Warm-up as you log in

Write the pseudo code for breadth first search and depth first search

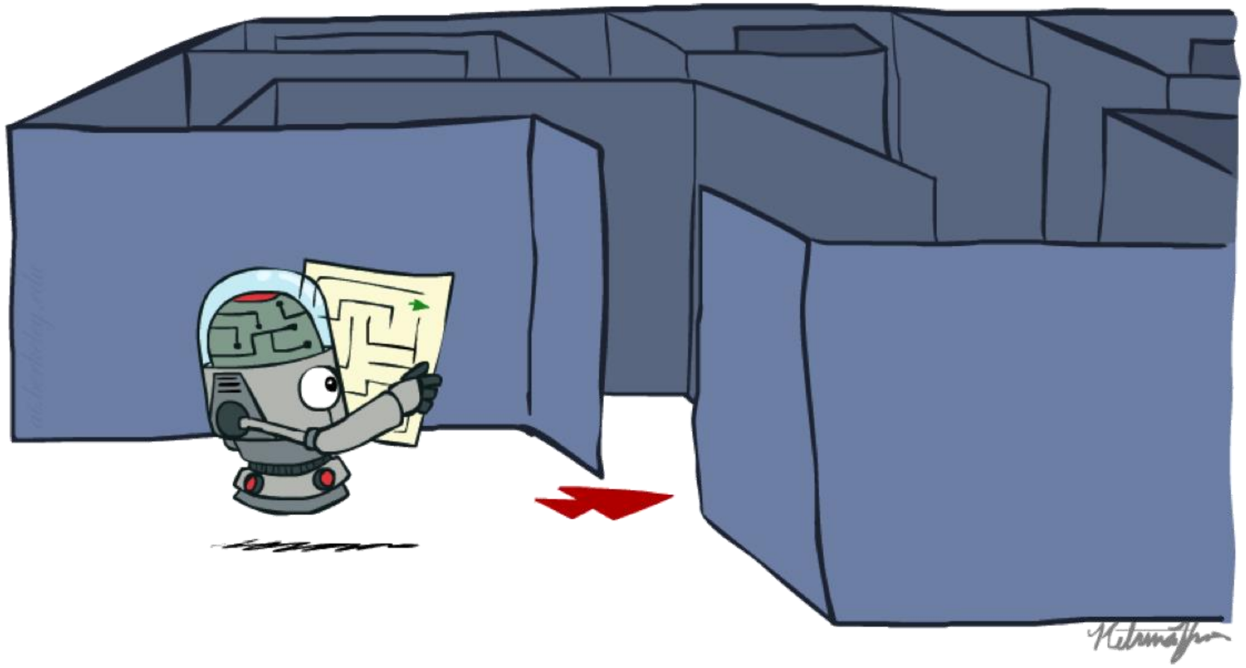
- Iterative version, not recursive

```
class TreeNode
    TreeNode[] children()
    boolean isGoal()

BFS(TreeNode start)…
DFS(TreeNode start)…
```

AI: Representation and Problem Solving

Agents and Search ← Planning



Instructor: Pat Virtue

Slide credits: CMU AI, http://ai.berkeley.edu

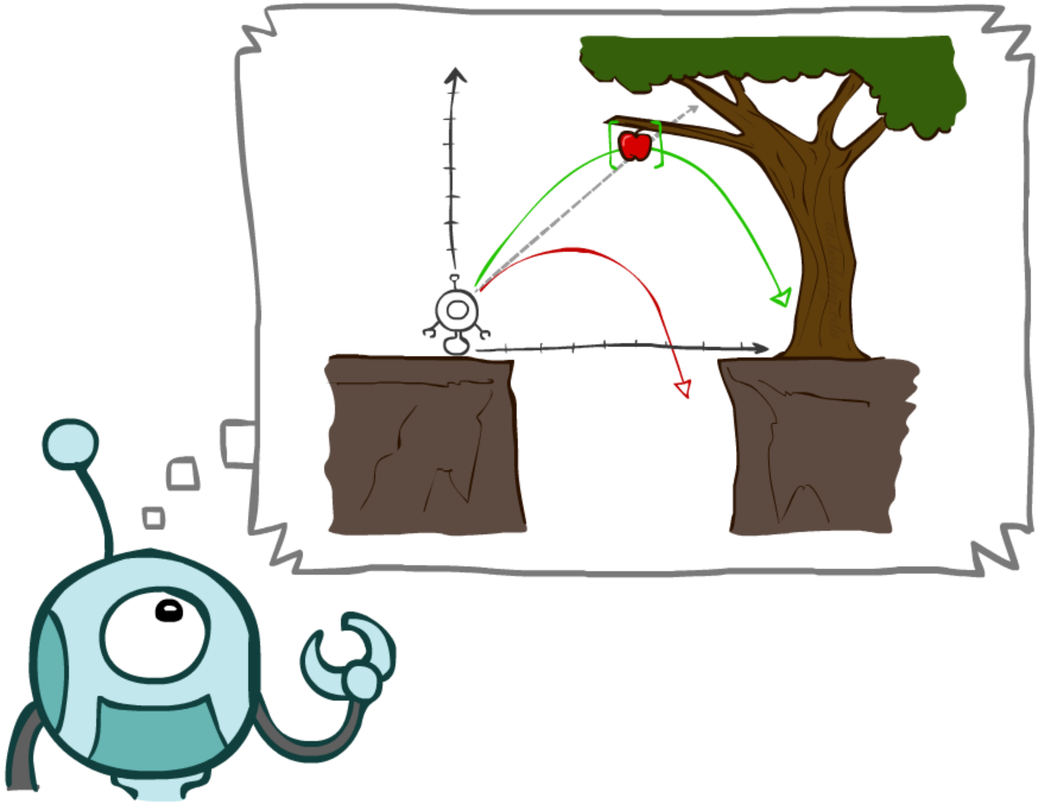
Outline

Agents and Environments

Search Problems

Uninformed Search Methods

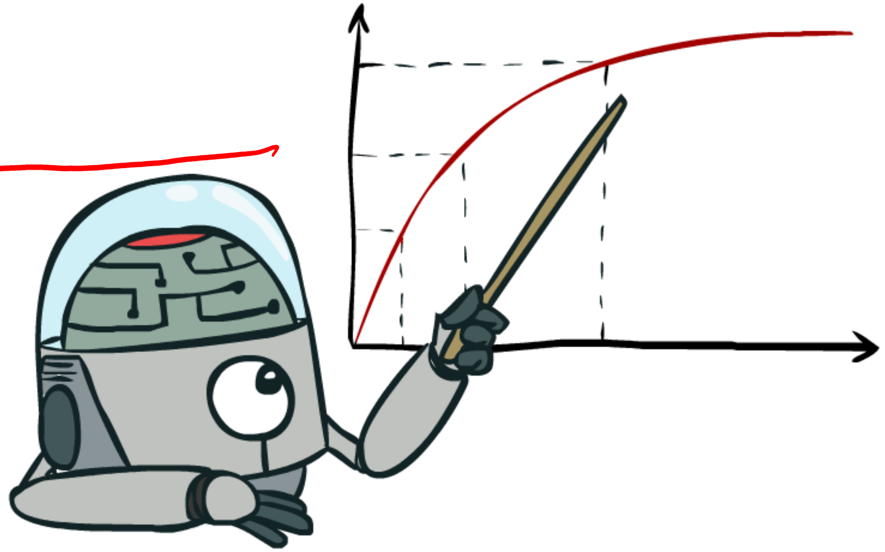
- Depth-First Search
- Breadth-First Search
- Uniform-Cost Search



Rationality, contd.

What is rational depends on:

- Performance measure
- Agent’s prior knowledge of environment
- Actions available to agent
- Percept sequence to date



Being rational means **maximizing your expected utility**

Rational Agents

Are rational agents ***omniscient***?

- No – they are limited by the available percepts

Are rational agents ***clairvoyant***?

- No – they may lack knowledge of the environment dynamics

Do rational agents ***explore*** and ***learn***?

- Yes – in unknown environments these are essential

So rational agents are not necessarily successful, but they are ***autonomous*** (i.e., transcend initial program)

Task Environment - PEAS

Performance measure

- -1 per step; +10 food; +500 win; -500 die; +200 hit scared ghost

Environment

- Pacman dynamics (incl ghost behavior)

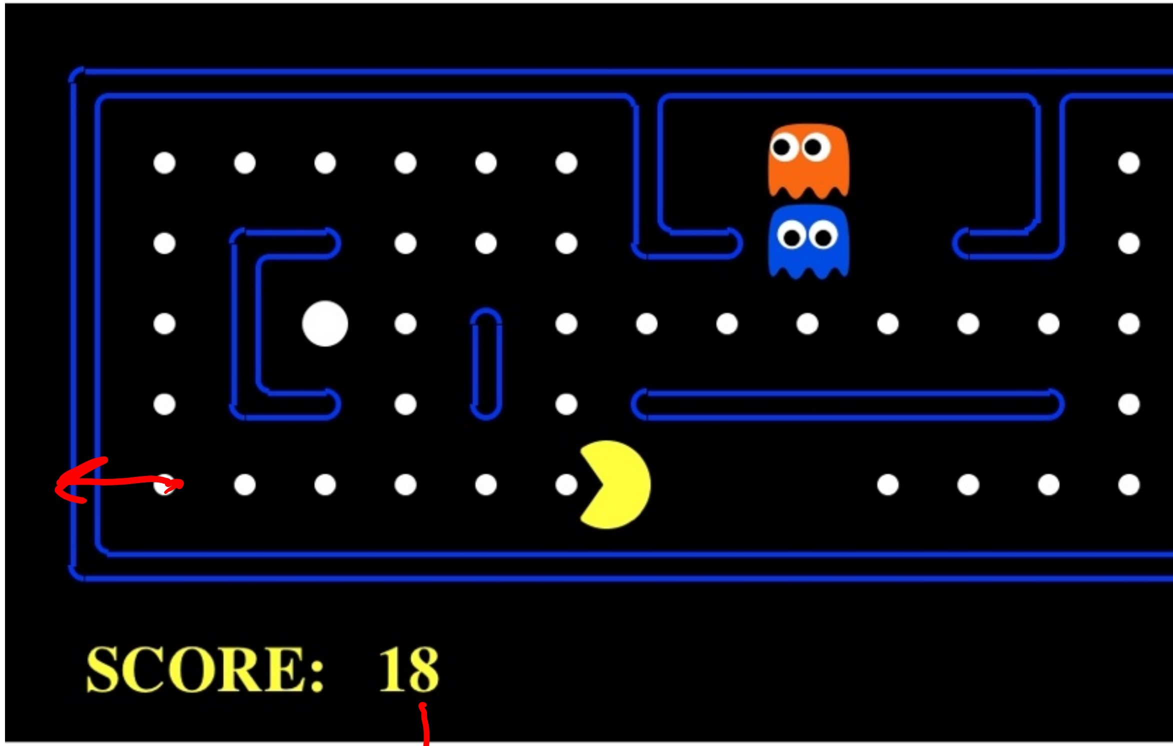
Actuators

NSEW

- North, South, East, West, (Stop)

Sensors

- Entire state is visible



PEAS: Automated Taxi

Performance measure

Alignment

- Income, happy customer, vehicle costs, fines, insurance premiums

Environment

- US streets, other drivers, customers

Actuators

- Steering, brake, gas, display/speaker

Sensors

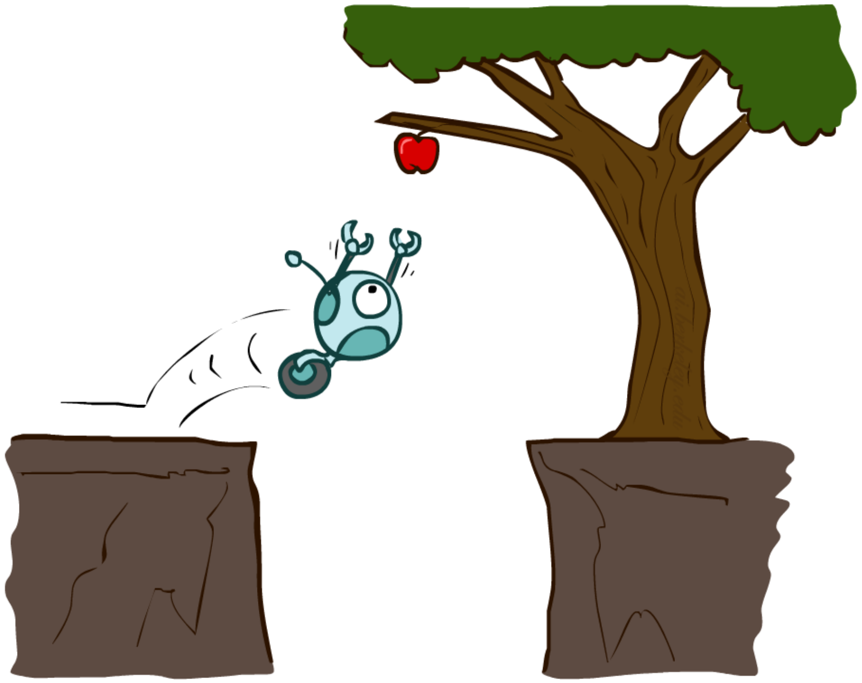
- Camera, radar, accelerometer, engine sensors, microphone

Image: http://nypost.com/2014/06/21/how-google-might-put-taxi-drivers-out-of-business/

Environment Types

	Pacman	Taxi
Fully or partially observable	Fully	Partially
Single agent or multi-agent	Multi	Multi
Deterministic or stochastic	Stoch.	Stoch.
Static or dynamic	(in 281) Static → turn-taking	Dynamic
Discrete or continuous	Discrete	Cont.

Reflex Agents



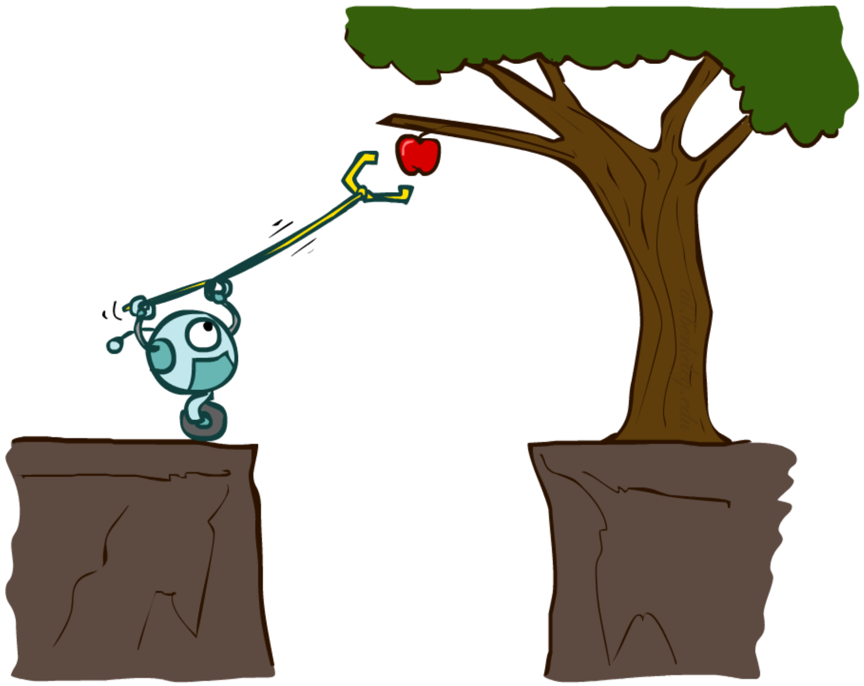
Reflex agents:

- Choose action based on current percept (and maybe memory)
- May have memory or a model of the world's current state
- Do not consider the future consequences of their actions
- Consider how the world IS

Can a reflex agent be rational?

[Demo: reflex optimal (L2D1)]
[Demo: reflex optimal (L2D2)]

Agents that Plan Ahead



Planning agents:

- Decisions based on ***predicted consequences*** of actions
- Must have a ***transition model***: how the world evolves in response to actions
- Must formulate a goal
- Consider how the world WOULD BE

Spectrum of deliberativeness:

- Generate complete, optimal plan offline, then execute
- Generate a simple, greedy plan, start executing, replan when something goes wrong

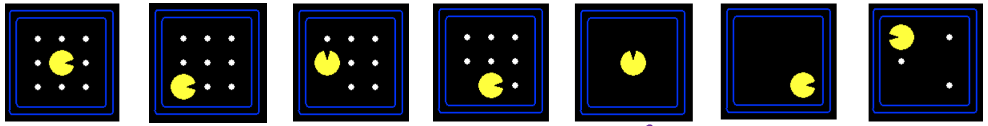
Search Problems

Search Problems

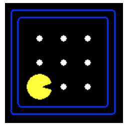
State representation (x, y, bool x9)

A search problem consists of:

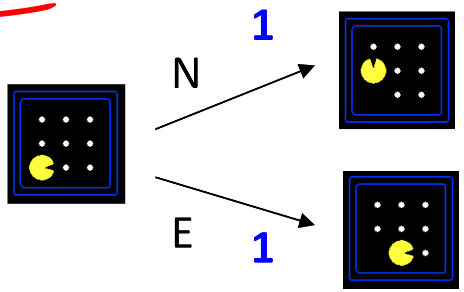
- A state space



- For each state, a set Actions(s) of allowable actions



- A transition model Result(s,a)



