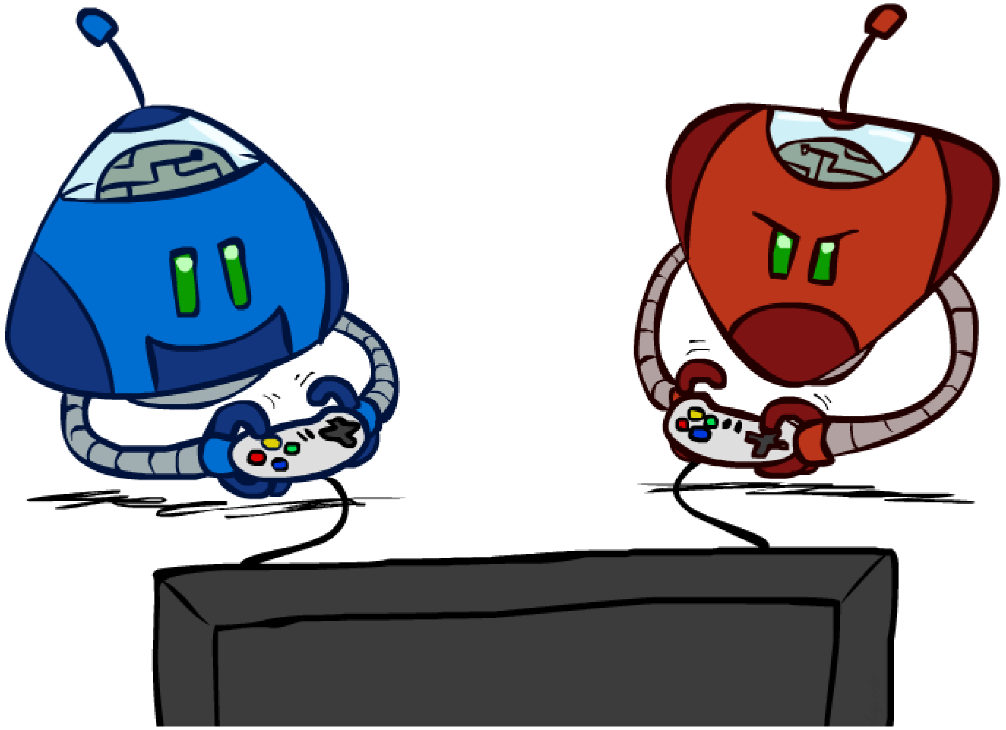
AI: Representation and Problem Solving

Adversarial Search



Instructor: Pat Virtue

Slide credits: CMU AI, http://ai.berkeley.edu

Outline

- History / Overview
- Zero-Sum Games (Minimax)
- Evaluation Functions
- Search Efficiency (α-β Pruning)
- Games of Chance (Expectimax)

Game Playing State-of-the-Art

α/β pruning

Checkers:

- 1950: First computer player.
- 1959: Samuel's self-taught program.
- 1994: First computer world champion: Chinook ended 40-year-reign of human champion Marion Tinsley using complete 8-piece endgame.
- 2007: Checkers solved! Endgame database of 39 trillion states

