Warm-up as You Log In

Given

- Set `actions` (persistent/static)
- Set `states` (persistent/static)
- Function `T(s,a,s_prime)`

Write the pseudo code for:

- `function V(s) return value`

that implements:

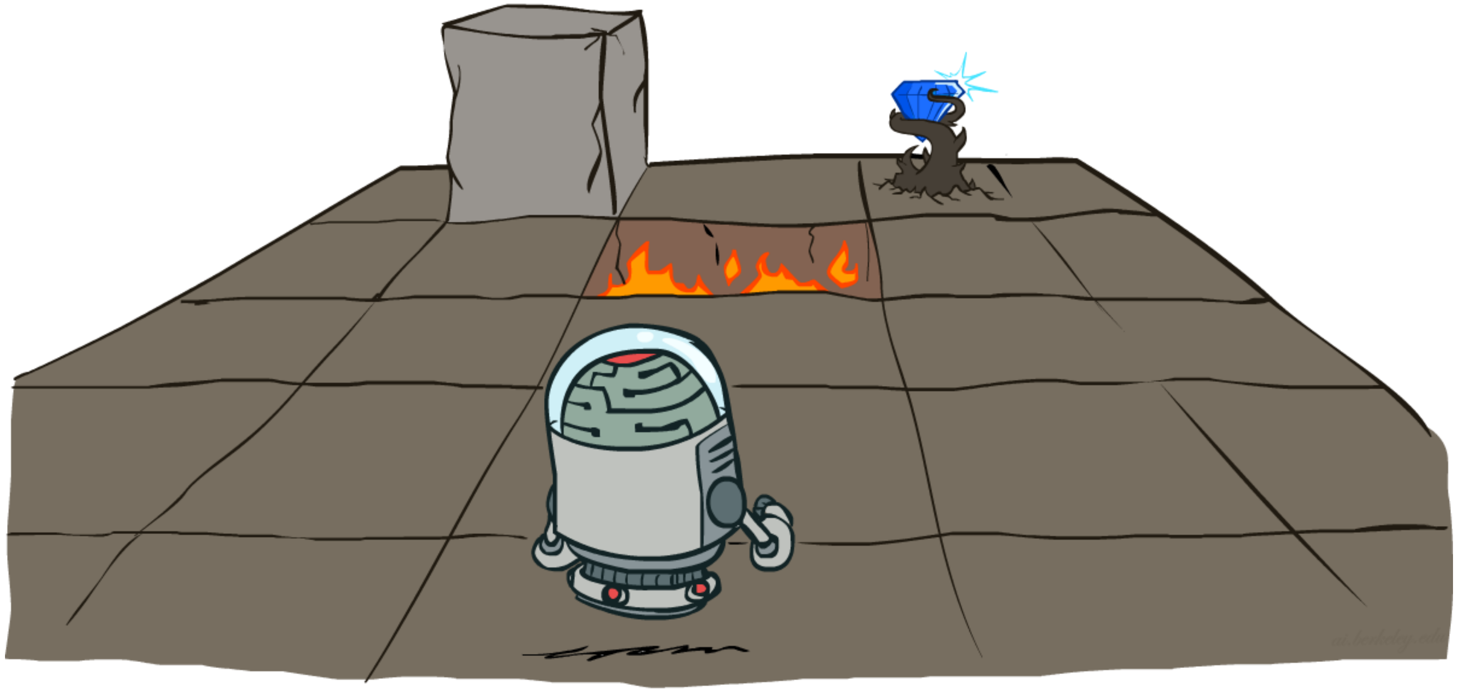
$$V(s) = \max_{a \in actions} \sum_{s' \in states} T(s,a,s')V(s')$$

Finish up Graph Plan

Jump to previous slides

AI: Representation and Problem Solving

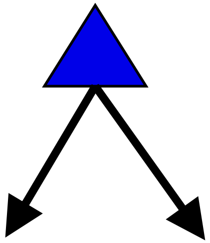
Markov Decision Processes



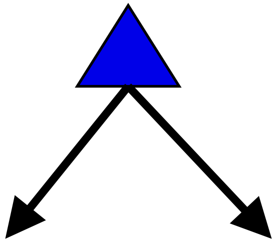
Instructor: Pat Virtue

Slide credits: CMU AI and http://ai.berkeley.edu

Minimax Notation



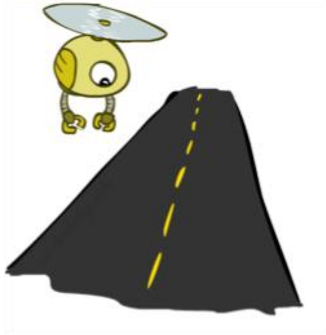
$$V(s) = \max_{a} V(s'),$$
$$\text{where } s' = result(s, a)$$