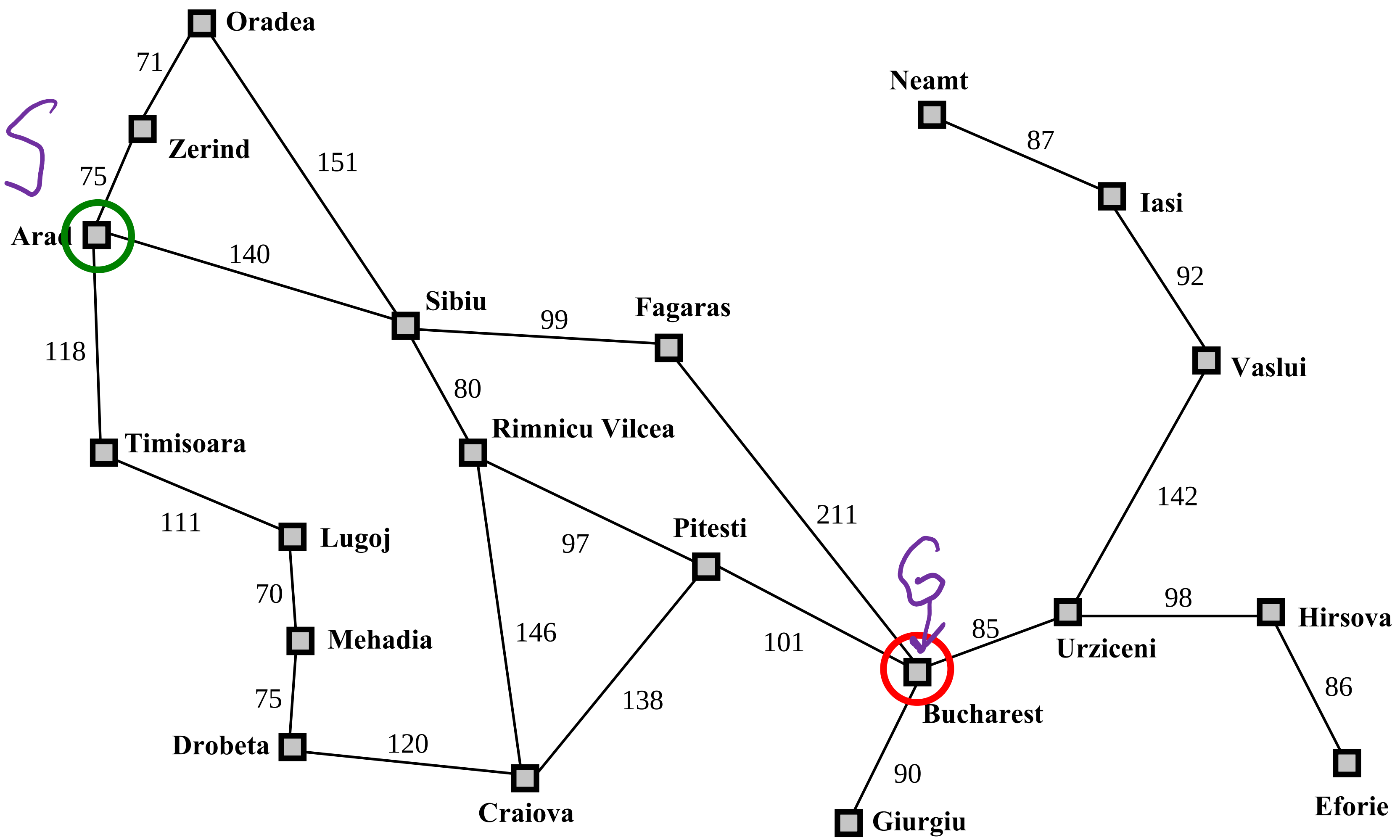
- A step cost function c(s,a,s')
- A start state and a goal test

A solution is a sequence of actions (a plan) which transforms the start state to a goal state

Search Problems Are Models

Example: Travelling in Romania

state: city



State space:

- Cities

Actions:

- Go to adjacent city

Transition model

- Result(A, Go(B)) = B

Step cost

- Distance along road link

Start state:

- Arad

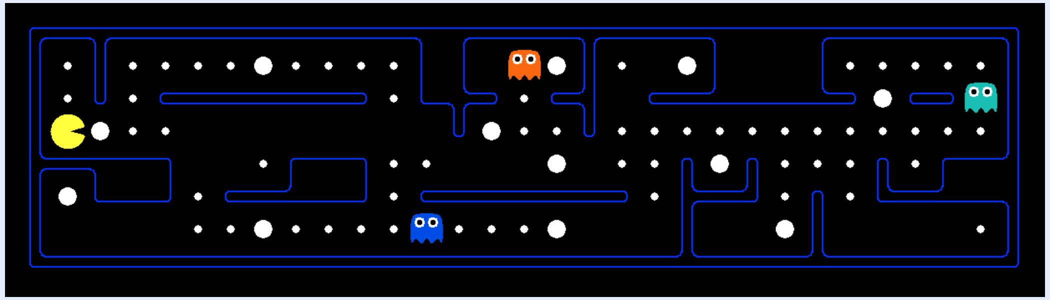
Goal test:

- Is state == Bucharest?

Solution?

What's in a State Space?

The real world state includes every last detail of the environment



A search state abstracts away details not needed to solve the problem

- Problem: Pathing
  - State representation: (x,y) location
  - Actions: NSEW
  - Transition model: update location
  - Goal test: is (x,y)=END
- Problem: Eat-All-Dots
  - State representation: {(x,y), dot booleans}
  - Actions: NSEW
  - Transition model: update location and possibly a dot boolean
  - Goal test: dots all false

Maze
one dot

No ghost

State Space Sizes?

1–120   12   12

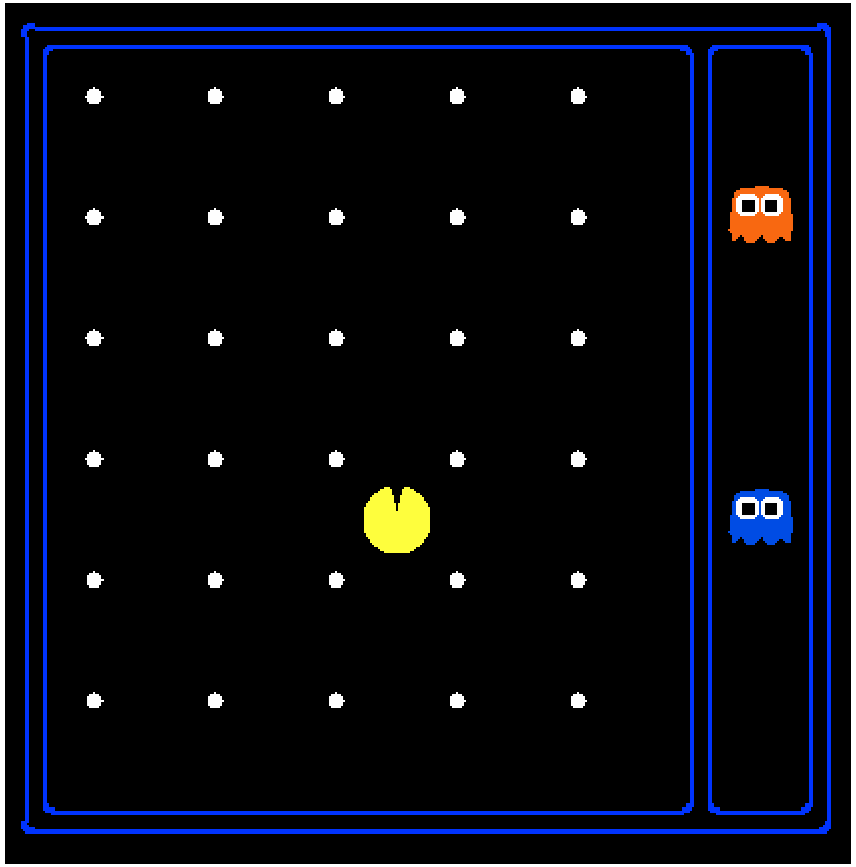
$(p, f_1, \ldots f_{30}, g_1, g_2)$

World state:

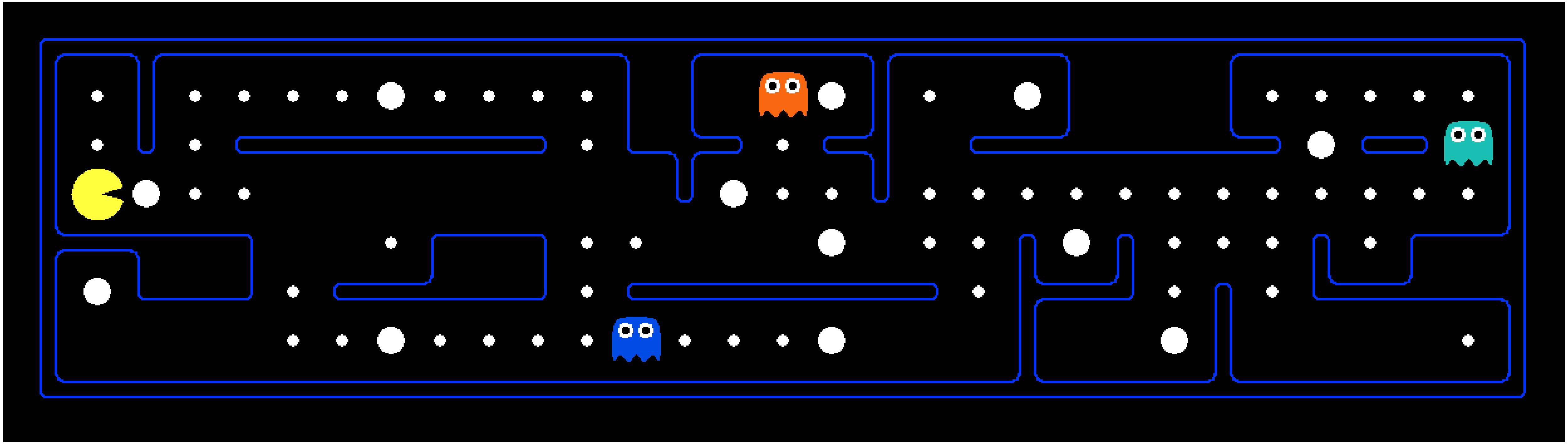
- Agent positions: 120
- Food count: 30
- Ghost positions: 12
- Agent facing: NSEW

How many

- World states?
  120x(2^{30})x(12^2)x4
- States for pathing?
  120
- States for eat-all-dots?
  120x(2^{30})



Safe Passage

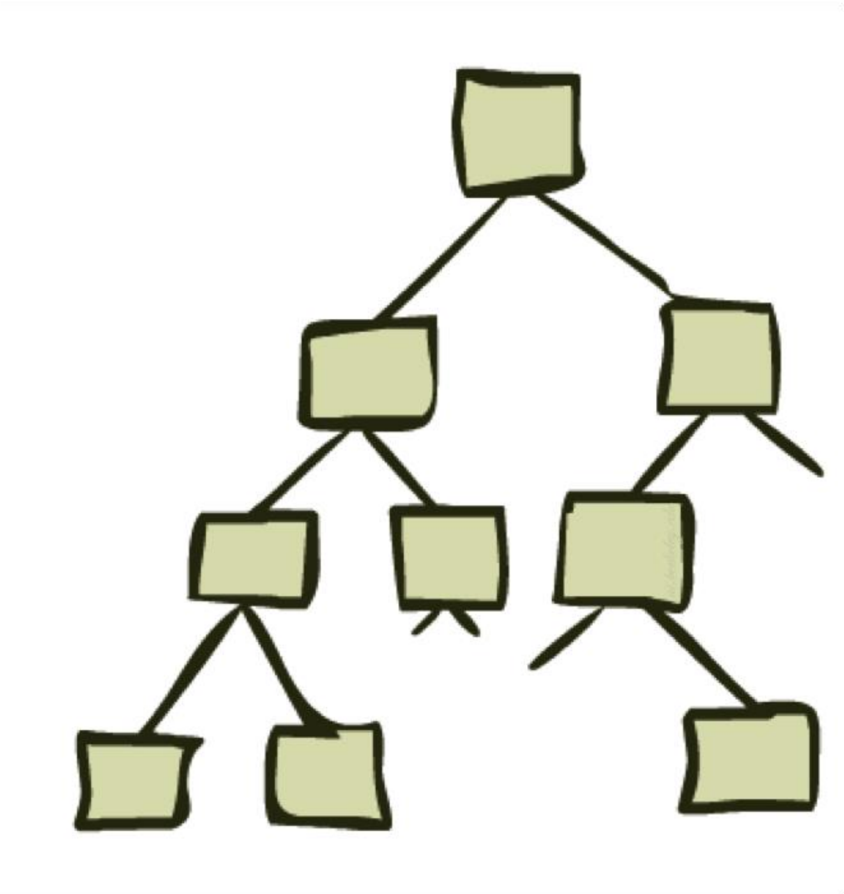


Problem: eat all dots while keeping the ghosts perma-scared

What does the state representation have to specify?

- (agent position, dot booleans, power pellet booleans, remaining scared time)

State Space Graphs and Search Trees



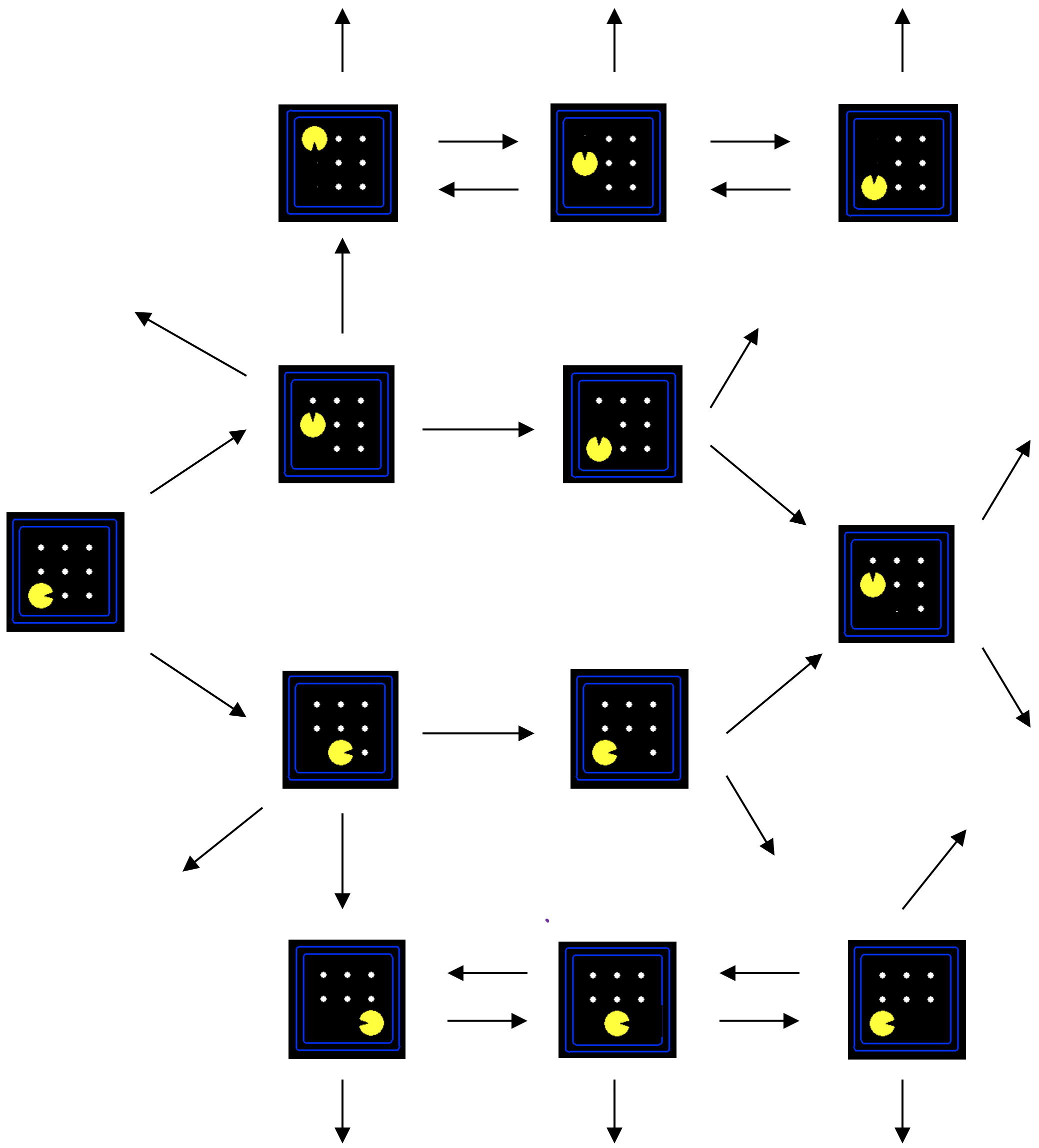
State Space Graphs

State space graph: A mathematical representation of a search problem

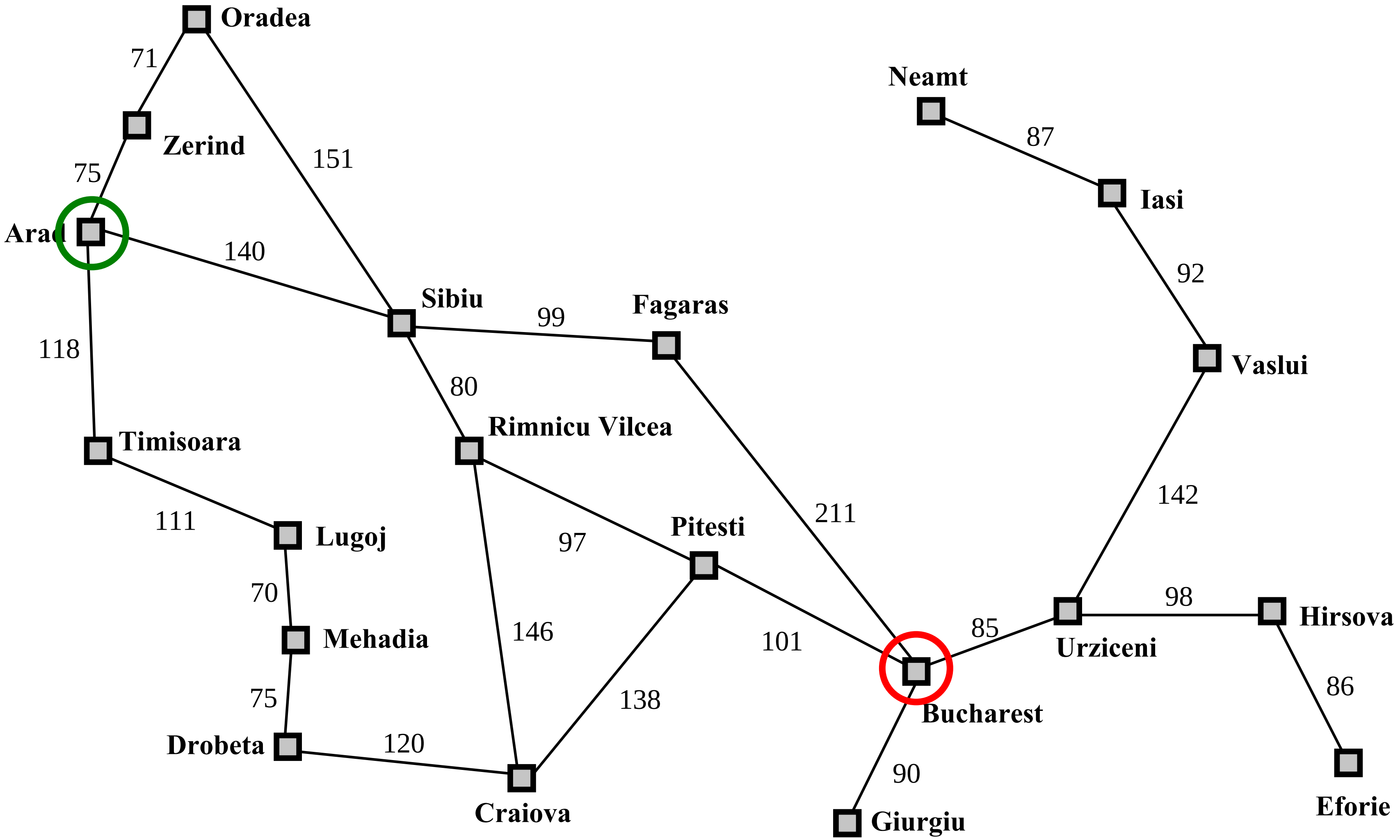
- Nodes are (abstracted) world configurations
- Arcs represent transitions resulting from actions
- The goal test is a set of goal nodes (maybe only one)

In a state space graph, each state occurs only once!

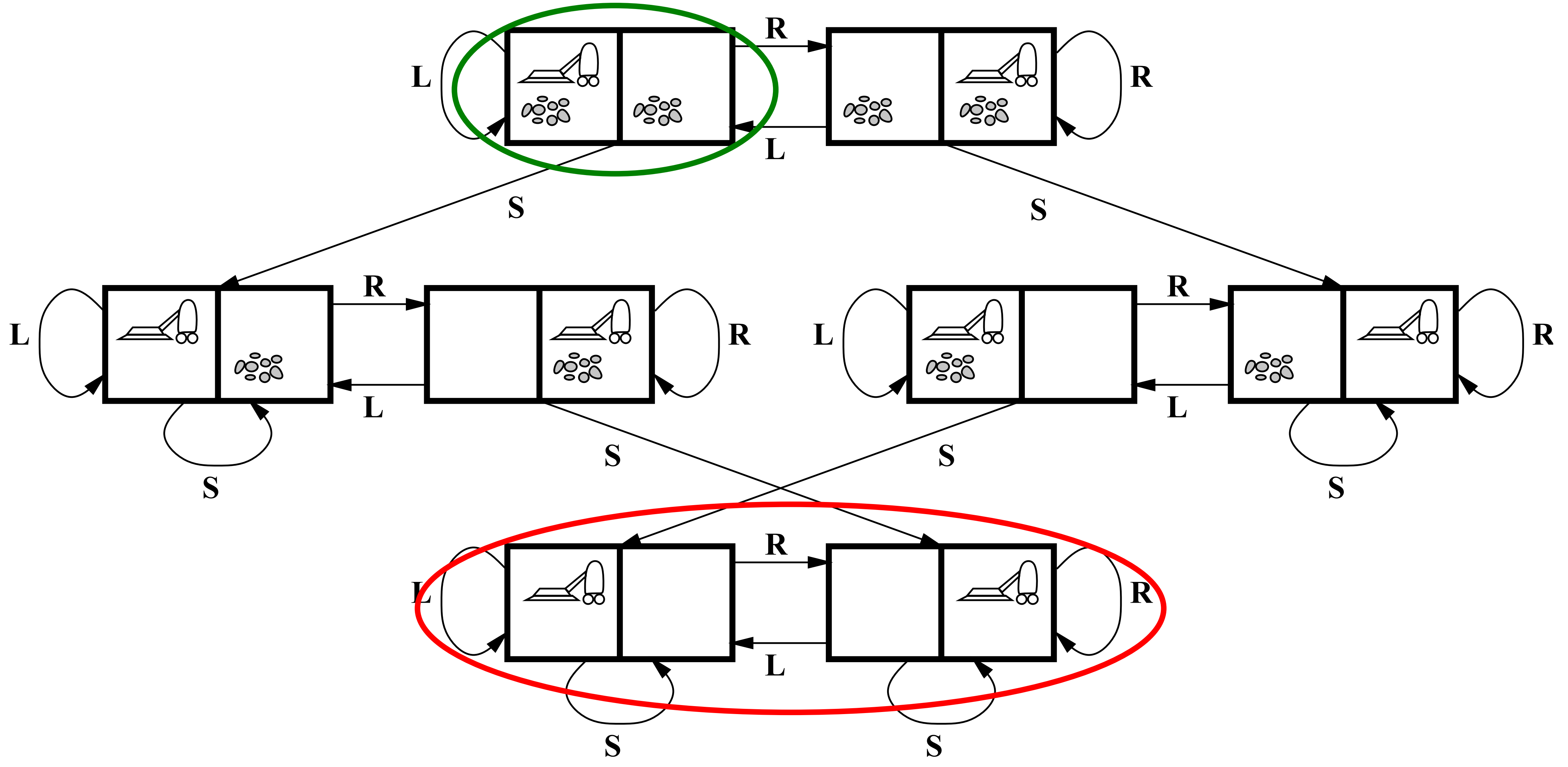
We can rarely build this full graph in memory (it's too big), but it's a useful idea



More Examples



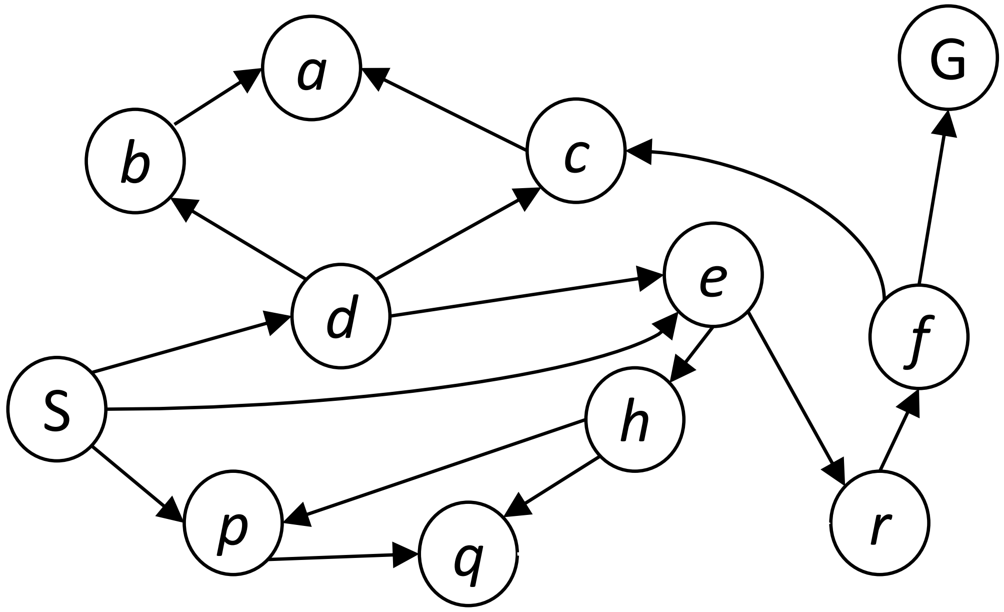
More Examples



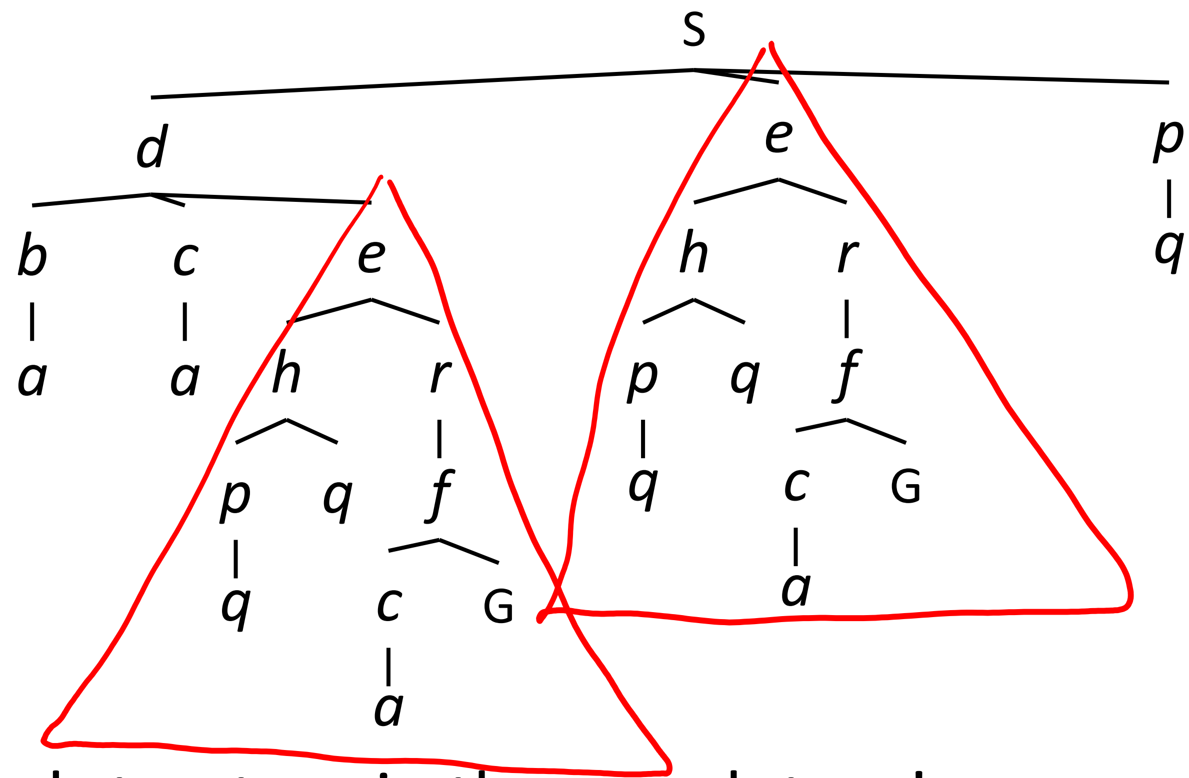
State Space Graphs vs. Search Trees

We build a search tree by traversing various paths in a state space graph, beginning from a specific start state.

State space graph



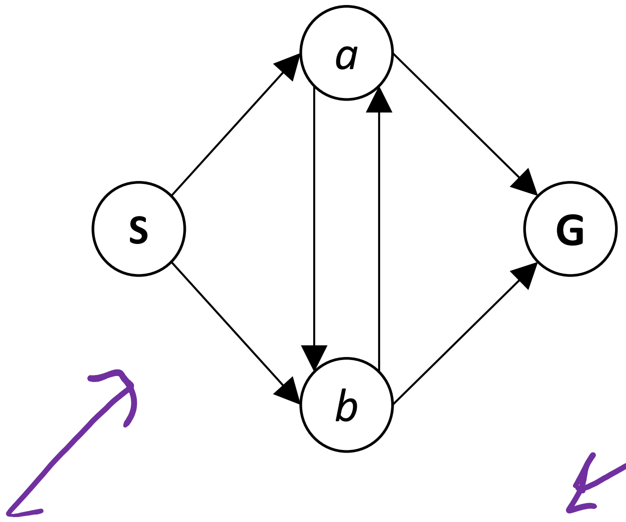
Resulting search tree

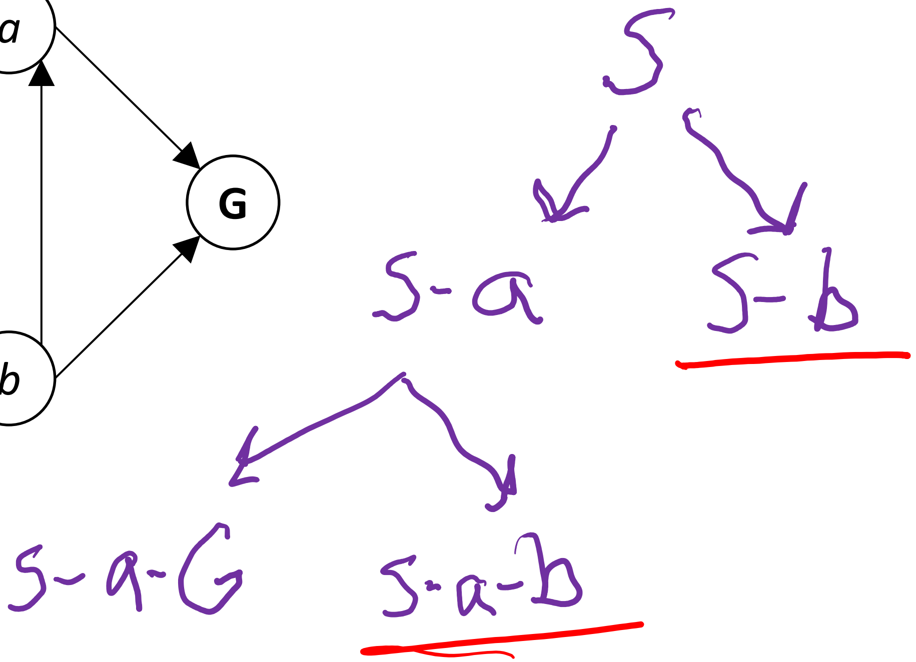


Important: Lots of repeated structure in the search tree!

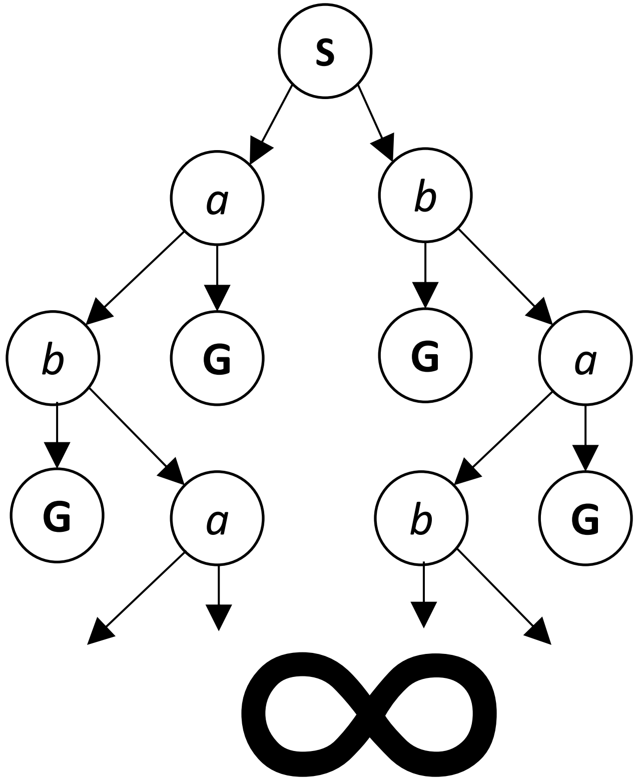
State Space Graphs vs. Search Trees

Consider this 4-state graph:



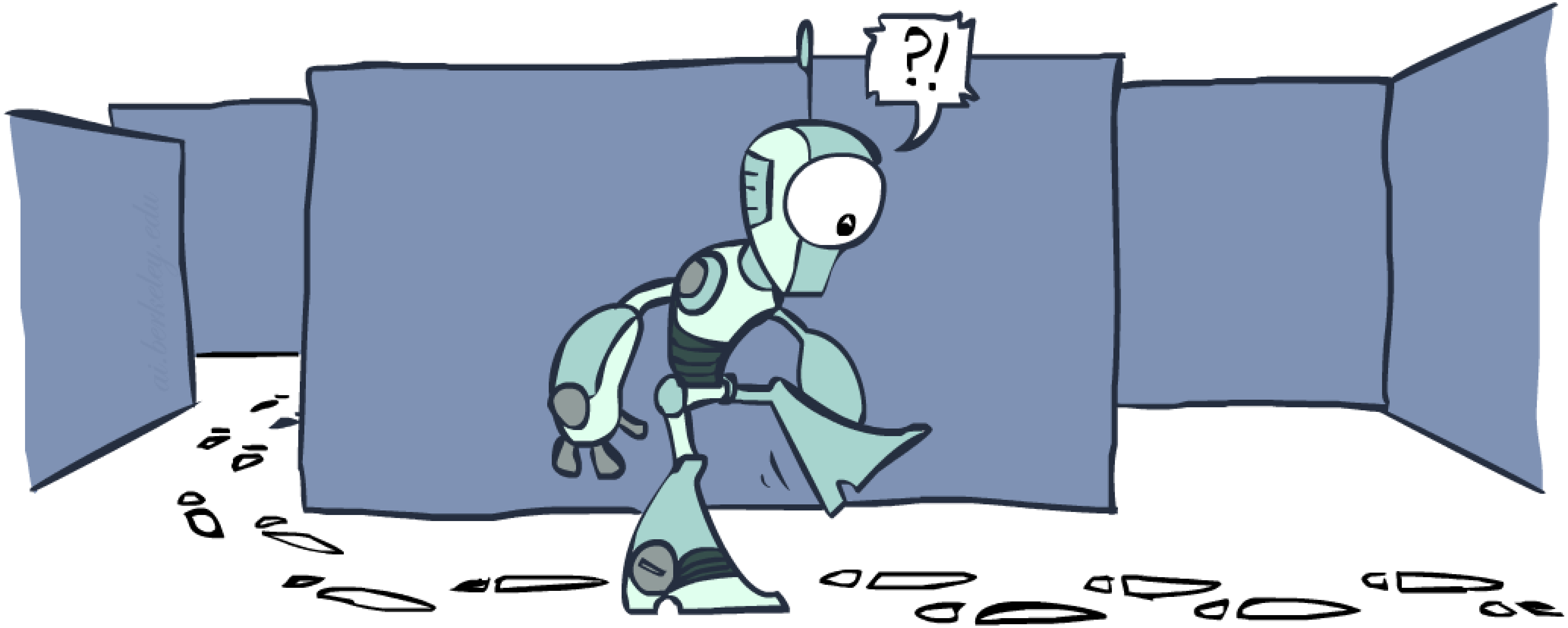


How big is its search tree (from S)?



Important: Lots of repeated structure in the search tree!

