15-281 Artificial Intelligence HW9

INSTRUCTIONS

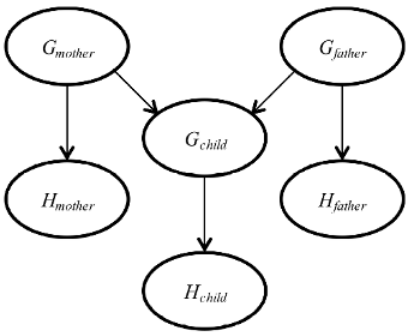
- **Due: Monday, November 18, 2024 at 10:00 PM EDT.** Remember that you may use up to 2 slip days for the written homework making the last day to submit **Wednesday November 20, 2024 at 10:00 PM EDT**.
- **Format:** Write your answers in the `yoursolution.tex` file and compile a pdf (preferred) or you can type directly on the blank pdf. Make sure that your answers are within the dedicated regions for each question/part. If you do not follow this format, we may deduct points. Handwritten solutions are not acceptable and may lead to lost points.
- **How to submit:** Submit a pdf with your answers on Gradescope. Log in and click on our class 15-281, click on the HW10 assignment, and upload your pdf containing your answers.
- **Policy:** See the course website for homework policies and academic integrity.

Name	
Andrew ID	
Hours to complete?	◯ (0, 2] hours ◯ (2, 3] hours ◯ (3, 4] hours ◯ (4, 5] hours ◯ (5, 6] hours ◯ (6, 7] hours ◯ (7, 8] hours ◯ > 8 hours

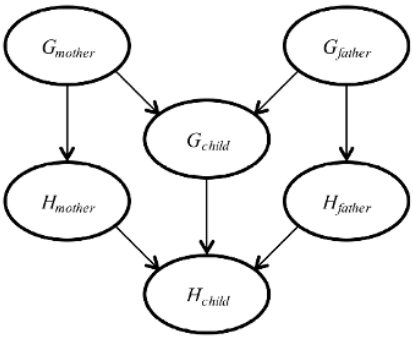
Q1. [10 pts] An Independence Dad Joke

Let H_x be a random variable denoting the handedness of an individual x, with possible values l or r. A common hypothesis is that left- or right-handedness is inherited by a simple mechanism; that is, perhaps there is a gene G_x, also with values l or r, and perhaps actual handedness turns out mostly the same (with some probability s) as the gene an individual possesses. Furthermore, perhaps the gene itself is equally likely to be inherited from either of an individual's parents, with a small nonzero probability m of a random mutation flipping the handedness.

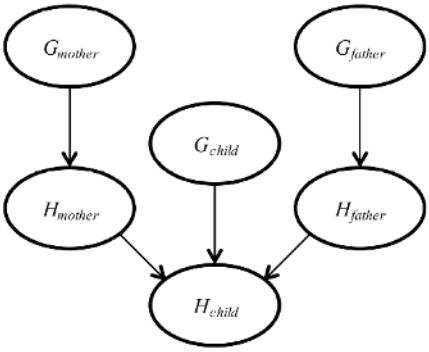
The following three images are possible models involving the genes G and handednesses H.



(a)



(b)



(c)

(a) [3 pts] Which of the three networks above claim that $P(G_{father}, G_{mother}, G_{child}) = P(G_{father})P(G_{mother})P(G_{child})$? Select all that apply.

☐ (a) ☐ (b) ☐ (c)

(b) [3 pts] Which of the three networks make independence claims that are consistent with the hypothesis about the inheritance of handedness? Select all that apply.

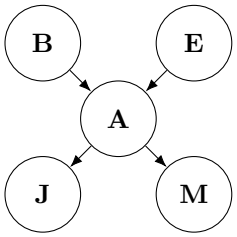
☐ (a) ☐ (b) ☐ (c)

(c) [4 pts] Which of the three networks is the most direct transcription of the hypothesis? Choose only one answer.

◯ (a) ◯ (b) ◯ (c)

Q2. [39 pts] Variable Elimination Nation

Suppose you are given a Bayes net with the same variables and structure as the alarm Bayes net from lecture (reproduced below), with the conditional probability tables given below.