Heuristic

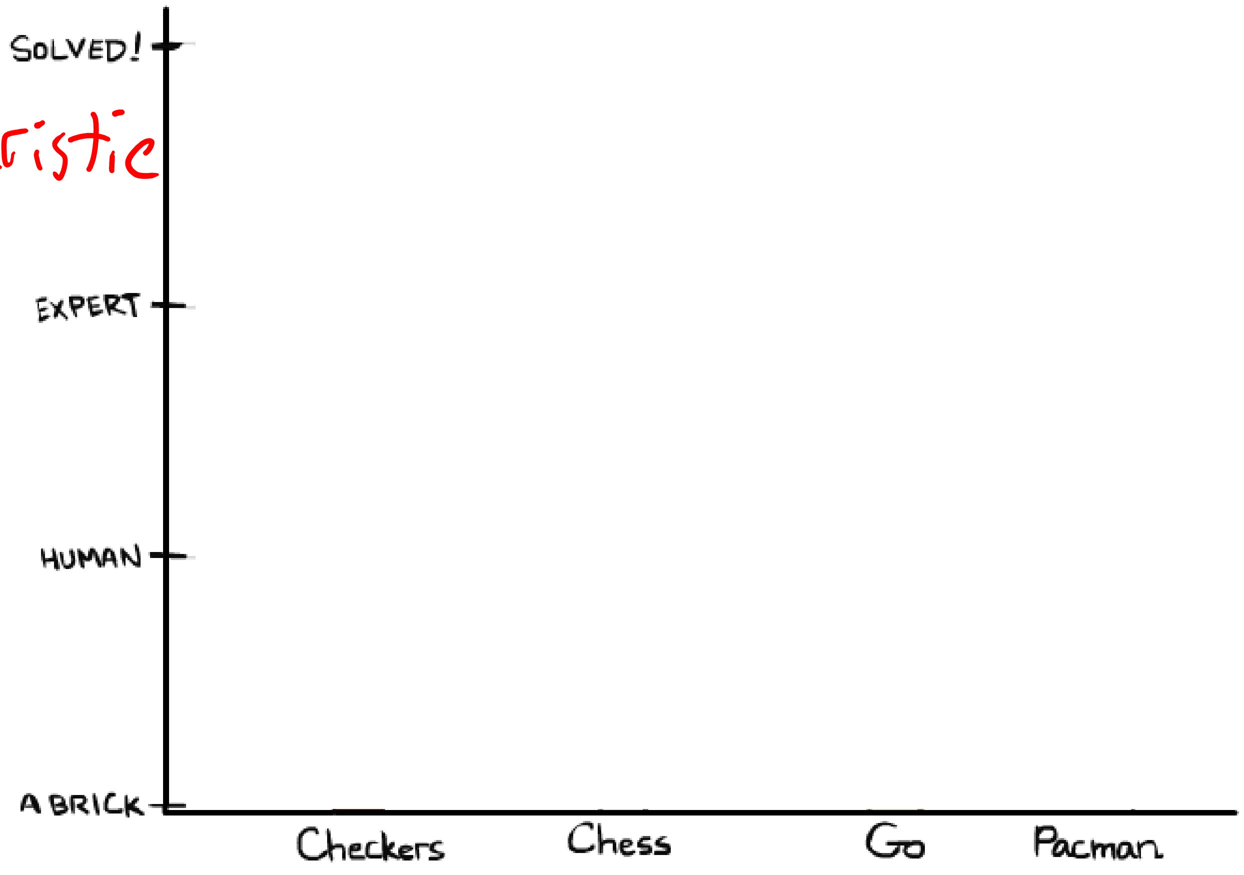
Chess:

- 1945-1960: Zuse, Wiener, Shannon, Turing, Newell & Simon, McCarthy.
- 1960s onward: gradual improvement under "standard model"
- 1997: special-purpose chess machine Deep Blue defeats human champion Gary Kasparov in a six-game match. Deep Blue examined 200M positions per second and extended some lines of search up to 40 ply. Current programs running on a PC rate > 3200 (vs 2870 for Magnus Carlsen).

300

Go:

- 1968: Zobrist's program plays legal Go, barely (b>300!)
- 2005-2014: Monte Carlo tree search enables rapid advances: current programs beat strong amateurs, and professionals with a 3-4 stone handicap.



Game Playing State-of-the-Art

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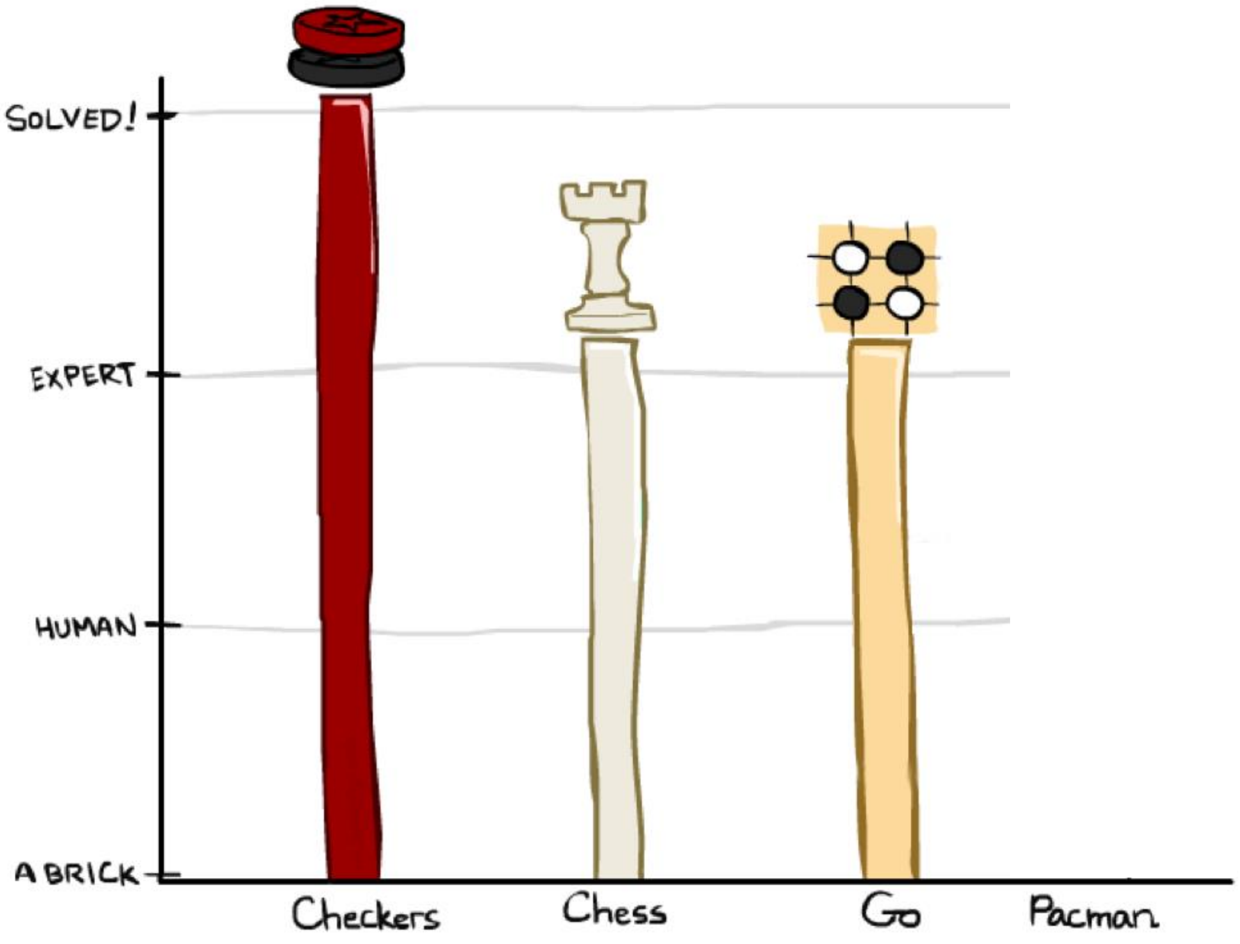
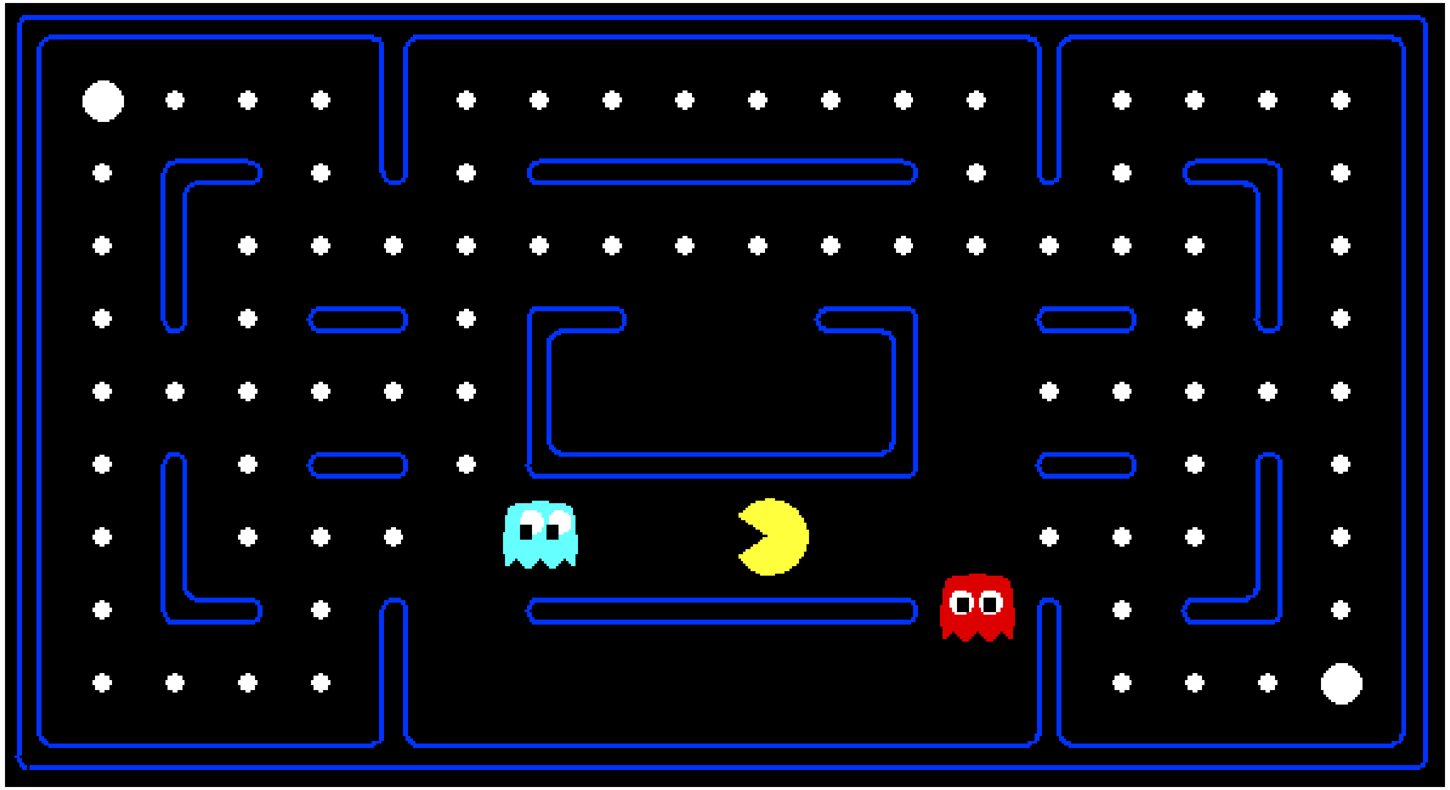
- 2015: AlphaGo from DeepMind beats Lee Sedol



Behavior from Computation



[Demo: mystery pacman (L6D1)]

Types of Games

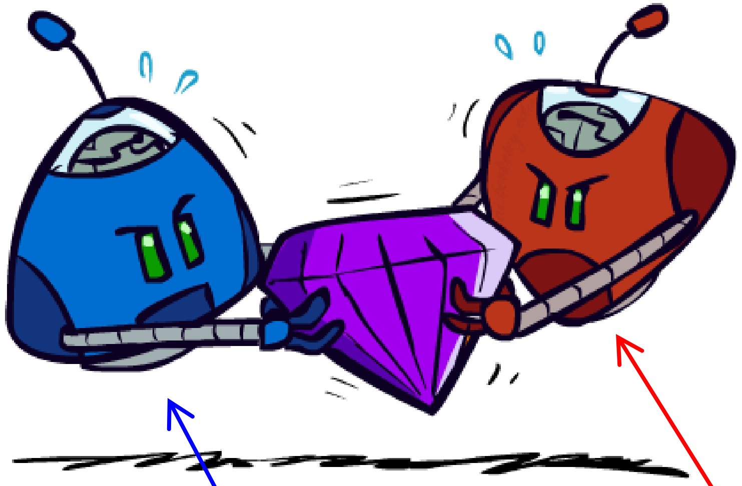
Many different kinds of games!

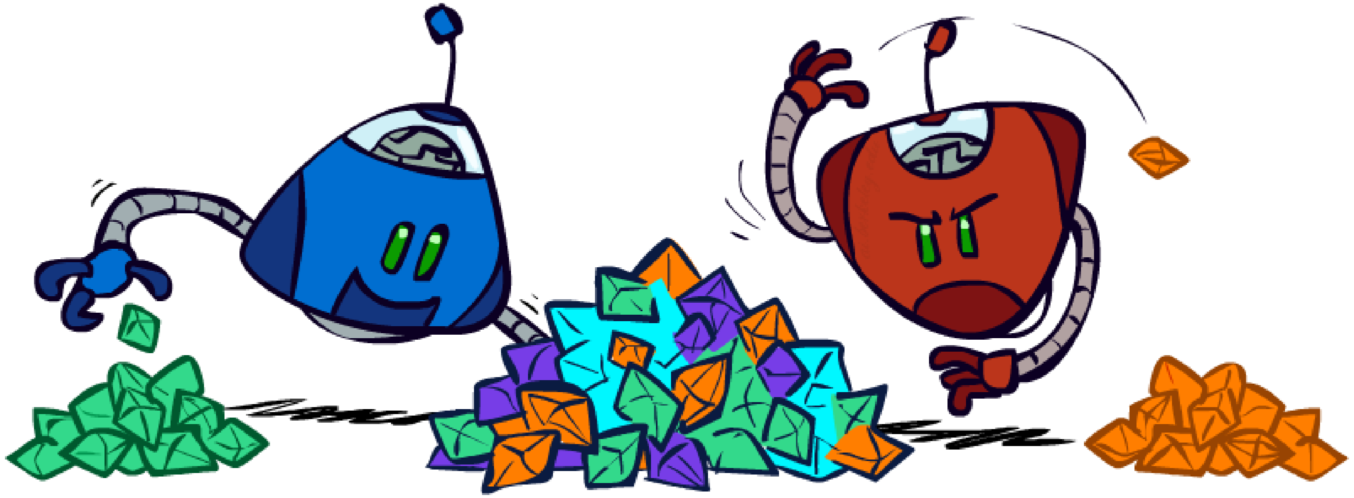
Axes:

- Deterministic or stochastic?
- Perfect information (fully observable)?
- One, two, or more players?
- Turn-taking or simultaneous?
- Zero sum?

Want algorithms for calculating a ***contingent plan*** (a.k.a. strategy or policy) which recommends a move for every possible eventuality

Zero-Sum Games





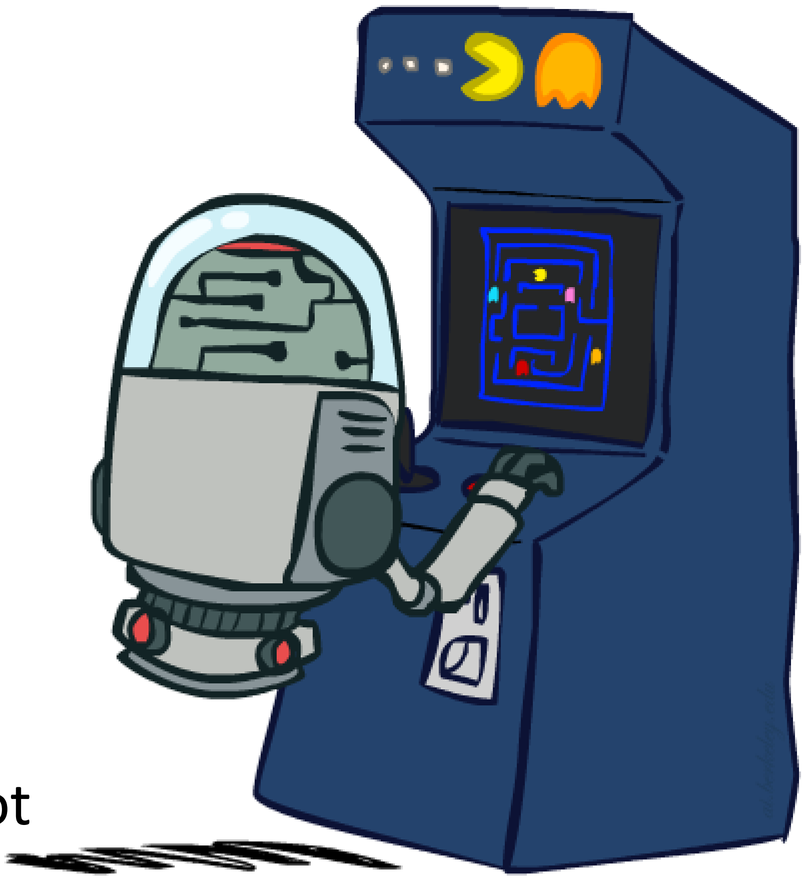
- Zero-Sum Games
  - Agents have ***opposite*** utilities
  - Pure competition:
    - One ***maximizes***, the other ***minimizes***
- General Games
  - Agents have ***independent*** utilities
  - Cooperation, indifference, competition, shifting alliances, and more are all possible

“Standard” Games

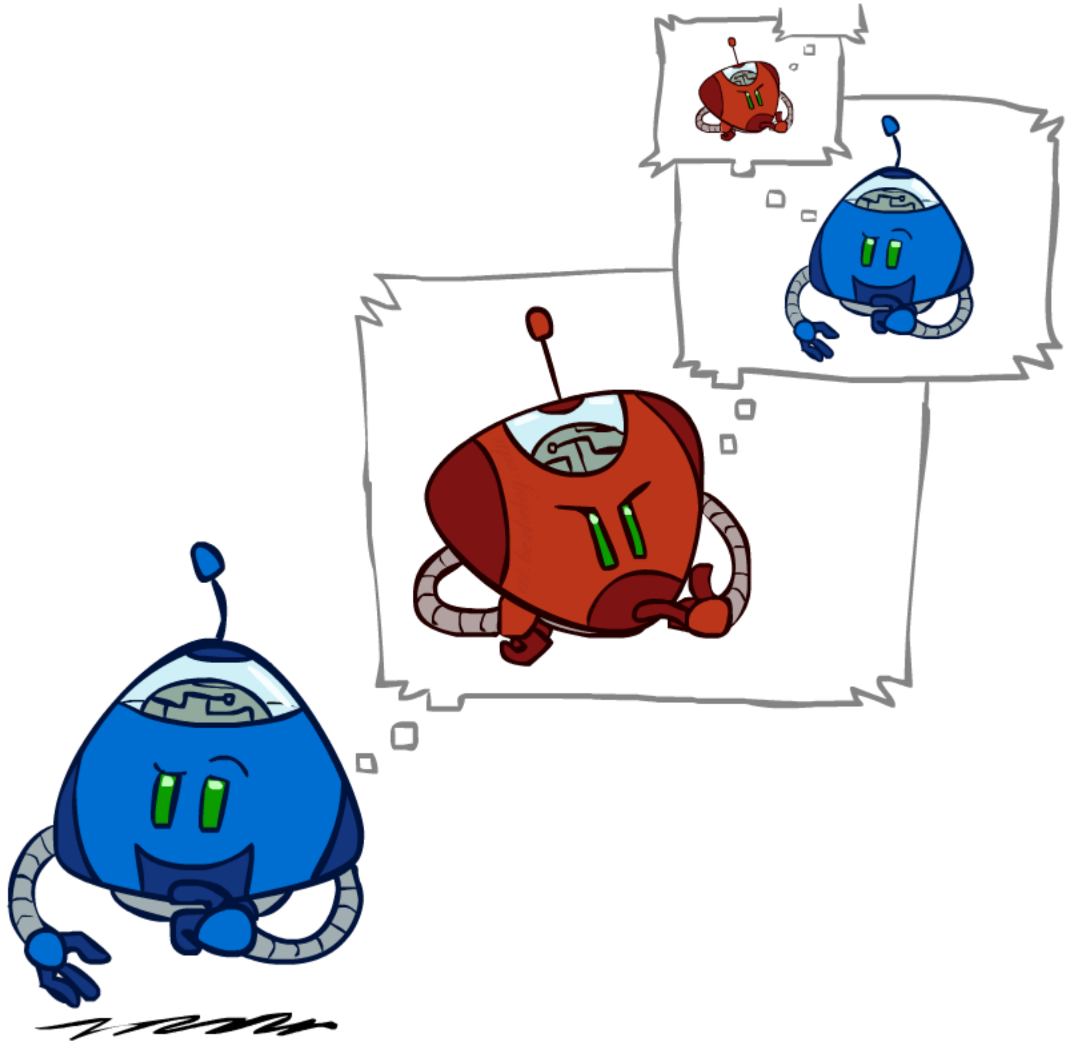
Standard games are deterministic, observable, two-player, turn-taking, zero-sum