Tree Search vs Graph Search



```
function TREE_SEARCH(problem) returns a solution, or failure
                                                DFS   BFS
    initialize the frontier as a specific work list (stack, queue, priority queue)
    add initial state of problem to frontier    LIFO  FIFO
    loop do
        if the frontier is empty then
            return failure
        choose a node and remove it from the frontier
        if the node contains a goal state then
            return the corresponding solution

        for each resulting child from node
            add child to the frontier
```

function GRAPH_SEARCH(problem) returns a solution, or failure

    initialize the explored set to be empty

    initialize the frontier as a specific work list (stack, queue, priority queue)

    add initial state of problem to frontier

    loop do

        if the frontier is empty then

            return failure

        choose a node and remove it from the frontier

        if the node contains a goal state then

            return the corresponding solution

        add the node state to the explored set ←

        for each resulting child from node

            if the child state is not already in the frontier or explored set then

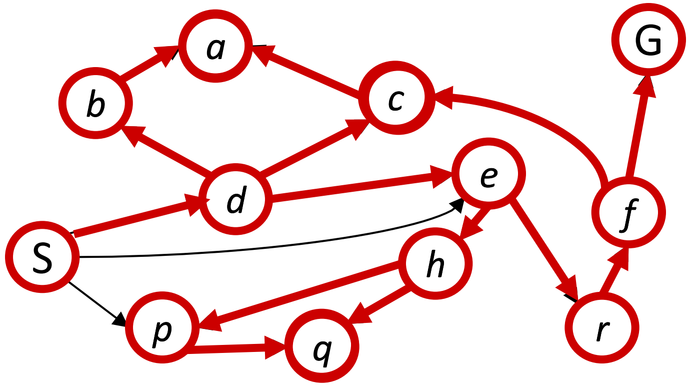
                add child to the frontier

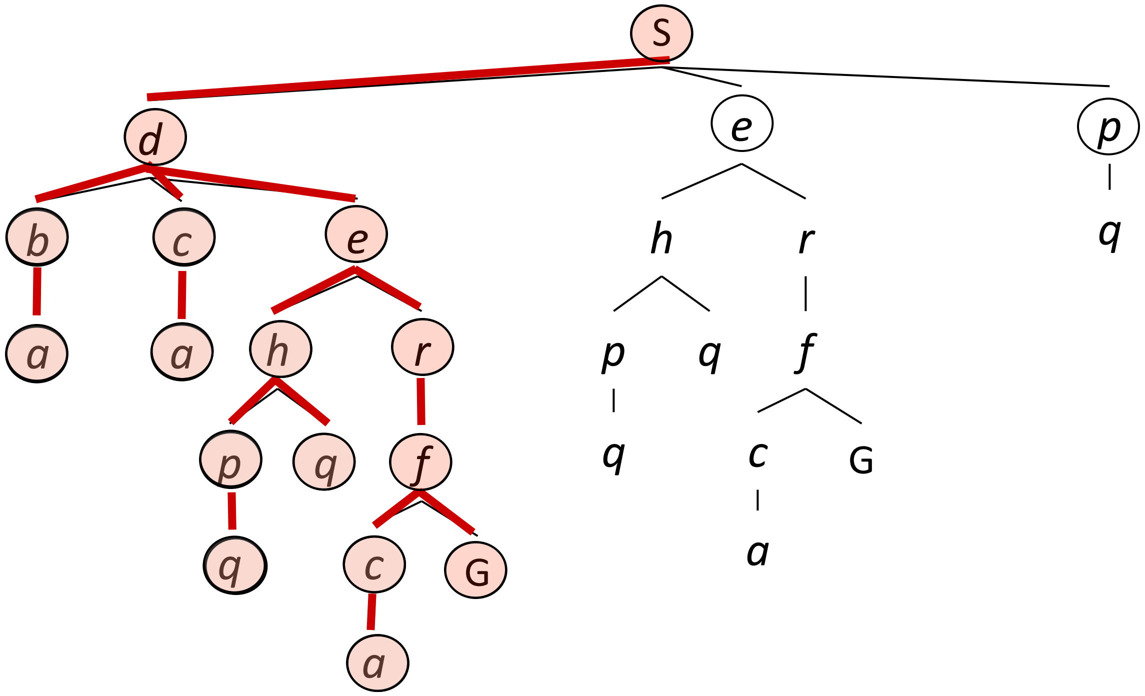
frontier	explored
node	
S-A-B	B

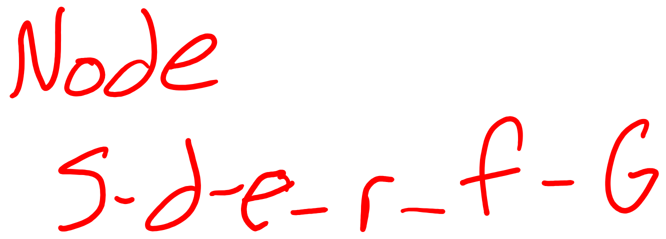
Depth-First (Tree) Search

Strategy: expand a deepest node first

Implementation:
Frontier is a LIFO stack





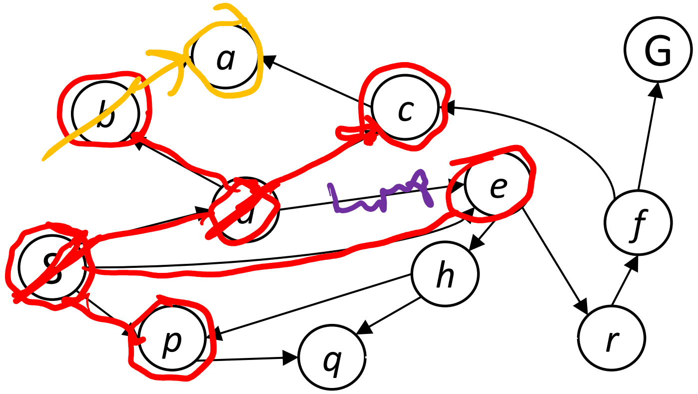


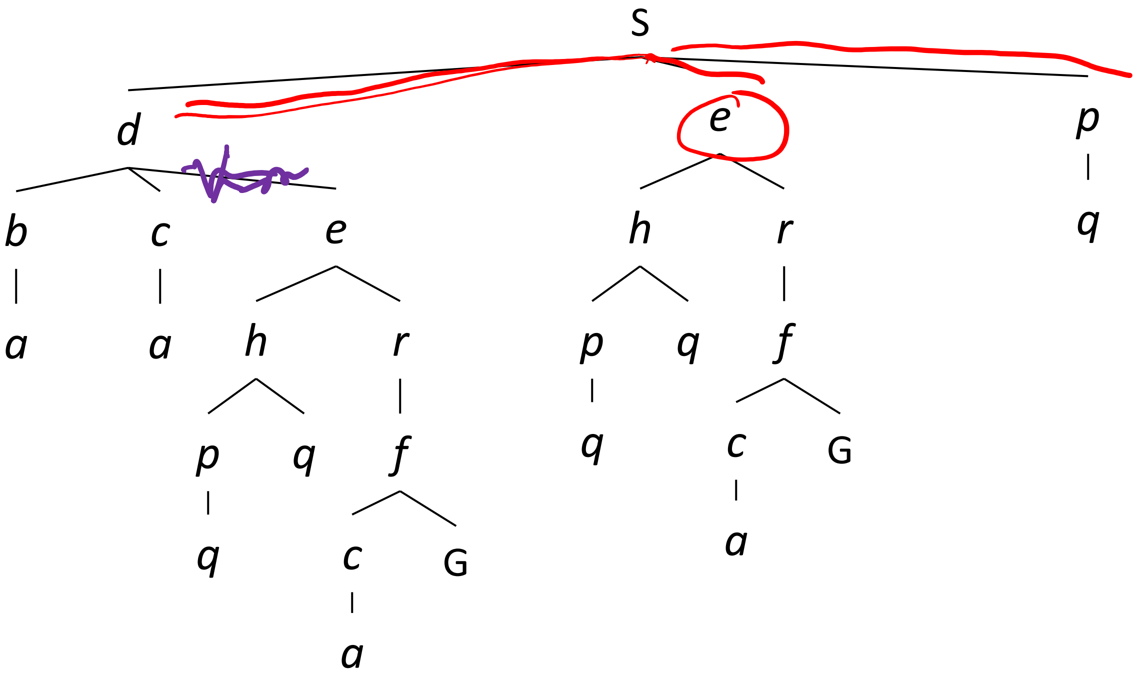
Depth-First (Graph) Search

Strategy: expand a deepest node first

Implementation:
Frontier is a LIFO stack

Explored set prevents loops and repeated work





Front
S
S-p
S-e
S-d
S-d-e
S-d-c
S-d-b
S-d-b-a

Exp
S
d
b

Poll 1

function GRAPH-SEARCH(problem) returns a solution, or failure

    initialize the explored set to be empty

    initialize the frontier as a specific work list (stack, queue, priority queue)

    add initial state of problem to frontier

    loop do

        if the frontier is empty then

            return failure

        choose a node and remove it from the frontier

        if the node contains a goal state then

            return the corresponding solution

        add the node state to the explored set

        for each resulting child from node

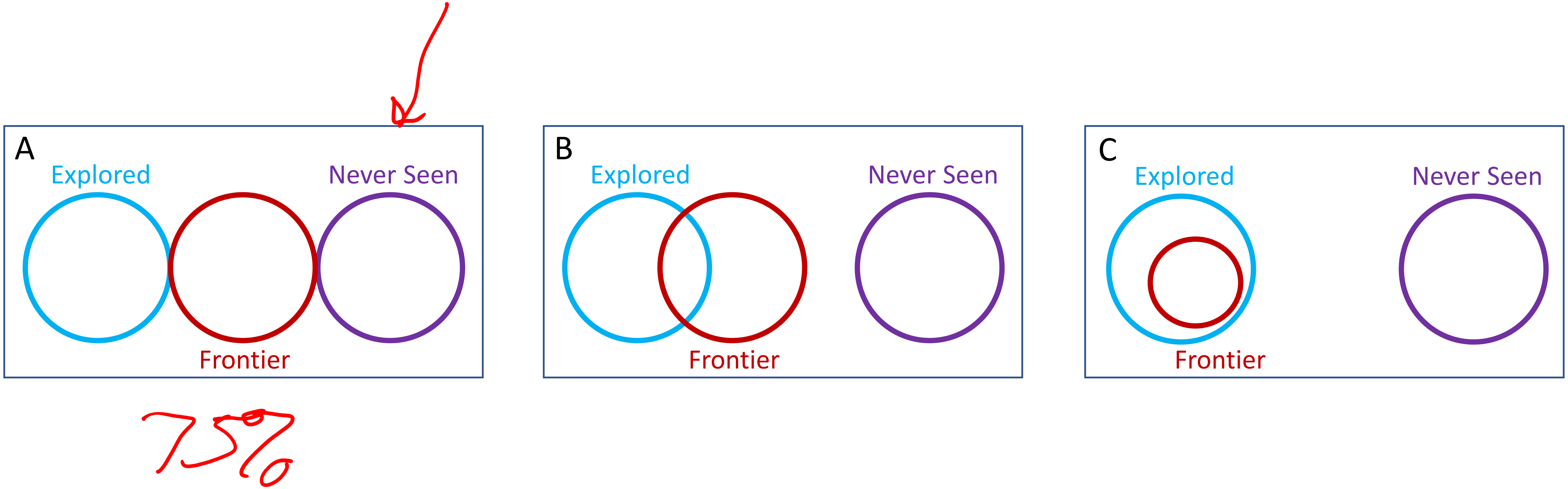
            if the child state is not already in the frontier or explored set then

                add child to the frontier

Poll 1

What is the relationship between these sets of states after each loop iteration in GRAPH_SEARCH?

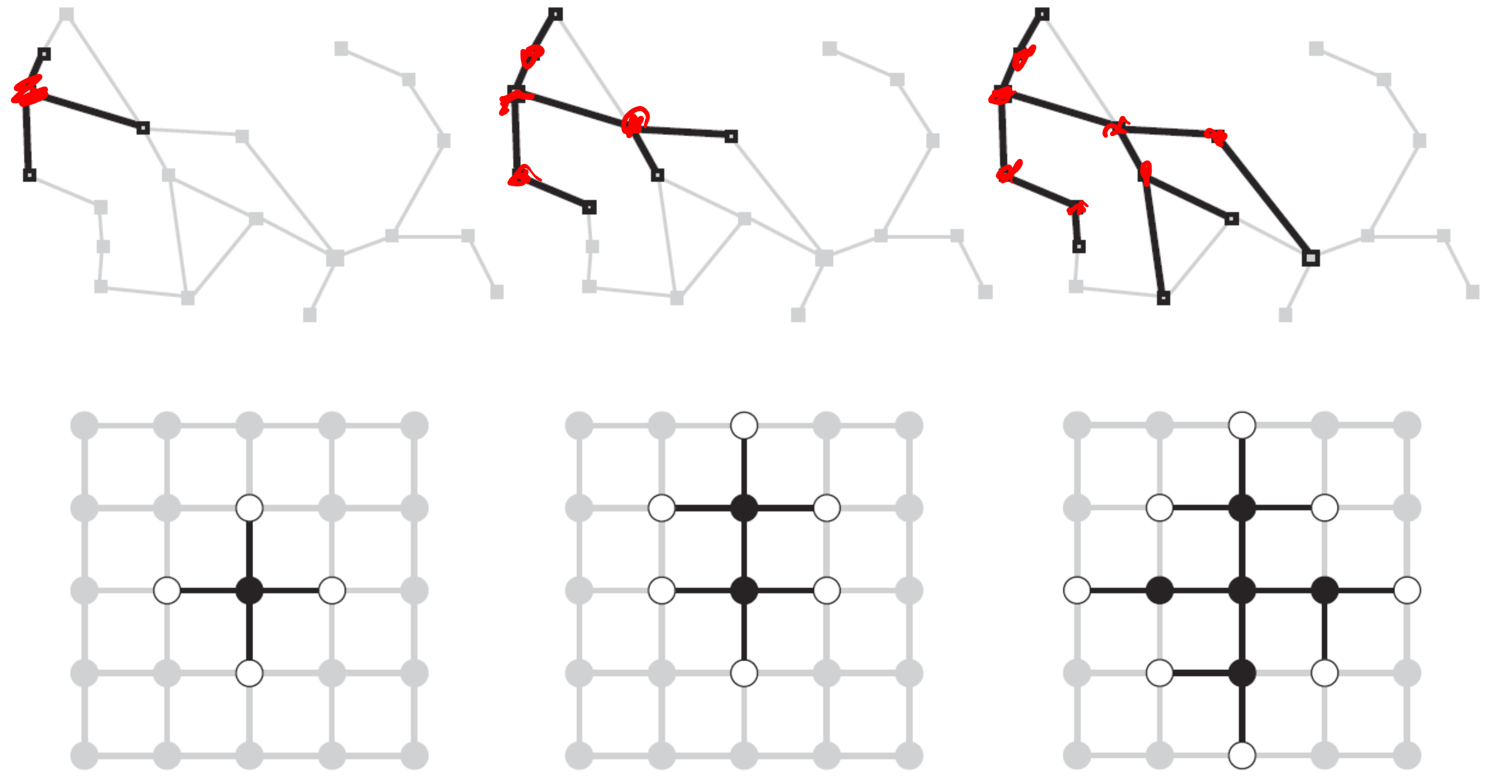
(Loop invariants!!!)



Graph Search

This graph search algorithm overlays a tree on a graph

The frontier states separate the explored states from never seen states



Images: AIMA, Figure 3.8, 3.9

A Note on Implementation

Nodes have

state, parent, action, path-cost

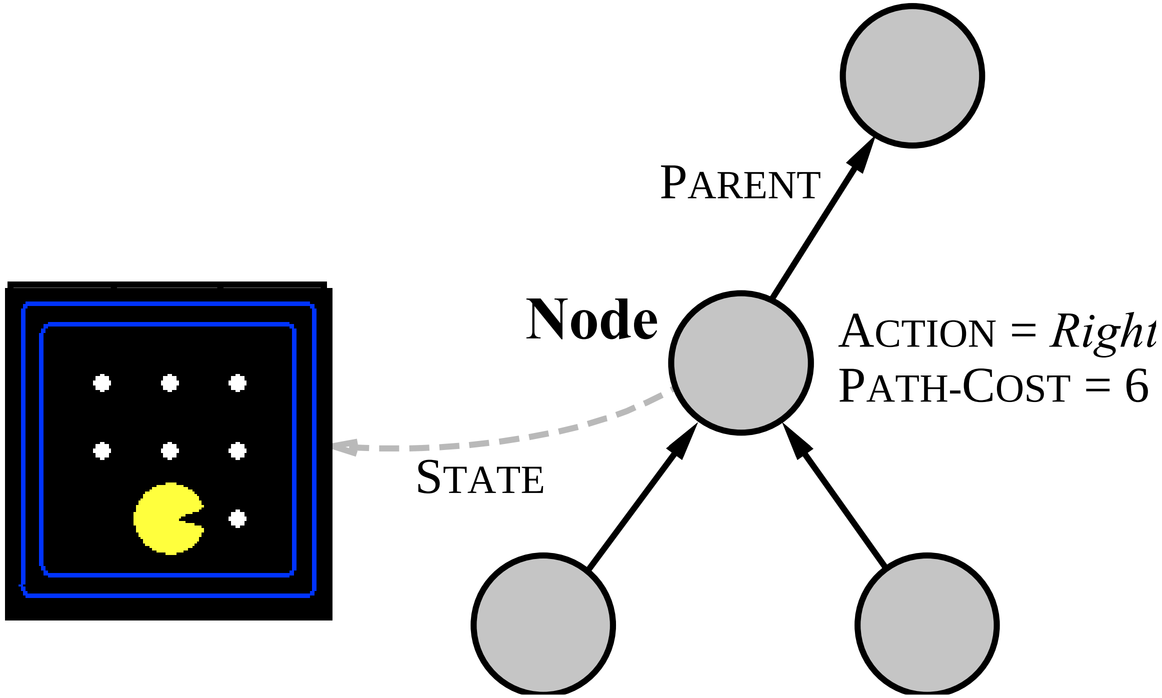
A child of parent_node by action *a* has:

state = result(parent_node.state, *a*)

parent = parent_node

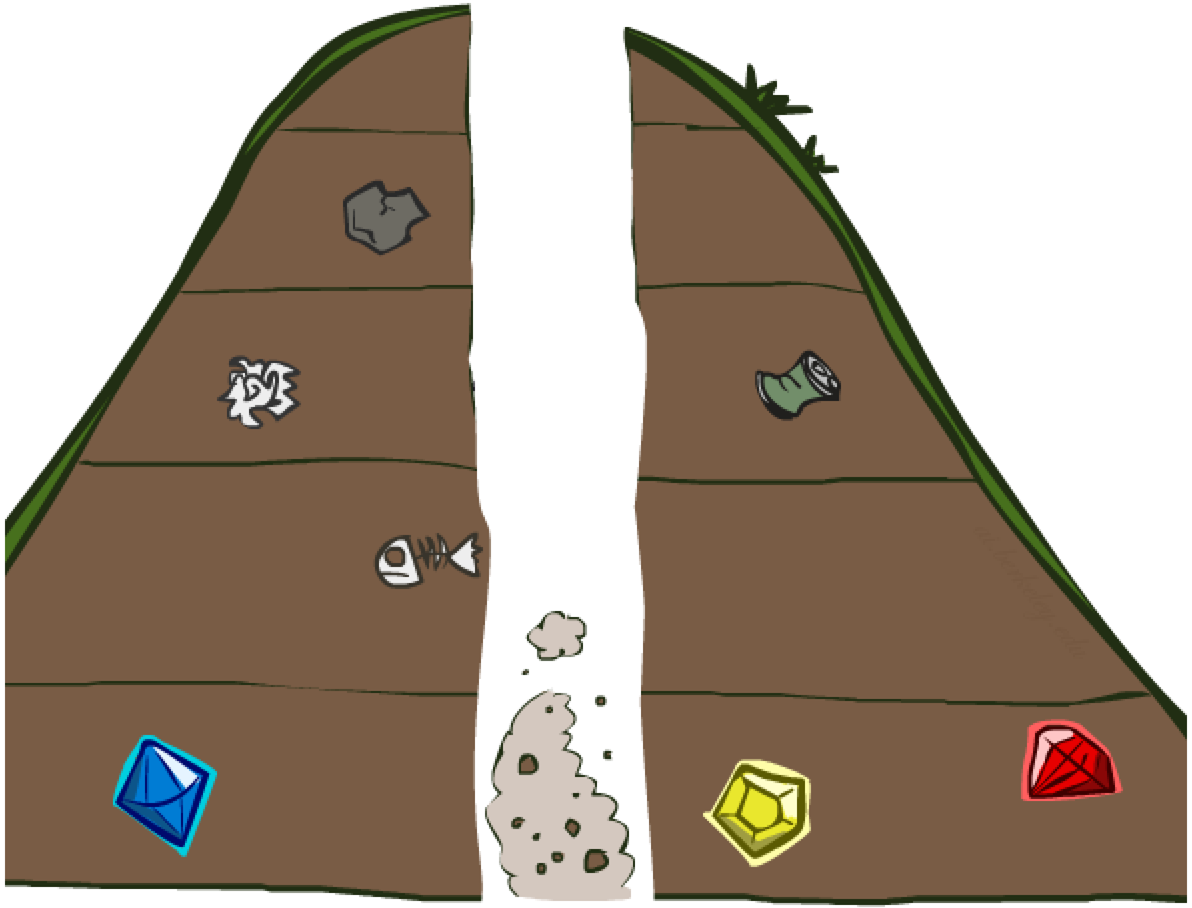
action = *a*

path-cost = parent_node.path_cost + step_cost(parent_ node.state, *a*, self.state)