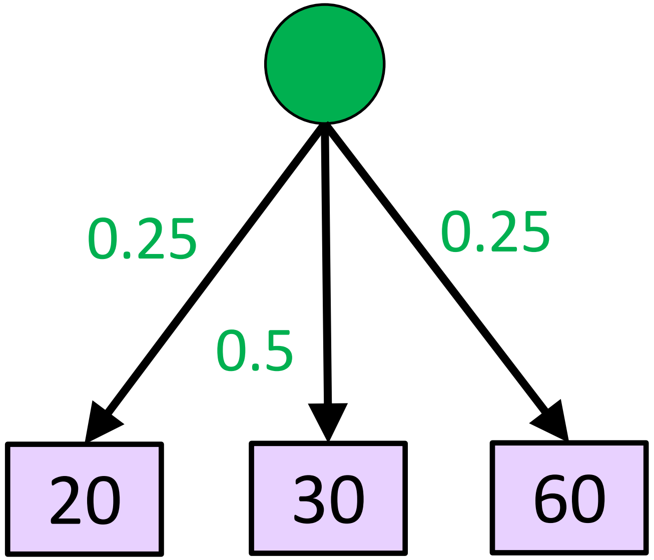


$$\hat{a} = \operatorname*{argmax}_{a} V(s'),$$
$$\text{where } s' = result(s, a)$$