Game formulation:

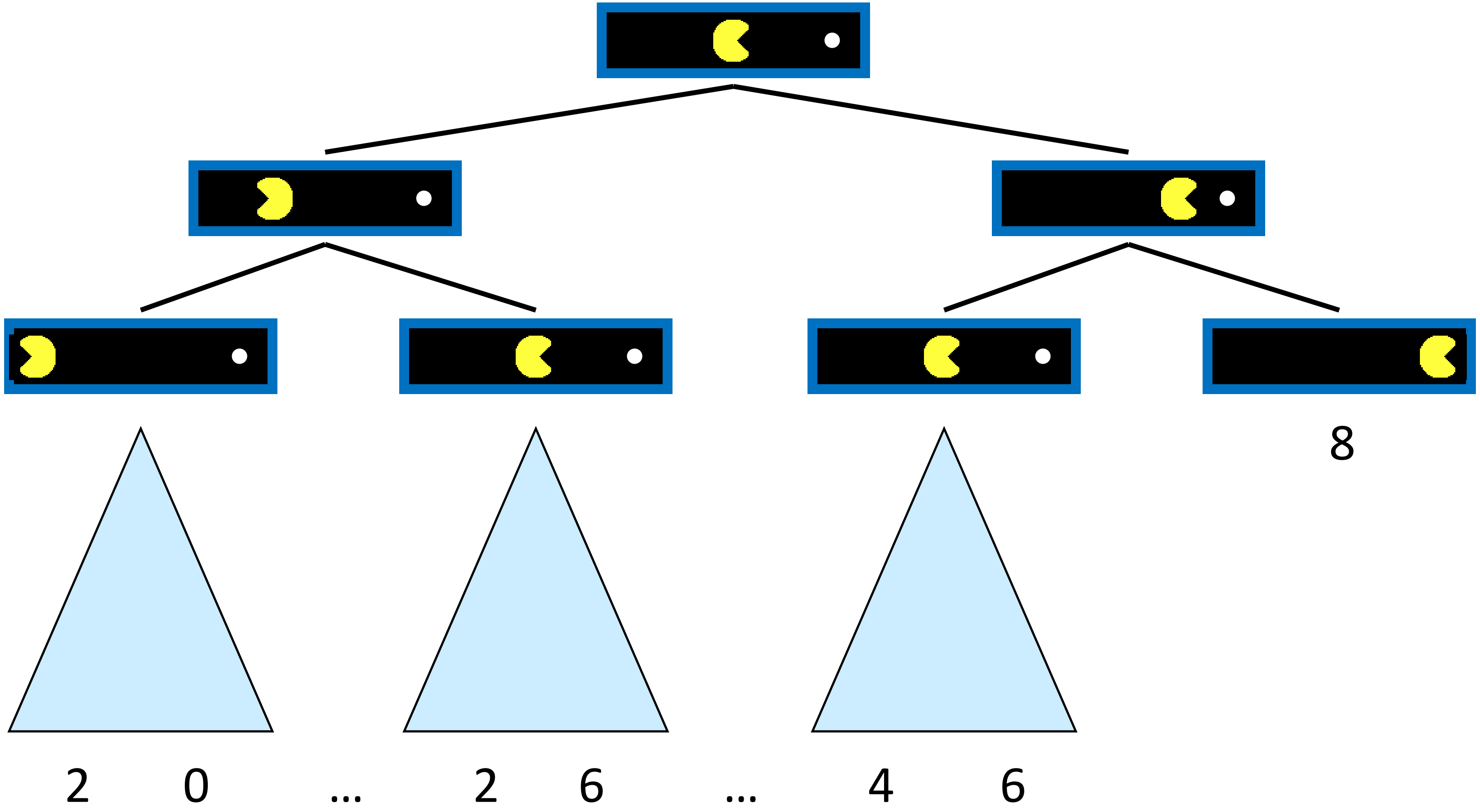
- Initial state: s_0
- Players: Player(s) indicates whose move it is
- Actions: Actions(s) for player on move
- Transition model: Result(s,a)
- Terminal test: Terminal-Test(s)
- Terminal values: Utility(s,p) for player p
  - Or just Utility(s) for player making the decision at root