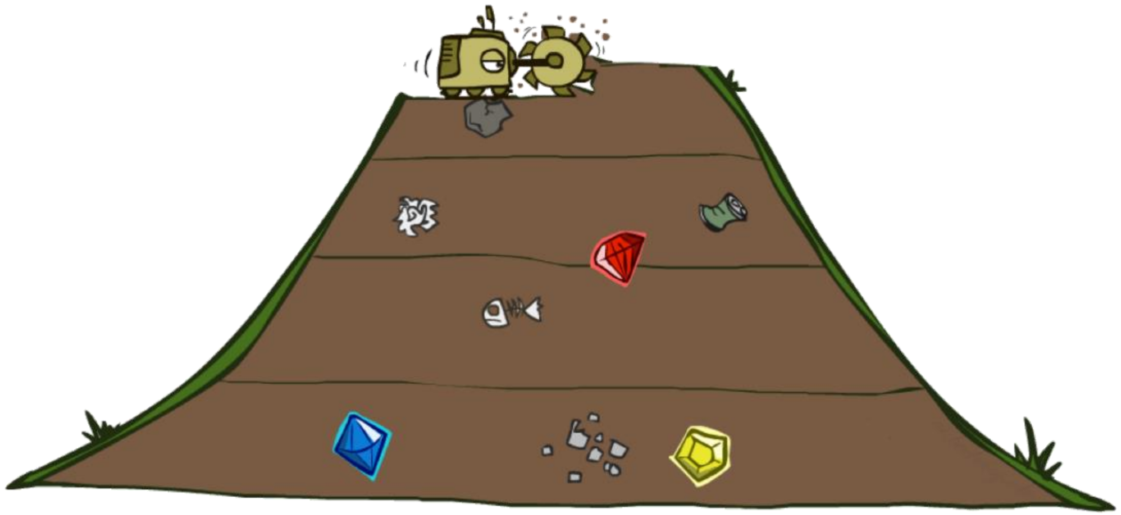


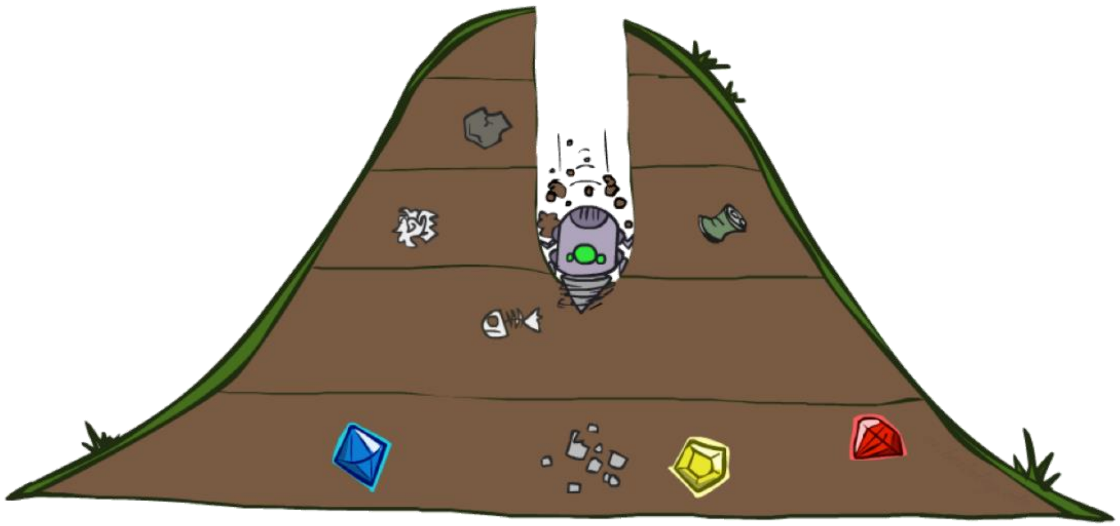
Extract solution by tracing back parent pointers, collecting actions

Search Algorithm Properties



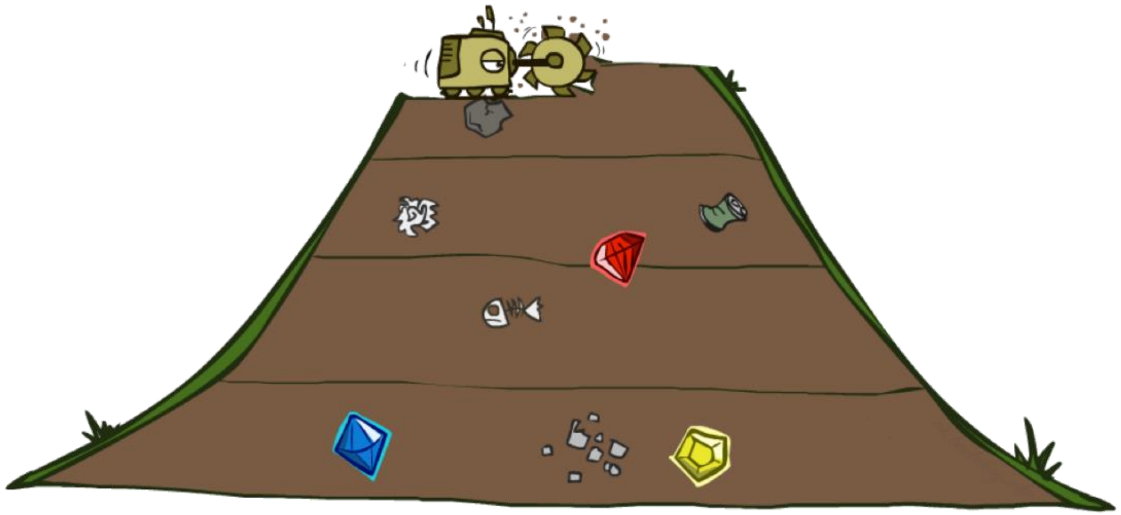
BFS vs DFS

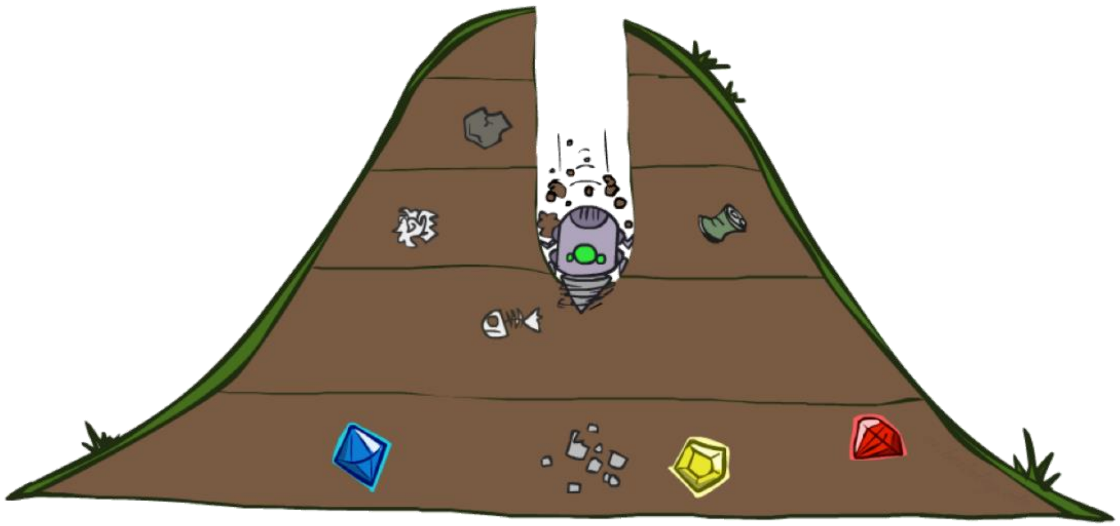




BFS vs DFS

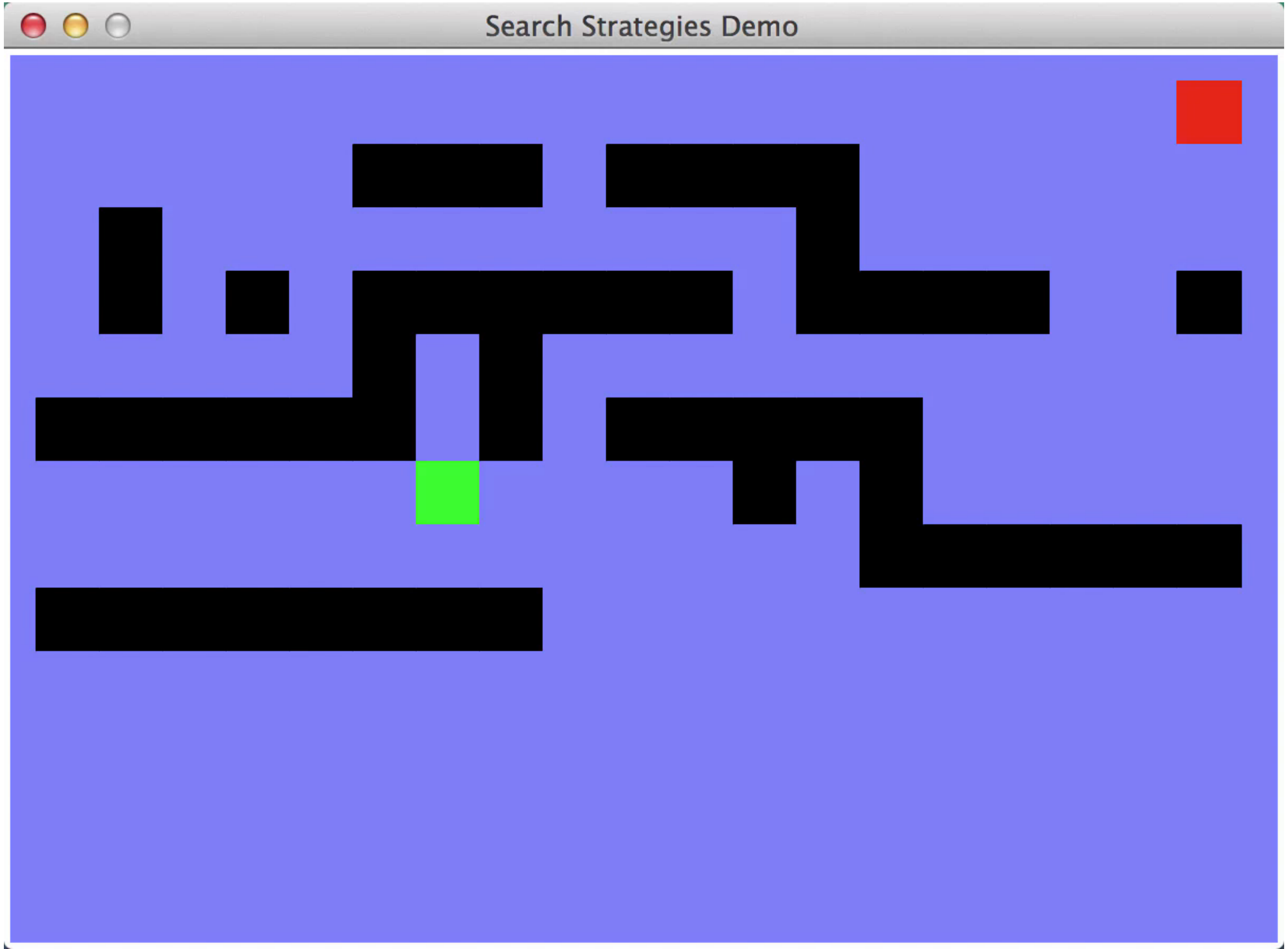
Is the following demo using BFS or DFS



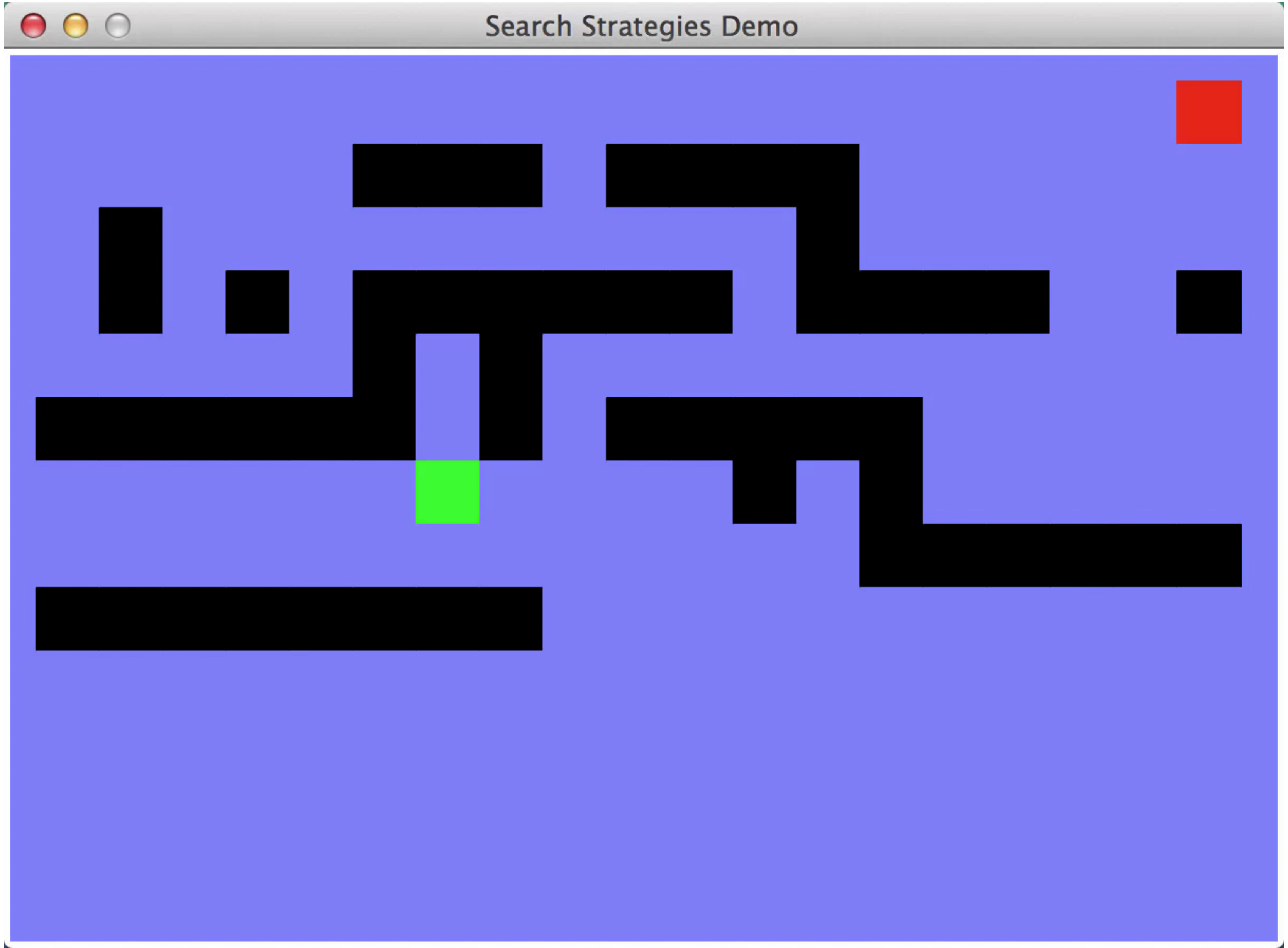


[Demo: dfs/bfs maze water (L2D6)]

Video of Demo Maze Water DFS/BFS (part 1)



Video of Demo Maze Water DFS/BFS (part 2)



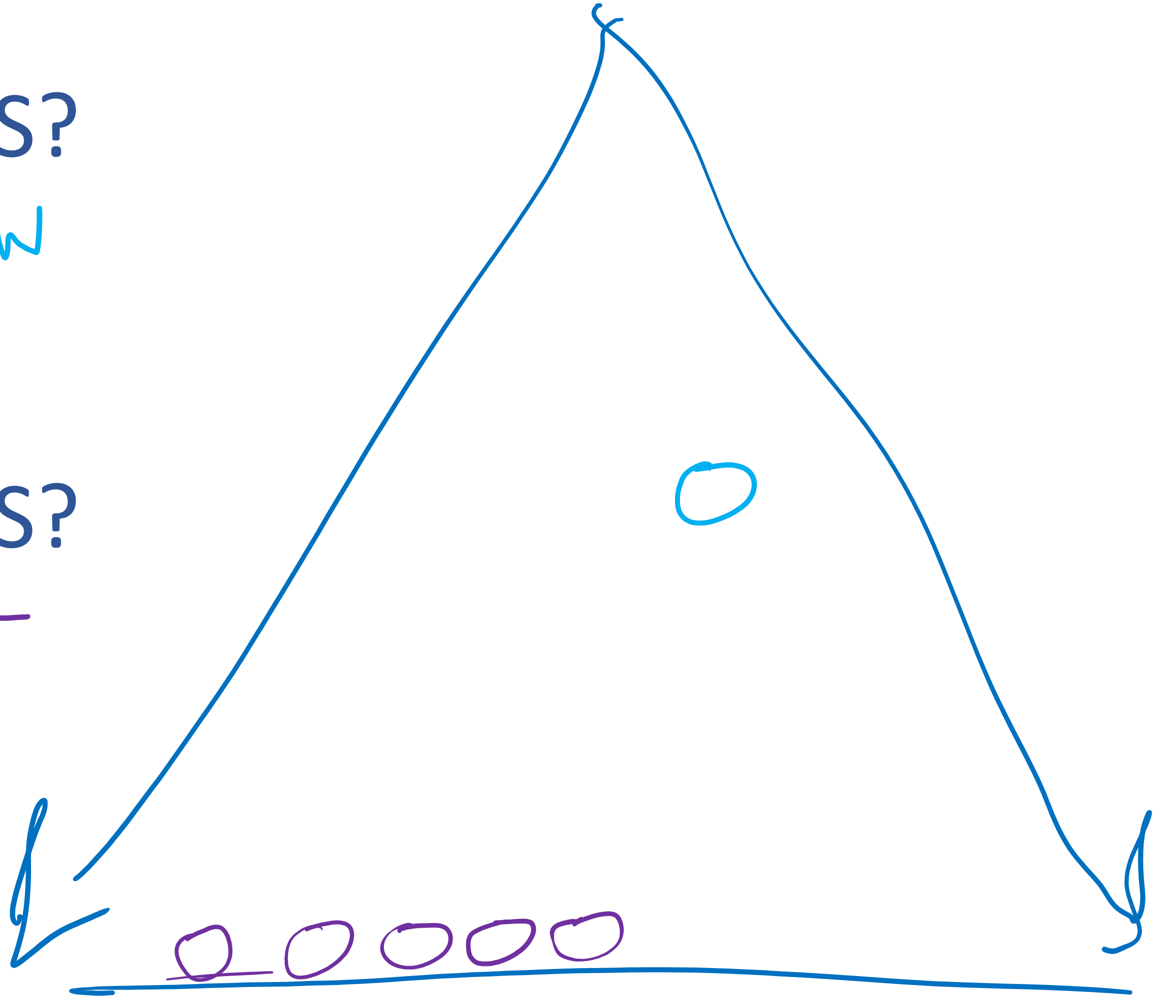
BFS vs DFS

When will BFS outperform DFS?