Expectations

Time:	20 min		30 min		60 min
	x	+	x	+	x
Probability:	0.25		0.50		0.25





Max node notation

$$V(s) = \max_{a} V(s'),$$

where $s' = result(s, a)$

Chance node notation

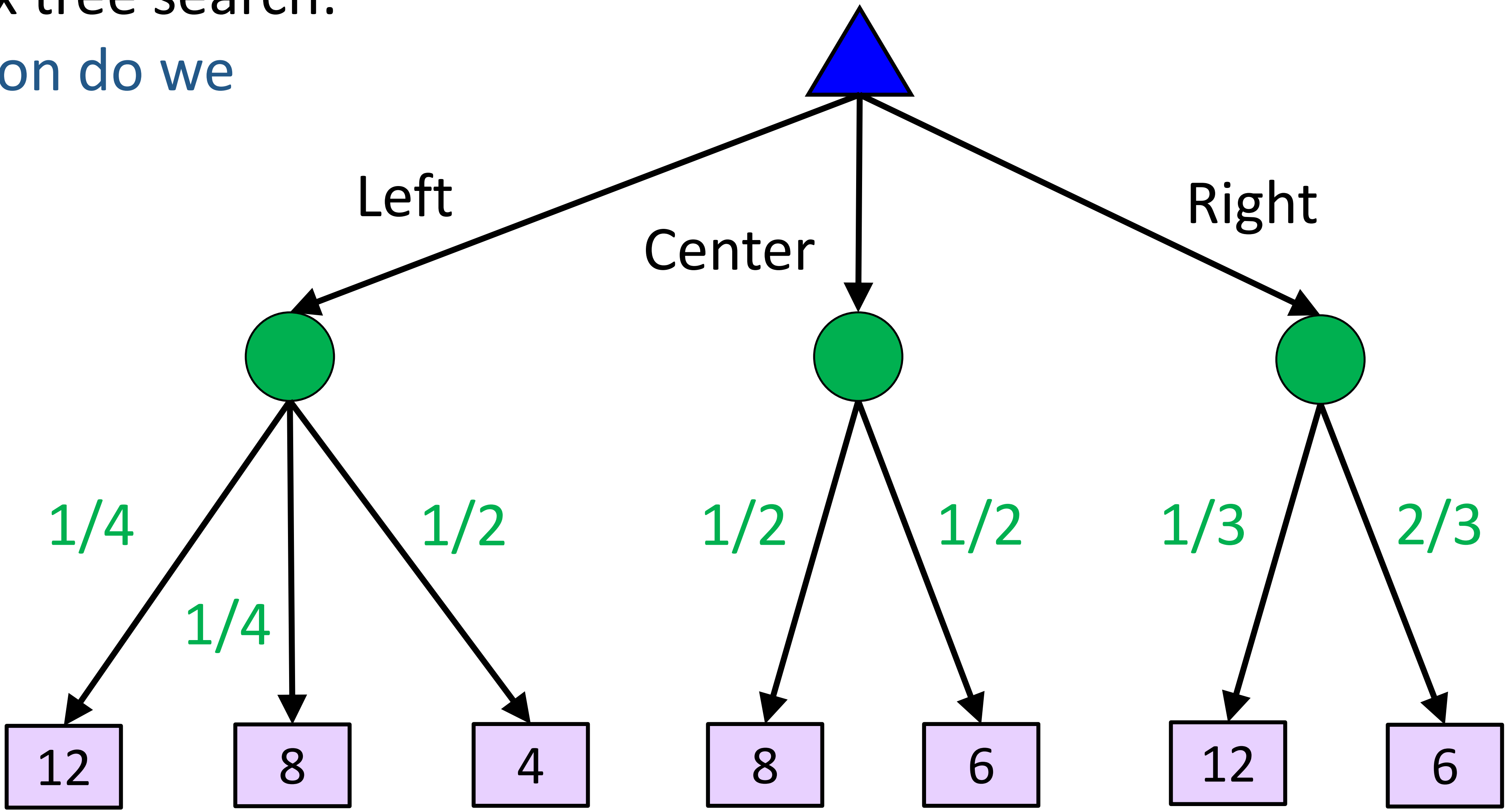
$$V(s) = \sum_{s'} P(s')\, V(s')$$

Previous Poll

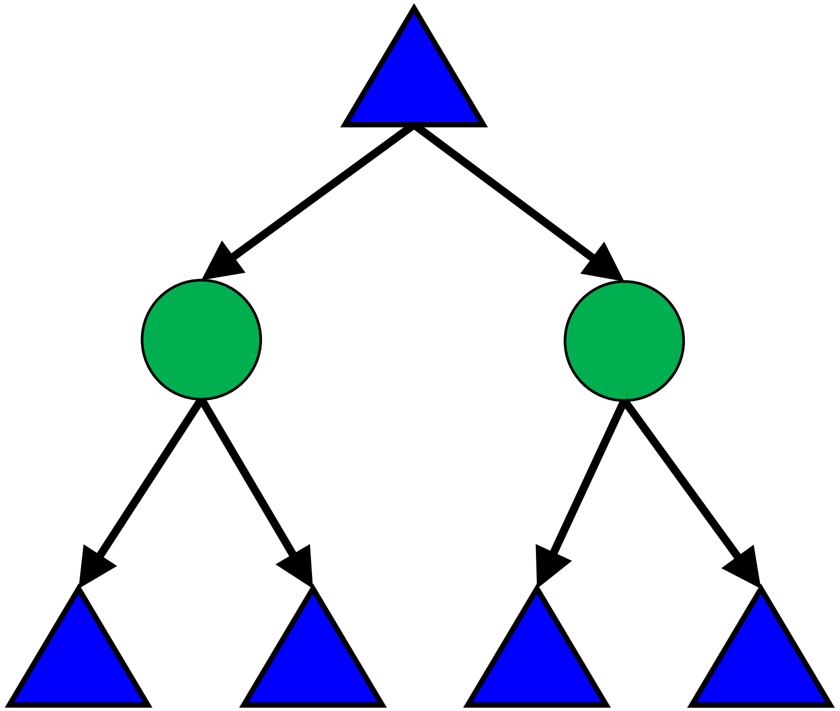
Expectimax tree search:

Which action do we choose?

A) Left
B) Center
C) Right
D) Eight



Expectimax Notation



$$V(s) = \max_{a} \sum_{s'} P(s'|s,a)\, V(s')$$

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MDP Notation

Standard expectimax: $V(s) = \max_a \sum_{s'} P(s'|s,a)V(s')$

Bellman equations: $V^*(s) = \max_a \sum_{s'} P(s'|s,a)[R(s,a,s') + \gamma V^*(s')]$

Value iteration: $V_{k+1}(s) = \max_a \sum_{s'} P(s'|s,a)[R(s,a,s') + \gamma V_k(s')], \quad \forall\, s$

Q-iteration: $Q_{k+1}(s,a) = \sum_{s'} P(s'|s,a)[R(s,a,s') + \gamma \max_{a'} Q_k(s',a')], \quad \forall\, s,a$

Policy extraction: $\pi_V(s) = \operatorname{argmax}_a \sum_{s'} P(s'|s,a)[R(s,a,s') + \gamma V(s')], \quad \forall\, s$

Policy evaluation: $V^{\pi}_{k+1}(s) = \sum_{s'} P(s'|s,\pi(s))[R(s,\pi(s),s') + \gamma V^{\pi}_k(s')], \quad \forall\, s$

Policy improvement: $\pi_{new}(s) = \operatorname{argmax}_a \sum_{s'} P(s'|s,a)[R(s,a,s') + \gamma V^{\pi_{old}}(s')], \quad \forall\, s$