B	$P(B)$
$+b$	0.25
$-b$	0.75

E	$P(E)$
$+e$	0.15
$-e$	0.85

A	B	E	$P(A \mid B, E)$
$+a$	$+b$	$+e$	0.8
$-a$	$+b$	$+e$	0.2
$+a$	$+b$	$-e$	0.6
$-a$	$+b$	$-e$	0.4
$+a$	$-b$	$+e$	0.6
$-a$	$-b$	$+e$	0.4
$+a$	$-b$	$-e$	0.1
$-a$	$-b$	$-e$	0.9

J	A	$P(J \mid A)$
$+j$	$+a$	0.8
$-j$	$+a$	0.2
$+j$	$-a$	0.1
$-j$	$-a$	0.9

M	A	$P(M \mid A)$
$+m$	$+a$	0.65
$-m$	$+a$	0.35
$+m$	$-a$	0.4
$-m$	$-a$	0.6

In this problem, you'll be applying the variable elimination algorithm to the query $P(B \mid +j,\ +m)$. You will have to eliminate the variables E and A, in that order. For each variable, write:

(i) The variables involved in the resulting factor (e.g., after eliminating X the resulting factor might be $f_1(Y, Z)$, so the variables involved are Y, Z). You should not include constant values (ie. the given evidence).

(ii) The summation to calculate the factor (e.g., $f_1(Y, Z) = \sum_x P(Y)P(x \mid Y)P(Z \mid x)$)

(iii) The numeric values in the factor table

(a) [10 pts] Eliminating E:

(i) Variables:

(ii) Summation:

(iii) Factor table:

(b) [10 pts] Eliminating A:

(i) Variables:	**(ii) Summation:**
(iii) Factor table:	

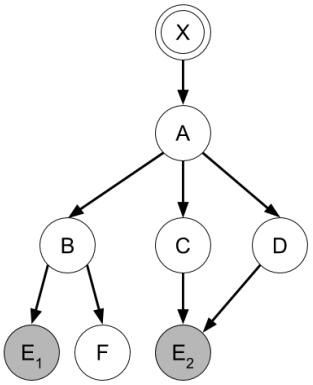
(c) [12 pts] After eliminating E and A above, you must multiply the remaining factors to produce yet another factor table. Specify the variables associated with the resulting factor, and fill out the values in the corresponding factor table.

(i) Variables:	**(ii) Factor table:**

Now, find the normalizing constant to make this factor the probability distribution we want, $P(B \mid +j, +m)$. Write out the values of this normalized probability table.

(iii) Constant (α or Z):	**(iv) Probability table:**

Now consider the Bayes net below. Suppose we are trying to compute the query $P(X \mid e_1, e_2)$. Assume all variables are binary.



(d) [2 pts] Suppose we choose to eliminate variables in the order A, C, B, D, F.
Of the factors resulting from summing out over each of these variables, which factor has the most entries in its corresponding table? How many entries are in its table? Assume that we have separate entries for pairs of numbers even if we know sum to one (e.g., we would store both $P(X = +x)$ and $P(X = -x)$).

(i) Factor:	**(ii) # entries:**

(e) [5 pts] An optimal variable elimination ordering is one which minimizes the sum of the sizes of factors generated.

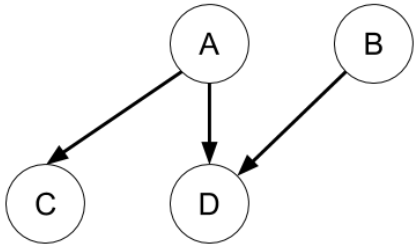
Fill in the table below with an optimal variable elimination ordering. For each variable, include the resulting factor and the number of entries in its table, again assuming that we separately store pairs of numbers which sum to one.

Variable	Factor	# Entries

Q3. [26 pts] Irrelevance Criteria

A variable in a Bayes net is said to be *irrelevant* to a given query if we could remove the variable from the Bayes net without changing the answer to the query. For example, in the Bayes net below, the variable D is irrelevant to the query $\mathbf{P}(B)$, because we already have the $\mathbf{P}(B)$ table. On the other hand, B is *not* irrelevant to the query $\mathbf{P}(D)$, because $\mathbf{P}(B)$ is needed to compute $\mathbf{P}(D)$.

Consider the Bayes net below:



(a) [6 pts] Suppose we are making the query $\mathbf{P}(A \mid D = d)$. Prove that C is irrelevant to this query using the following steps:

(i) Write the full joint distribution as a product of the CPTs in the Bayes net.

