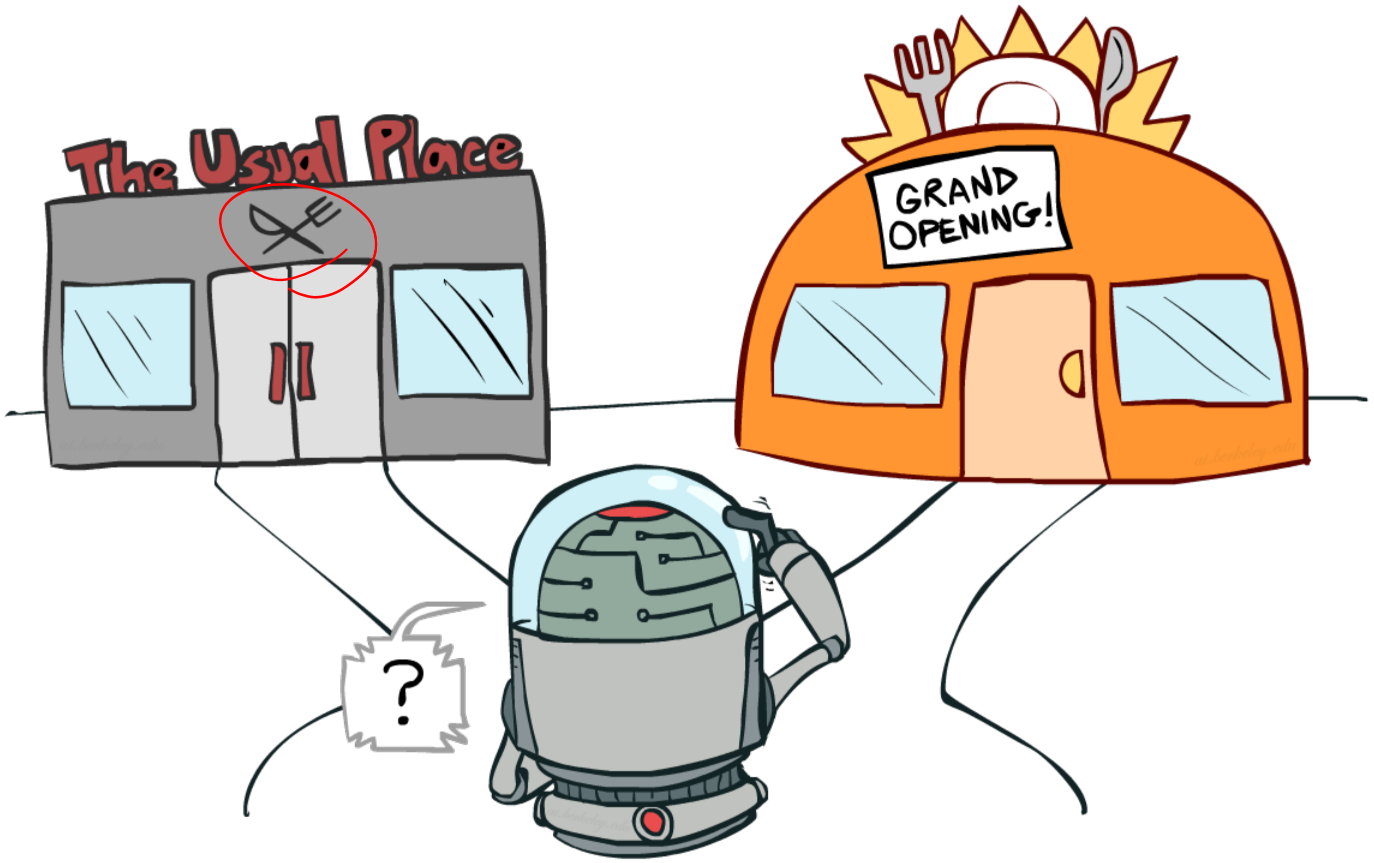
Unknown MDP: Model-Based

Goal	Technique
Compute V*, Q*, π*	VI/PI on approx. MDP
Evaluate a fixed policy π	PE on approx. MDP

Unknown MDP: Model-Free

Goal	Technique
Compute V*, Q*, π*	Q-learning
Evaluate a fixed policy π	TD/Value Learning

Exploration vs. Exploitation



How to Explore?

Several schemes for forcing exploration

- Simplest: random actions (ε-greedy)
  - Every time step, flip a coin
  - With (small) probability ε, act randomly
  - With (large) probability 1-ε, act on current policy

- Problems with random actions?

How to Explore?

Several schemes for forcing exploration

- Simplest: random actions (ε-greedy)
  - Every time step, flip a coin
  - With (small) probability ε, act randomly
  - With (large) probability 1-ε, act on current policy

- Problems with random actions?
  - You do eventually explore the space, but keep thrashing around once learning is done
  - One solution: lower ε over time
  - Another solution: exploration functions

Exploration Functions

When to explore?

- Random actions: explore a fixed amount
- Better idea: explore areas whose badness is not (yet) established, eventually stop exploring

Exploration function

- Takes a value estimate u and a visit count n, and returns an optimistic utility, e.g.

Q value

$$f(u,n) = \underline{u} + \boxed{k/(n+1)}$$

Regular Q-Update: $Q(s,a) = Q(s,a) + \alpha\,[r + \gamma \max_{a'} Q(s',a') - Q(s,a)]$

Modified Q-Update: $Q(s,a) = Q(s,a) + \alpha\,[r + \gamma \max_{a'} \boxed{f(\underline{Q(s',a')}, \underline{N(s',a')})} - Q(s,a)]$

- Note: this propagates the "bonus" back to states that lead to unknown states as well!

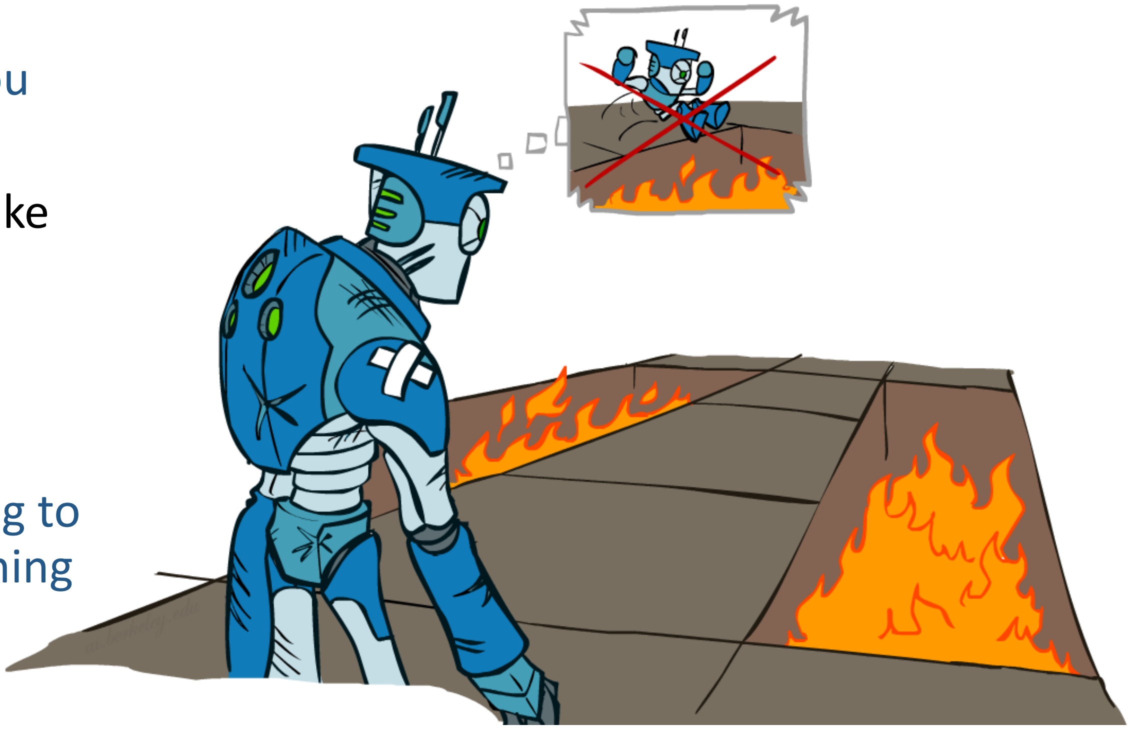
Regret

Even if you learn the optimal policy, you still make mistakes along the way!

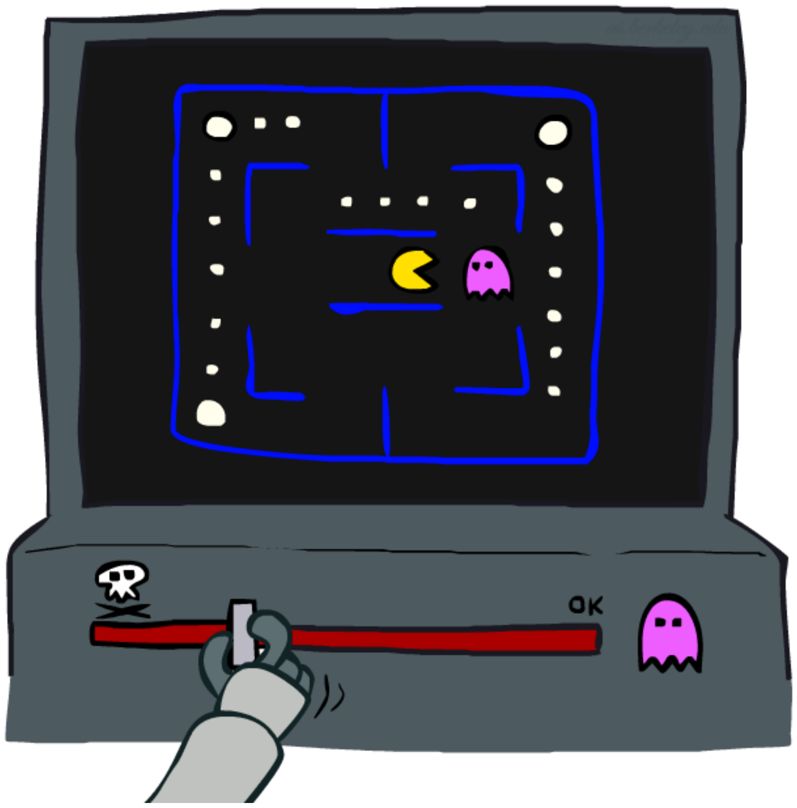
Regret is a measure of your total mistake cost: the difference between your (expected) rewards, including youthful suboptimality, and optimal (expected) rewards

Minimizing regret goes beyond learning to be optimal – it requires optimally learning to be optimal

Example: random exploration and exploration functions both end up optimal, but random exploration has higher regret