MDP Notation

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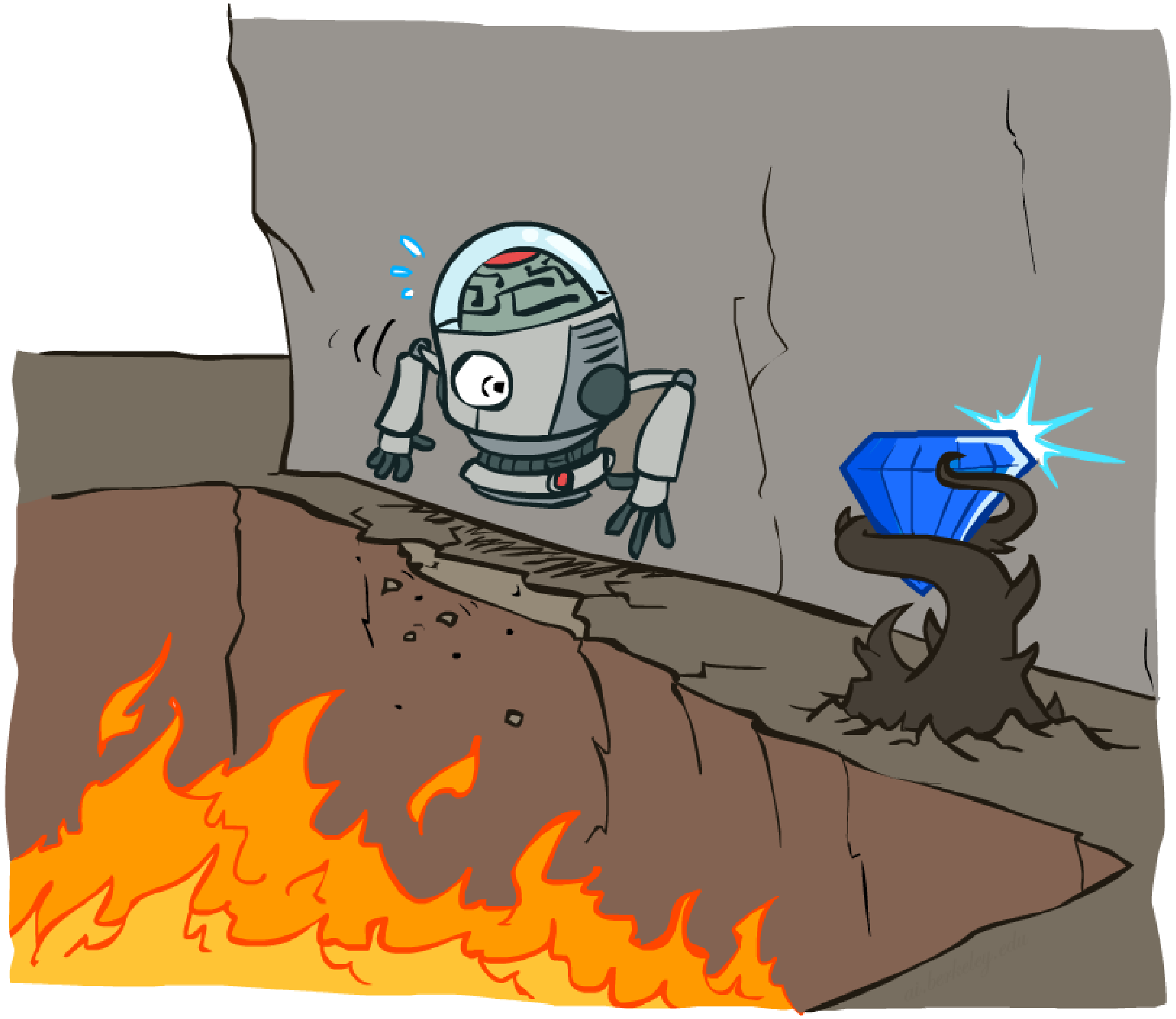
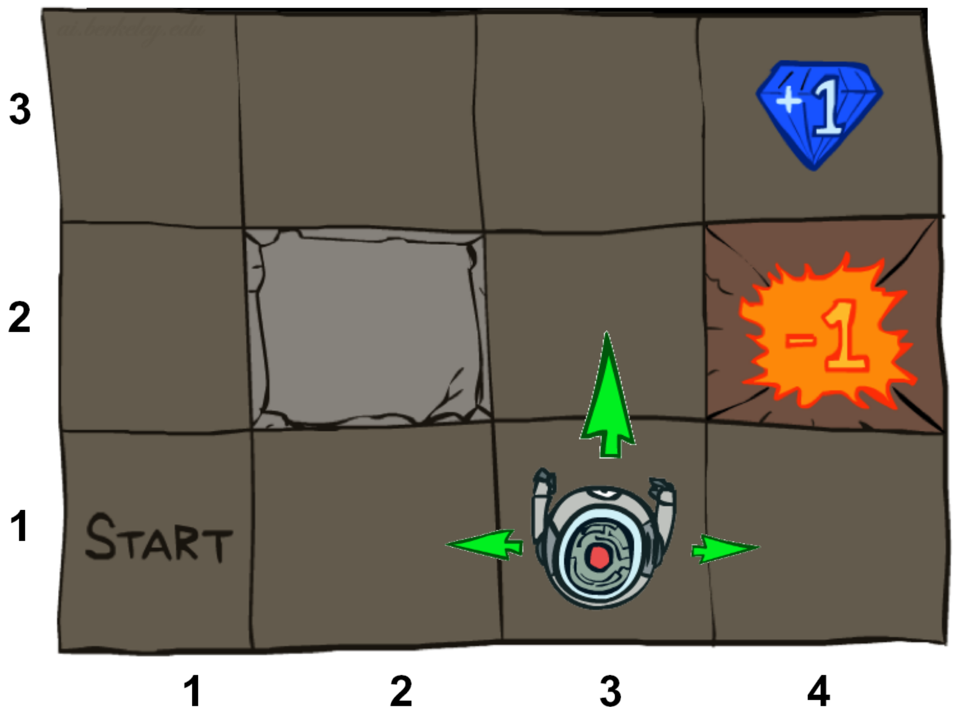
Non-Deterministic Search



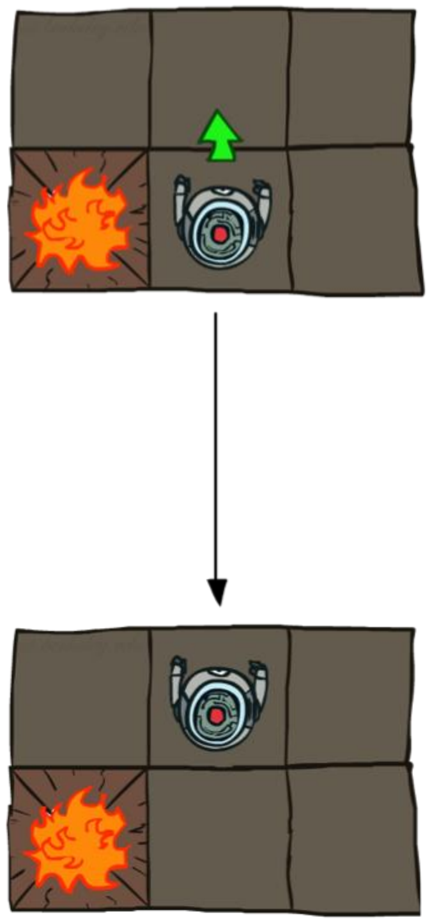
Example: Grid World

- A maze-like problem
  - The agent lives in a grid
  - Walls block the agent's path
- Noisy movement: actions do not always go as planned
  - 80% of the time, the action North takes the agent North (if there is no wall there)
  - 10% of the time, North takes the agent West; 10% East
  - If there is a wall in the direction the agent would have been taken, the agent stays put
- The agent receives rewards each time step
  - Small "living" reward each step (can be negative)
  - Big rewards come at the end (good or bad)
- Goal: maximize sum of rewards