Optimal goal is shallow

When will DFS outperform BFS?

Optimal goal(s) are at max depth



Search Algorithm Properties

Complete: Guaranteed to find a solution if one exists?

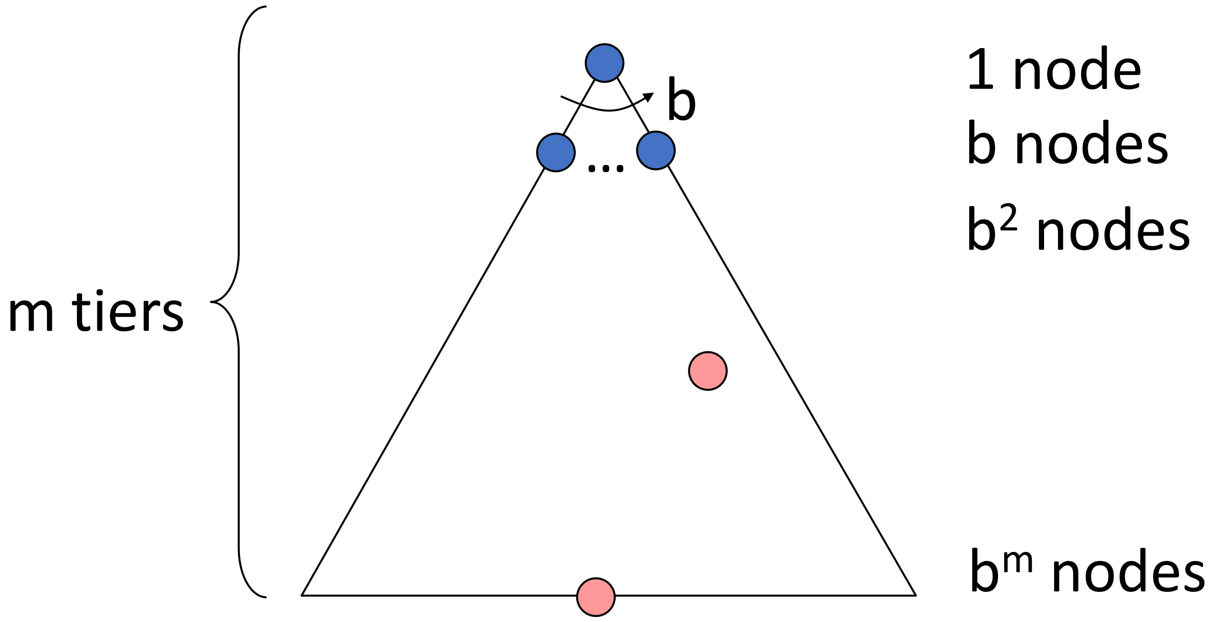
Optimal: Guaranteed to find the least cost path?

Time complexity?

Space complexity?

Cartoon of search tree:

- b is the branching factor
- m is the maximum depth
- solutions at various depths



Number of nodes in entire tree?

- $1 + b + b^2 + \ldots. b^m = O(b^m)$

Search Algorithm Properties

Complete: Guaranteed to find a solution if one exists?

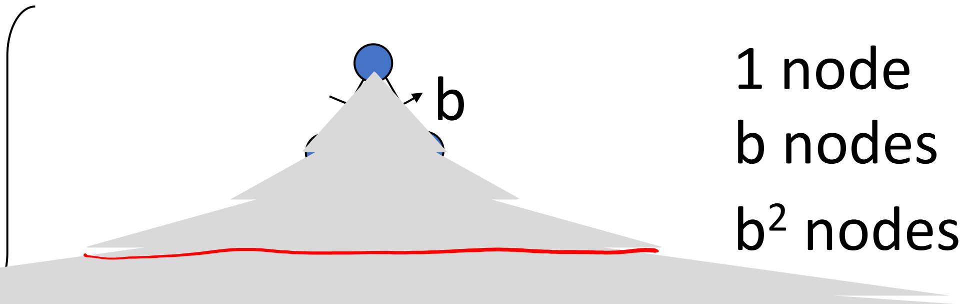
Optimal: Guaranteed to find the least cost path?

Time complexity?

Space complexity?

Cartoon of search tree:

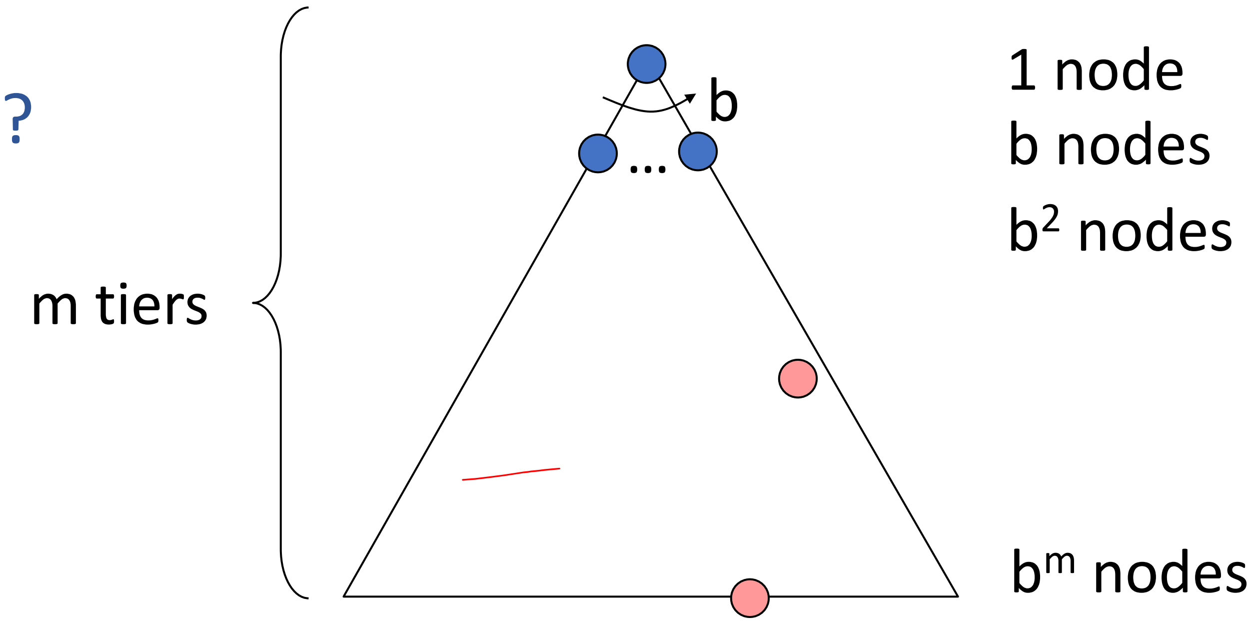
- b is the branching factor



Poll 2

Are these the properties for BFS or DFS?

- Takes $O(b^m)$ time
- Uses $O(bm)$ space on frontier
- Complete with graph search
- Not optimal unless all goals are in the same level (and the same step cost everywhere)



Depth-First Search (DFS) Properties

What nodes does DFS expand?

- Some left prefix of the tree.
- Could process the whole tree!
- If m is finite, takes time $O(b^m)$

How much space does the frontier take?

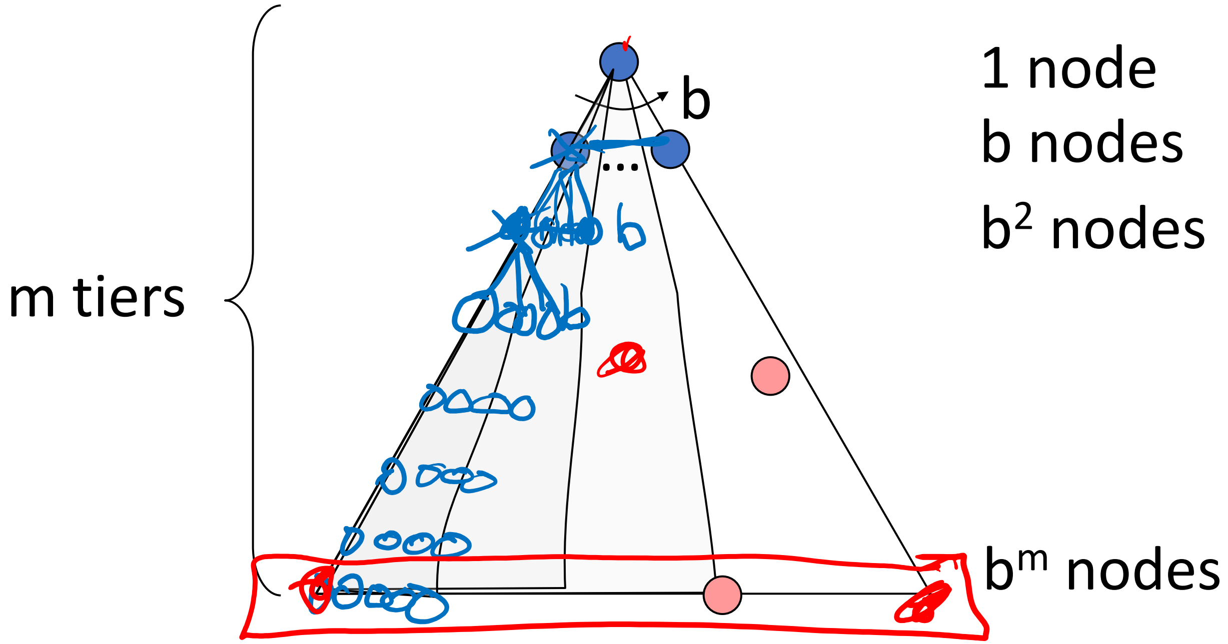
- Only has siblings on path to root, so $O(bm)$

Is it complete?

- m could be infinite, so only if we prevent cycles (graph search)

Is it optimal?

- No, it finds the "leftmost" solution, regardless of depth or cost



Breadth-First Search (BFS) Properties

What nodes does BFS expand?

- Processes all nodes above shallowest solution
- Let depth of shallowest solution be s
- Search takes time $O(b^s)$

How much space does the frontier take?

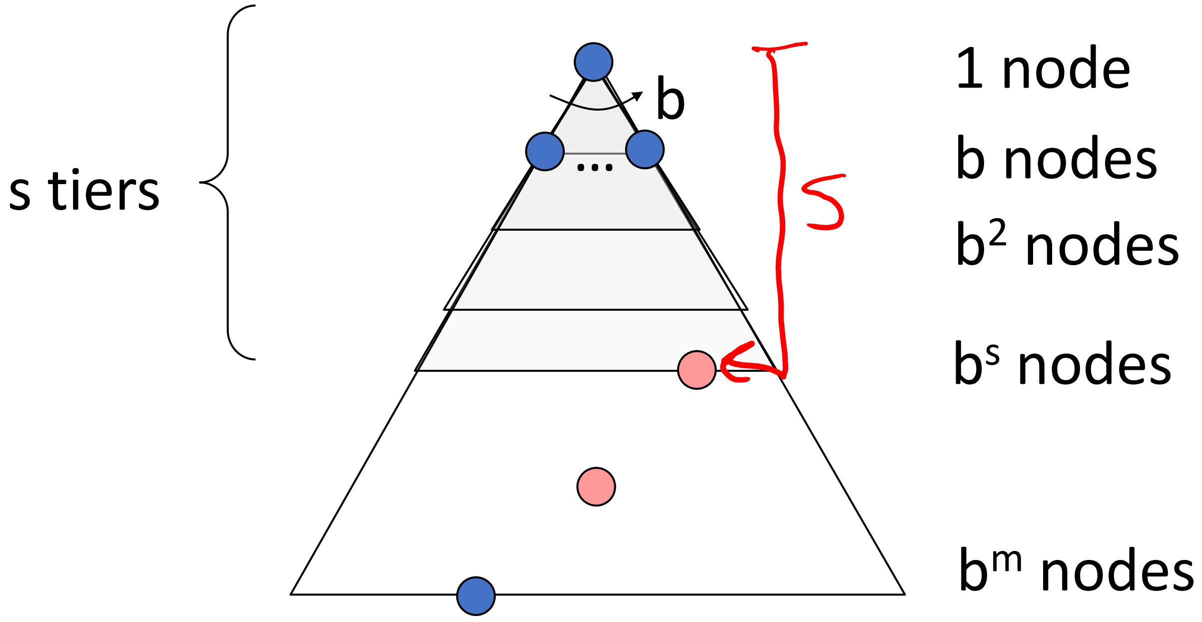
- Has roughly the last tier, so $O(b^s)$

Is it complete?

- s must be finite if a solution exists, so yes!

Is it optimal?

- Only if costs are all the same (more on costs later)



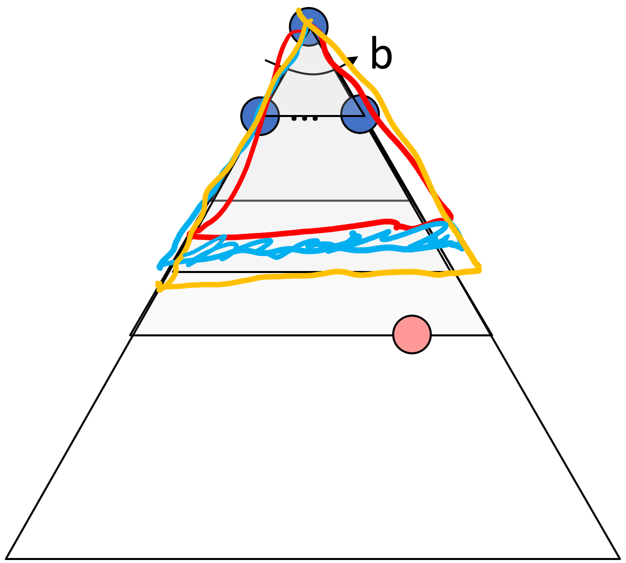
Iterative Deepening

Idea: get DFS's space advantage with BFS's time / shallow-solution advantages

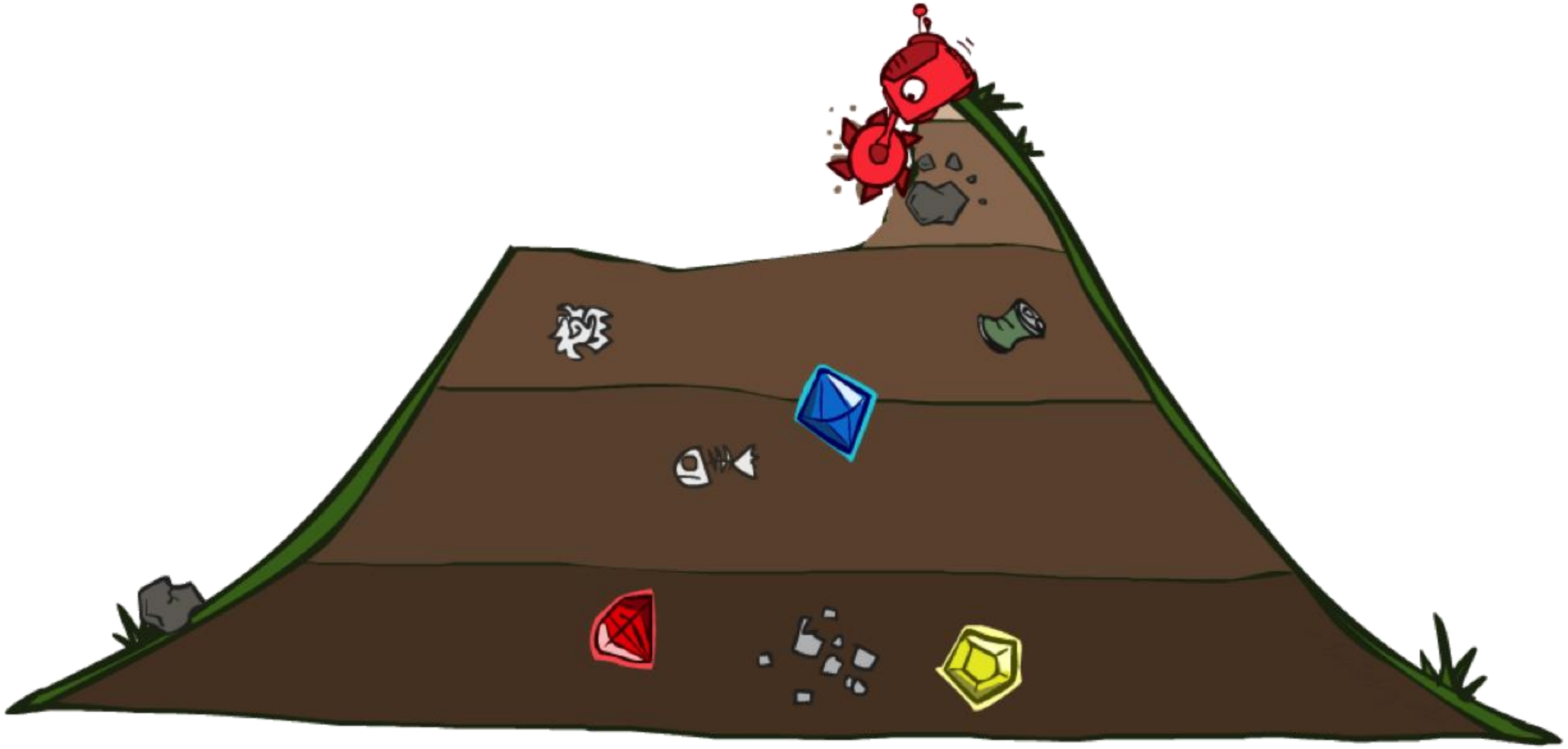
- Run a DFS with depth limit 1. If no solution…
- Run a DFS with depth limit 2. If no solution…
- Run a DFS with depth limit 3. …..

Isn't that wastefully redundant?