(ii) Sum over this product and normalize to get $\mathbf{P}(A \mid d)$. Make sure you write out the full equations.

(iii) Explain why this expression does not depend on the variable C.

(b) [6 pts] Now suppose we are making the query $\mathbf{P}(C \mid D = d)$. Execute the first two steps in part (a) for this query, and then argue why B is *not* irrelevant.

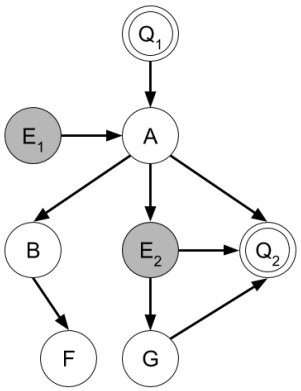
The *ancestor criterion* says that any node which is not an ancestor of a query or evidence variable is irrelevant.

In the Bayes net below, query variables are indicated by a double circle and evidence variables are shaded in.

(c) [6 pts] Cross out all the nodes that are irrelevant to this query according to the ancestor criterion.

For your submission to this problem, you may do one of the following:

- Draw/annotate on top of the existing images in the pdf.
- Edit the `figures/q5c.png` image file to add markings.

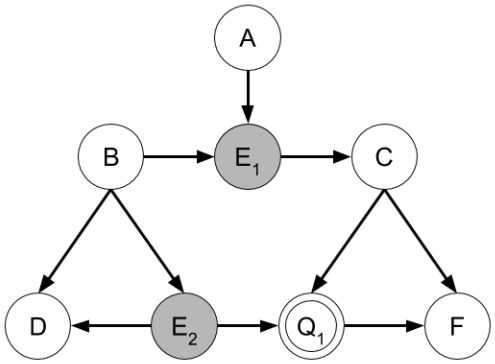


The moral graph of a Bayes net is an undirected graph containing all of the same connections as the original Bayes net, plus edges that connect variables which shared a child in the original Bayes net. Another criterion of irrelevance says that X is irrelevant to the query $\mathbf{P}(Q_1 \dots Q_n \mid e_1 \dots e_n)$ if in the moral graph, every path between a query variable Q_i and X goes through some evidence variable e_j (i.e., X is *m-separated* from the query variables given the evidence variables).

(d) [6 pts] For the following Bayes net, **draw in** the additional edges found in the moral graph **and cross out** all the variables that are irrelevant to the query according to the m-separation criterion.

For your submission to this problem, you may do one of the following:

- Draw/annotate on top of the existing images in the pdf.
- Edit the `figures/q5d.png` image file to add markings.



(e) [2 pts] Which variables *are not* considered irrelevant by the ancestor criterion but *are* considered irrelevant by the m-separation criterion?

Variables:

Q4. [15 pts] Ethics

Read this article about using Bayes Nets in a criminal case:
`https://crimesciencejournal.biomedcentral.com/articles/10.1186/s40163-016-0057-6`

Figure 3 of the paper presents a diagram of the Bayes Net along with the corresponding CPTs used by the authors to analyze a specific appeals case.

(a) [10 pts]

(i) [5 pts] What is $P(e_2 \mid a2_p)$?

Answer:

(ii) [5 pts] What is $P(s \mid a1_d, a2_d)$?

Answer:

(b) [5 pts] How were the probabilities above determined? Limit your response to two sentences.

Answer:

Q5. [10 pts] Bayes Nets IRL

Draw a Bayes Net representing some aspect of your life. For example, you could try to relate different parts of your life as a student like doing homework, attending lecture, playing pacman, getting sleep, playing sports, cooking, partying, etc. Another approach is to think about how a Bayes Net could be applied to a favorite hobby of yours. Feel free to deviate from this, we want you to get as creative as you can. :)

TO GET FULL POINTS, you must include at least one set of variables A, B, C such that $A \perp\!\!\!\perp B \mid C$, and one set of variables X, Y, Z such that $X \not\perp\!\!\!\perp Y \mid Z$. You don't have to name the variables those names, of course.

(a) [4 pts] Draw your Bayes Net. **Please include at least 5 nodes.**

Answer:

(b) [6 pts]

(i) [3 pts] Write one conditional independence relationship between variables (ex. $A \perp\!\!\!\perp B \mid C$) :

Answer:

(ii) [3 pts] Write a conditional independence assumption of two variables given a third that we CANNOT guarantee to be true given your bayes net.

Answer: