Announcements

Assignments:

- HW2 (written)
  - Due Tuesday 1/31, 10 pm
- P1: Search
  - Due Monday 2/6, 10pm
  - Working in pairs is suggested but not required

Polls

- Don’t worry if you miss a few
- Talk to Stephanie if you are systematically missing polls

Announcements

Recitation

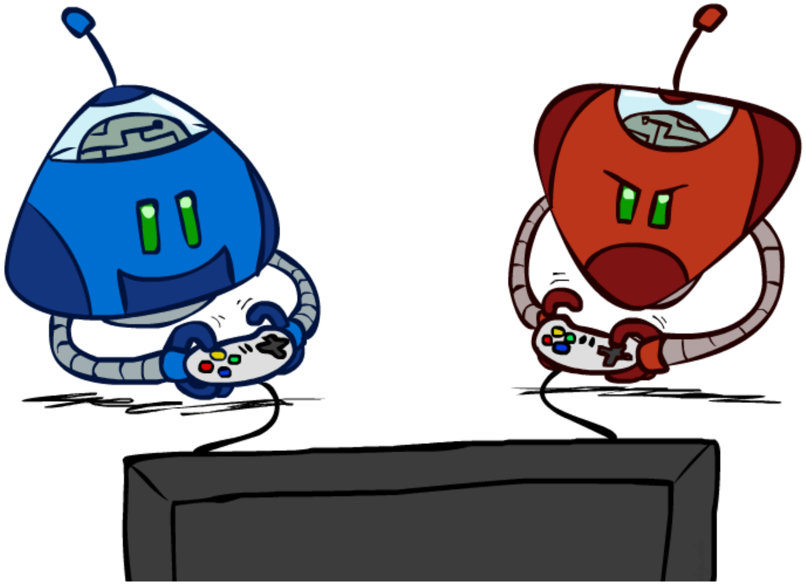
- Join any recitation you want this week
- Stay tuned to Diderot for post about informally changing section

More coming on Diderot

- Recitation change form
- Student info survey

AI: Representation and Problem Solving

Adversarial Search



Instructor: Stephanie Rosenthal

Slide credits: CMU AI, http://ai.berkeley.edu

Outline

History / Overview

Zero-Sum Games (Minimax)

Evaluation Functions

Search Efficiency (α-β Pruning)

Games of Chance (Expectimax)

Game Playing State-of-the-Art

Checkers:

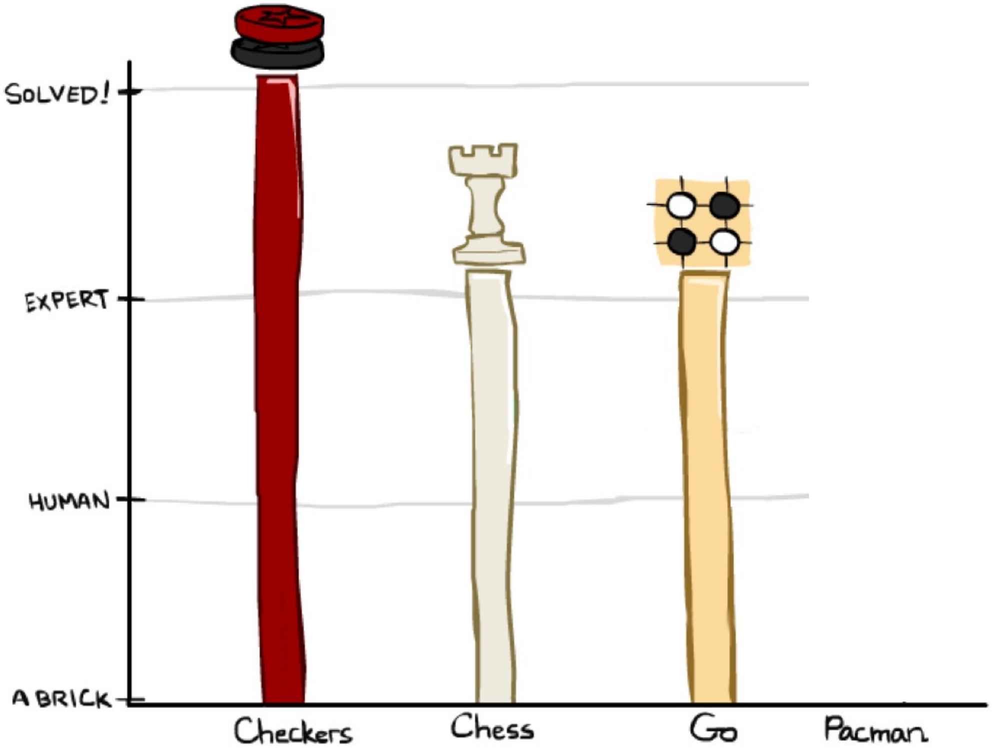
- 1950: First computer player.
- 1959: Samuel's self-taught program.
- 1994: First computer world champion: Chinook ended 40-year-reign of human champion Marion Tinsley using complete 8-piece endgame.
- 2007: Checkers solved! Endgame database of 39 trillion states

Chess:

- 1945-1960: Zuse, Wiener, Shannon, Turing, Newell & Simon, McCarthy.
- 1960s onward: gradual improvement under "standard model"
- 1997: special-purpose chess machine Deep Blue defeats human champion Gary Kasparov in a six-game match. Deep Blue examined 200M positions per second and extended some lines of search up to 40 ply. Current programs running on a PC rate > 3200 (vs 2870 for Magnus Carlsen).

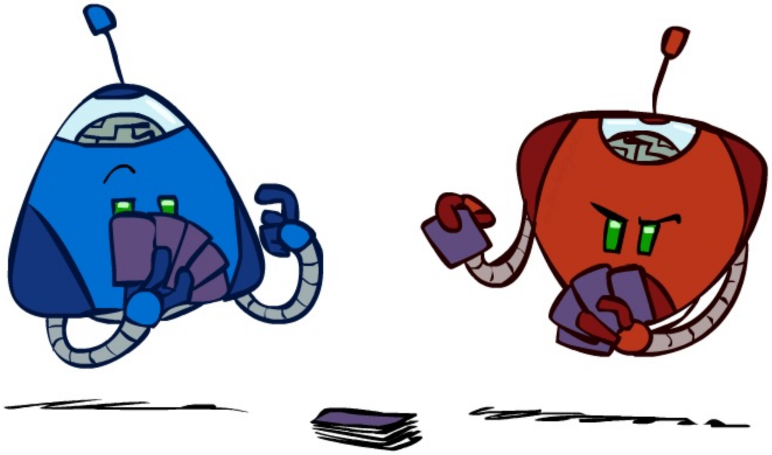
Go:

- 1968: Zobrist's program plays legal Go, barely (b>300!)
- 2005-2014: Monte Carlo tree search enables rapid advances: current programs beat strong amateurs, and professionals with a 3-4 stone handicap.
- 2015: AlphaGo from DeepMind beats Lee Sedol



Types of Games

Many different kinds of games!

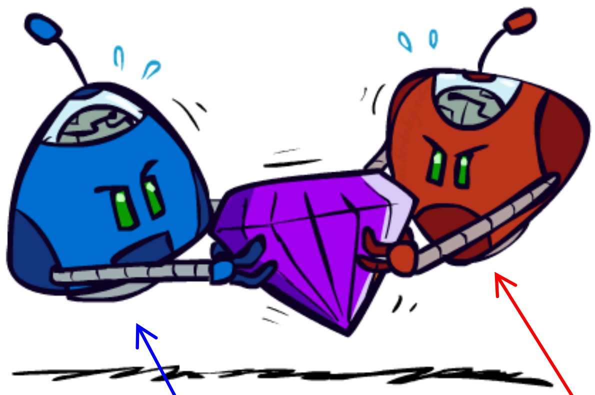


Axes:

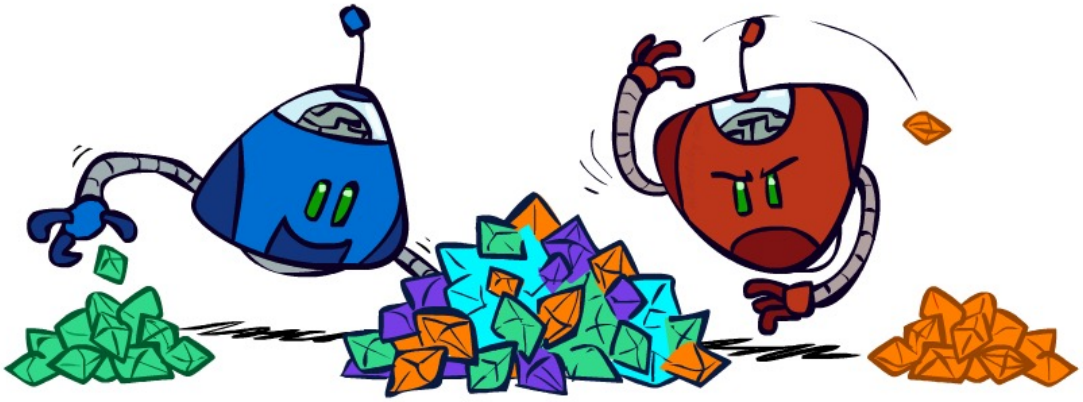
- Deterministic or stochastic?
- Perfect information (fully observable)?
- One, two, or more players?
- Turn-taking or simultaneous?
- Zero sum?

Want algorithms for calculating a ***contingent plan*** (a.k.a. strategy or policy) which recommends a move for every possible eventuality

Zero-Sum Games



- Zero-Sum Games
  - Agents have ***opposite*** utilities
  - Pure competition:
    - One ***maximizes***, the other ***minimizes***



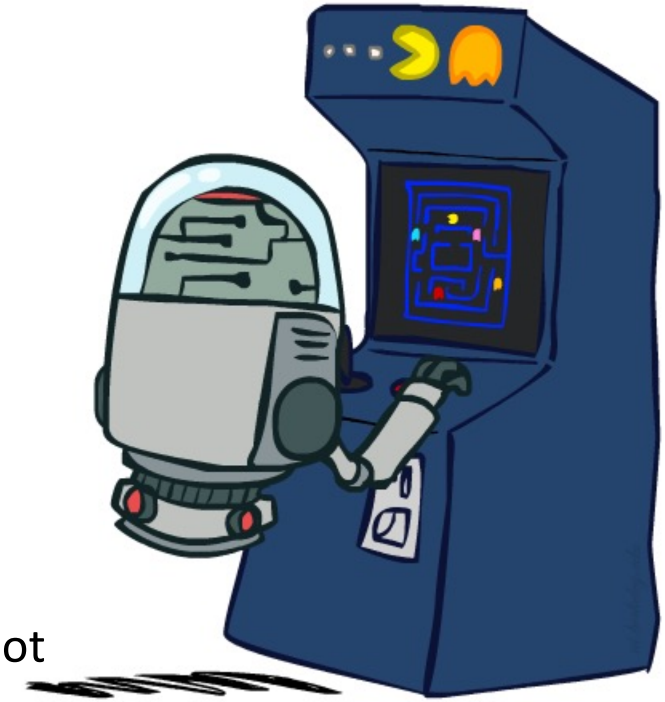
- General Games
  - Agents have ***independent*** utilities
  - Cooperation, indifference, competition, shifting alliances, and more are all possible

“Standard” Games

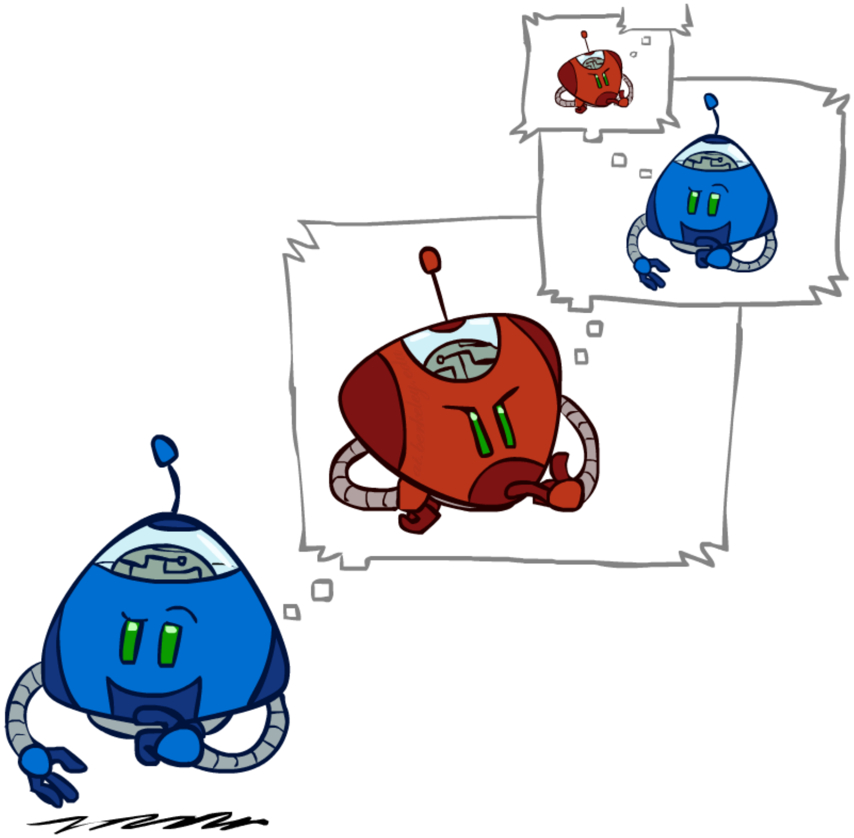
Standard games are deterministic, observable, two-player, turn-taking, zero-sum

Game formulation:

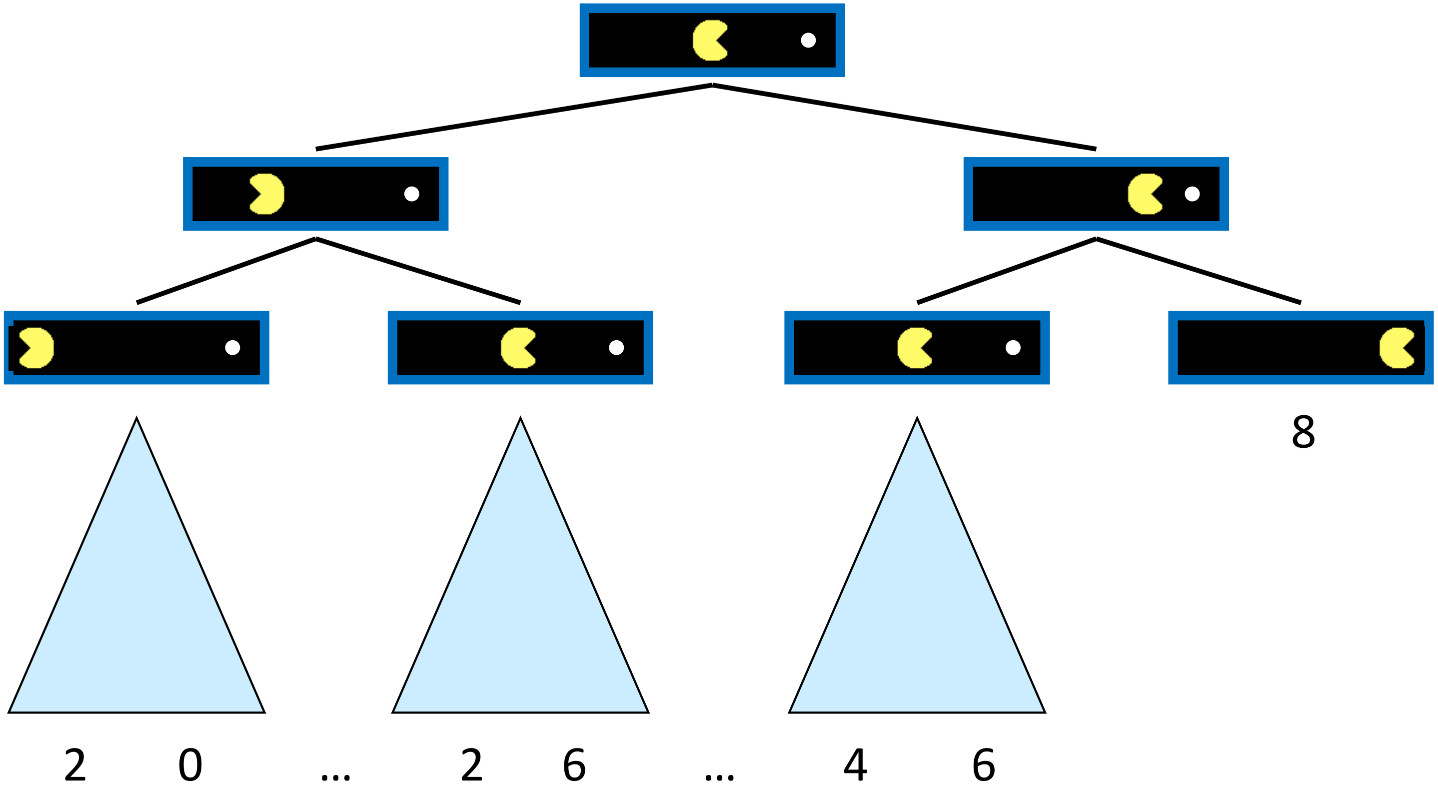
- Initial state: s_0
- Players: Player(s) indicates whose move it is
- Actions: Actions(s) for player on move
- Transition model: Result(s,a)
- Terminal test: Terminal-Test(s)
- Terminal values: Utility(s,p) for player p
  - Or just Utility(s) for player making the decision at root



Adversarial Search



Single-Agent Trees

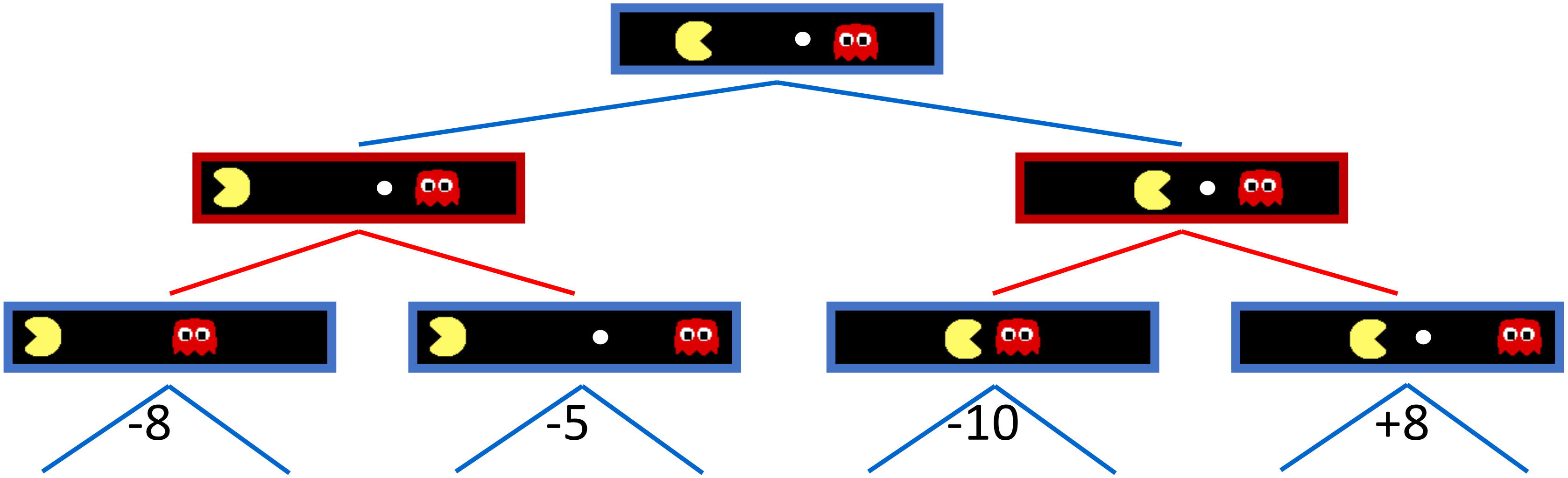


Minimax

States

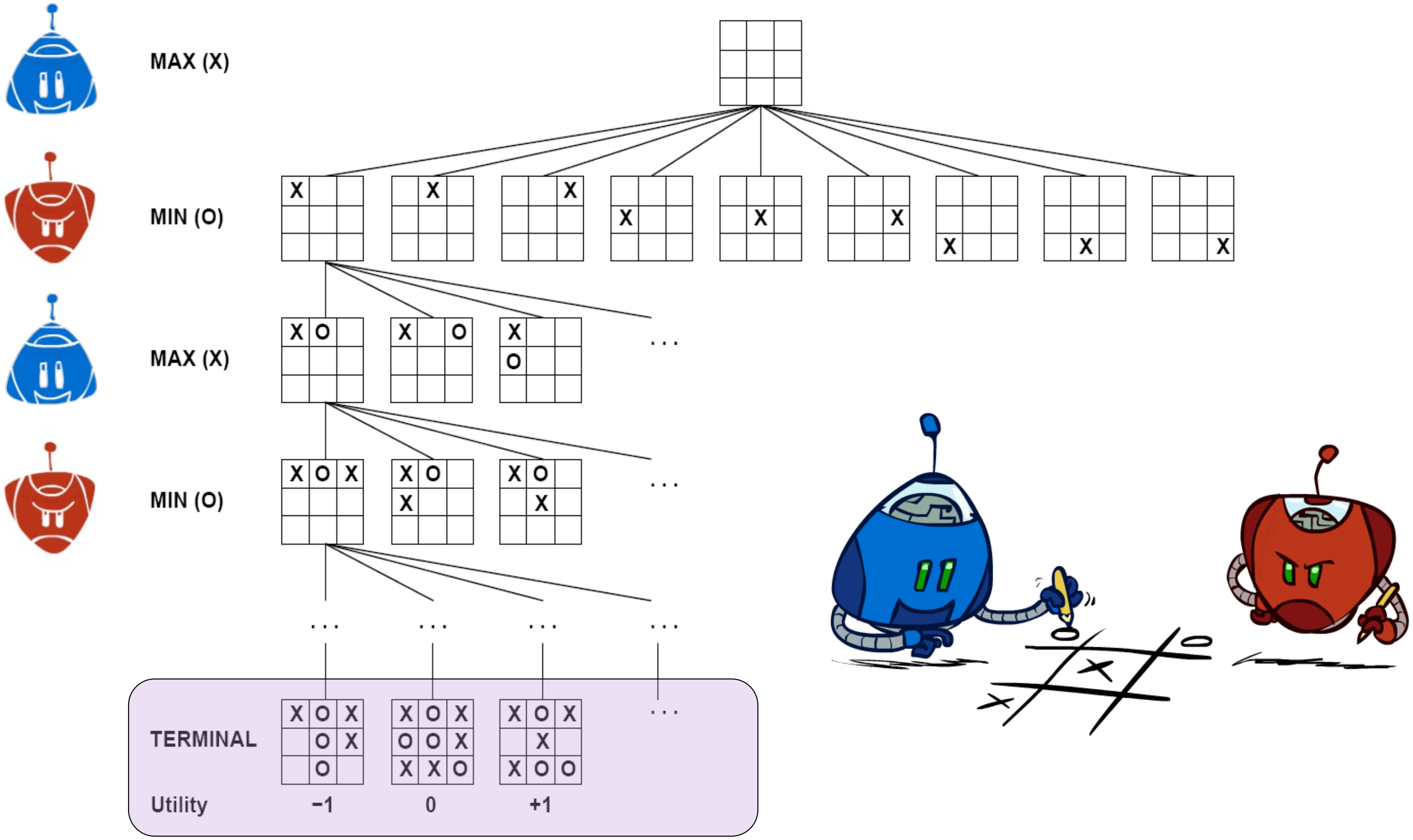
Actions

Values



Minimax

- States
- Actions
- Values



Minimax Code

```
def max_value(state):

    if state.is_leaf:
        return state.value
    # TODO Also handle depth limit

    best_value = -10000000

    for action in state.actions:
        next_state = state.result(action)

        next_value = min_value(next_state)

        if next_value > best_value:
            best_value = next_value

    return best_value

def min_value(state):
```

Poll 1 (+ worksheet Poll 2 and 3 for Q1a/b)

What is the minimax value at the root?