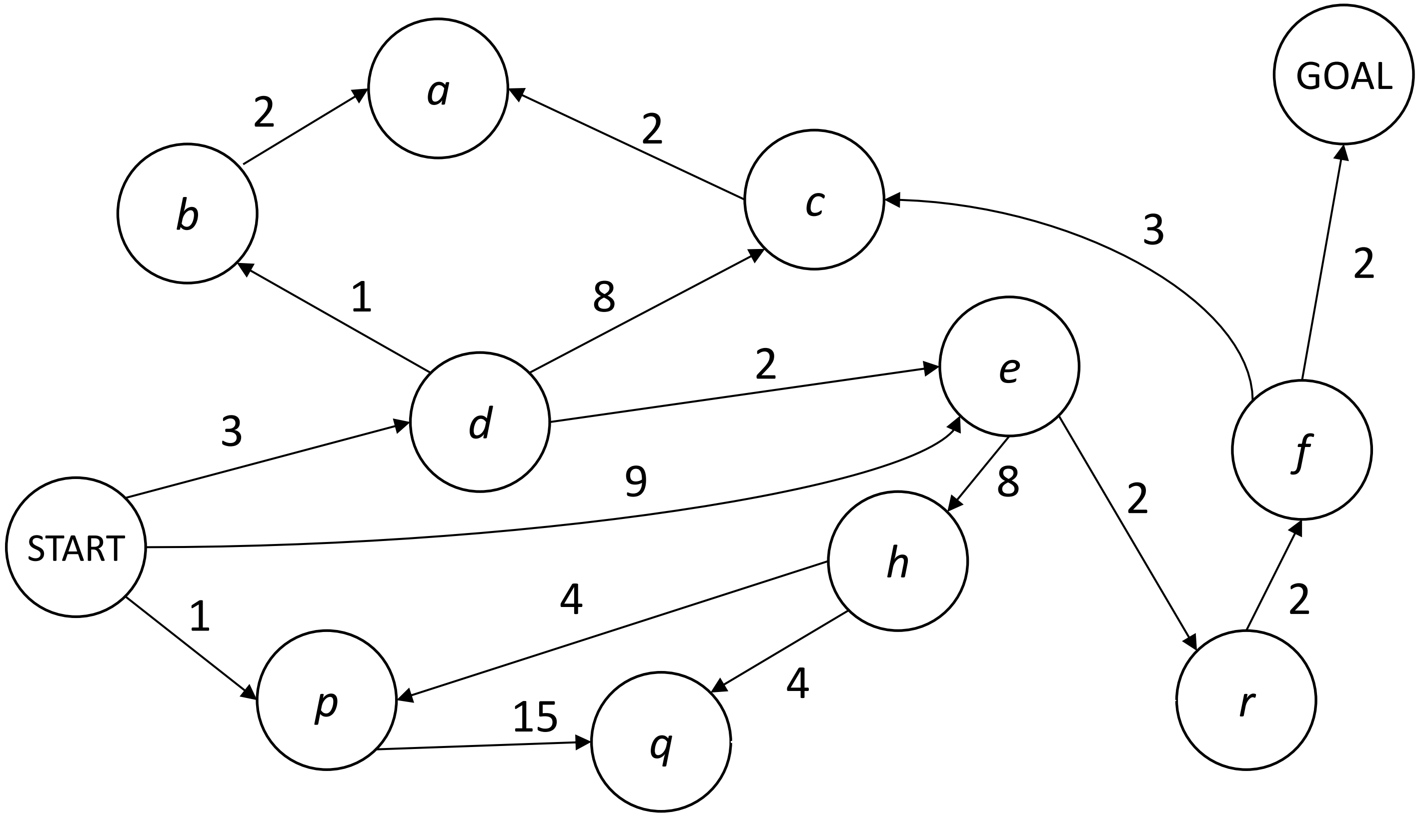
- Generally most work happens in the lowest level searched, so not so bad!



Uniform Cost Search



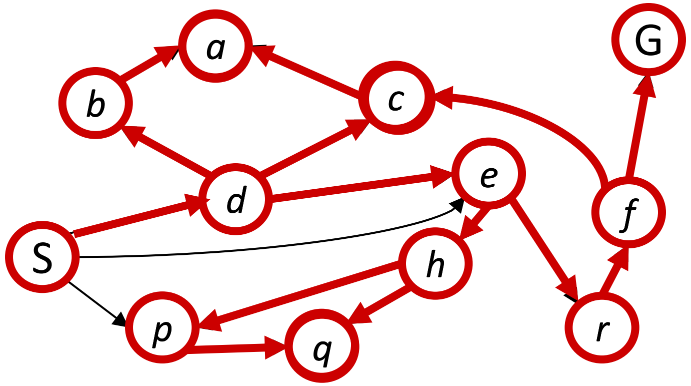
Finding a Least-Cost Path

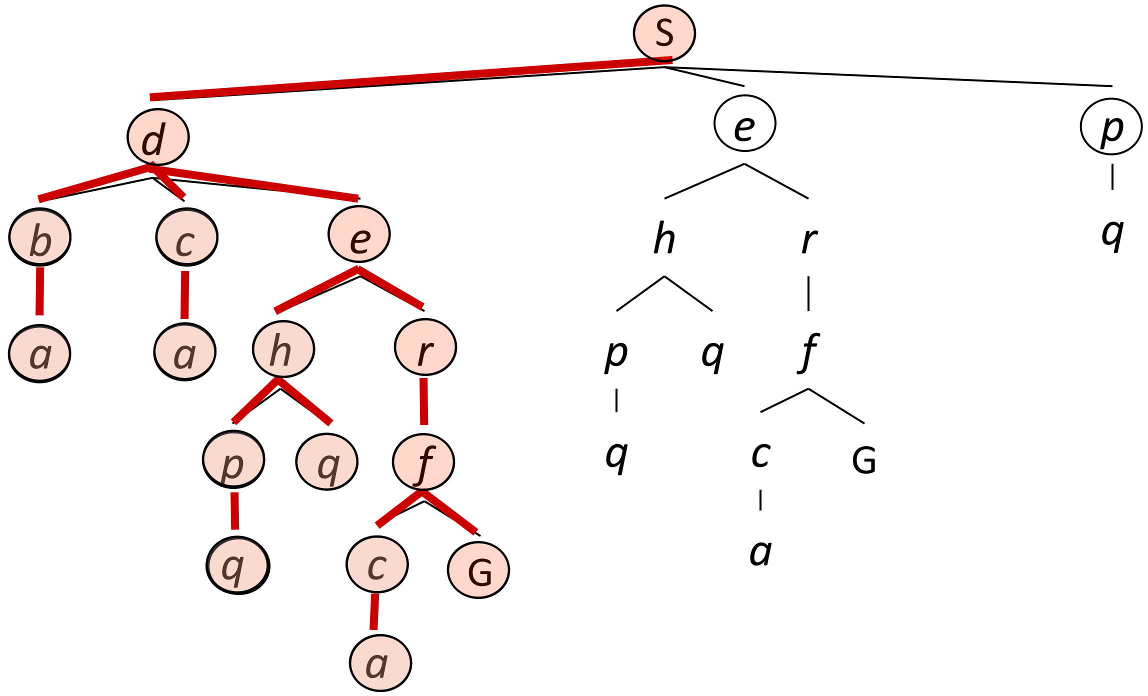


Depth-First (Tree) Search

Strategy: expand a deepest node first

Implementation:
Frontier is a LIFO stack

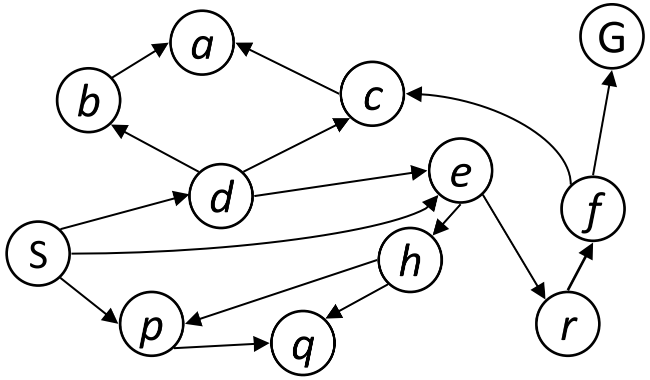


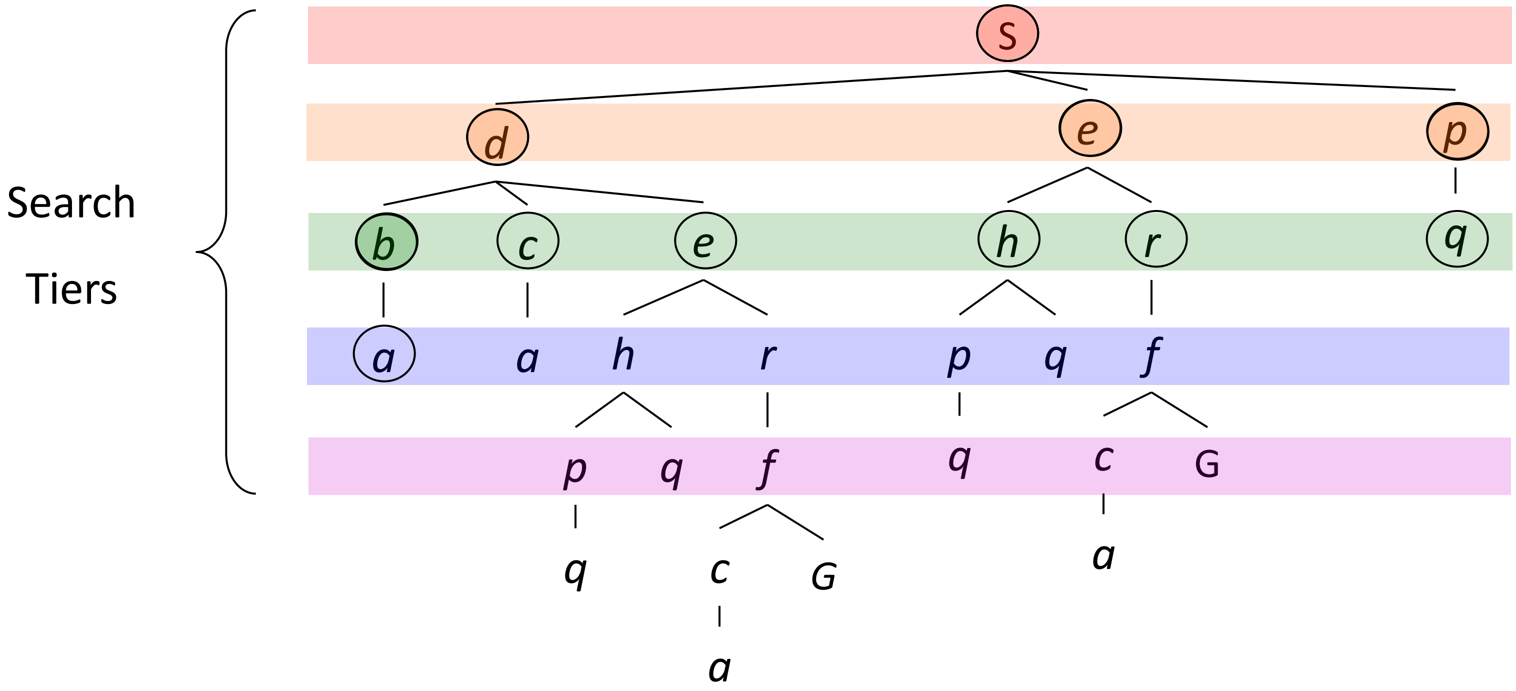


Breadth-First (Tree) Search

Strategy: expand a shallowest node first

Implementation:
Frontier is a FIFO queue



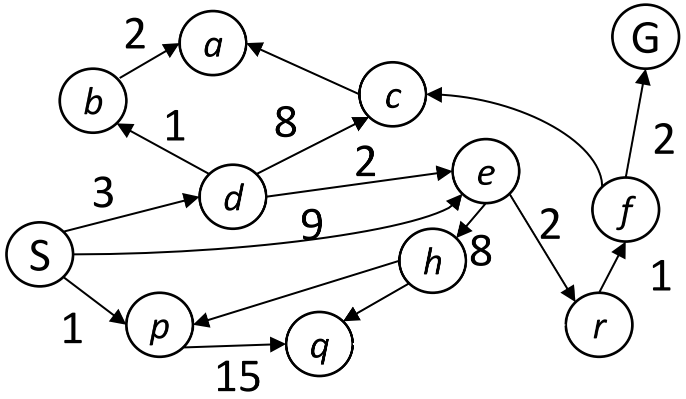


Uniform Cost (Tree) Search

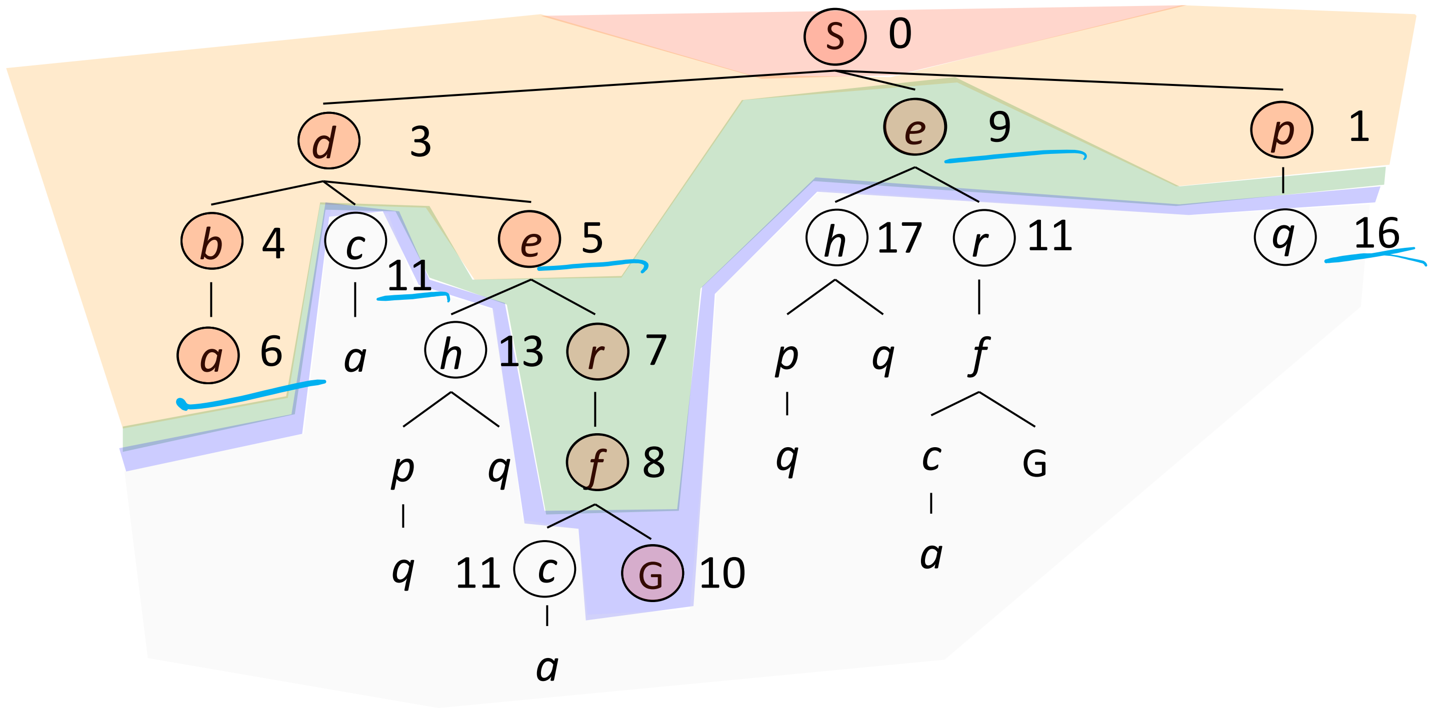
Strategy: expand a cheapest node first:

Frontier is a priority queue (priority: cumulative cost)

g(node)



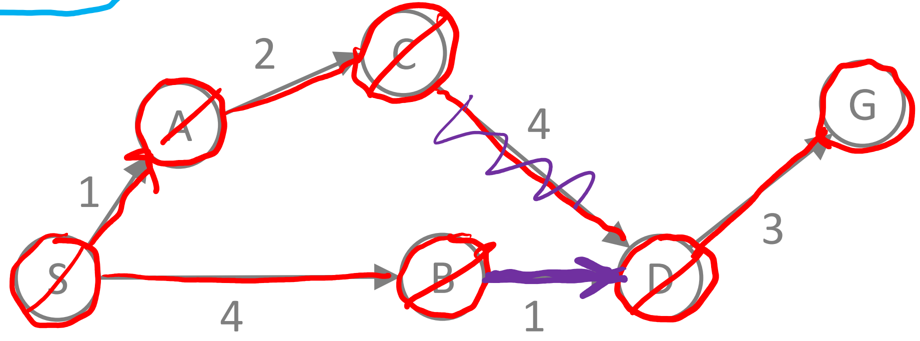
Cost contours



```
function GRAPH_SEARCH(problem) returns a solution, or failure
    initialize the explored set to be empty
    initialize the frontier as a specific work list (stack, queue, priority queue)
    add initial state of problem to frontier
    loop do
        if the frontier is empty then
            return failure
        choose a node and remove it from the frontier
        if the node contains a goal state then
            return the corresponding solution
        add the node state to the explored set
        for each resulting child from node
            if the child state is not already in the frontier or explored set then
                add child to the frontier
```

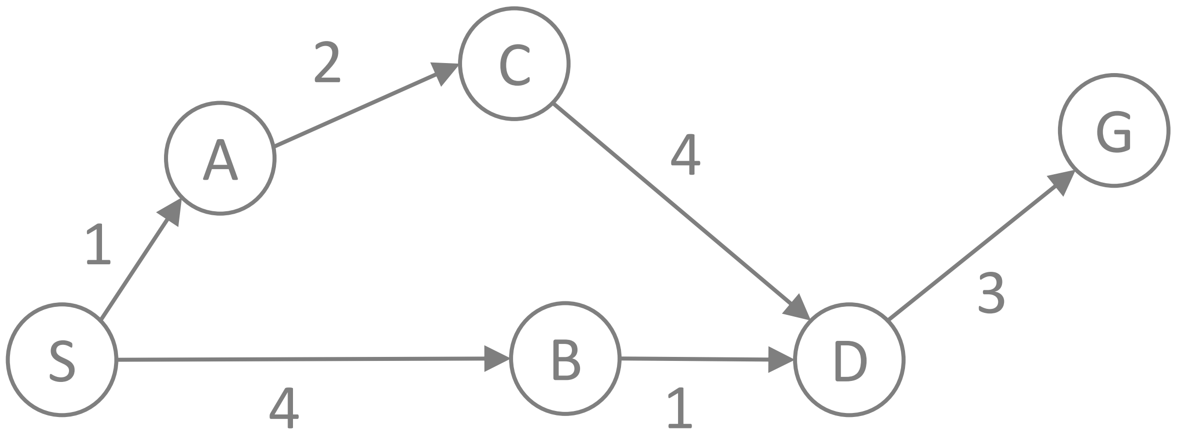
```
function UNIFORM-COST-SEARCH(problem) returns a solution, or failure
    initialize the explored set to be empty
    initialize the frontier as a priority queue using node path_cost as the priority
    add initial state of problem to frontier with path_cost = 0
    loop do
        if the frontier is empty then
            return failure
        choose a node and remove it from the frontier
        if the node contains a goal state then
            return the corresponding solution
        add the node state to the explored set
        for each resulting child from node
            if the child state is not already in the frontier or explored set then
                add child to the frontier
            else if the child is already in the frontier with higher path_cost then
                replace that frontier node with child
```

g(n)



