Overview: MDPs and Reinforcement Learning

Known MDP: Offline Solution

Goal	Technique
Compute V^*, Q^*, π^*	Value / policy iteration
Evaluate a fixed policy π	Policy evaluation

Unknown MDP: Model-Based

Goal	Technique
Compute V^*, Q^*, π^*	VI/PI on approx. MDP
Eval fixed policy π	PE on approx. MDP

Unknown MDP: Model-Free

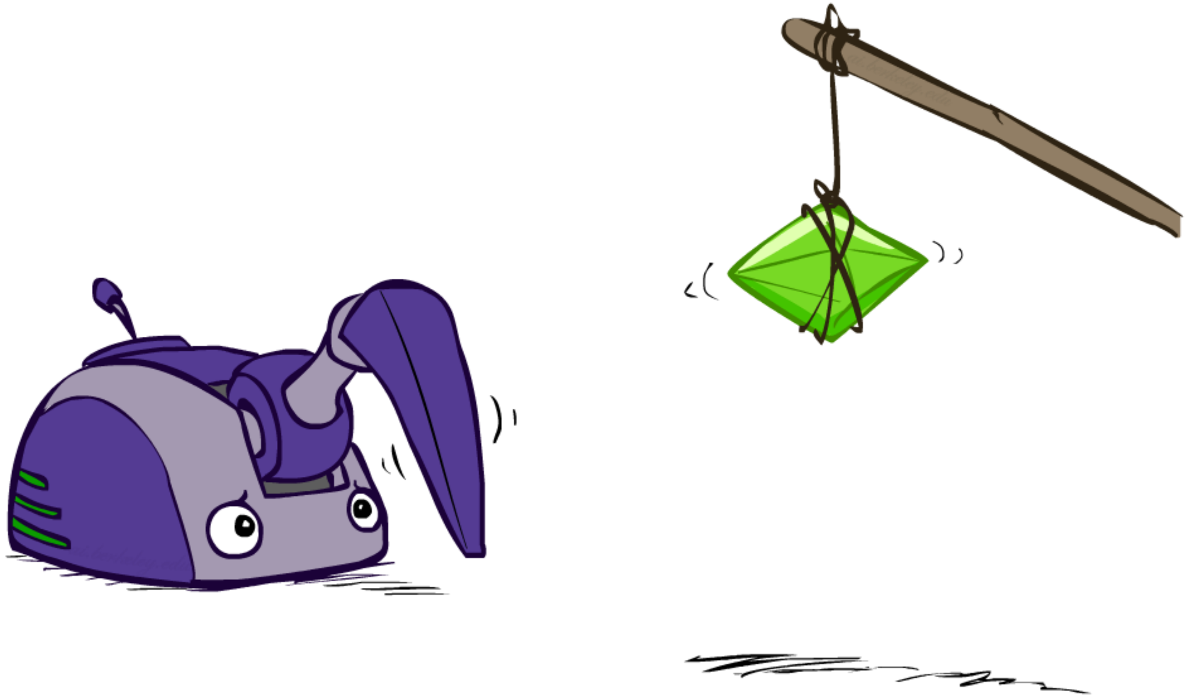
Goal	Technique
Compute V^*, Q^*, π^*	Q-learning
Eval fixed policy π	TD/Value Learning

Wrap up MDPs

Switch to MDP II slides

AI: Representation and Problem Solving

Reinforcement Learning



Instructor: Pat Virtue

Slide credits: CMU AI and http://ai.berkeley.edu

MDP Notation

Standard expectimax: $V(s) = \max_a \sum_{s'} P(s'|s,a)V(s')$

Bellman equations: $V^*(s) = \max_a \sum_{s'} P(s'|s,a)[R(s,a,s') + \gamma V^*(s')]$

Value iteration: $V_{k+1}(s) = \max_a \sum_{s'} P(s'|s,a)[R(s,a,s') + \gamma V_k(s')], \quad \forall\, s$

Q-iteration: $Q_{k+1}(s,a) = \sum_{s'} P(s'|s,a)[R(s,a,s') + \gamma \max_{a'} Q_k(s',a')], \quad \forall\, s,a$

Policy extraction: $\pi_V(s) = \operatorname{argmax}_a \sum_{s'} P(s'|s,a)[R(s,a,s') + \gamma V(s')], \quad \forall\, s$

Policy evaluation: $V_{k+1}^{\pi}(s) = \sum_{s'} P(s'|s,\pi(s))[R(s,\pi(s),s') + \gamma V_k^{\pi}(s')], \quad \forall\, s$

Policy improvement: $\pi_{new}(s) = \operatorname{argmax}_a \sum_{s'} P(s'|s,a)[R(s,a,s') + \gamma V^{\pi_{old}}(s')], \quad \forall\, s$

Poll (unused)

Which of the following are used in policy iteration?

A. Value iteration: $V_{k+1}(s) = \max_a \sum_{s'} P(s'|s,a)[R(s,a,s') + \gamma V_k(s')]\,, \quad \forall\, s$

B. Q-iteration: $Q_{k+1}(s,a) = \sum_{s'} P(s'|s,a)[R(s,a,s') + \gamma \max_{a'} Q_k(s',a')]\,, \quad \forall\, s,a$

C. Policy extraction: $\pi_V(s) = \operatorname{argmax}_a \sum_{s'} P(s'|s,a)[R(s,a,s') + \gamma V(s')]\,, \quad \forall\, s$

D. Policy evaluation: $V^{\pi}_{k+1}(s) = \sum_{s'} P(s'|s,\pi(s))[R(s,\pi(s),s') + \gamma V^{\pi}_k(s')]\,, \quad \forall\, s$

E. Policy improvement: $\pi_{new}(s) = \operatorname{argmax}_a \sum_{s'} P(s'|s,a)[R(s,a,s') + \gamma V^{\pi_{old}}(s')]\,, \quad \forall\, s$

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C. Policy extraction: $\pi_V(s) = \operatorname{argmax}_a \sum_{s'} P(s'|s,a)[R(s,a,s') + \gamma V(s')]\,, \quad \forall\, s$

✓ D. Policy evaluation: $V_{k+1}^{\pi}(s) = \sum_{s'} P(s'|s,\pi(s))[R(s,\pi(s),s') + \gamma V_k^{\pi}(s')]\,, \quad \forall\, s$

✓ E. Policy improvement: $\pi_{new}(s) = \operatorname{argmax}_a \sum_{s'} P(s'|s,a)[R(s,a,s') + \gamma V^{\pi_{old}}(s')]\,, \quad \forall\, s$

Poll (unused)

Rewards may depend on any combination of *state*, *action*, *next state*.

Which of the following are valid formulations of the Bellman equations?

A. $V^*(s) = \max_a \sum_{s'} P(s'|s,a)[R(s,a,s') + \gamma V^*(s')]$

B. $V^*(s) = R(s) + \gamma \max_a \sum_{s'} P(s'|s,a) V^*(s')$

C. $V^*(s) = \max_a [R(s,a) + \gamma \sum_{s'} P(s'|s,a) V^*(s')$

D. $Q^*(s,a) = R(s,a) + \gamma \sum_{s'} P(s'|s,a) \max_{a'} Q^*(s',a')$

Poll (unused)

Rewards may depend on any combination of *state*, *action*, *next state*.

Which of the following are valid formulations of the Bellman equations?

✓ A. $V^*(s) = \max_a \sum_{s'} P(s'|s,a)[R(s,a,s') + \gamma V^*(s')]$

✓ B. $V^*(s) = R(s) + \gamma \max_a \sum_{s'} P(s'|s,a) V^*(s')$

✓ C. $V^*(s) = \max_a [R(s,a) + \gamma \sum_{s'} P(s'|s,a) V^*(s')$

✓ D. $Q^*(s,a) = R(s,a) + \gamma \sum_{s'} P(s'|s,a) \max_{a'} Q^*(s',a')$

Reinforcement learning

What if we didn't know $P(s'|s,a)$ and $R(s,a,s')$?

Value iteration: $$V_{k+1}(s) = \max_a \sum_{s'} \sout{P(s'|s,a)}[\sout{R(s,a,s')} + \gamma V_k(s')], \quad \forall\, s$$

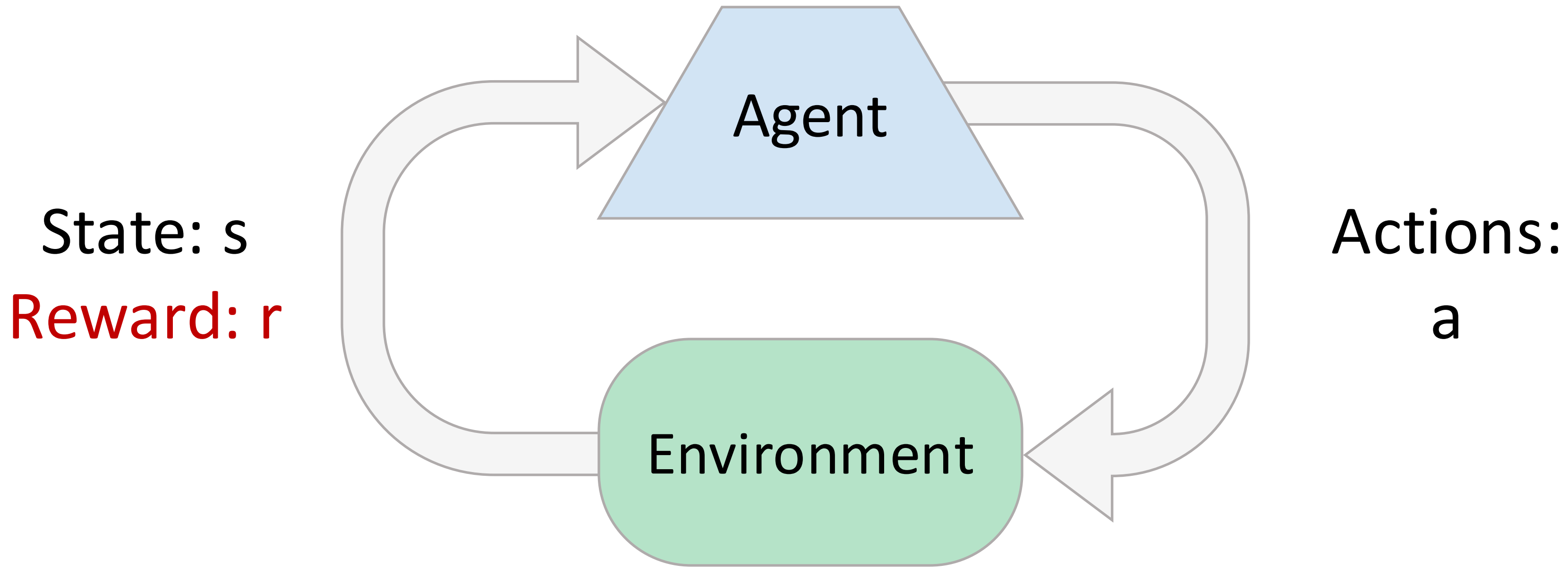
Q-iteration: $$Q_{k+1}(s,a) = \sum_{s'} \sout{P(s'|s,a)}[\sout{R(s,a,s')} + \gamma \max_{a'} Q_k(s',a')], \quad \forall\, s,a$$

Policy extraction: $$\pi_V(s) = \operatorname{argmax}_a \sum_{s'} \sout{P(s'|s,a)}[\sout{R(s,a,s')} + \gamma V(s')], \quad \forall\, s$$

Policy evaluation: $$V^{\pi}_{k+1}(s) = \sum_{s'} \sout{P(s'|s,\pi(s))}[\sout{R(s,\pi(s),s')} + \gamma V^{\pi}_k(s')], \quad \forall\, s$$

Policy improvement: $$\pi_{new}(s) = \operatorname{argmax}_a \sum_{s'} \sout{P(s'|s,a)}[\sout{R(s,a,s')} + \gamma V^{\pi_{old}}(s')], \quad \forall\, s$$

Reinforcement Learning



Basic idea:

- All learning is based on observed samples of rewards and next states!
- Receive feedback in the form of rewards
- Must (learn to) act so as to maximize expected rewards

Example: Learning to Walk

Initial

[Kohl and Stone, ICRA 2004]

[Video: AIBO WALK – initial]

Example: Learning to Walk

Finished

[Kohl and Stone, ICRA 2004]

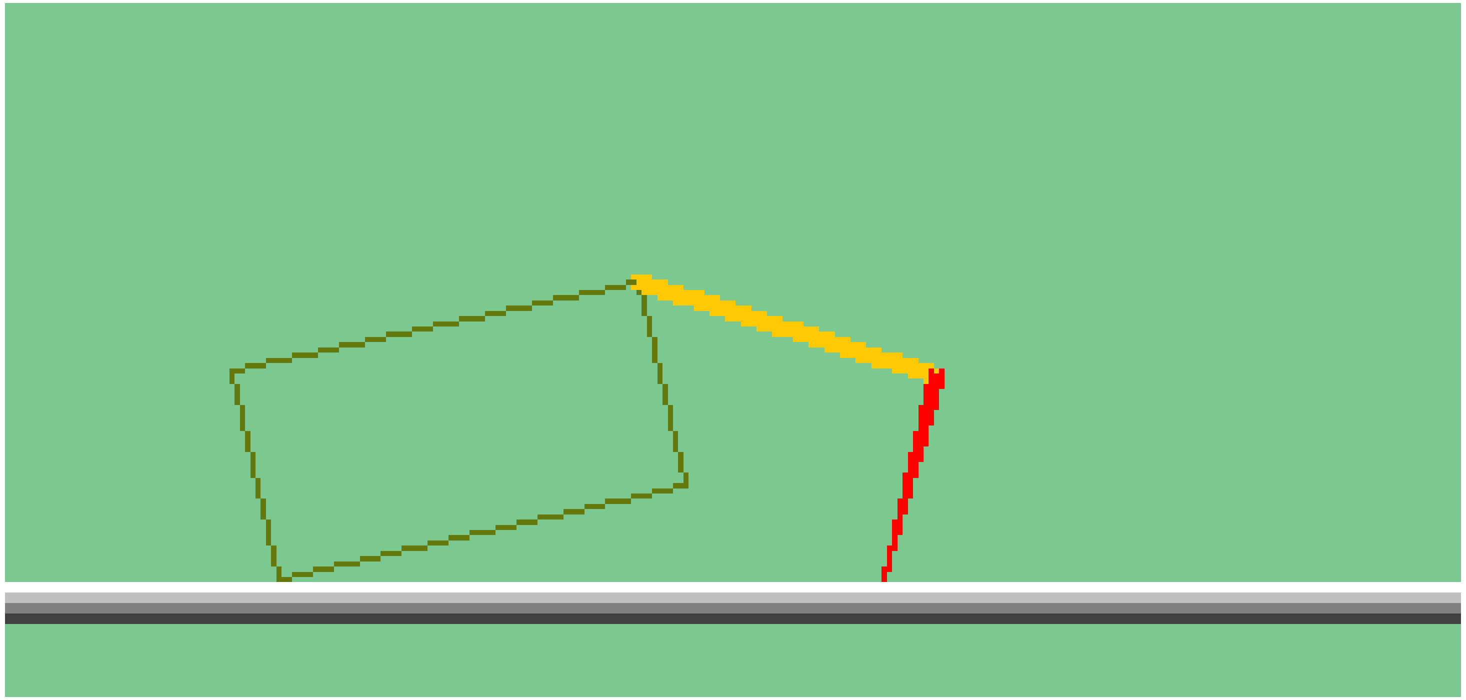
[Video: AIBO WALK – finished]

Example: Toddler Robot

[Tedrake, Zhang and Seung, 2005]

[Video: TODDLER – 40s]

The Crawler!



[Demo: Crawler Bot (L10D1)] [You, in Project 3]

Demo Crawler Bot

Reinforcement Learning

Still assume a Markov decision process (MDP):

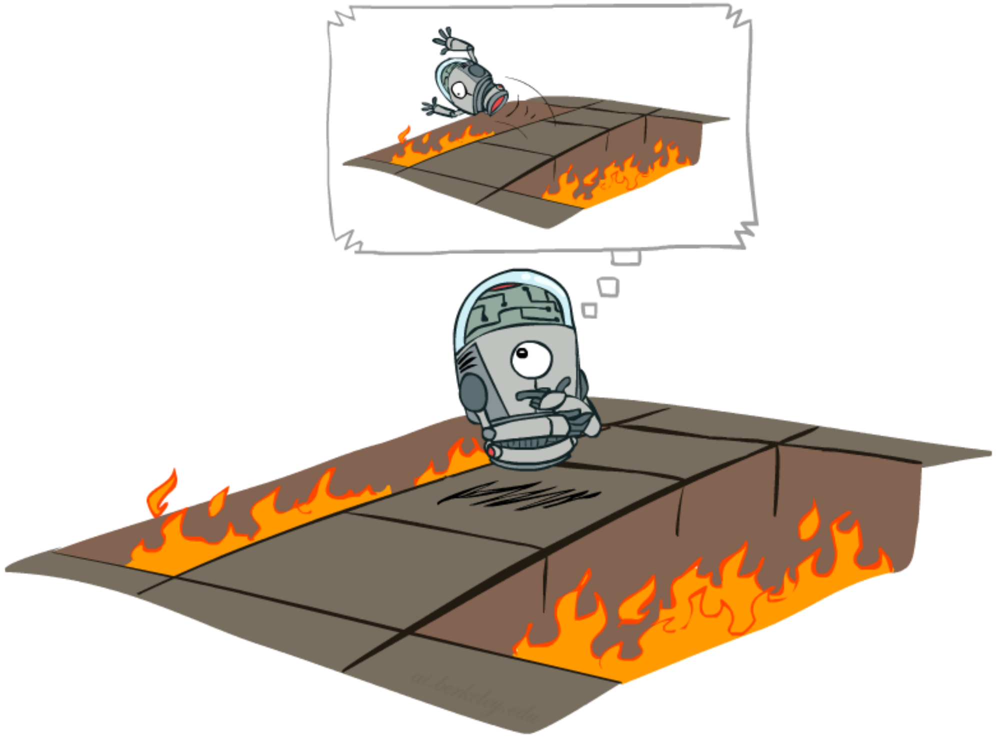
- A set of states $s \in S$
- A set of actions (per state) A
- A model T(s,a,s')
- A reward function R(s,a,s')

Still looking for a policy $\pi(s)$

New twist: don't know T or R

- I.e. we don't know which states are good or what the actions do
- Must actually try actions and states out to learn

Offline (MDPs) vs. Online (RL)



Offline Solution
(Known MDP)

Online Learning
(Unknown MDP)

Overview: MDPs and Reinforcement Learning

Known MDP: Offline Solution

Value Iteration / Policy Iteration

Unknown MDP: Online Learning

Model-Based

Estimate MDP T(s,a,s') and R(s,a,s') from samples of environment

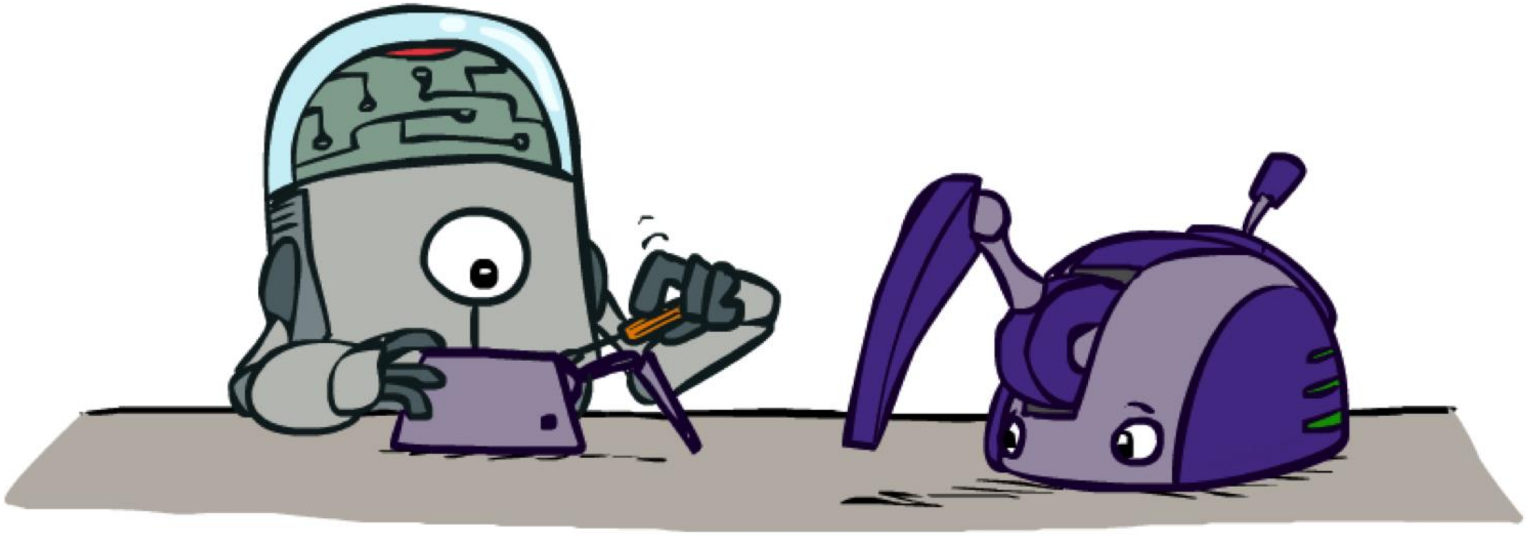
Model-Free

Passive Reinforcement Learning

- Direct Evaluation (simple)
- TD Learning

Active Reinforcement Learning

- Q-Learning



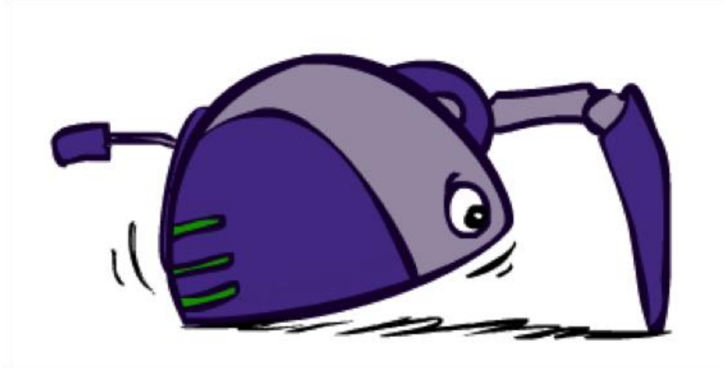
Online Learning

Model-based Learning

Model-Based Learning

Model-Based Idea:

- Learn an approximate model based on experiences
- Solve for values as if the learned model were correct



Step 1: Learn empirical MDP model

- Count outcomes s' for each s, a
- Normalize to give an estimate of $\widehat{T}(s, a, s')$
- Discover each $\widehat{R}(s, a, s')$ when we experience (s, a, s')

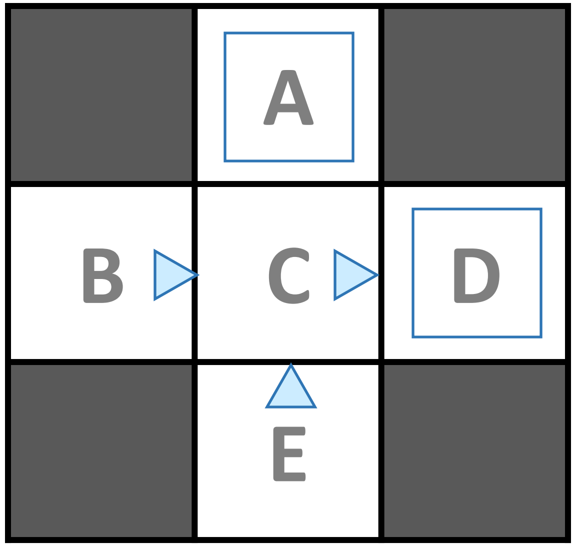
Step 2: Solve the learned MDP

- For example, use value iteration, as before

Example: Model-Based Learning

Episode: a sequence of states actions and rewards sampled from the environment

Input Policy π



Assume: $\gamma = 1$

Observed Episodes (Training)

Episode 1

B, east, C, -1
C, east, D, -1
D, exit, x, +10

Episode 2

B, east, C, -1
C, east, D, -1
D, exit, x, +10

Episode 3

E, north, C, -1
C, east, D, -1
D, exit, x, +10

Episode 4

E, north, C, -1
C, east, A, -1
A, exit, x, -10

Learned Model

$$\hat{T}(s, a, s')$$

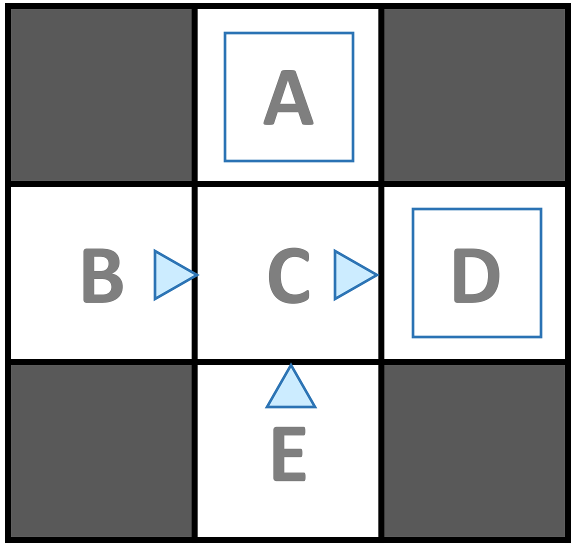
T(B, east, C) =
T(C, east, D) =
T(C, east, A) =
…

$$\hat{R}(s, a, s')$$

R(B, east, C) =
R(C, east, D) =
R(D, exit, x) =
…

Example: Model-Based Learning

Input Policy π



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D, exit, x, +10

Episode 4

E, north, C, -1
C, east, A, -1
A, exit, x, -10

Learned Model

$\hat{T}(s, a, s')$

T(B, east, C) = 1.00
T(C, east, D) = 0.75
T(C, east, A) = 0.25
…

$\hat{R}(s, a, s')$

R(B, east, C) = -1
R(C, east, D) = -1
R(D, exit, x) = +10
…

Example: Expected Age

Goal: Compute expected age of 15-281 students

Known P(A)

$$E[A] = \sum_a P(a) \cdot a \qquad = 0.35 \times 20 + \ldots$$

Without P(A), instead collect samples [a_1, a_2, … a_N]

Unknown P(A): "Model Based"

$$\hat{P}(a) = \frac{\text{num}(a)}{N}$$

$$E[A] \approx \sum_a \hat{P}(a) \cdot a$$

Unknown P(A): "Model Free"

$$E[A] \approx \frac{1}{N} \sum_i a_i$$

Why does this work? Because samples appear with the right frequencies.

Overview: MDPs and Reinforcement Learning

Known MDP: Offline Solution

Value Iteration / Policy Iteration

Unknown MDP: Online Learning

Model-Based

Estimate MDP T(s,a,s') and R(s,a,s') from samples of environment

Model-Free

Passive Reinforcement Learning

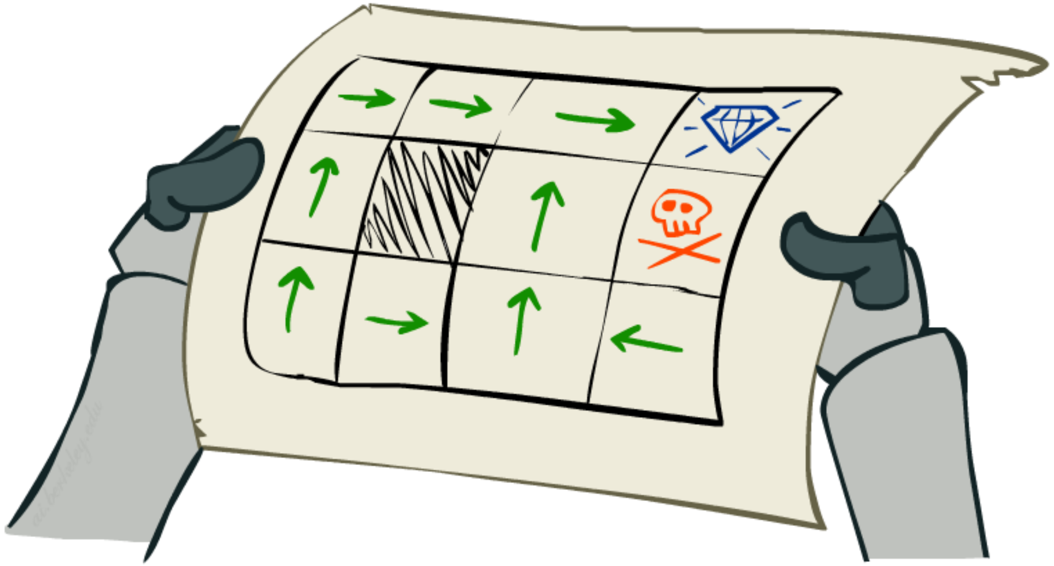
- Direct Evaluation (simple)
- TD Learning

Active Reinforcement Learning

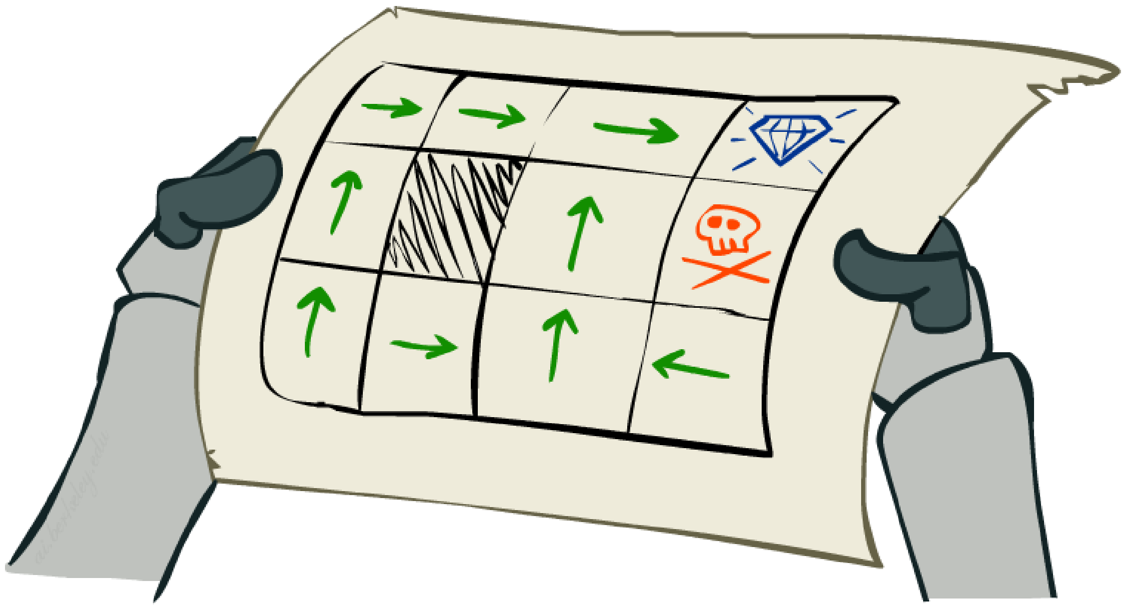
- Q-Learning

Online Learning

Model-free Learning
Passive Reinforcement Learning



Passive Reinforcement Learning



Simplified task: policy evaluation

- Input: a fixed policy $\pi(s)$
- You don’t know the transitions T(s,a,s’)
- You don’t know the rewards R(s,a,s’)
- Goal: learn the state values

In this case:

- Learner is “along for the ride”
- No choice about what actions to take
- Just execute the policy and learn from experience
- This is NOT offline planning! You actually take actions in the world.

Simple Passive Learning: Direct Evaluation

Goal: Compute values for each state under π

Idea: Average together observed sample values

- Act according to π
- Every time you visit a state, write down what the sum of discounted rewards turned out to be
- Average those samples

This is called direct evaluation

Simple Passive Learning: Direct Evaluation

Goal: Compute values for each state under π

Idea: Average together observed sample values

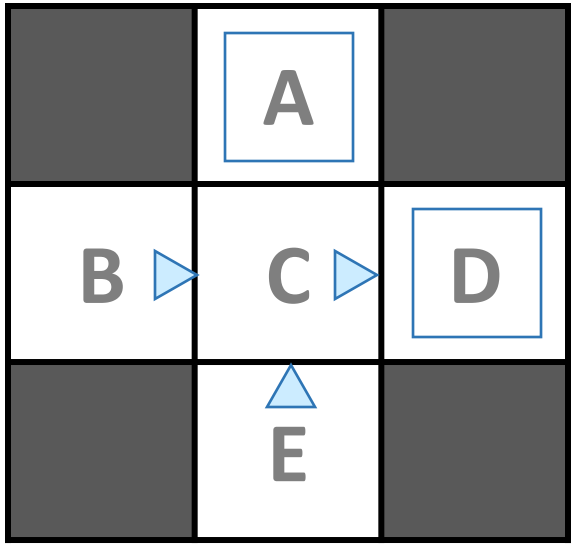
- Act according to π
- Every time you visit a state, write down what the sum of discounted rewards turned out to be
- Average those samples

Pieces Available	Take 1	Take 2
2	0%	**100%**
3	2%	0%
4	**75%**	2%
5	4%	**68%**
6	5%	6%
7	**60%**	5%

This is called direct evaluation

Example: Direct Evaluation

Input Policy π



Assume: $\gamma = 1$

Observed Episodes (Training)

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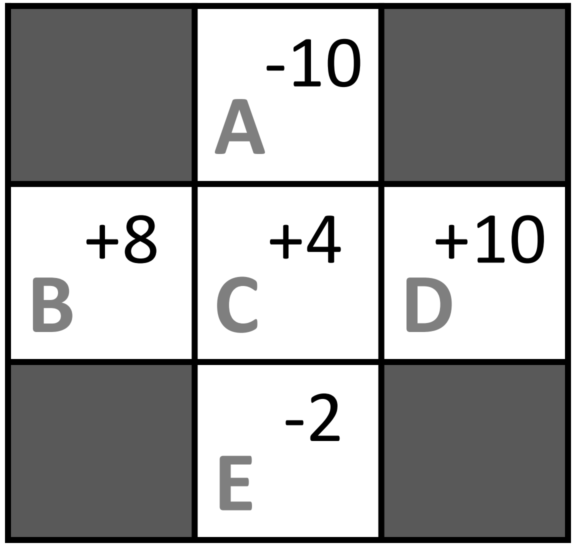
E, north, C, -1
C, east, A, -1
A, exit, x, -10

Output Values



Problems with Direct Evaluation

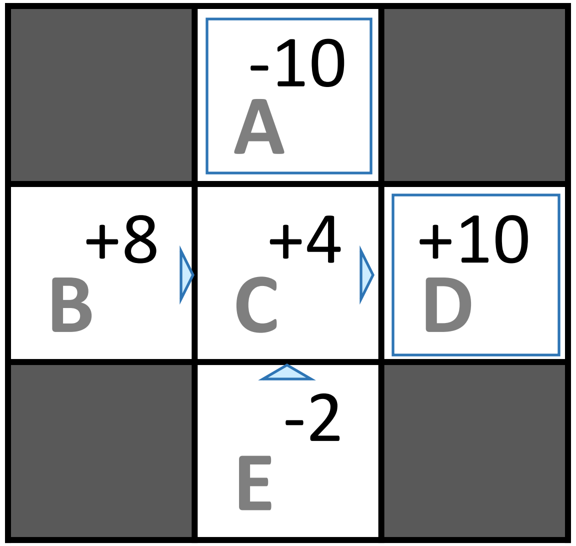
What's good about direct evaluation?

- It's easy to understand
- It doesn't require any knowledge of T, R
- It eventually computes the correct average values, using just sample transitions

What bad about it?

- It wastes information about state connections
- Each state must be learned separately
- So, it takes a long time to learn

Output Values



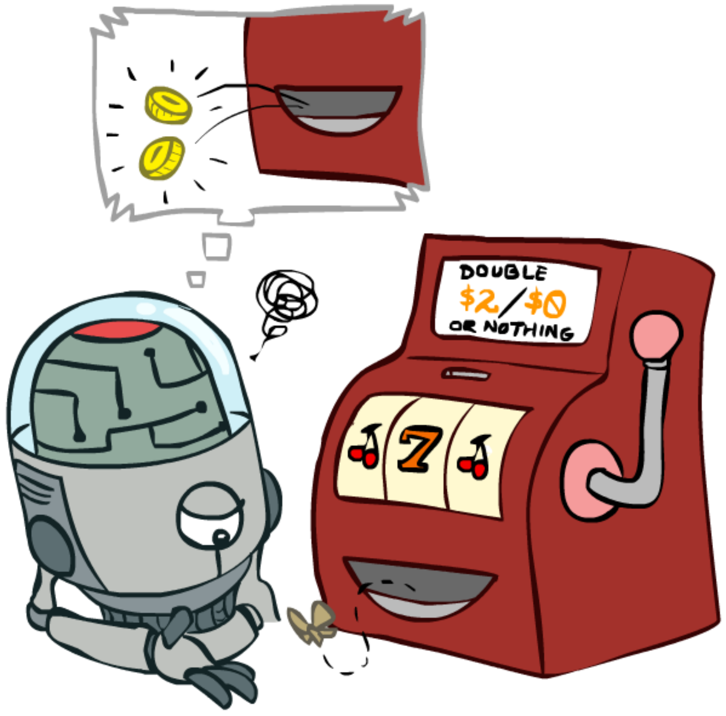
If B and E both go to C under this policy, how can their values be different?

Online Learning

Model-free Learning

Passive Reinforcement Learning

Temporal Difference Learning



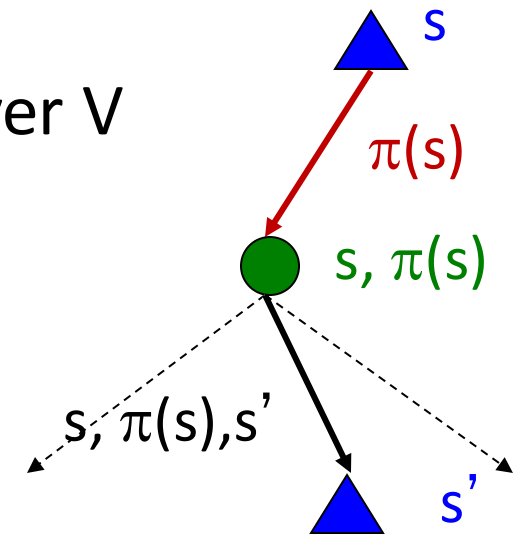
Why Not Use Policy Evaluation?

Simplified Bellman updates calculate V for a fixed policy:

- Each round, replace V with a one-step-look-ahead layer over V

$$V_0^\pi(s) = 0$$

$$V_{k+1}^\pi(s) \leftarrow \sum_{s'} T(s,\pi(s),s')[R(s,\pi(s),s') + \gamma V_k^\pi(s')]$$



- This approach fully exploited the connections between the states
- Unfortunately, we need T and R to do it!

Key question: How can we do this update to V without knowing T and R?

- In other words, how to we take a weighted average without knowing the weights?

Sample-Based Policy Evaluation?

We want to improve our estimate of V by computing these averages:

$$V_{k+1}^{\pi}(s) \leftarrow \sum_{s'} T(s,\pi(s),s')[R(s,\pi(s),s') + \gamma V_k^{\pi}(s')]$$

Idea: Take samples of outcomes s' (by doing the action!) and average

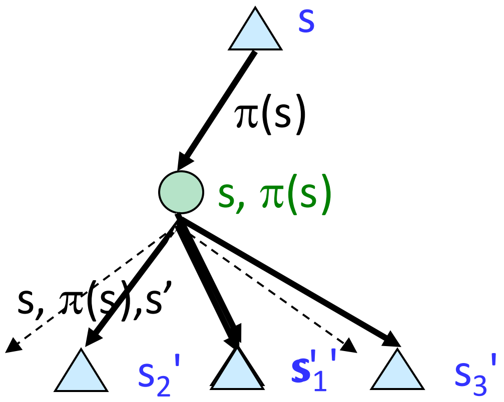
$$sample_1 = R(s,\pi(s),s_1') + \gamma V_k^{\pi}(s_1')$$

$$sample_2 = R(s,\pi(s),s_2') + \gamma V_k^{\pi}(s_2')$$

$$\ldots$$

$$sample_n = R(s,\pi(s),s_n') + \gamma V_k^{\pi}(s_n')$$

$$V_{k+1}^{\pi}(s) \leftarrow \frac{1}{n}\sum_i sample_i$$

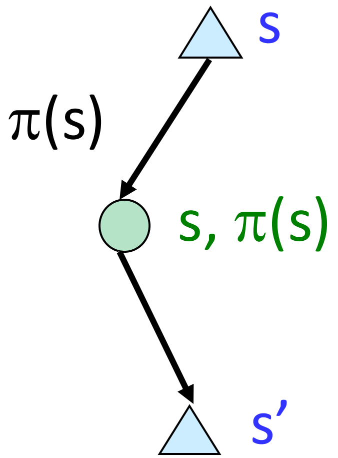


Almost! But we can't rewind time to get sample after sample from state s.

Temporal Difference Learning

Big idea: learn from every experience!

- Update V(s) each time we experience a transition (s, a, s’, r)
- Likely outcomes s’ will contribute updates more often



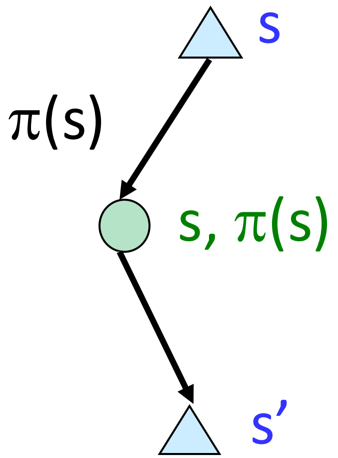
Temporal difference learning of values

- Policy still fixed, still doing evaluation!
- Move values toward value of whatever successor occurs: running average

Sample of V(s): $$sample = R(s, \pi(s), s') + \gamma V^{\pi}(s')$$

Update to V(s):

Temporal Difference Learning



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Temporal difference learning of values

- Policy still fixed, still doing evaluation!
- Move values toward value of whatever successor occurs: running average

Sample of V(s): $sample = R(s, \pi(s), s') + \gamma V^{\pi}(s')$

Update to V(s): $V^{\pi}(s) \leftarrow (1 - \alpha)V^{\pi}(s) + (\alpha)sample$

Same update: $V^{\pi}(s) \leftarrow V^{\pi}(s) + \alpha(sample - V^{\pi}(s))$

Exponential Moving Average

Exponential moving average

- The running interpolation update: $\bar{x}_n = (1-\alpha) \cdot \bar{x}_{n-1} + \alpha \cdot x_n$

- Makes recent samples more important:

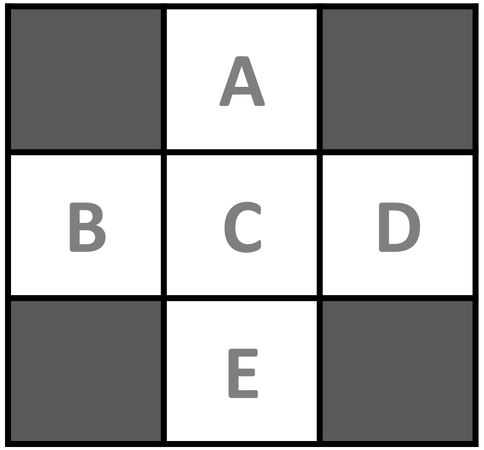
$$\bar{x}_n = \frac{x_n + (1-\alpha) \cdot x_{n-1} + (1-\alpha)^2 \cdot x_{n-2} + \ldots}{1 + (1-\alpha) + (1-\alpha)^2 + \ldots}$$

- Forgets about the past (distant past values were wrong anyway)

Decreasing learning rate (alpha) can give converging averages

Example: Temporal Difference Learning

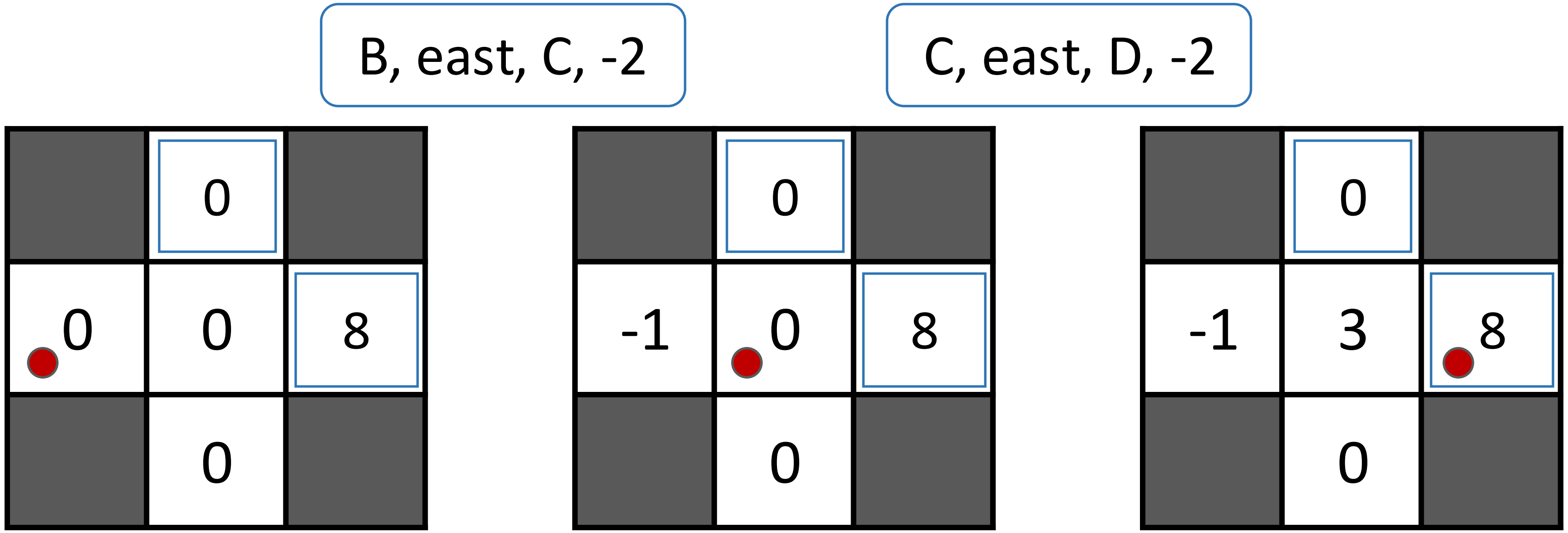
States



Assume: $\gamma = 1$,

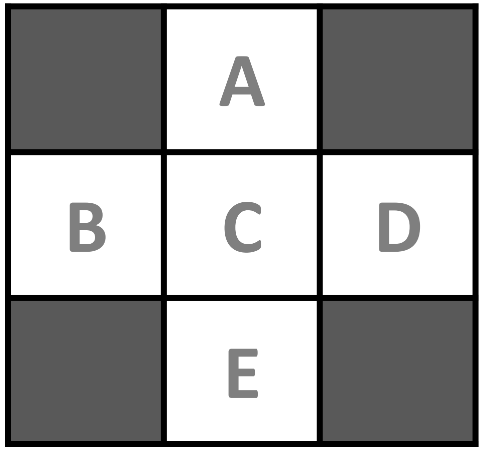
$\alpha = 1/2$

Observed Transitions



Example: Temporal Difference Learning

States

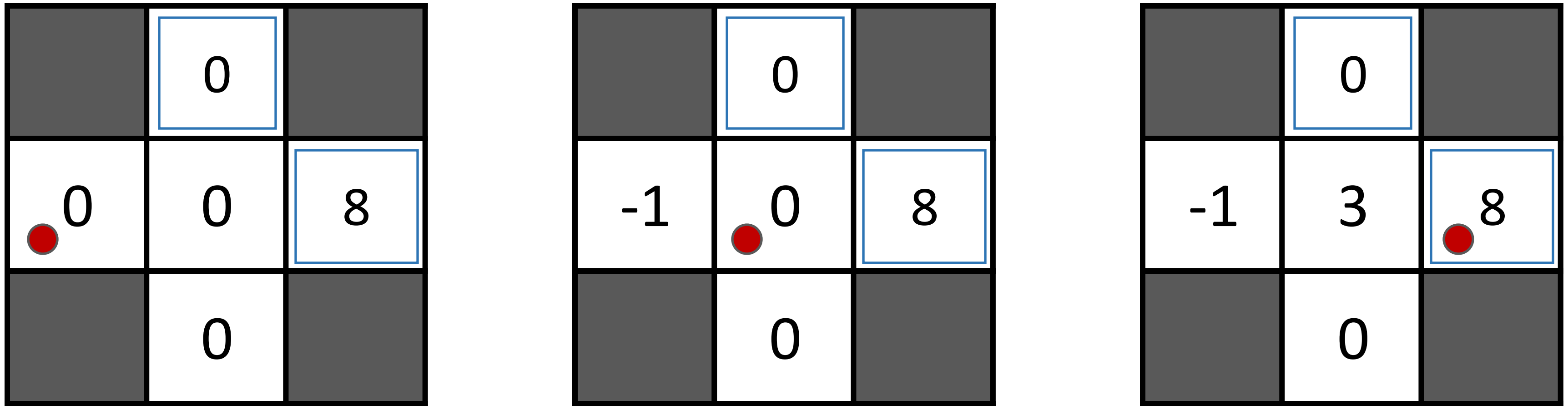


Assume: $\gamma = 1$,
$\alpha = 1/2$

Observed Transitions

B, east, C, -2

C, east, D, -2



$$V^{\pi}(s) \leftarrow (1-\alpha)V^{\pi}(s) + \alpha\left[R(s,\pi(s),s') + \gamma V^{\pi}(s')\right]$$

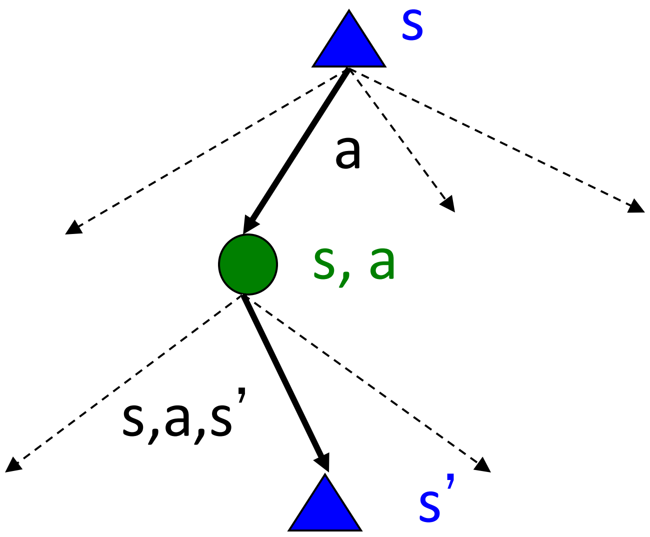
Problems with TD Value Learning

TD value leaning is a model-free way to do policy evaluation, mimicking Bellman updates with running sample averages

However, if we want to turn values into a (new) policy, we're sunk:

$$\pi(s) = \arg\max_a Q(s,a)$$

$$Q(s,a) = \sum_{s'} T(s,a,s')\left[R(s,a,s') + \gamma V(s')\right]$$



Idea: learn Q-values, not values

Makes action selection model-free too!