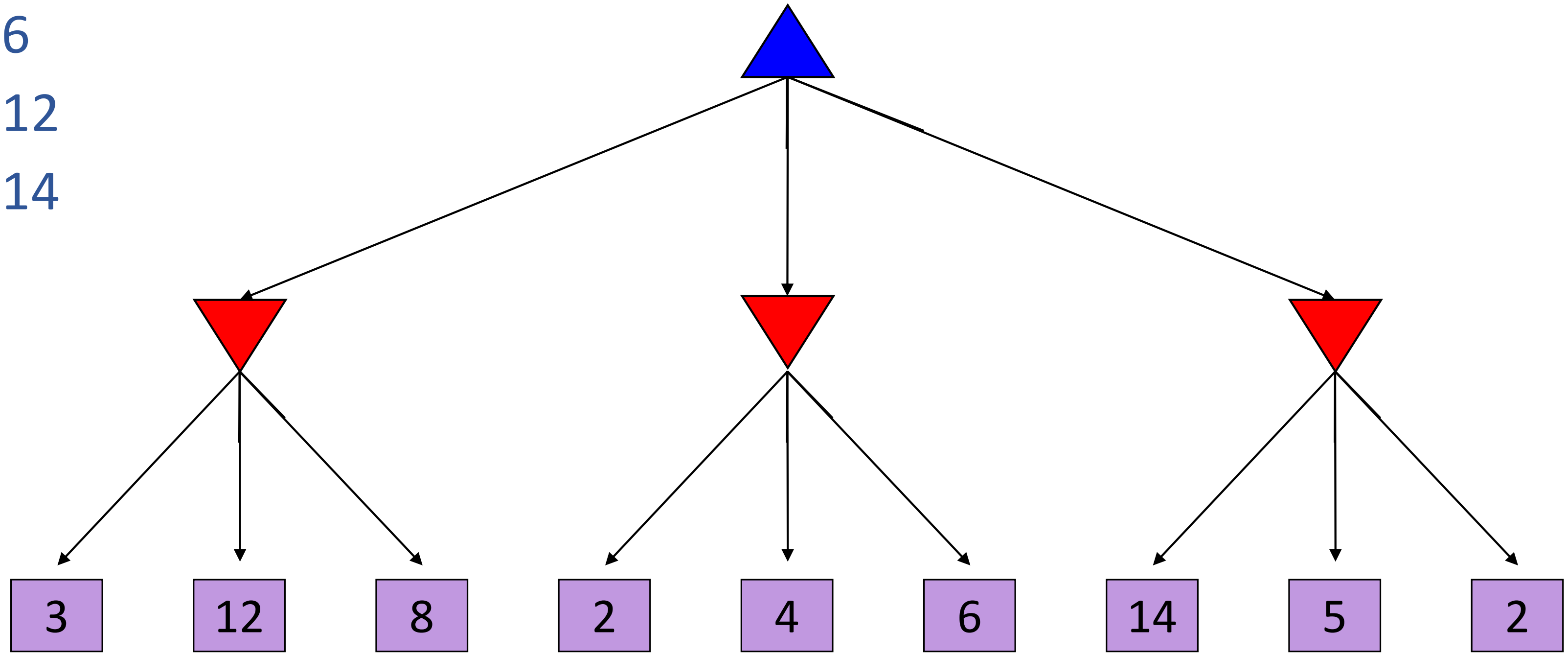
A) 2

B) 3

C) 6

D) 12

E) 14



Poll 1

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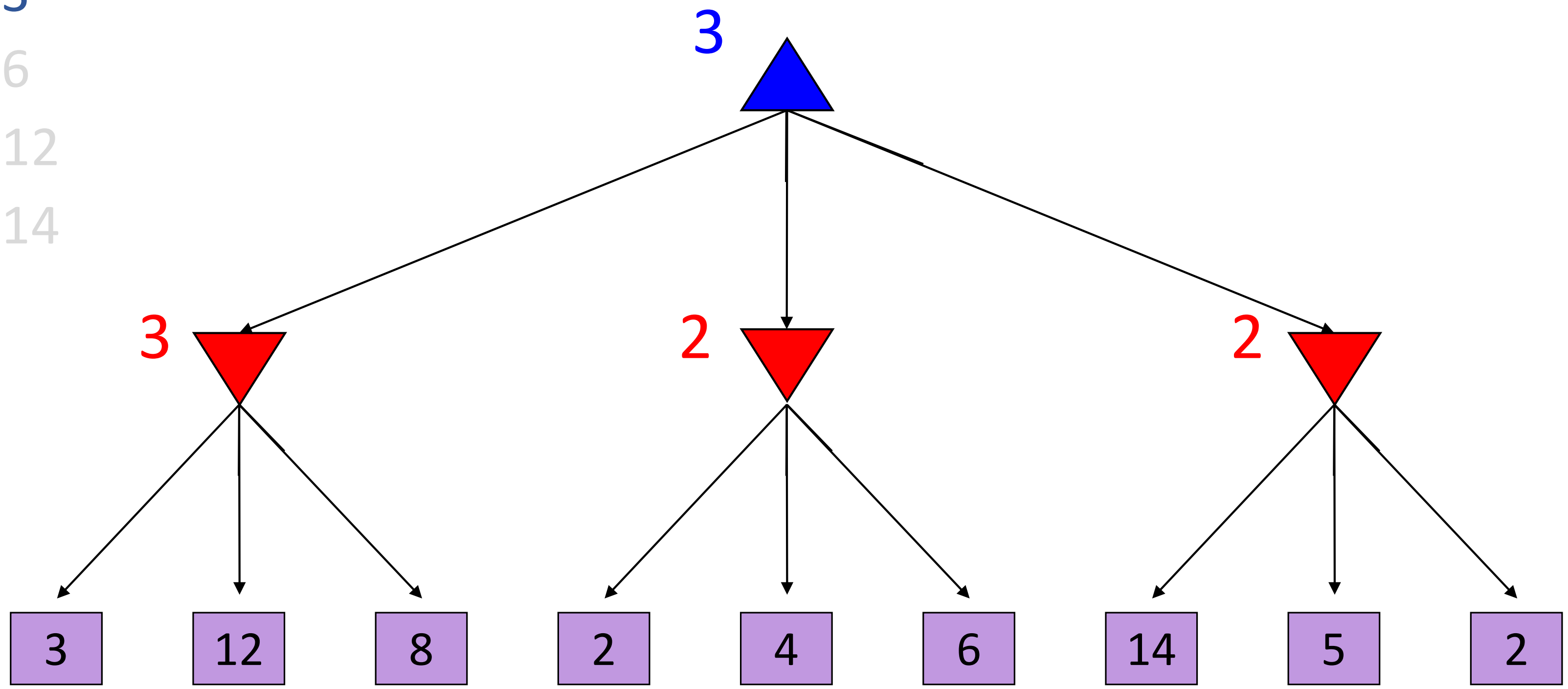

Minimax Notation

```
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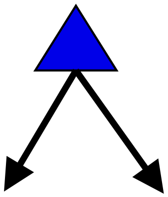
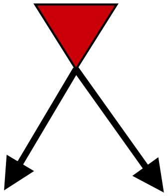
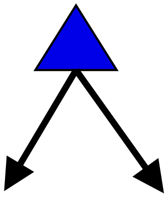


$$V(s) = \max_{a} V(s'),$$

$$\text{where } s' = result(s, a)$$

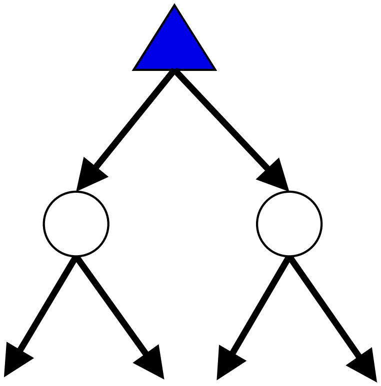


Minimax Notation



$$V(s) = \max_{a} V(s'),$$

where $s' = result(s, a)$



$$\hat{a} = \operatorname*{argmax}_{a} V(s'),$$

where $s' = result(s, a)$

Generic Game Tree Pseudocode

```
function minimax_decision( state )
    return argmax_{a in state.actions} value( state.result(a) )

function value( state )
    if state.is_leaf
        return state.value

    if state.player is MAX
        return max_{a in state.actions} value( state.result(a) )

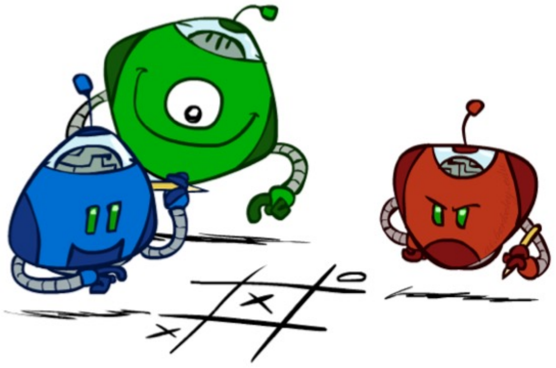
    if state.player is MIN
        return min_{a in state.actions} value( state.result(a) )
```

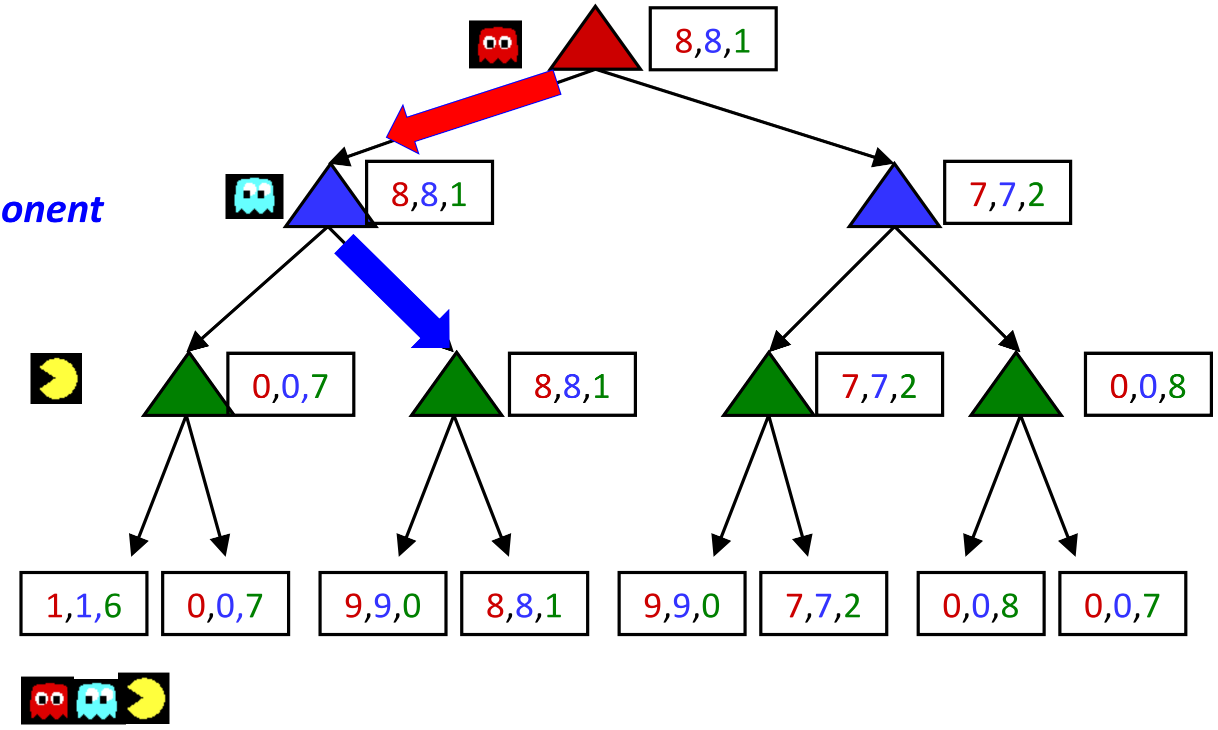
Generalized minimax

What if the game is not zero-sum, or has multiple players?

Generalization of minimax:

- Terminals have ***utility tuples***
- Node values are also utility tuples
- ***Each player maximizes its own component***
- Can give rise to cooperation and competition dynamically…





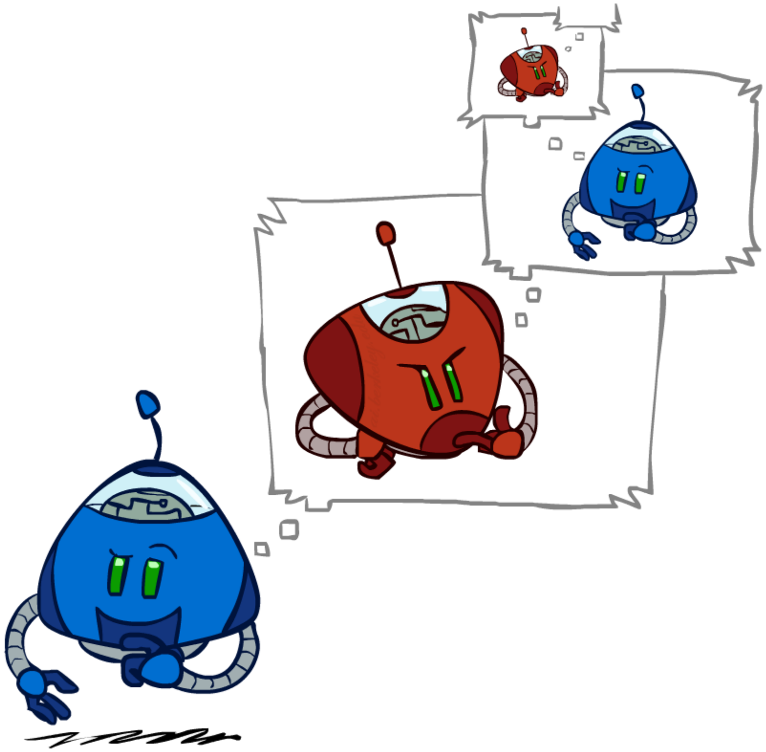
Minimax Efficiency

How efficient is minimax?

- Just like (exhaustive) DFS
- Time: $O(b^m)$
- Space: $O(bm)$

Example: For chess, $b \approx 35$, $m \approx 100$

  - Exact solution is completely infeasible
  - Humans can't do this either, so how do we play chess?
  - Bounded rationality – Herbert Simon



Resource Limits

Resource Limits

Problem: In realistic games, cannot search to leaves!

Solution 1: Bounded lookahead

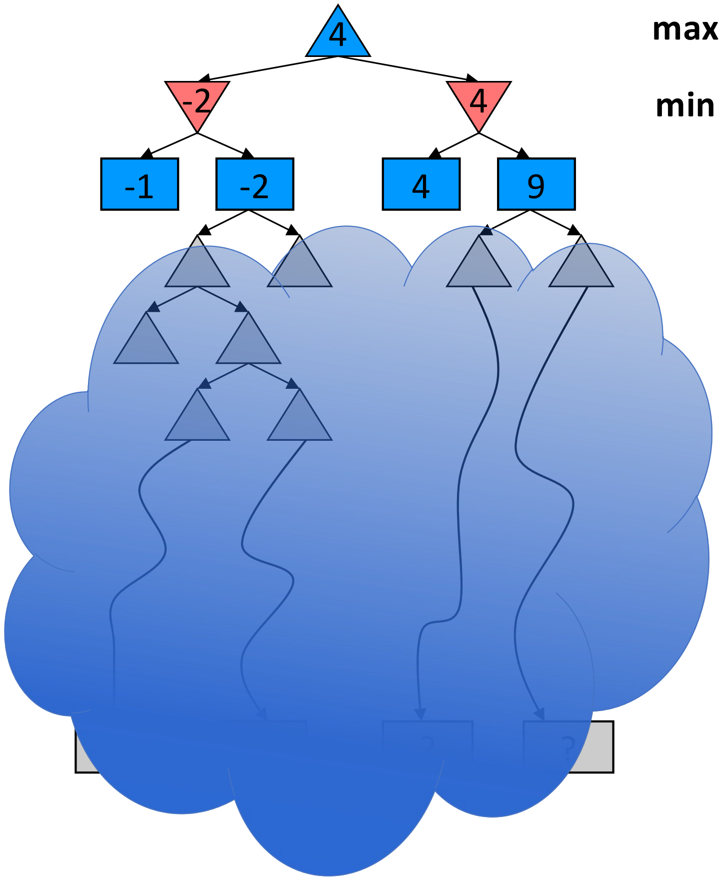
- Search only to a preset ***depth limit*** or ***horizon***
- Use an ***evaluation function*** for non-terminal positions

Guarantee of optimal play is gone

More plies make a BIG difference

Example:

- Suppose we have 100 seconds, can explore 10K nodes / sec
- So can check 1M nodes per move
- For chess, b=~35 so reaches about depth 4 – not so good



Depth Matters

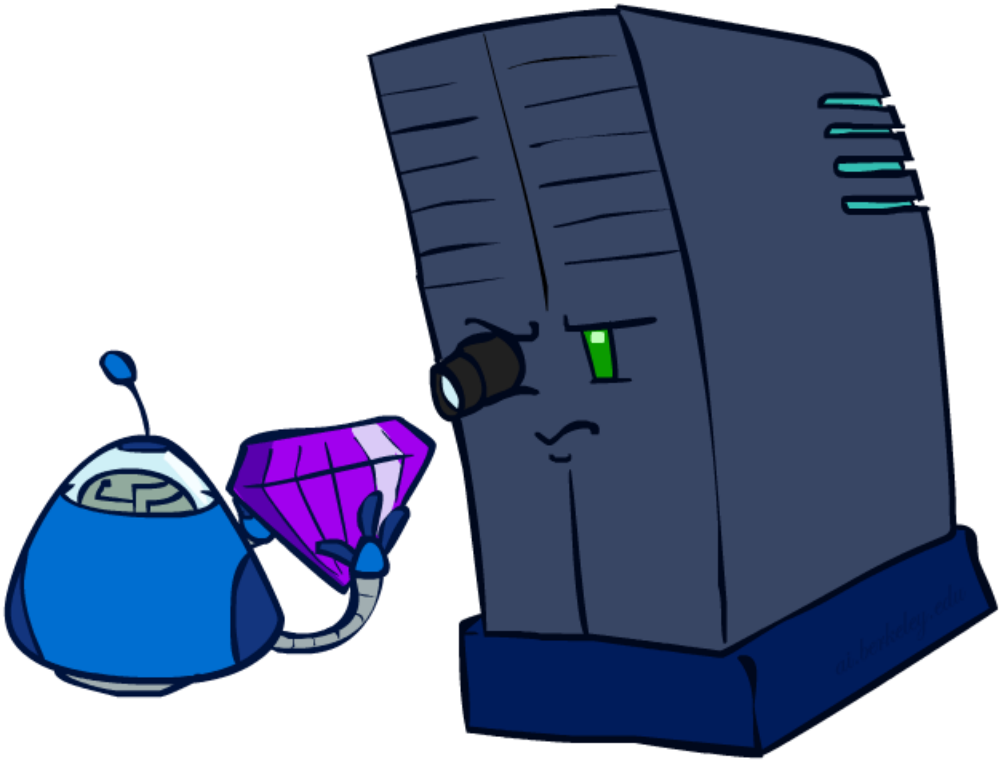
Evaluation functions are always imperfect

Deeper search => better play (usually)

Or, deeper search gives same quality of play with a less accurate evaluation function

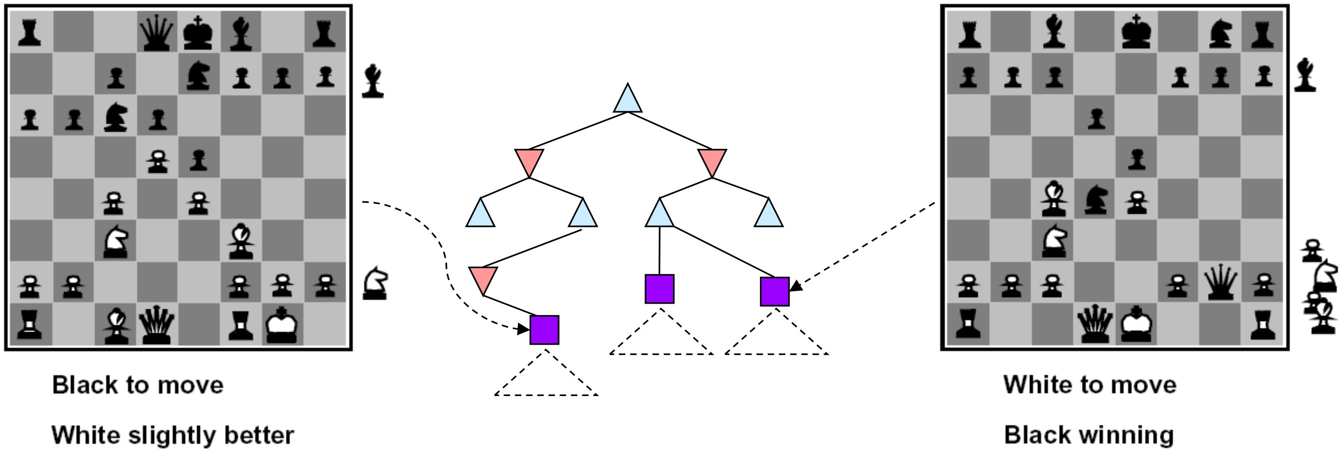
An important example of the tradeoff between complexity of features and complexity of computation

Evaluation Functions



Evaluation Functions

Evaluation functions score non-terminals in depth-limited search

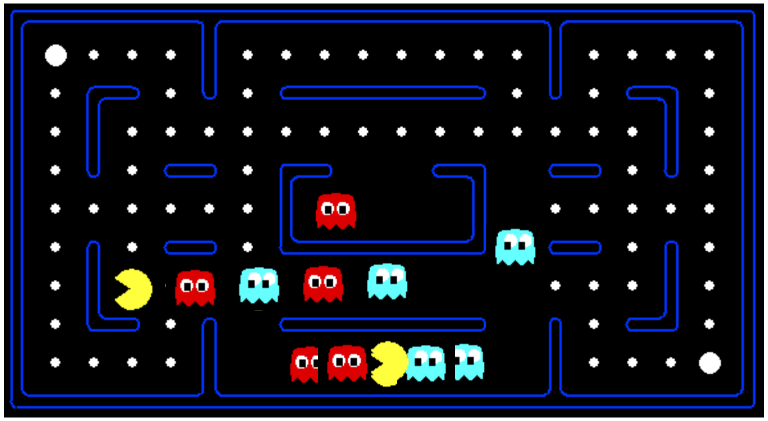


Ideal function: returns the actual minimax value of the position

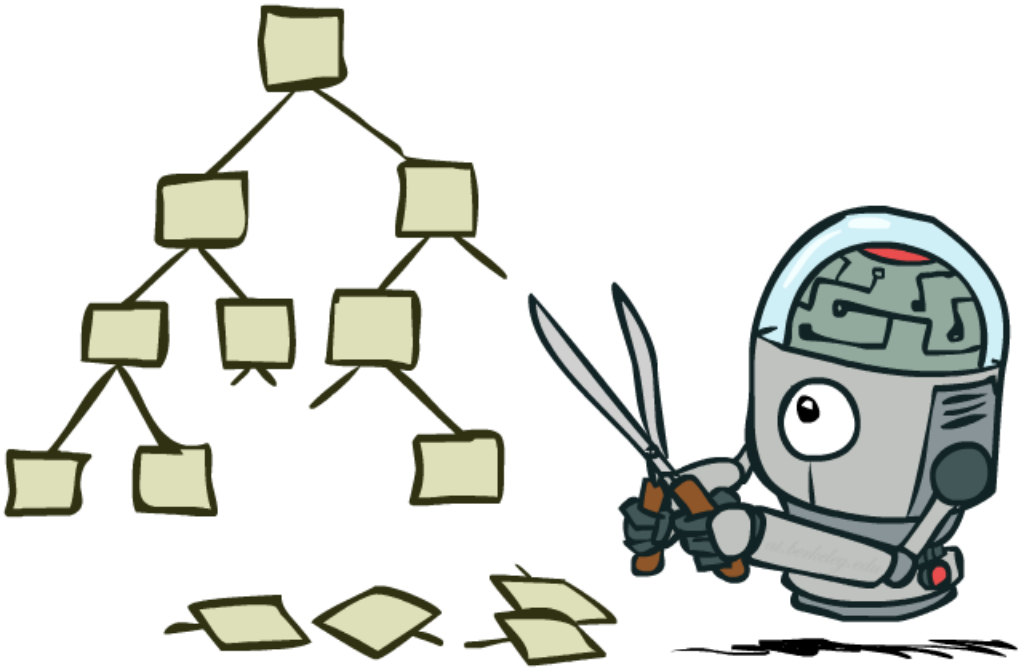
In practice: typically weighted linear sum of features:

- EVAL(s) = $w_1 f_1(s) + w_2 f_2(s) + \dots + w_n f_n(s)$
- E.g., $w_1 = 9$, $f_1(s)$ = (num white queens – num black queens), etc.

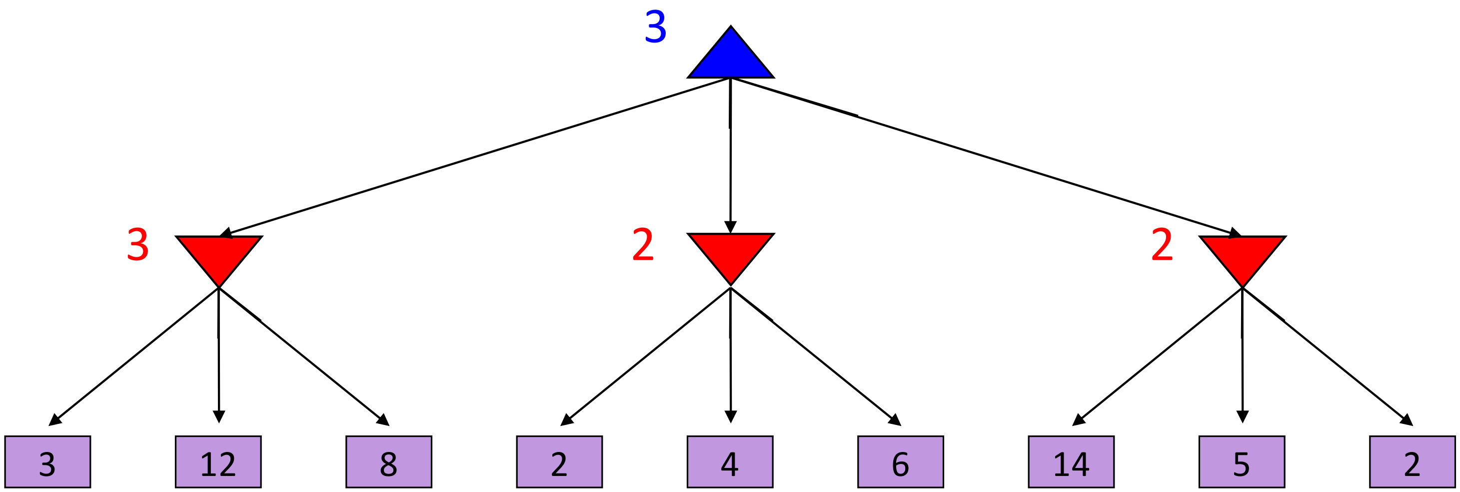
Evaluation for Pacman



Game Tree Pruning

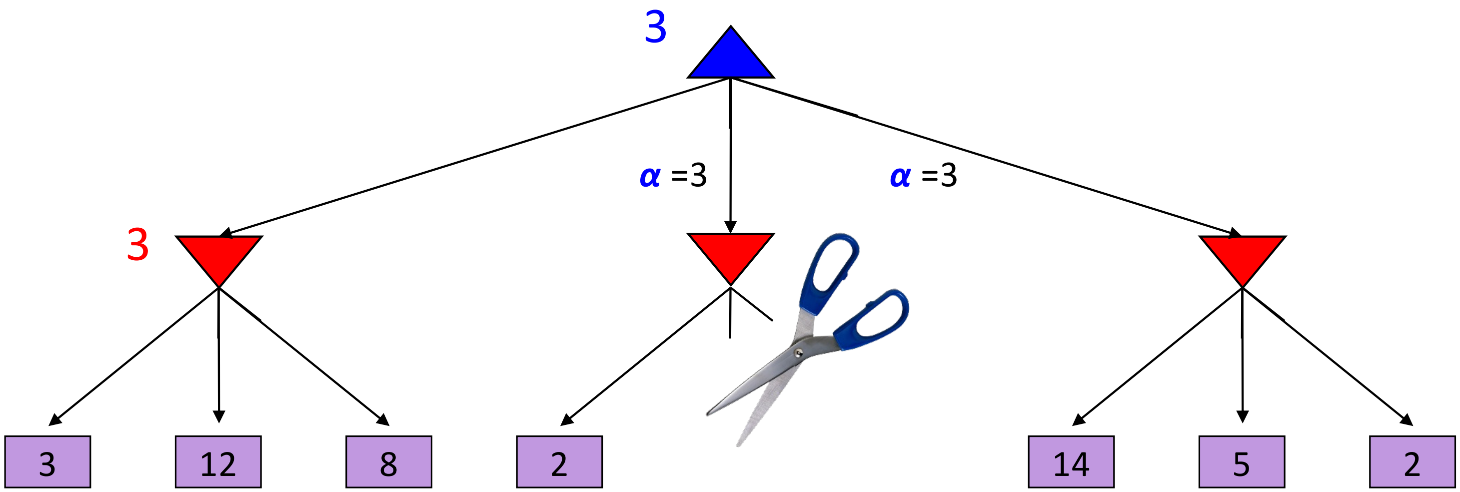


Minimax Example



Alpha-Beta Example

α = best option so far from any MAX node on this path



The order of generation matters: more pruning is possible if good moves come first

Alpha-Beta Implementation

α: MAX's best option on path to root
β: MIN's best option on path to root

```
def max-value(state, α, β):
    initialize v = -∞
    for each successor of state:
        v = max(v, value(successor, α, β))
        if v ≥ β
            return v
        α = max(α, v)
    return v
```

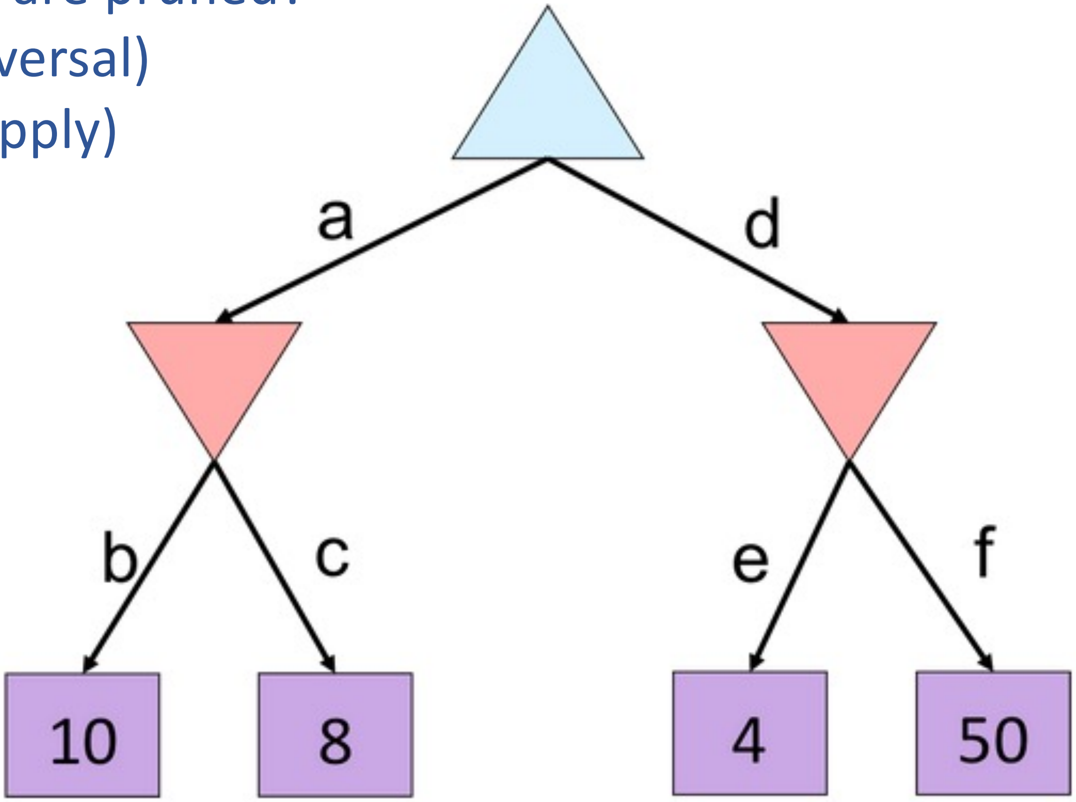
```
def min-value(state , α, β):
    initialize v = +∞
    for each successor of state:
        v = min(v, value(successor, α, β))
        if v ≤ α
            return v
        β = min(β, v)
    return v
```

On your own

Which branches are pruned?

(Left to right traversal)

(Select all that apply)



Poll 4

Which branches are pruned?

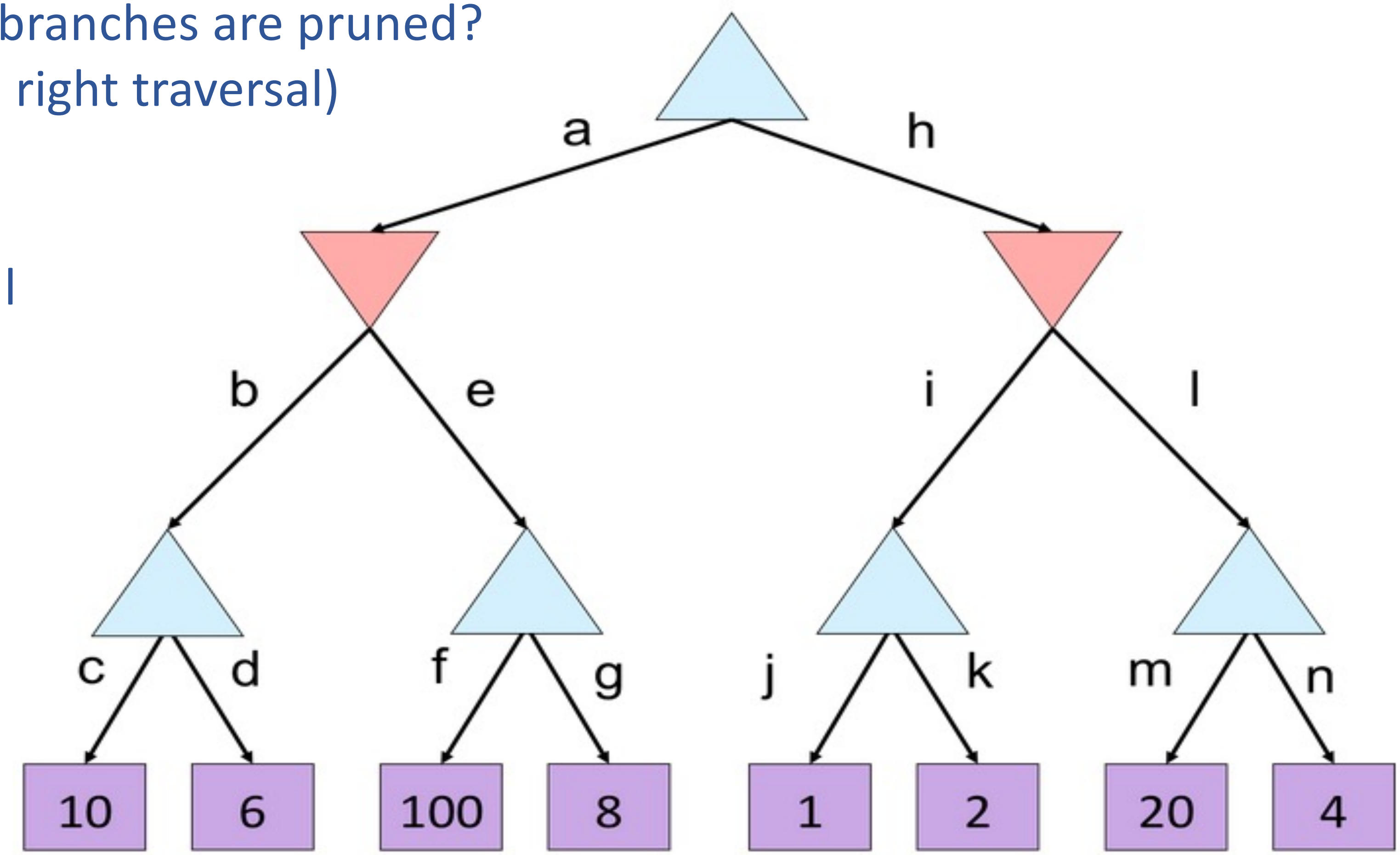
(Left to right traversal)

A) e, l

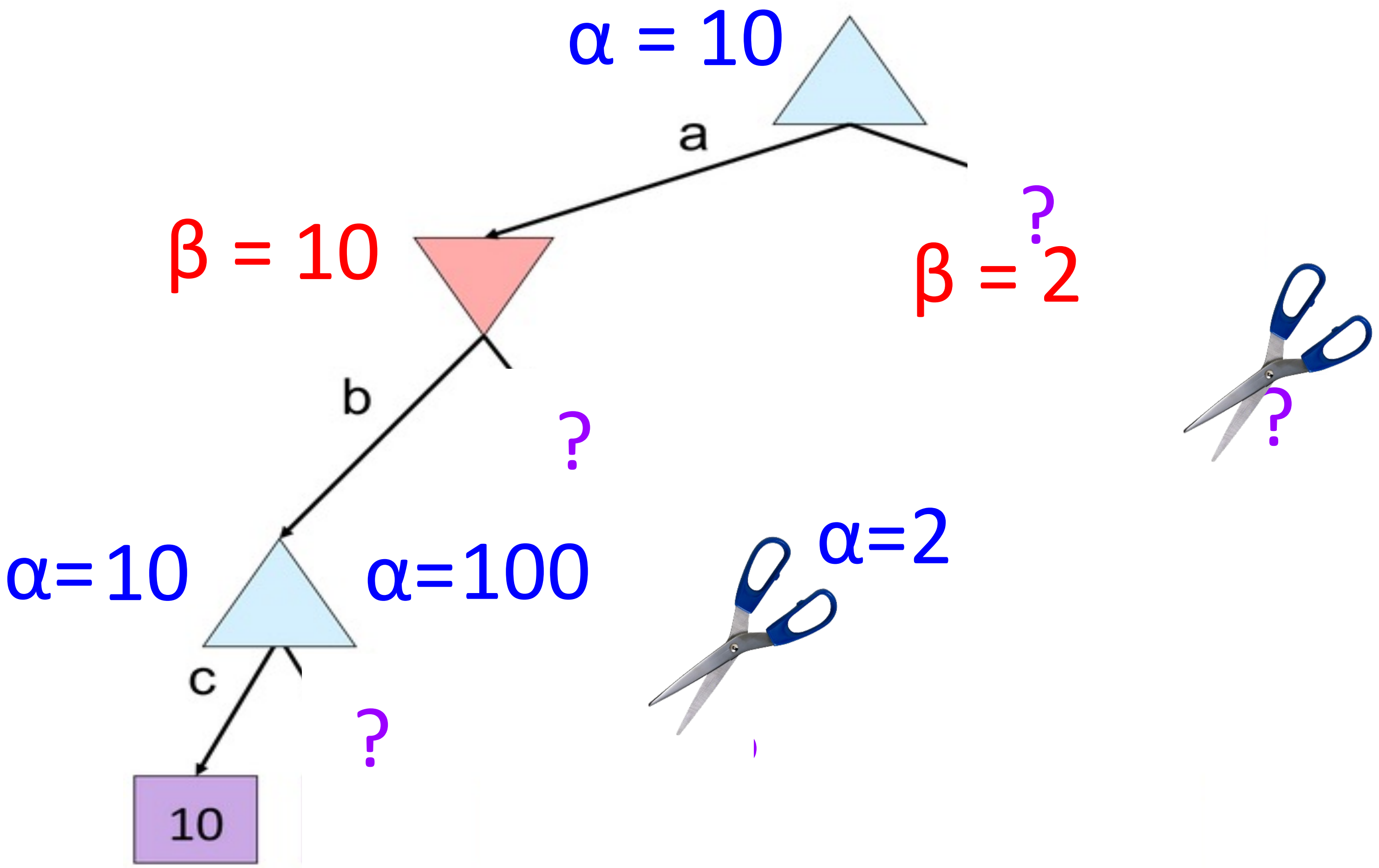
B) g, l

C) g, k, l

D) g, n



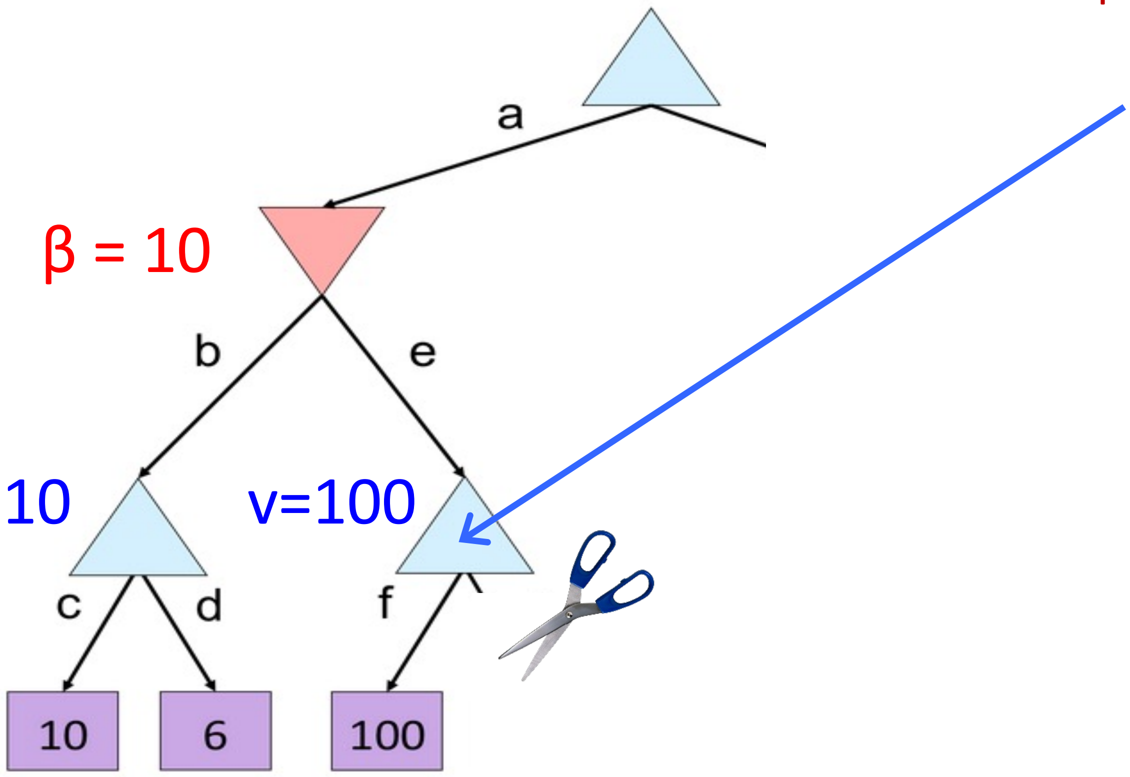
Poll 4



Alpha-Beta Code

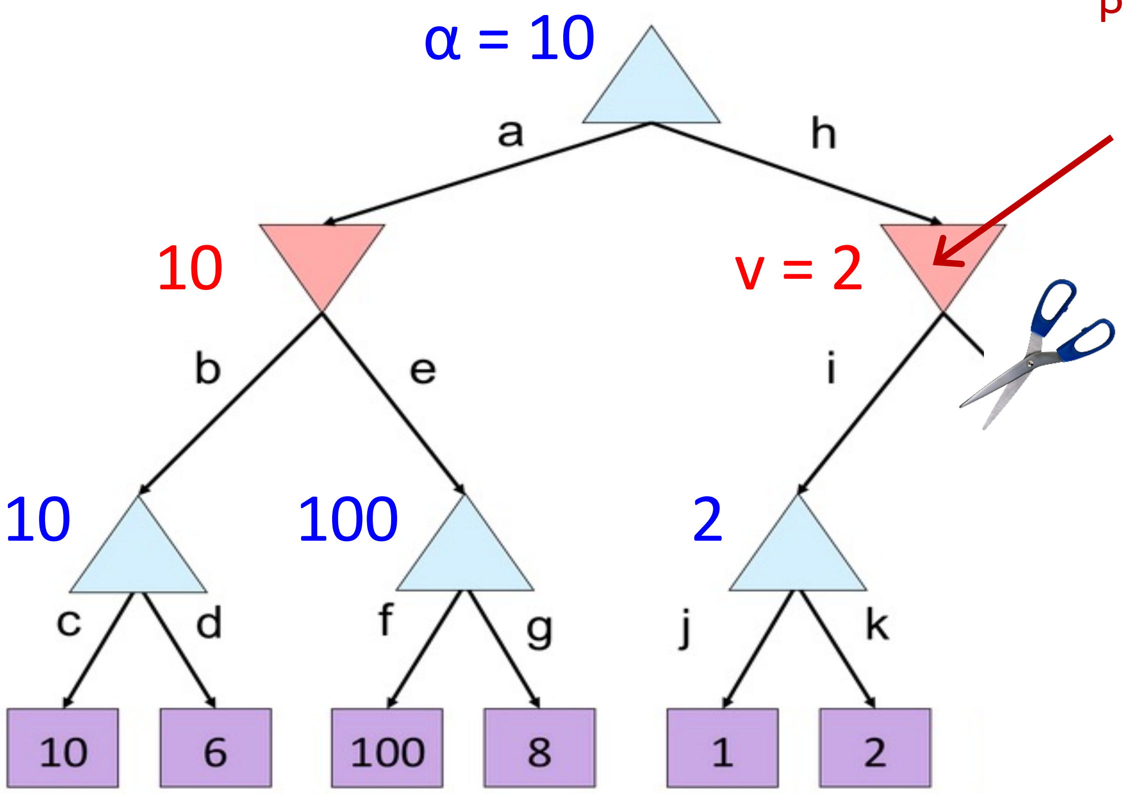
α: MAX's best option on path to root

β: MIN's best option on path to root



```
def max-value(state, $\alpha$, $\beta$):
    initialize $v = -\infty$
    for each successor of state:
        $v = \max(v, \text{value}(\text{successor}, \alpha, \beta))$
        if $v \geq \beta$
            return v
        $\alpha = \max(\alpha, v)$
    return v
```

Alpha-Beta Code



α: MAX’s best option on path to root

β: MIN’s best option on path to root

```
def min-value(state , α, β):
    initialize v = +∞
    for each successor of state:
        v = min(v, value(successor, α, β))
        if v ≤ α
            return v
        β = min(β, v)
    return v
```

Alpha-Beta Pruning Properties

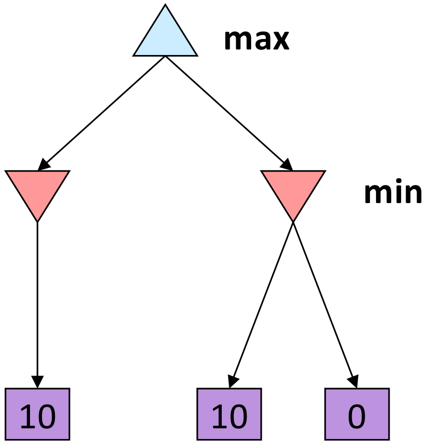
Theorem: This pruning has ***no effect*** on minimax value computed for the root!

Good child ordering improves effectiveness of pruning

- Iterative deepening helps with this

With “perfect ordering”:

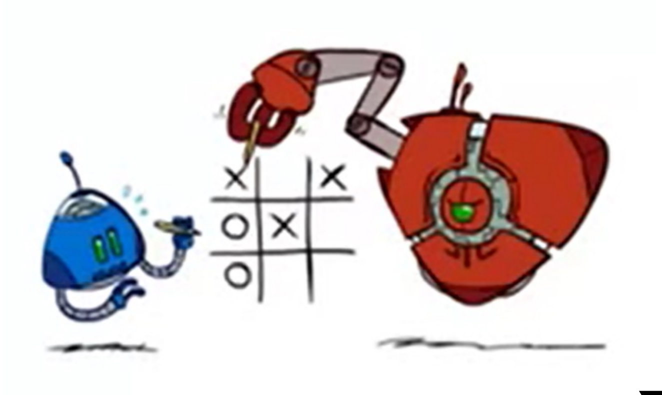
- Time complexity drops to $O(b^{m/2})$
- Doubles solvable depth!
- 1M nodes/move => depth=8, respectable

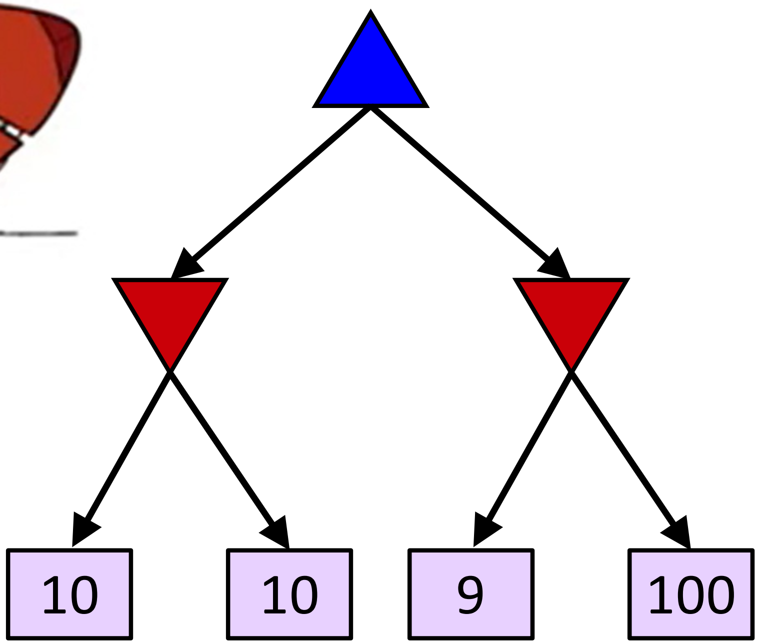


This is a simple example of metareasoning (computing about what to compute)

Modeling Assumptions

Know your opponent





Modeling Assumptions

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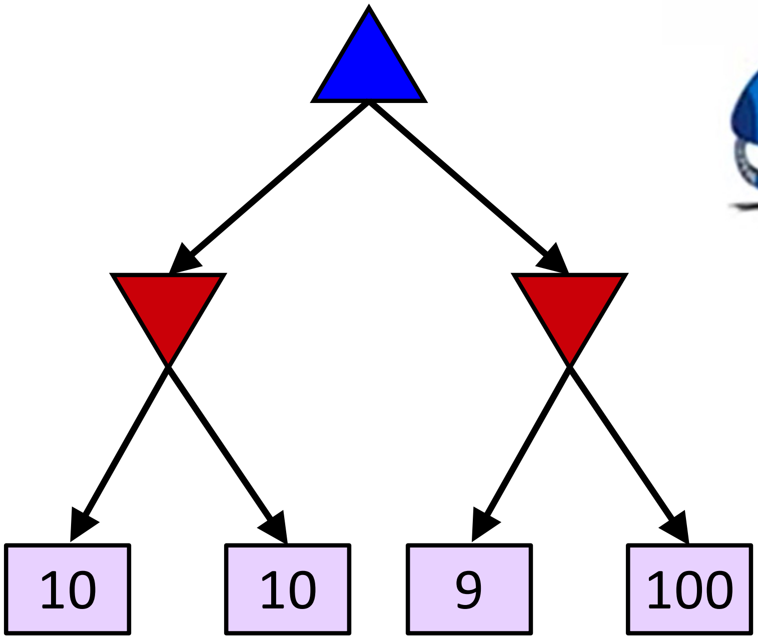


Modeling Assumptions

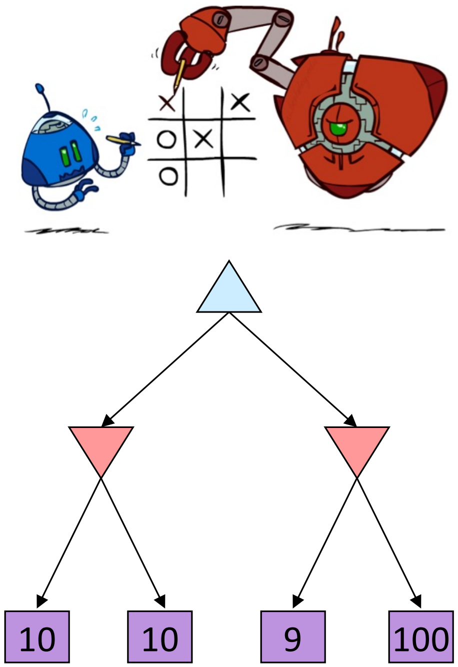
Dangerous Pessimism

Assuming the worst case when it's not likely

Dangerous Optimism

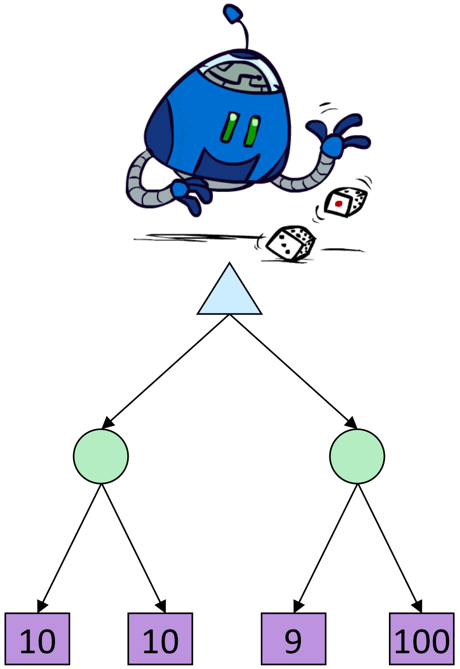
Assuming chance when the world is adversarial

Chance outcomes in trees



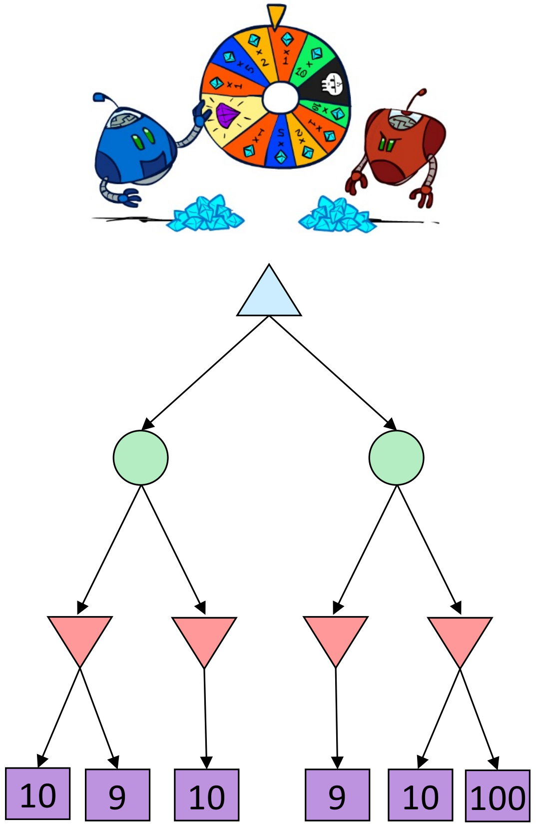
Tictactoe, chess

Minimax



Tetris, investing

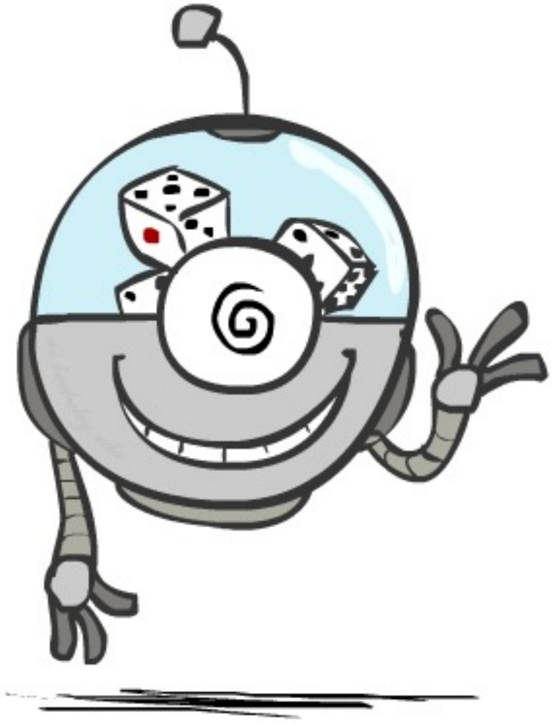
Expectimax



Backgammon, Monopoly

Expectiminimax

Probabilities



Probabilities

A random variable represents an event whose outcome is unknown

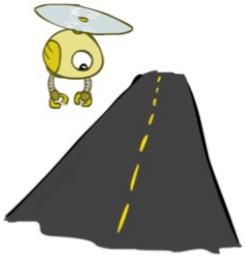
A probability distribution is an assignment of weights to outcomes

Example: Traffic on freeway

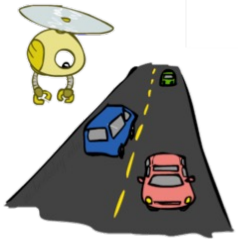
- Random variable: T = whether there’s traffic
- Outcomes: T in {none, light, heavy}
- Distribution:

  P(T=none) = 0.25, P(T=light) = 0.50, P(T=heavy) = 0.25

Probabilities over all possible outcomes sum to one



0.25



0.50

0.25

Expected Value

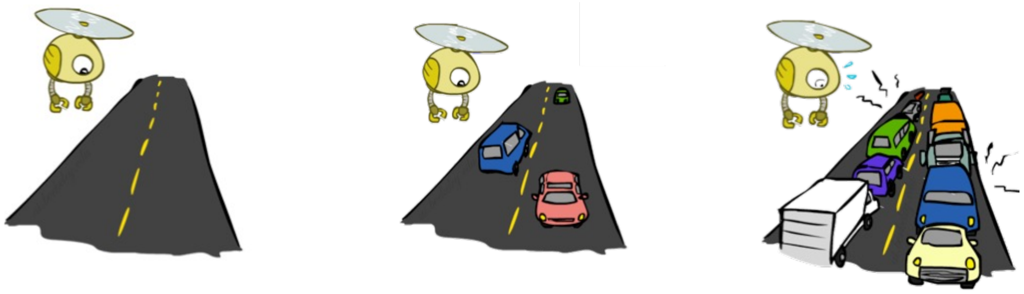
Expected value of a function of a random variable:

Average the values of each outcome,
weighted by the probability of that outcome

Example: How long to get to the airport?

Time:	20 min		30 min		60 min
	x	+	x	+	x
Probability:	0.25		0.50		0.25

35 min



Expectations

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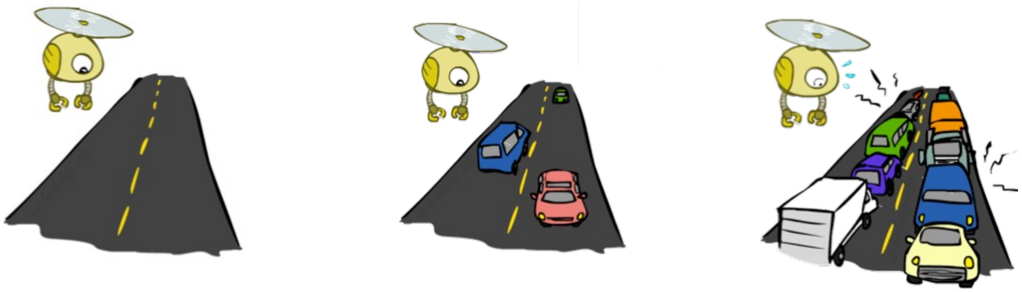
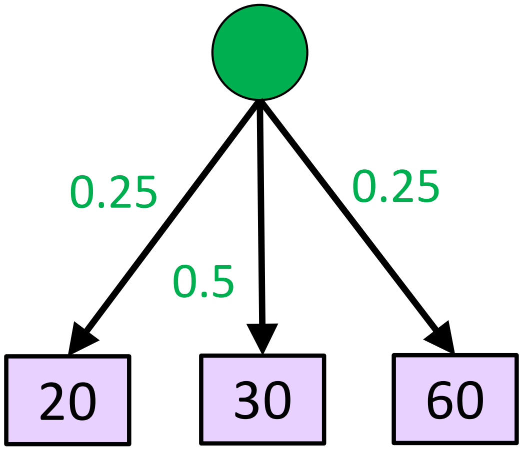




Max node notation

$$V(s) = \max_{a} V(s'),$$

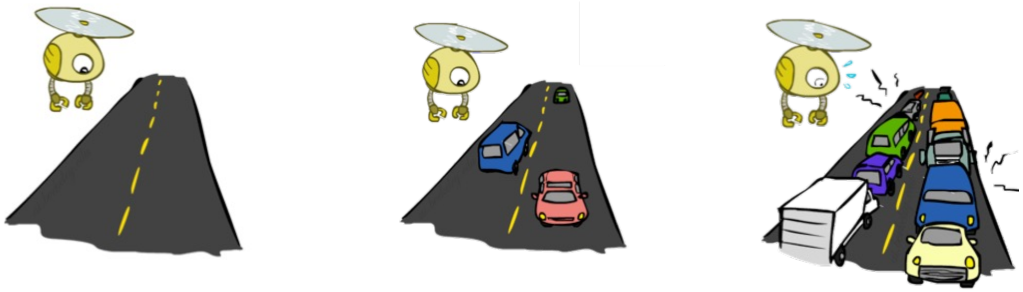
where $s' = result(s, a)$

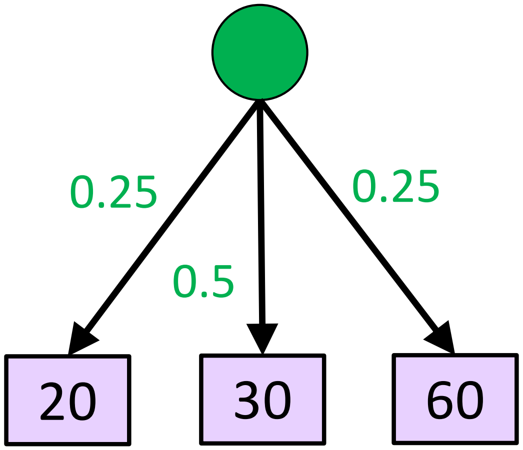
Chance node notation

$$V(s) =$$

Expectations

Time:	20 min		30 min		60 min
	x	+	x	+	x
Probability:	0.25		0.50		0.25





Max node notation

$$V(s) = \max_{a} V(s'),$$

where $s' = result(s, a)$

Chance node notation

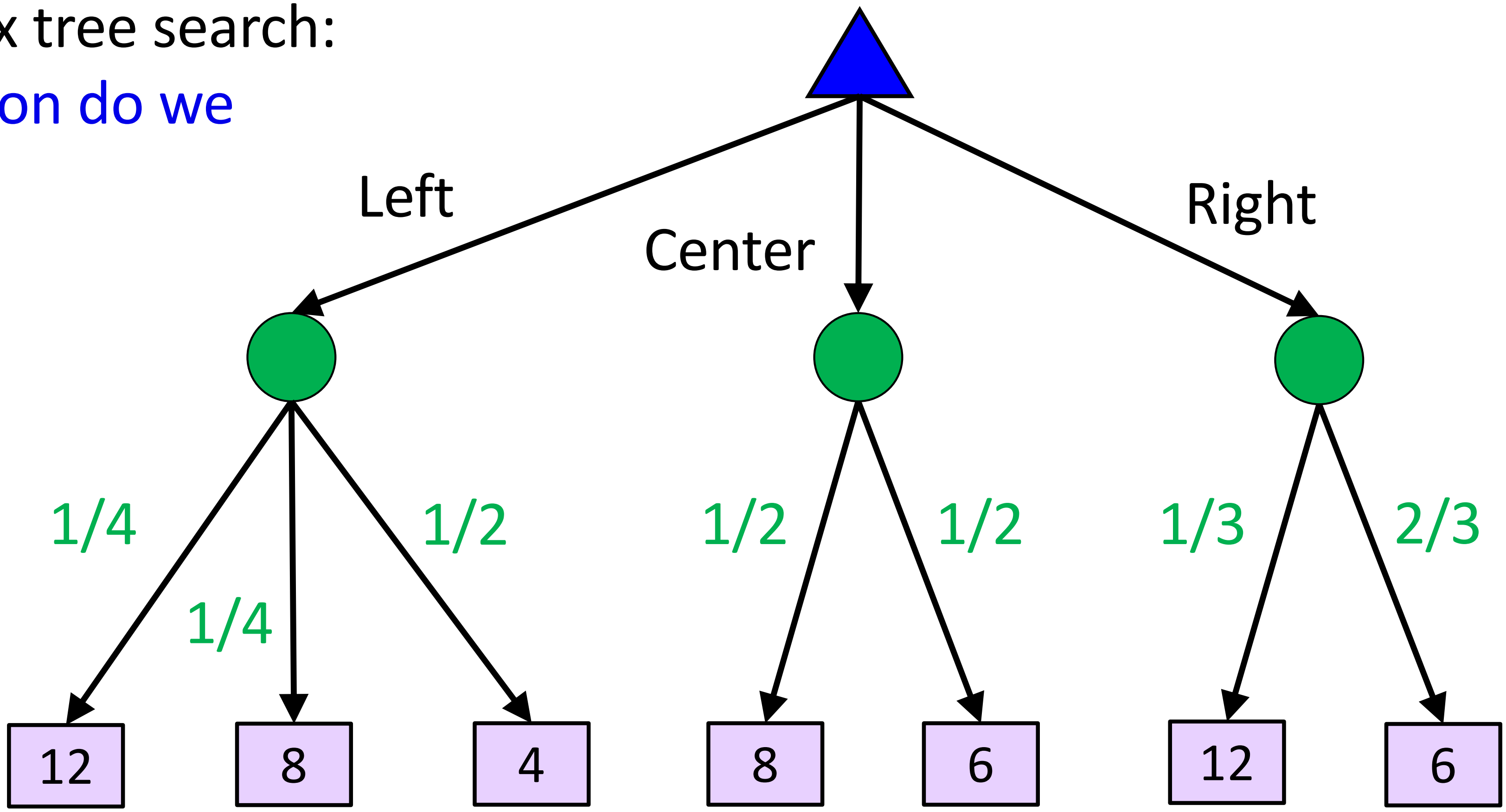
$$V(s) = \sum_{s'} P(s')\, V(s')$$

On your own…

Expectimax tree search:

Which action do we choose?

A: Left
B: Center
C: Right
D: Eight



On your own…

Expectimax tree search:

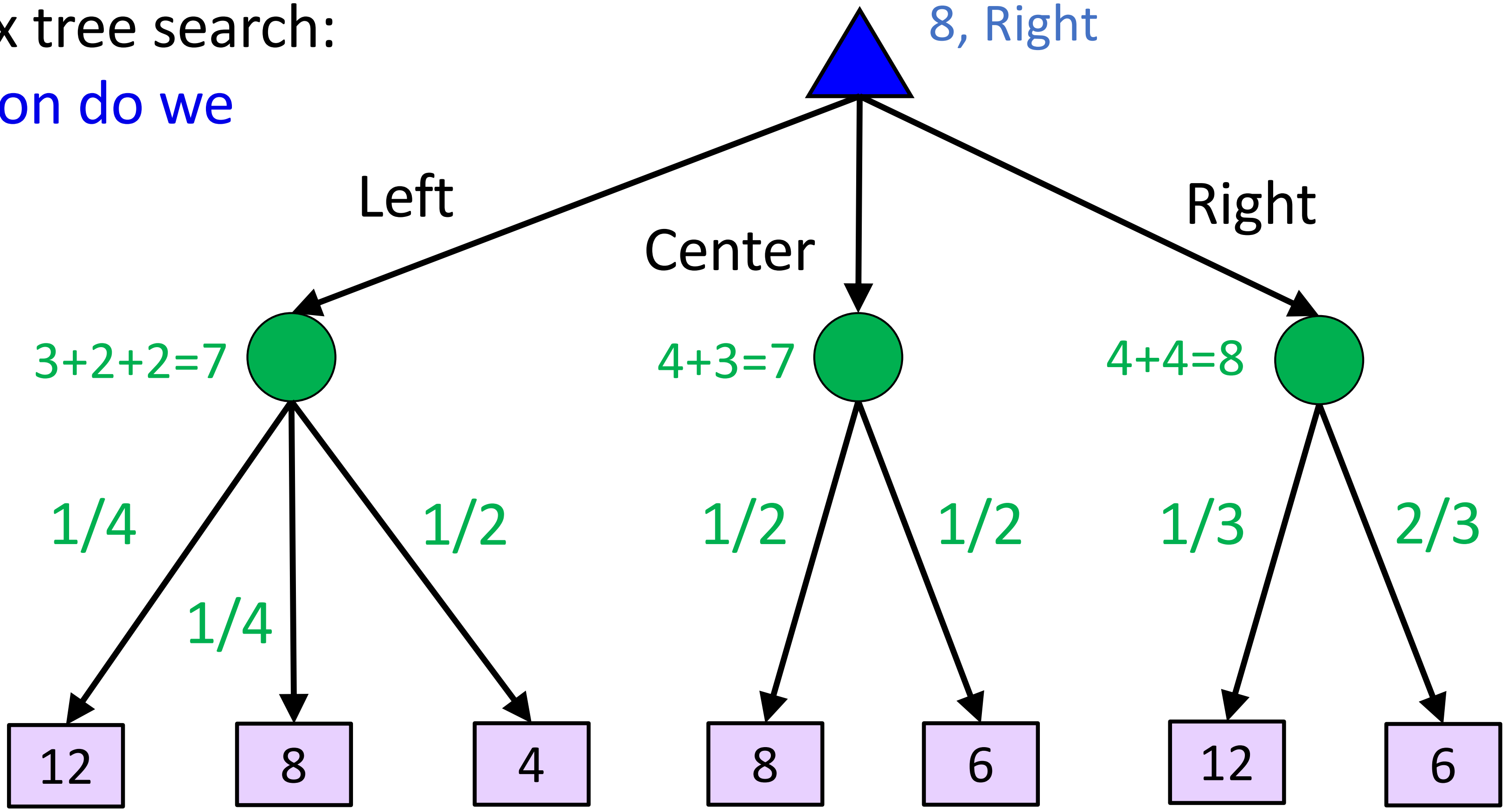
Which action do we choose?

A: Left

B: Center

C: Right

D: Eight



Expectimax Code

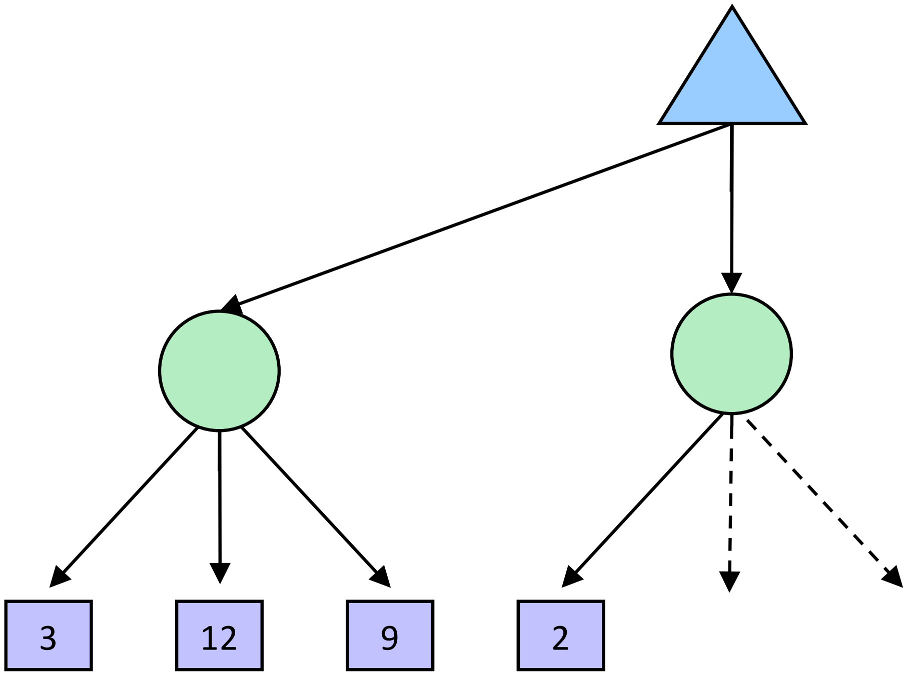
```
function value( state )
    if state.is_leaf
        return state.value

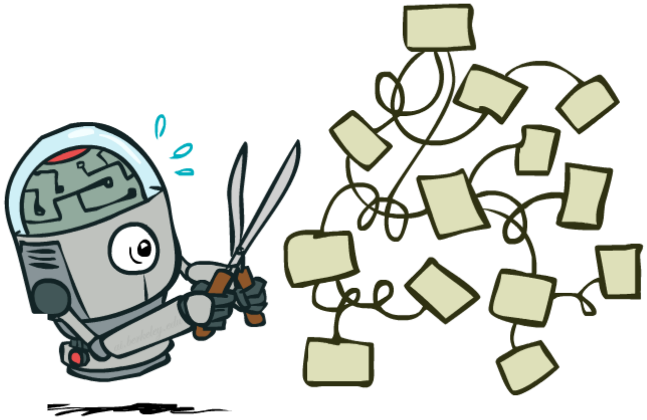
    if state.player is MAX
        return $\max_{\text{a in state.actions}}$ value( state.result(a) )

    if state.player is MIN
        return $\min_{\text{a in state.actions}}$ value( state.result(a) )

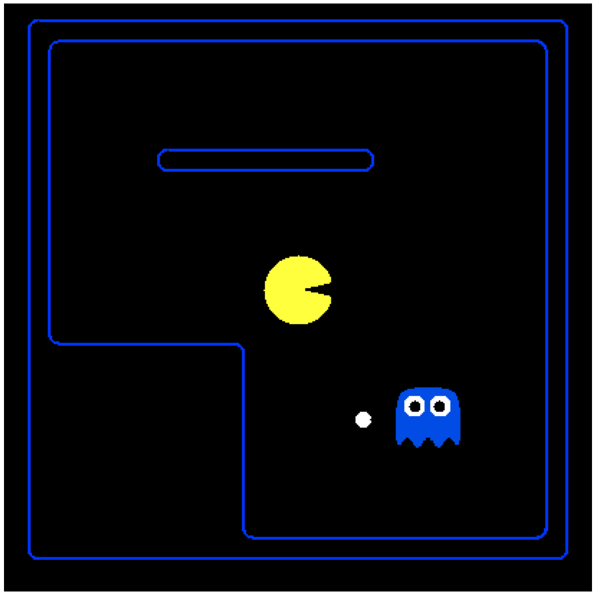
    if state.player is CHANCE
        return $\text{sum}_{\text{s in state.next\_states}}$ P( s ) * value( s )
```

Expectimax Pruning?





Modeling Assumptions



	Minimax Ghost	Random Ghost
Minimax Pacman		
Expectimax Pacman		

Results from playing 5 games

In Class Activity Demo – Connect 4

Q1c – practice alpha-beta pruning *on your own*

Q2 – apply minimax and evaluation functions (heuristics) to Connect 4

Summary

Games require decisions when optimality is impossible

- Bounded-depth search and approximate evaluation functions

Games force efficient use of computation

- Alpha-beta pruning

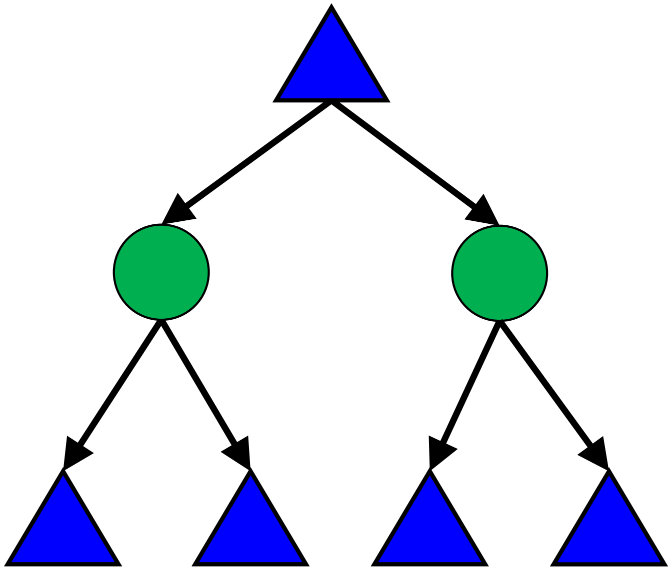
Game playing has produced important research ideas

- Reinforcement learning (checkers)
- Iterative deepening (chess)
- Monte Carlo tree search (Go)
- Solution methods for partial-information games in economics (poker)

Video games present much greater challenges – lots to do!

- $b = 10^{500}$, $|S| = 10^{4000}$, $m = 10{,}000$

Preview: MDP/Reinforcement Learning Notation



$$V(s) = \max_{a} \sum_{s'} P(s')\, V(s')$$

Preview: MDP/Reinforcement Learning Notation

Standard expectimax: $V(s) = \max_a \sum_{s'} P(s'|s,a)V(s')$

Bellman equations: $V(s) = \max_a \sum_{s'} P(s'|s,a)[R(s,a,s') + \gamma V(s')]$

Value iteration: $V_{k+1}(s) = \max_a \sum_{s'} P(s'|s,a)[R(s,a,s') + \gamma V_k(s')], \quad \forall\, s$

Q-iteration: $Q_{k+1}(s,a) = \sum_{s'} P(s'|s,a)[R(s,a,s') + \gamma \max_{a'} Q_k(s',a')], \quad \forall\, s,a$

Policy extraction: $\pi_V(s) = \operatorname{argmax}_a \sum_{s'} P(s'|s,a)[R(s,a,s') + \gamma V(s')], \quad \forall\, s$

Policy evaluation: $V^{\pi}_{k+1}(s) = \sum_{s'} P(s'|s,\pi(s))[R(s,\pi(s),s') + \gamma V^{\pi}_k(s')], \quad \forall\, s$

Policy improvement: $\pi_{new}(s) = \operatorname{argmax}_a \sum_{s'} P(s'|s,a)[R(s,a,s') + \gamma V^{\pi_{old}}(s')], \quad \forall\, s$

Preview: MDP/Reinforcement Learning Notation

Standard expectimax: $V(s) = \max_a \sum_{s'} P(s'|s,a)V(s')$

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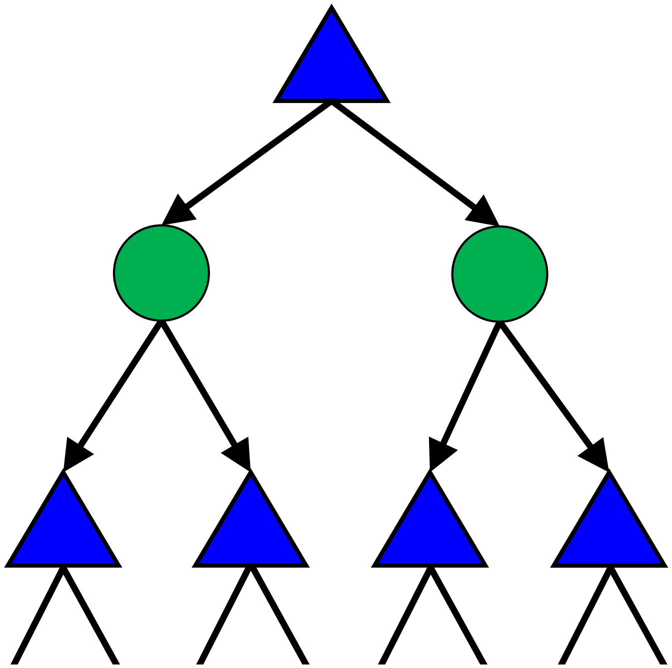
Why Expectimax?

Pretty great model for an agent in the world

Choose the action that has the: highest expected value



Bonus Question

Let’s say you know that your opponent is actually running a depth 1 minimax, using the result 80% of the time, and moving randomly otherwise

Question: What tree search should you use?

A: Minimax

B: Expectimax

C: Something completely different