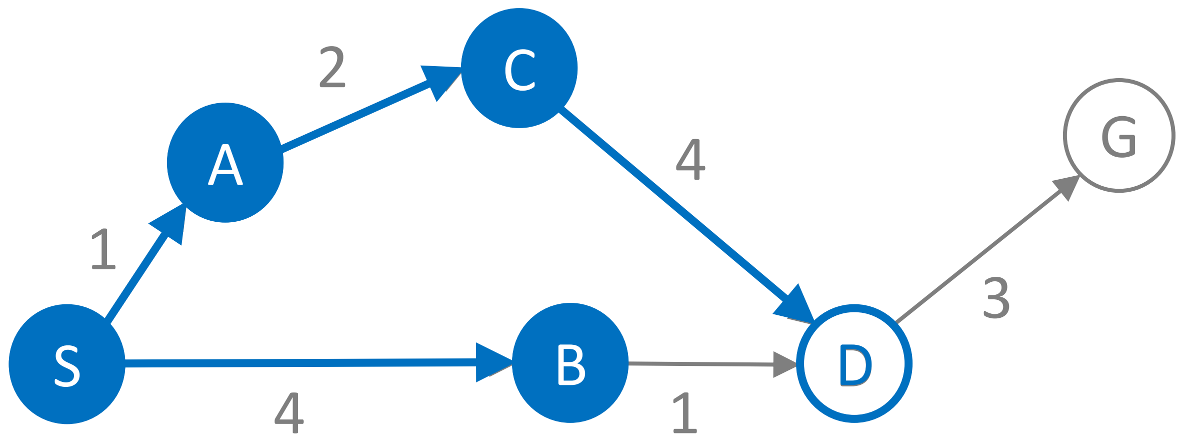
Frontier		Expl.
S:0	S-A-C:3	S
S-A:1	S-A-C-D:7	A
S-B:4	S-B-D:5	C
	S-B-D-G:8	B
		D

Walk-through UCS



Walk-through UCS

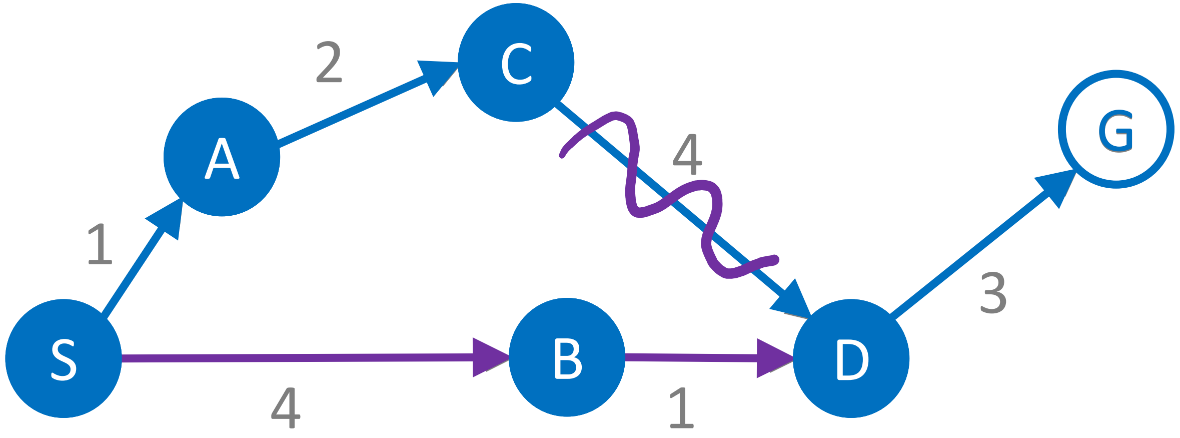
Frontier	Explored
~~S: 0~~	S
~~S-A: 1~~	A
~~S-B: 4~~	C
~~S-A-C: 3~~	B
S-A-C-D: 7	
S-B-D: 5 ??	



Walk-through UCS

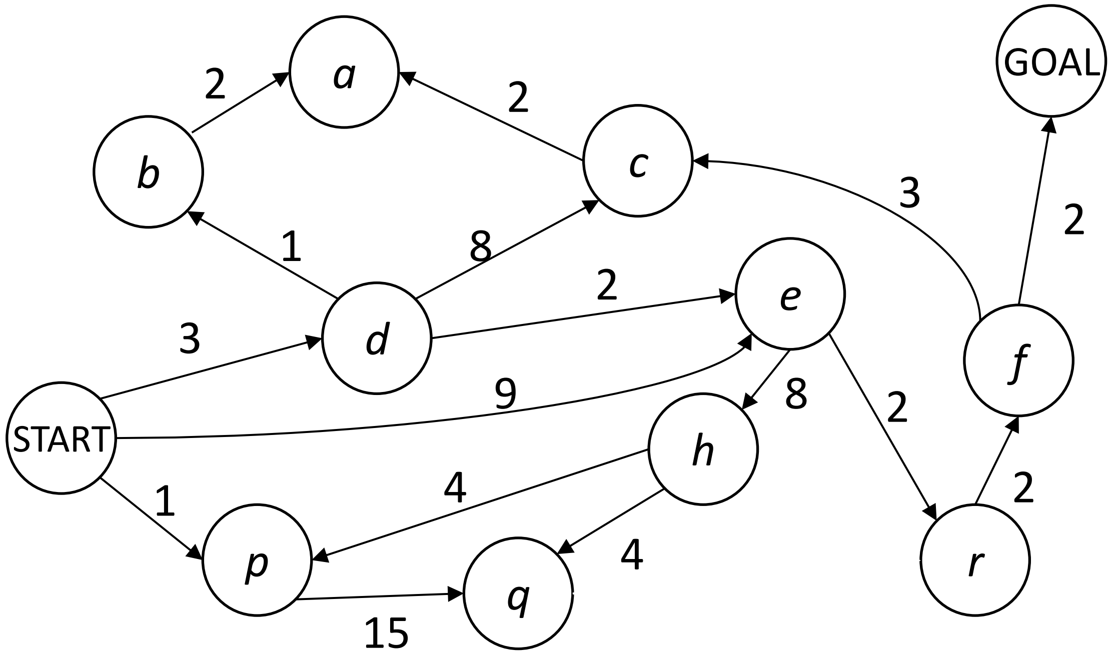
Frontier	Explored
~~S: 0~~	S
~~S-A: 1~~	A
~~S-B: 4~~	C
~~S-A-C: 3~~	B
S-A-C-D: 7	D
~~S-B-D: 5~~	
~~S-B-D-G: 8~~	

Replaced by better path to D!

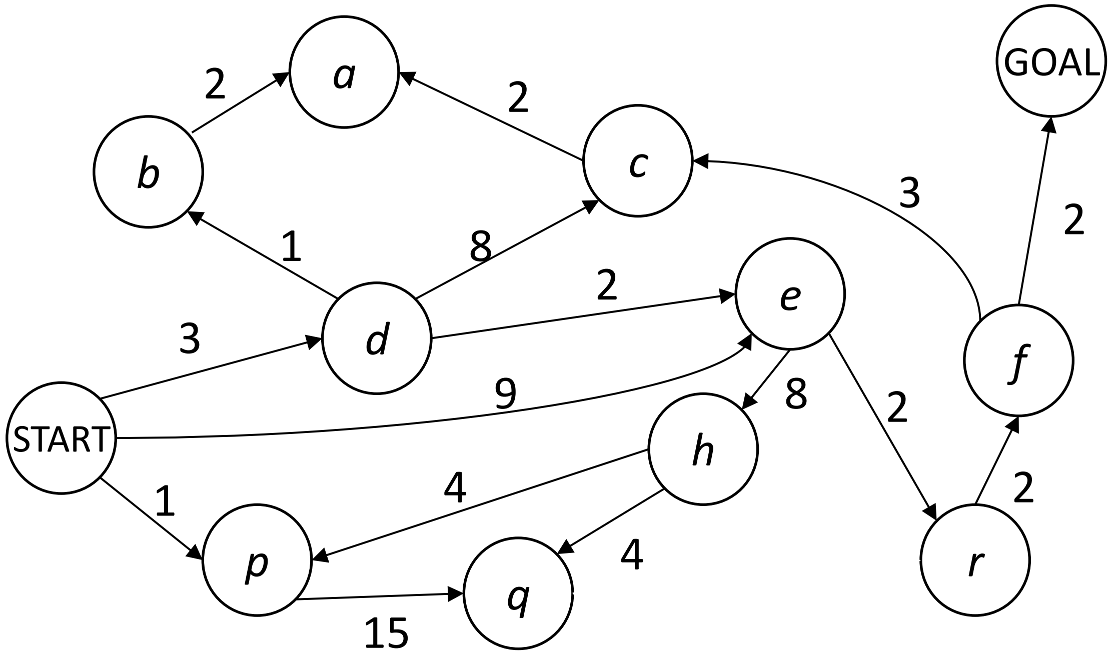


Result: S-B-D-G (path cost 8)

UCS: Another Example



UCS: Another Example



Frontier

~~S: 0~~

~~S-d: 3~~

S-e: 9

~~S-p: 1~~

S-p-q: 16

S-d-b: 4

S-d-c: 11

S-d-e: 5 ??

Explored

S

p

d

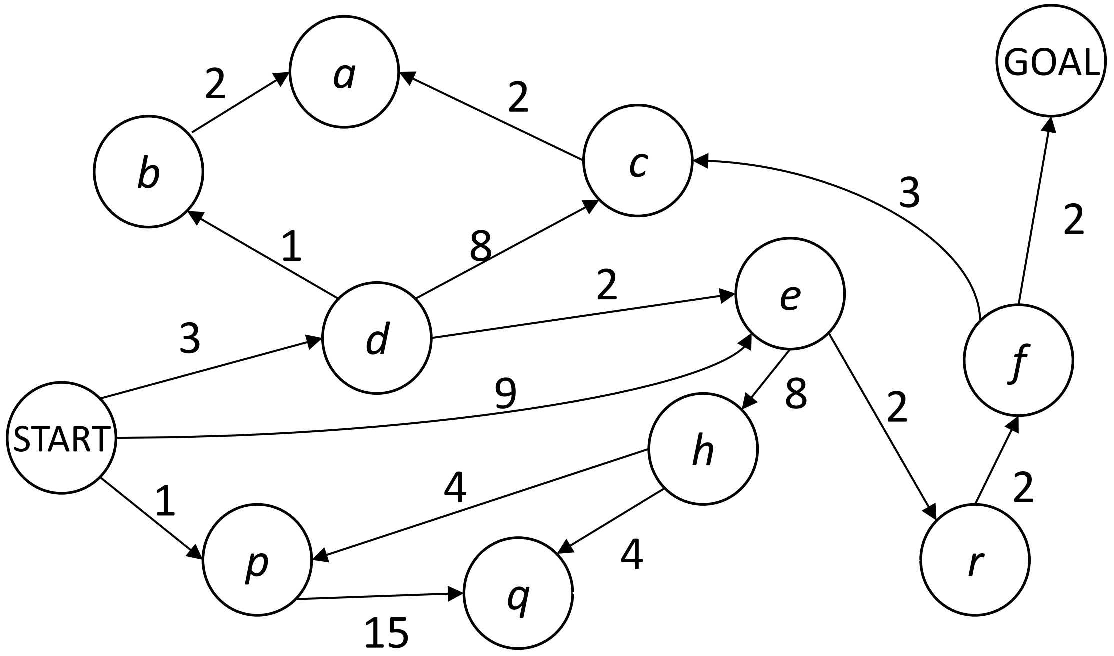
UCS: Another Example

Frontier

~~S: 0~~
~~S-d: 3~~
~~S-e: 9~~
~~S-p: 1~~
S-p-q: 16
~~S-d-b: 4~~
S-d-c: 11
~~S-d-e: 5~~
~~S-d-b-a: 6~~
S-d-e-h: 13
~~S-d-e-r: 7~~
~~S-d-e-r-f: 9~~
S-d-e-r-f-c: 12??
S-d-e-r-f-G: 11

Explored

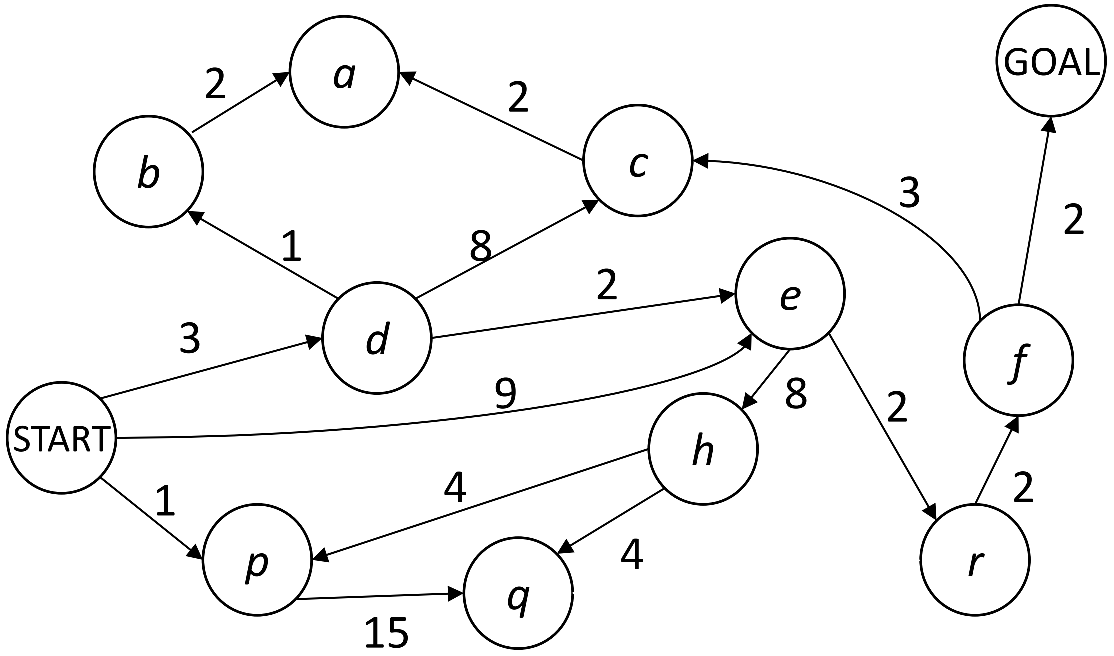
S
p
d
b
e
a
r
f



Add S-d-e-r-f-c: 12 to frontier?
→ No, there is a better path to c on the frontier, S-d-c: 11

I see G on the frontier. Are we done?
→ No, the goal test doesn't come until after we pop a node from the frontier

UCS: Another Example



Frontier

~~S: 0~~
~~S-d: 3~~
~~S-e: 9~~
~~S-p: 1~~
S-p-q: 16
~~S-d-b: 4~~
~~S-d-c: 11~~
~~S-d-e: 5~~
~~S-d-b-a: 6~~
S-d-e-h: 13
~~S-d-e-r: 7~~
~~S-d-e-r-f: 9~~
S-d-e-r-f-c: 12
~~S-d-e-r-f-G: 11~~

Explored

S
p
d
b
e
a
r
f
c

FYI: Breaking tie at cost 11 alphabetically

a is already on the explored set, so we don't consider adding S-d-c-a

Result: S-d-e-r-f-G with cost 11

Uniform Cost Search (UCS) Properties

What nodes does UCS expand?

- Processes all nodes with cost less than cheapest solution!
- If that solution costs C^* and arcs cost at least ε, then the "effective depth" is roughly C^*/ε
- Takes time $O(b^{C^*/\varepsilon})$ (exponential in effective depth)

How much space does the frontier take?

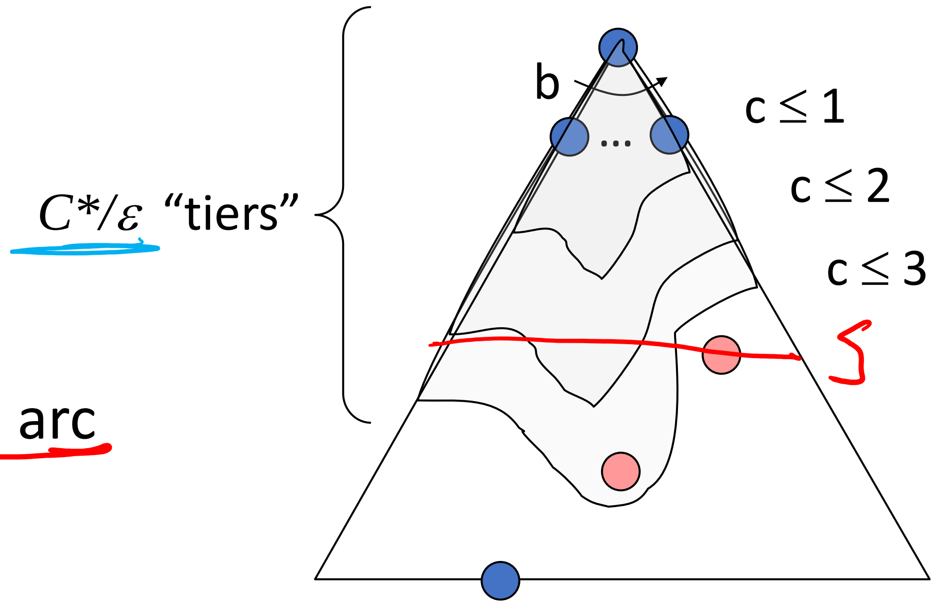
- Has roughly the last tier, so $O(b^{C^*/\varepsilon})$

Is it complete?

- Assuming best solution has a finite cost and minimum arc cost is positive, yes!

Is it optimal?

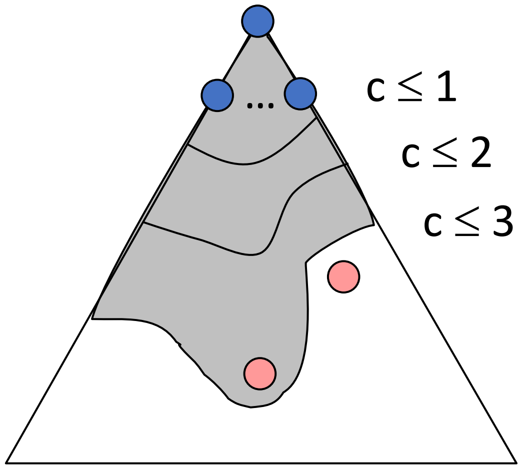
- Yes! (Proof next lecture via A*)



Uniform Cost Issues

Remember:

- UCS explores increasing cost contours

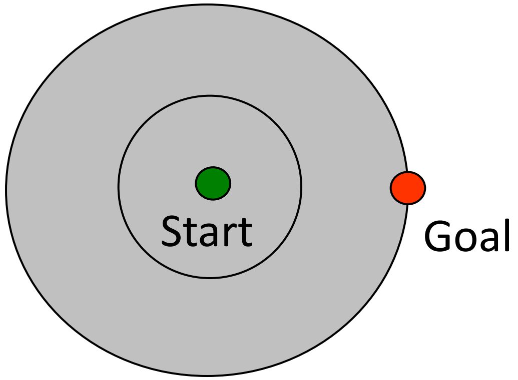


The good:

- UCS is complete and optimal!

The bad:

- Explores options in every "direction"
- No information about goal location



We'll fix that!