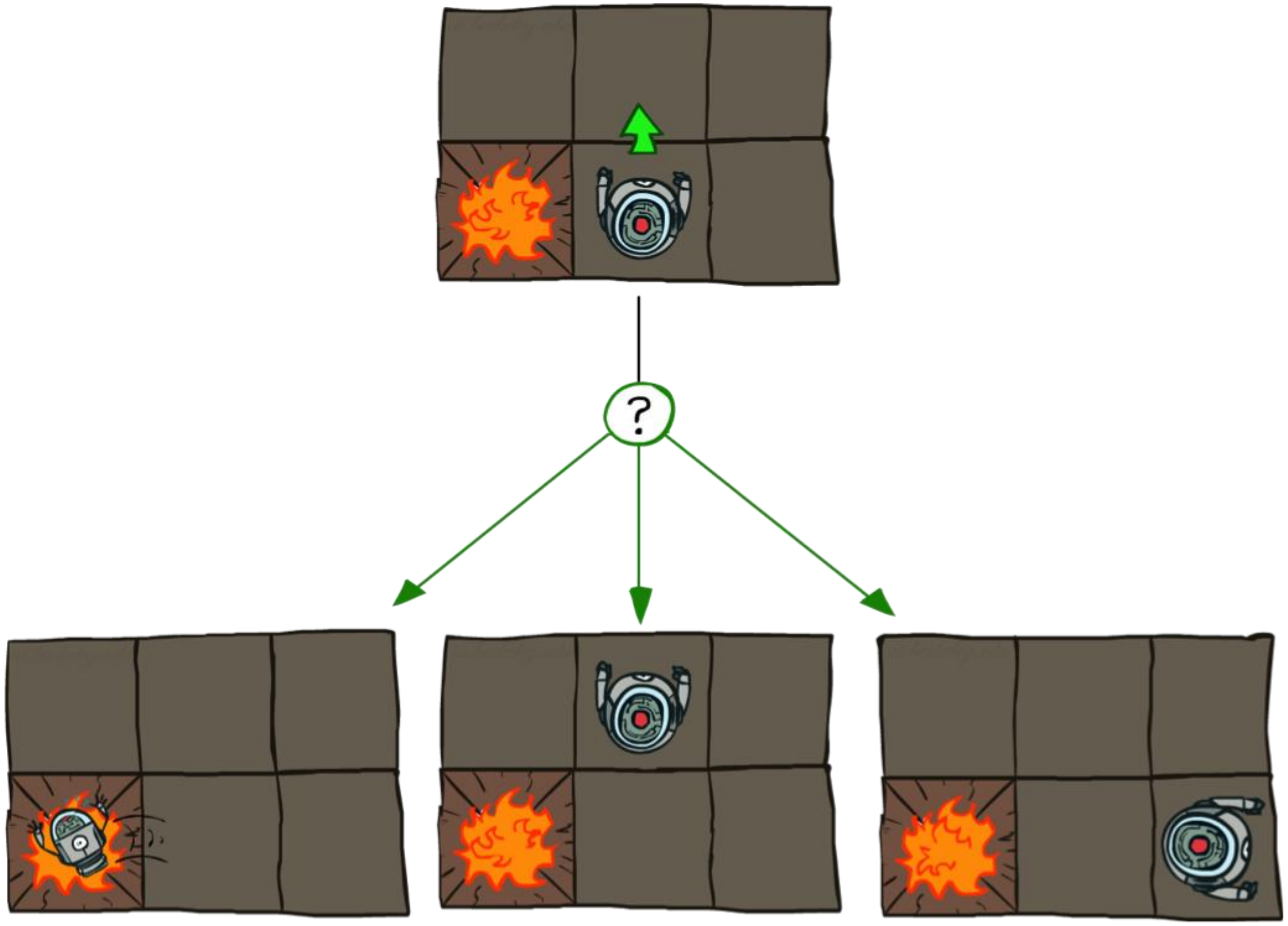


Grid World Actions

Deterministic Grid World



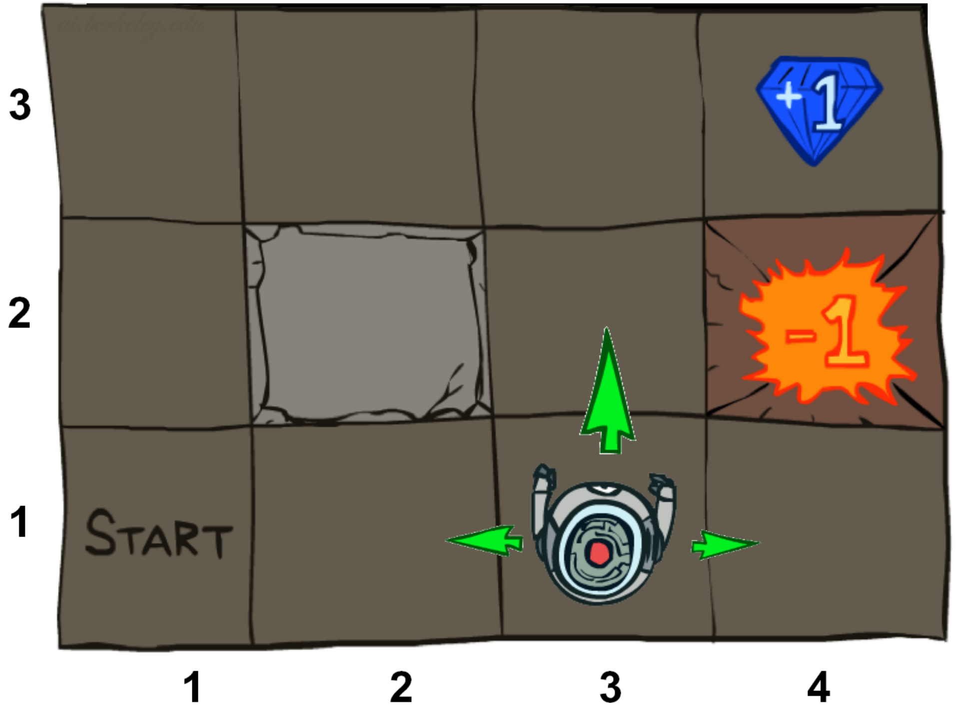
Stochastic Grid World



Markov Decision Processes

An MDP is defined by:

- A set of states $s \in S$
- A set of actions $a \in A$
- A transition function T(s, a, s')
  - Probability that a from s leads to s', i.e., P(s' | s, a)
  - Also called the model or the dynamics
- A reward function R(s, a, s')
  - Sometimes just R(s) or R(s')
- Maybe a terminal state



MDPs are non-deterministic search problems

- One way to solve them is with expectimax search
- We'll have a new tool soon

[Demo – gridworld manual intro (L8D1)]

Demo of Gridworld

What is Markov about MDPs?

“Markov” generally means that given the present state, the future and the past are independent

For Markov decision processes, “Markov” means action outcomes depend only on the current state

$$P(S_{t+1} = s' | S_t = s_t, A_t = a_t, S_{t-1} = s_{t-1}, A_{t-1}, \ldots S_0 = s_0)$$
$$=$$
$$P(S_{t+1} = s' | S_t = s_t, A_t = a_t)$$

This is just like search, where the successor function could only depend on the current state (not the history)

Andrey Markov
(1856-1922)