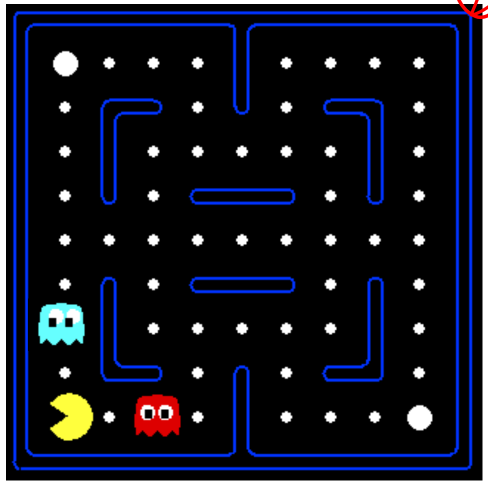


Approximate Q-Learning

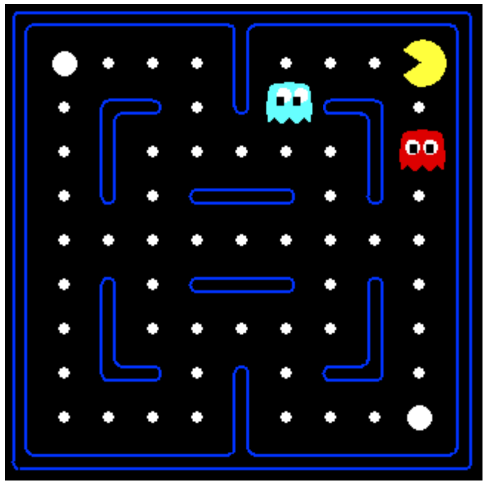


Example: Pacman

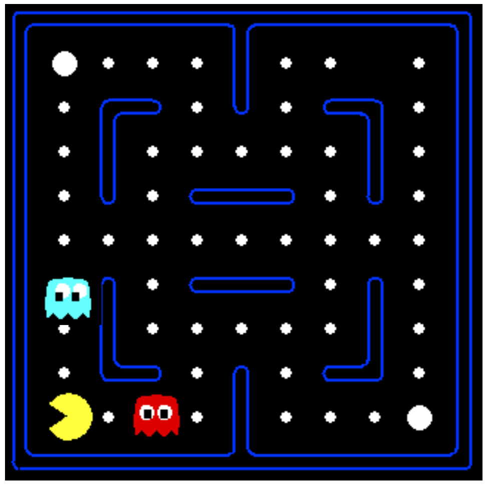
Let's say we discover through experience that this state is bad:



In naïve q-learning, we know nothing about this state:



Or even this one!



Generalizing Across States

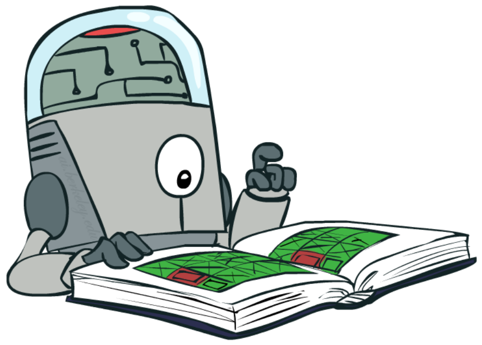
Basic Q-Learning keeps a table of all q-values

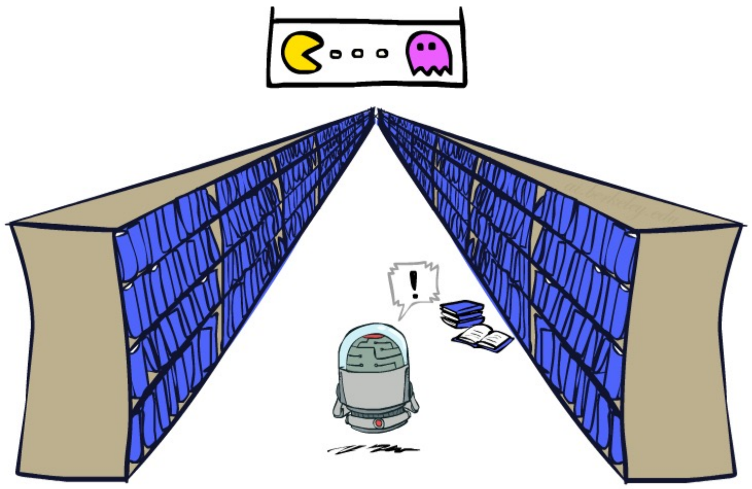
In realistic situations, we cannot possibly learn about every single state!

- Too many states to visit them all in training
- Too many states to hold the q-tables in memory

Instead, we want to generalize:

- Learn about some small number of training states from experience
- Generalize that experience to new, similar situations
- This is a fundamental idea in machine learning, and we'll see it over and over again





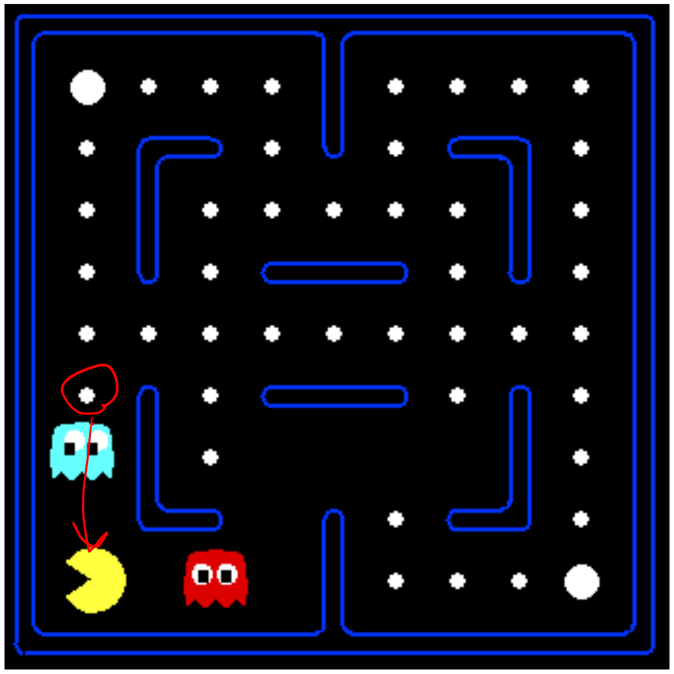
[demo – RL pacman]

Feature-Based Representations

$f_1 \quad f_2 \quad f_3 \quad f_4 \quad f_5$

Solution: describe a state using a vector of features (properties)

- Features are functions from states to real numbers (often 0/1) that capture important properties of the state
- Can also describe a q-state (s, a) with features (e.g. action moves closer to food)
- Example features:



Linear Value Functions

Using a feature representation, we can write a q function (or value function) for any state using a few weights:

- $V_w(s) = w_1 f_1(s) + w_2 f_2(s) + \ldots + w_n f_n(s)$
- $Q_w(s,a) = w_1 f_1(s,a) + w_2 f_2(s,a) + \ldots + w_n f_n(s,a)$

Advantage: our experience is summed up in a few powerful numbers

Disadvantage: states may share features but actually be very different in value!

Updating a linear value function

Original Q learning rule tries to reduce prediction error at s, a:

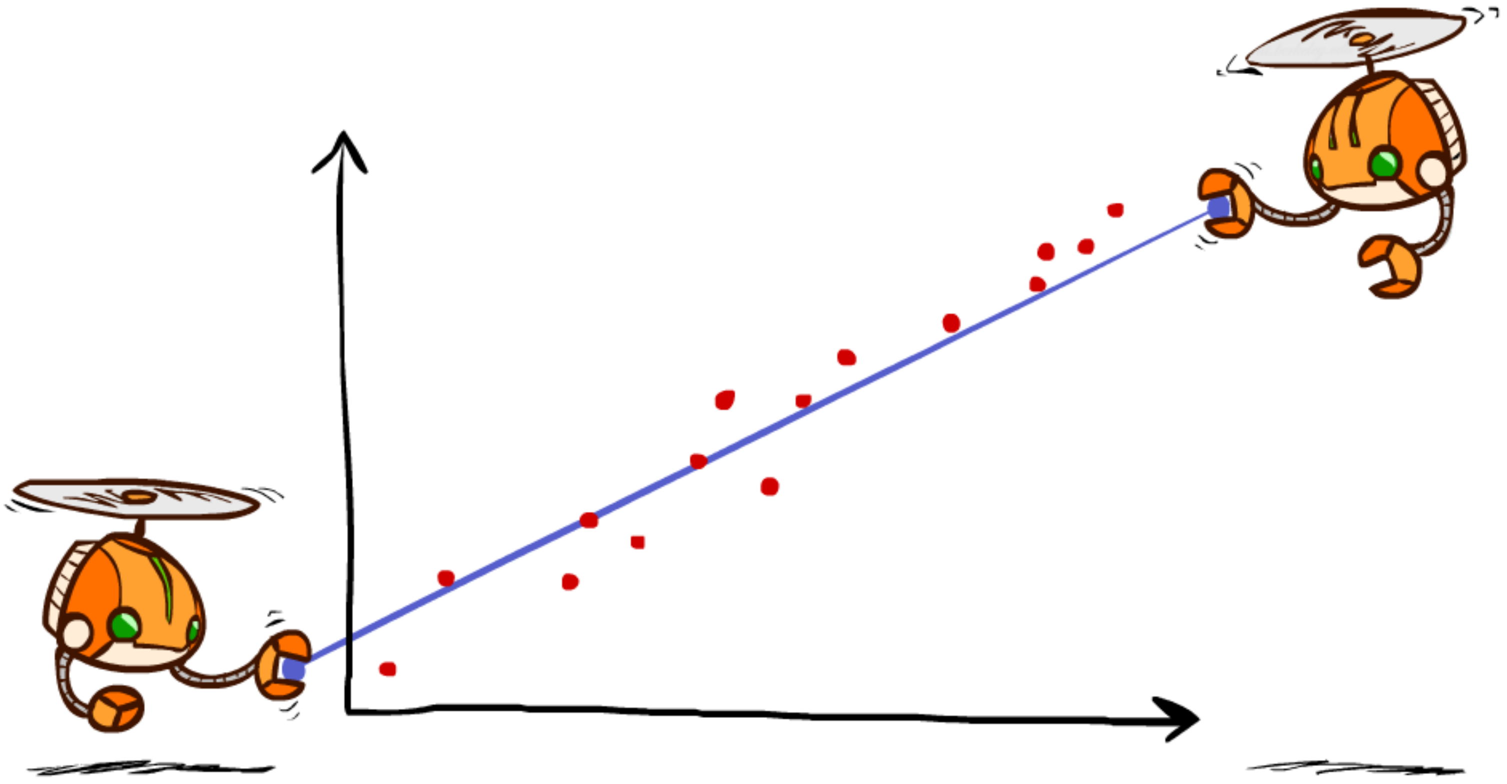
- $Q(s,a) \leftarrow Q(s,a) + \alpha[R(s,a,s') + \gamma \max_{a'} Q(s',a') - Q(s,a)]$

Instead, we update the weights to try to reduce the error at s, a:

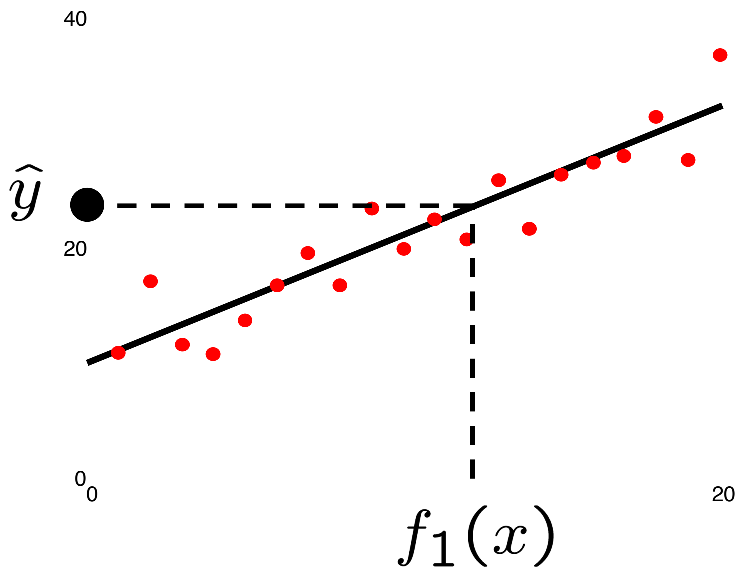
- $w_i \leftarrow ?$

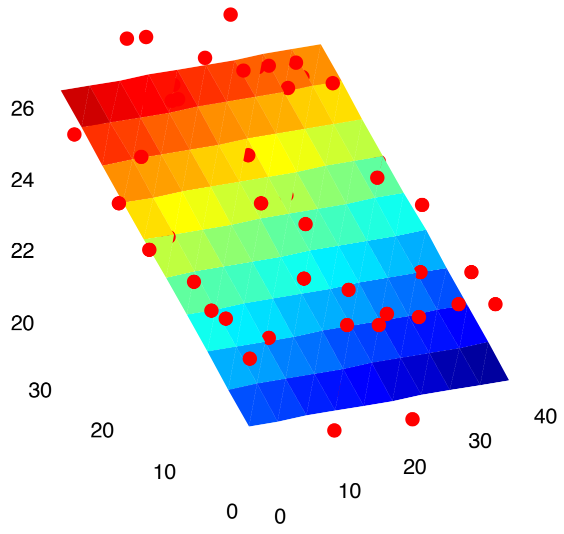
$$Q(s,a) = \sum_i \underline{w_i} f_i(s,a)$$

Detour: Minimizing Error and Least Squares



Linear Approximation: Regression





Prediction:

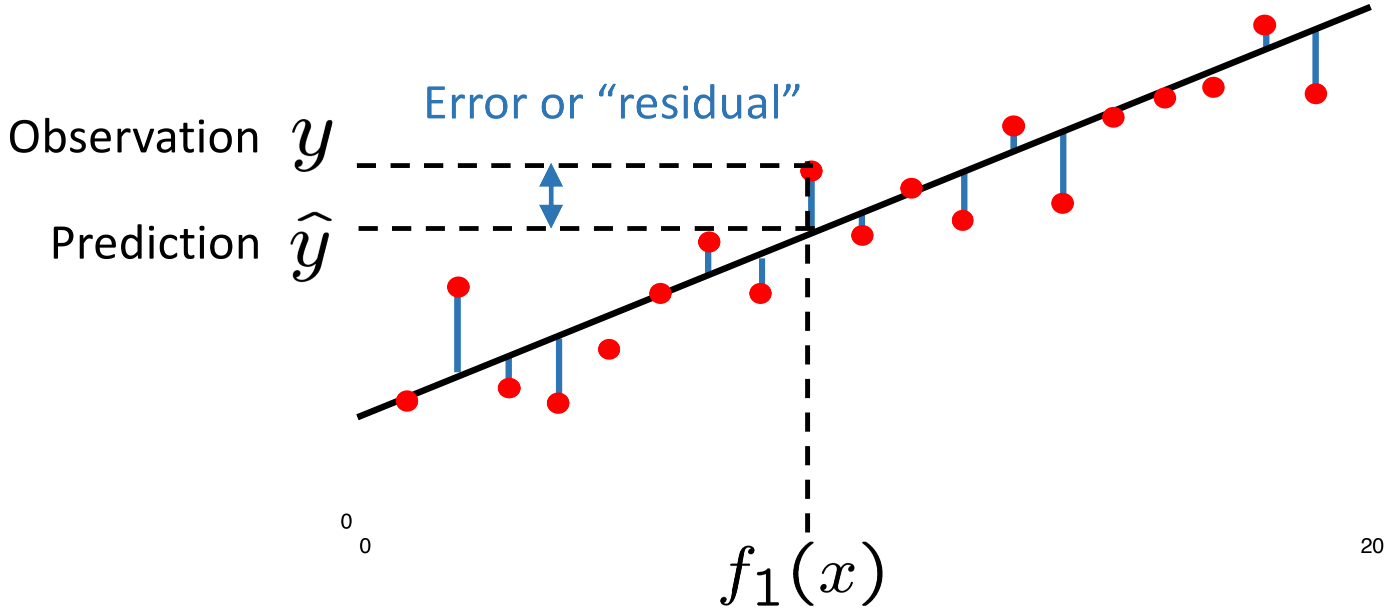
$$\widehat{y} = w_0 + w_1 f_1(x)$$

Prediction:

$$\widehat{y}_i = w_0 + w_1 f_1(x) + w_2 f_2(x)$$

Optimization: Least Squares

$$\text{total error} = \underline{\underline{\sum_i (y_i - \boxed{\hat{y_i}})^2}} = \sum_i \left(y_i - \sum_k w_k \underline{\underline{f_k(x_i)}} \right)^2 \rightarrow f_k(x_i)$$



Quick Calculus Quiz

$$Error(w) = \frac{1}{2}\big(y - wf(x)\big)^2$$

What is $\frac{dError}{dw}$?

Last time

$$Error(x) = \frac{1}{2}(y - x)^2$$

$$\frac{dError}{dx} = -(y - x)$$

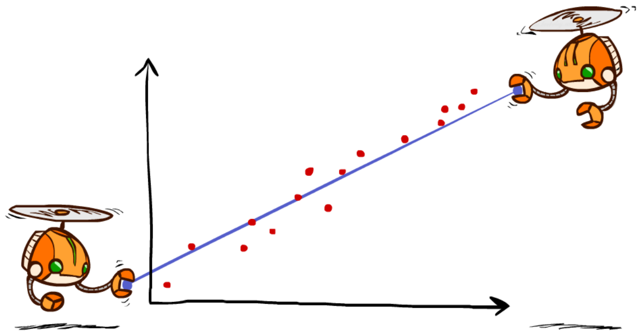
Minimizing Error

Imagine we had only one point x, with features f(x), target value y, and weights w:

$$\text{error}(w) = \frac{1}{2}\left(y - \sum_k w_k f_k(x)\right)^2$$

$$\frac{\partial\ \text{error}(w)}{\partial w_m} = -\left(y - \sum_k w_k f_k(x)\right) f_m(x)$$

$$\boxed{w_m \leftarrow w_m + \alpha \left(y - \sum_k w_k f_k(x)\right) f_m(x)}$$



Approximate q update explained:

$$w_m \leftarrow w_m + \alpha \left[r + \gamma \max_a Q(s', a') - Q(s, a) \right] f_m(s, a)$$

"target" "prediction"

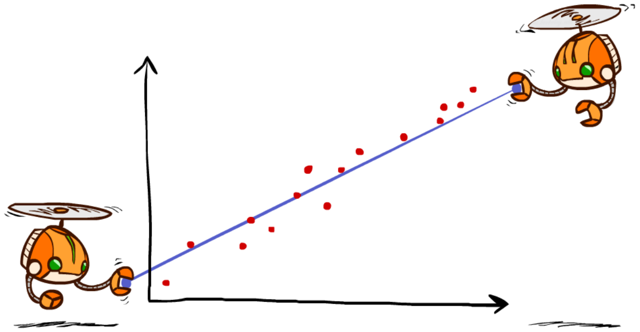
Minimizing Error

Imagine we had only one point x, with features f(x), target value y, and weights w:

$$\text{error}(w) = \frac{1}{2}\left(y - \sum_k w_k f_k(x)\right)^2$$

$$\frac{\partial \ \text{error}(w)}{\partial w_m} = -\left(y - \sum_k w_k f_k(x)\right) f_m(x)$$

$$w_m \leftarrow w_m + \alpha \left(y - \sum_k w_k f_k(x)\right) f_m(x)$$



Example: $Q(s,a) = 2f_1(s,a) + 3f_2(s,a)$

$$f_1(s,a) = 4, f_2(s,a) = 1, r_{sampled} = 3$$

$w_2 \leftarrow$

Updating a linear value function

Original Q learning rule tries to reduce prediction error at s, a:

- $Q(s,a) \leftarrow Q(s,a) + \alpha[(R(s,a,s') + \gamma \max_{a'} Q(s',a')) - Q(s,a)]$

Instead, we update the weights to try to reduce the error at s, a:

- $\mathrm{w_i} \leftarrow w_i + \alpha * f_i(s,a) * [(R(s,a,s') + \gamma \max_{a'} Q(s',a')) - Q(s,a)]$

Approximate Q-Learning

$$\boxed{Q(s,a) = w_1 f_1(s,a) + w_2 f_2(s,a) + \ldots + w_n f_n(s,a)}$$

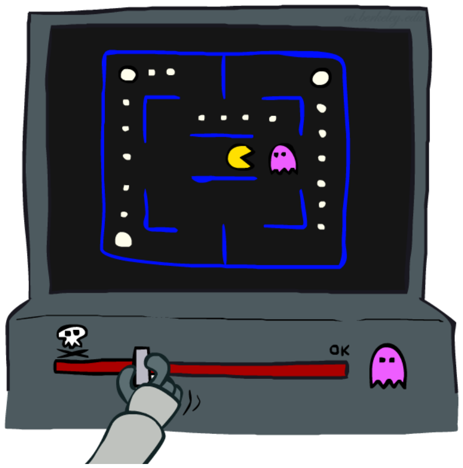
Q-learning with linear Q-functions:

$$\text{transition } = (s,a,r,s')$$

$$\text{difference} = \left[r + \gamma \max_{a'} Q(s',a')\right] - Q(s,a)$$

$$Q(s,a) \leftarrow Q(s,a) + \alpha\,[\text{difference}] \qquad \text{Exact Q's}$$

$$w_i \leftarrow w_i + \alpha\,[\text{difference}]\, f_i(s,a) \qquad \text{Approximate Q's}$$



Intuitive interpretation:

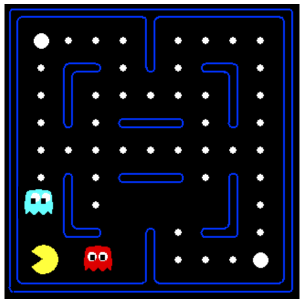
- Adjust weights of active features
- E.g., if something unexpectedly bad happens, blame the features that were on: disprefer all states with that state's features

Formal justification: online least squares

Example: Q-Pacman

$$Q(s,a) = 4.0 f_{DOT}(s,a) - 1.0 f_{GST}(s,a)$$

s



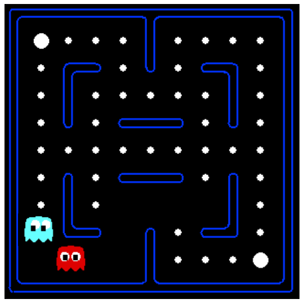
$f_{DOT}(s, \text{NORTH}) = 0.5$

$f_{GST}(s, \text{NORTH}) = 1.0$

$a = \text{NORTH}$

$r = -500$

s'



$Q(s, \text{NORTH}) = +1$

$Q(s', \cdot) = 0$

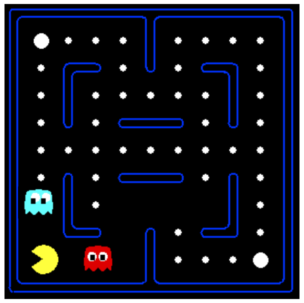
$r + \gamma \max_{a'} Q(s', a') = -500 + 0$ $\qquad \alpha = 0.004$

$w_i \leftarrow w_i + \alpha \, [\text{difference}] \, f_i(s,a)$

Example: Q-Pacman

$$Q(s,a) = 4.0 f_{DOT}(s,a) - 1.0 f_{GST}(s,a)$$

s



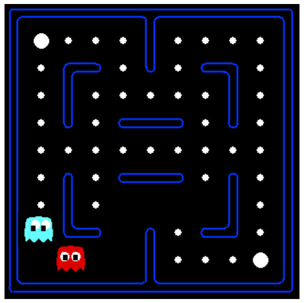
$f_{DOT}(s, \text{NORTH}) = 0.5$

$f_{GST}(s, \text{NORTH}) = 1.0$

$a = \text{NORTH}$

$r = -500$

s'



$Q(s, \text{NORTH}) = +1$

$r + \gamma \max_{a'} Q(s', a') = -500 + 0$

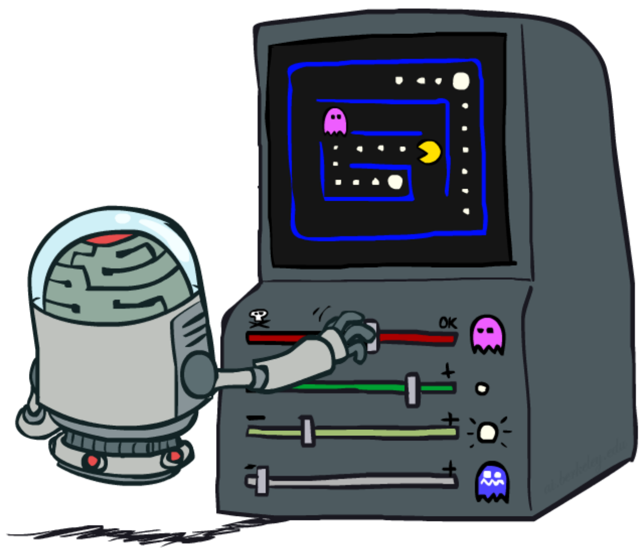
$Q(s', \cdot) = 0$

$\text{difference} = -501$ $\Rightarrow$ $w_{DOT} \leftarrow 4.0 + \alpha\,[-501]\,0.5$

$w_{GST} \leftarrow -1.0 + \alpha\,[-501]\,1.0$

$$Q(s,a) = 3.0 f_{DOT}(s,a) - 3.0 f_{GST}(s,a)$$

Recent Reinforcement Learning Milestones



TDGammon

1992 by Gerald Tesauro, IBM

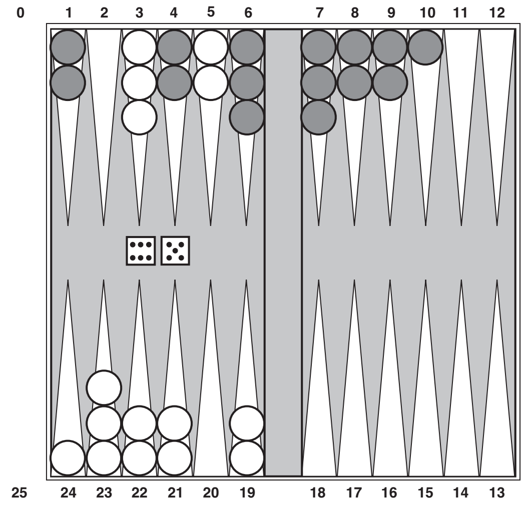
4-ply lookahead using V(s) trained from 1,500,000 games of self-play

3 hidden layers, ~100 units each

Input: contents of each location plus several handcrafted features

Experimental results:

- Plays approximately at parity with world champion
- Led to radical changes in the way humans play backgammon



Deep Q-Networks

sample = $r + \gamma \max_{a'} Q_{\mathbf{w}}(s',a')$

$Q_{\mathbf{w}}(s,a)$: Neural network

Deep Mind, 2015

Used a deep learning network to represent Q:

- Input is last 4 images (84x84 pixel values) plus score

49 Atari games, incl. Breakout, Space Invaders, Seaquest, Enduro

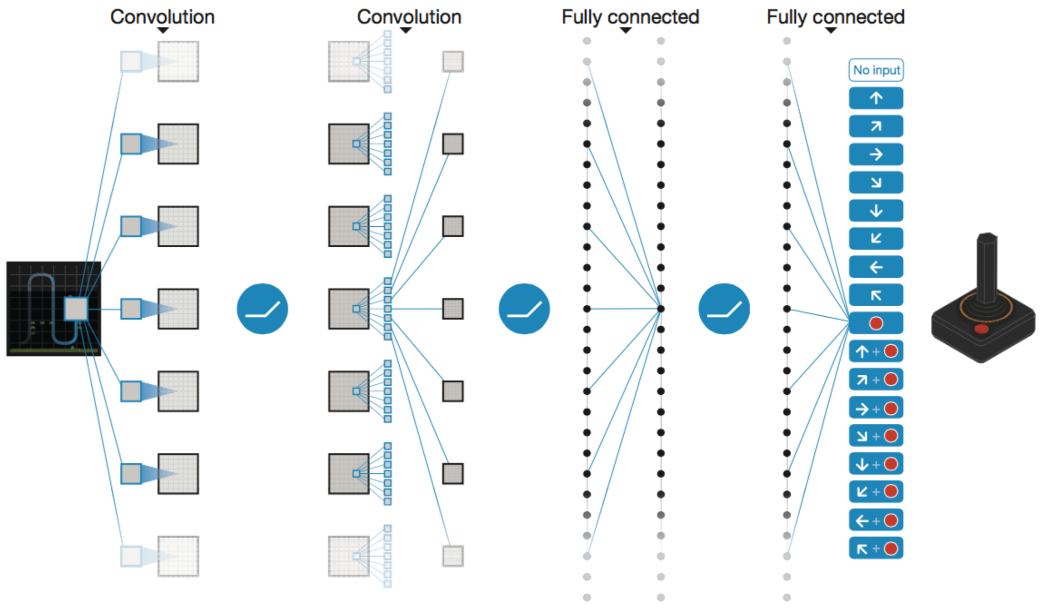
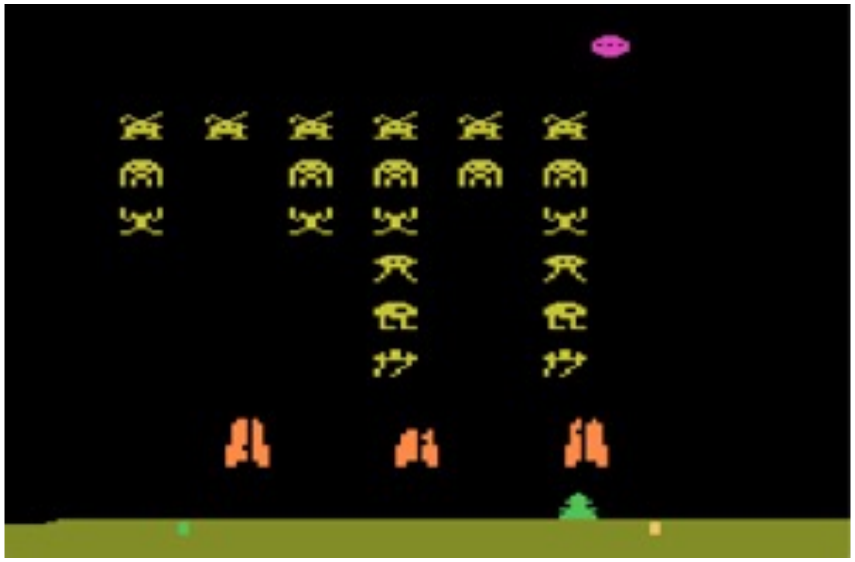


Image: Deep Mind



Images: Open AI, Atari

OpenAI Gym

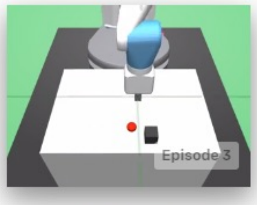
2016+

Benchmark problems for learning agents

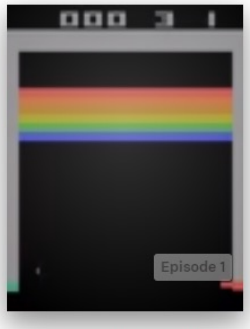
https://gym.openai.com/envs

Acrobot-v1
Swing up a two-link robot.

Ant-v2
Make a 3D four-legged robot walk.



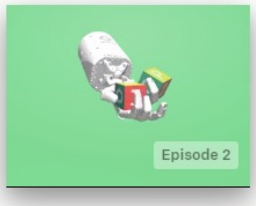
FetchPush-v0
Push a block to a goal position.



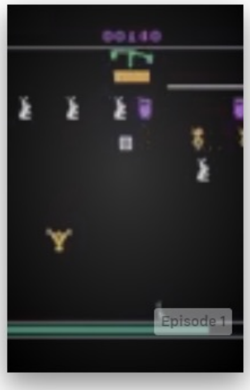
Breakout-ram-v0
Maximize score in the game Breakout, with RAM as input

MountainCarContinuous-v0
Drive up a big hill with continuous control.

Humanoid-v2
Make a 3D two-legged robot walk.



HandManipulateBlock-v0
Orient a block using a robot hand.



Carnival-v0
Maximize score in the game Carnival, with screen images as input

Images: Open AI

AlphaGo, AlphaZero

Deep Mind, 2016+

Autonomous Vehicles?