Policies

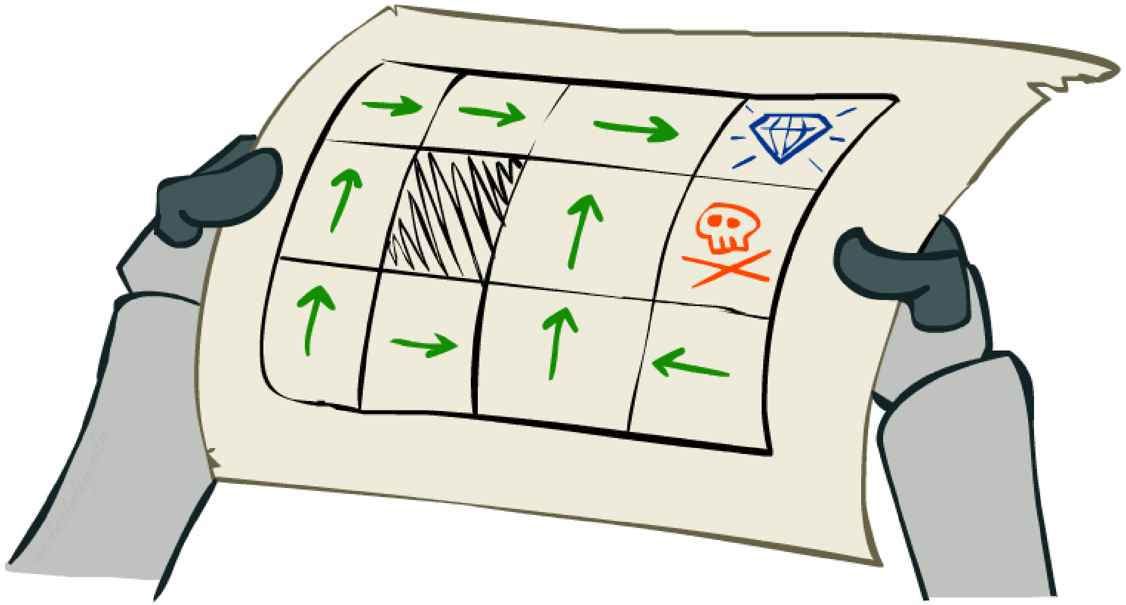
In deterministic single-agent search problems, we wanted an optimal plan, or sequence of actions, from start to a goal

For MDPs, we want an optimal policy $\pi^*: S \rightarrow A$

- A policy π gives an action for each state
- An optimal policy is one that maximizes expected utility if followed
- An explicit policy defines a reflex agent

Expectimax didn't compute entire policies

- It computed the action for a single state only



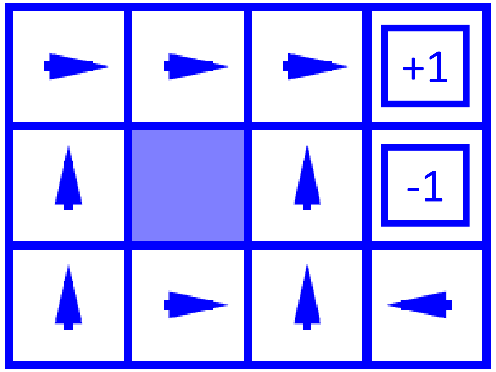
Optimal policy when R(s, a, s') = -0.03 for all non-terminals s

Poll 1

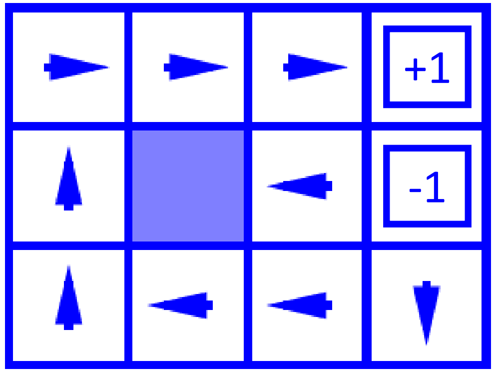
Which sequence of optimal policies matches the following sequence of living rewards:
{-0.01, -0.03, -0.04, -2.0}

I. {A, B, C, D}
II. {B, C, A, D}
III. {D, C, B, A}
IV. {D, A, C, B}

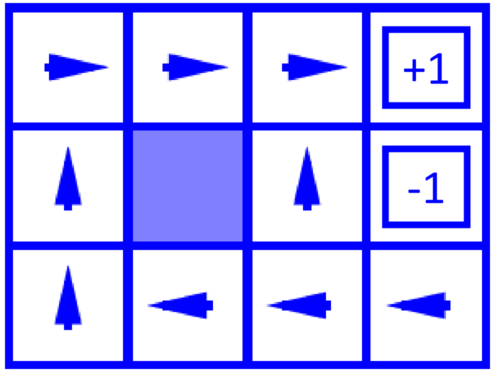
A)



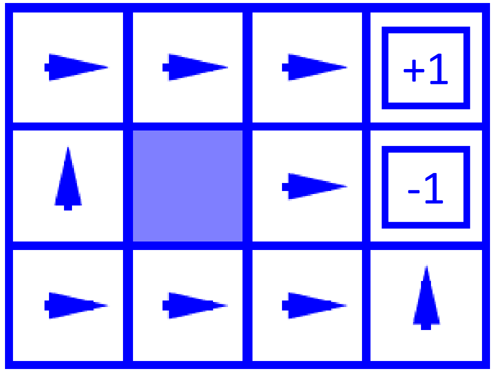
B)



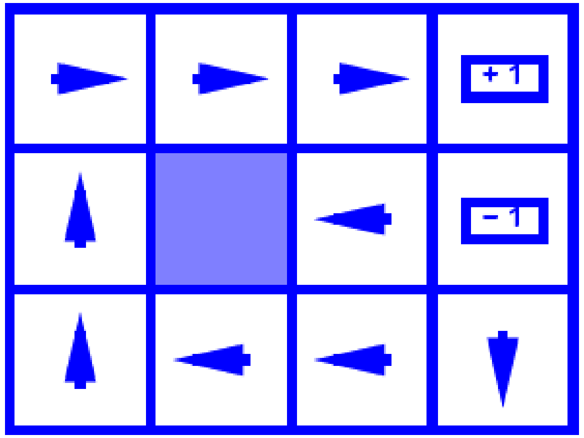
C)



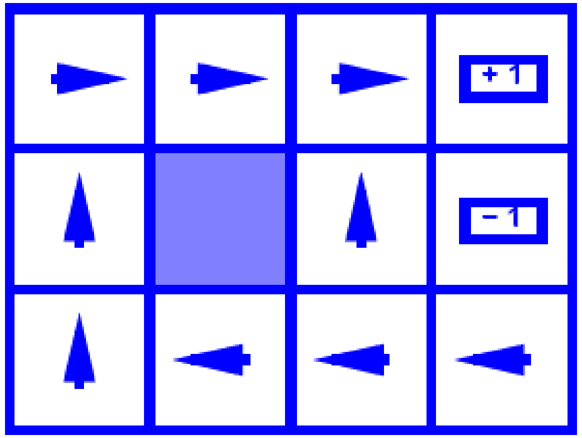
D)



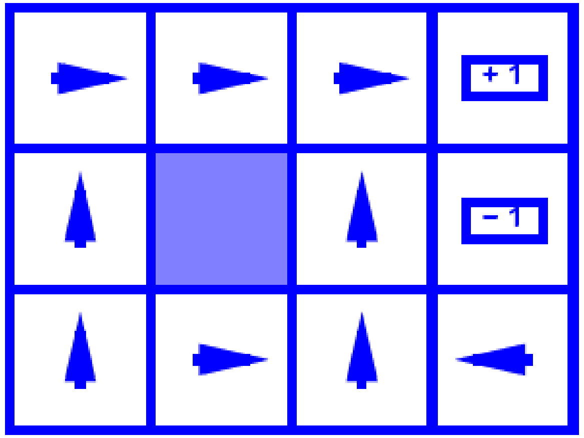
Optimal Policies



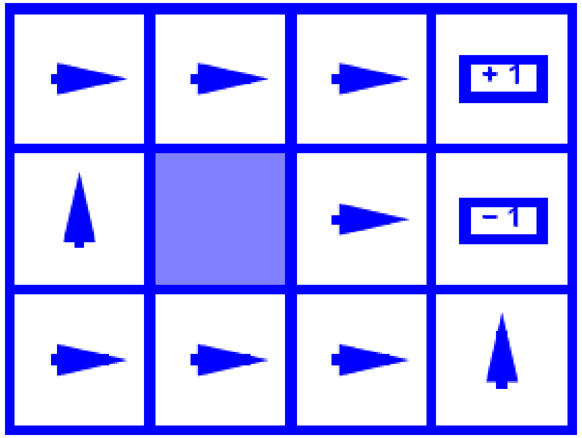
R(s) = -0.01



R(s) = -0.03

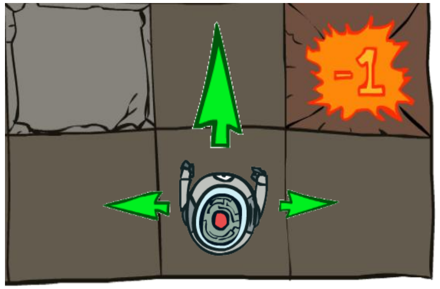


R(s) = -0.4



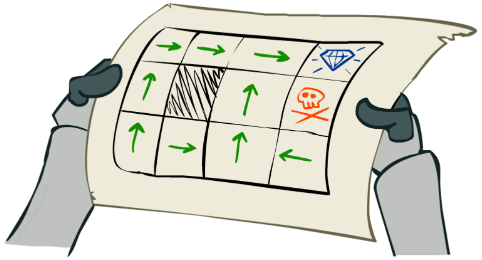
R(s) = -2.0

MDP Outline



MDP Setup

- Expectimax: State, actions, non-deterministic transition functions
  - Example: GridWorld
- Policies: Mapping states → actions



- Rewards
- Discounting, γ

Solving MDPs

- Method 1) Value iteration
- Method 2) Policy Iteration