Adversarial Search



Single-Agent Trees

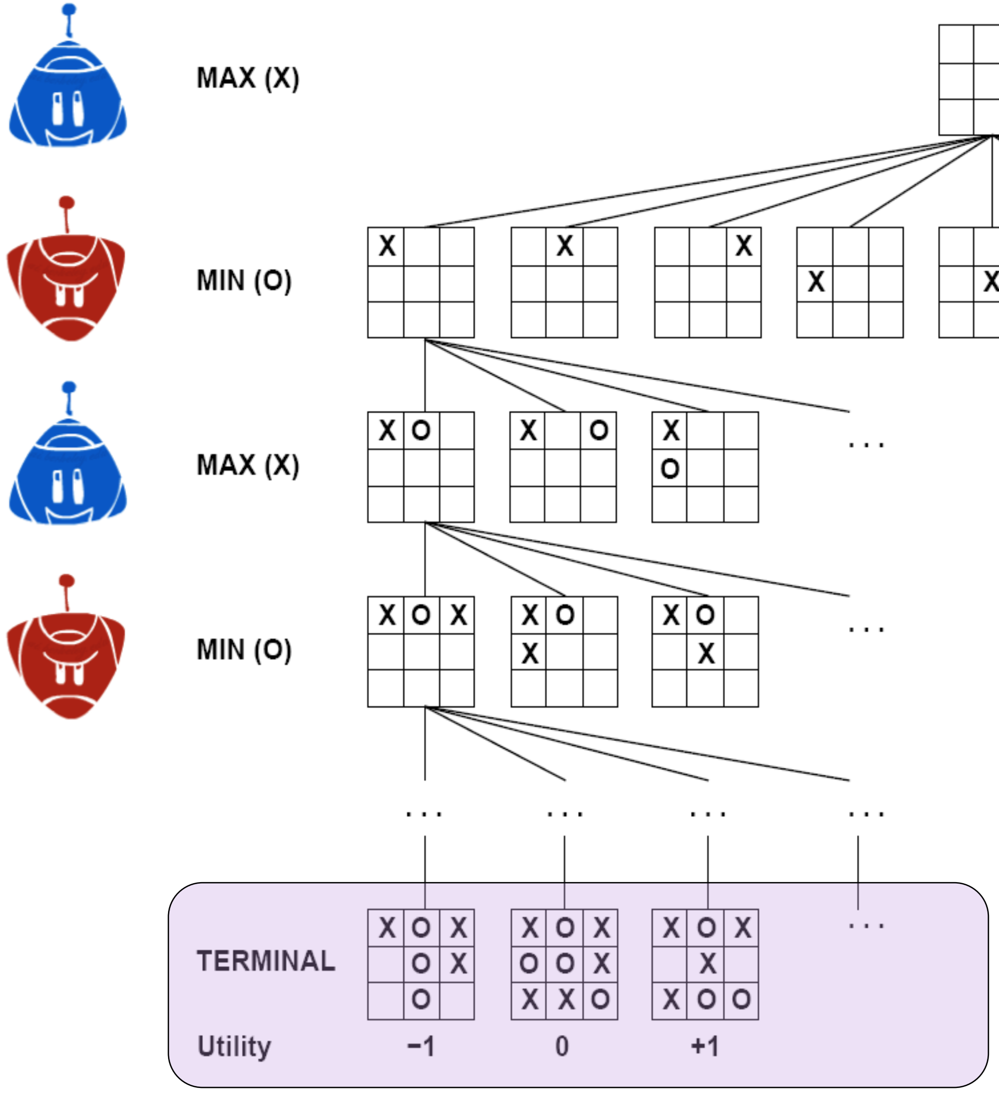


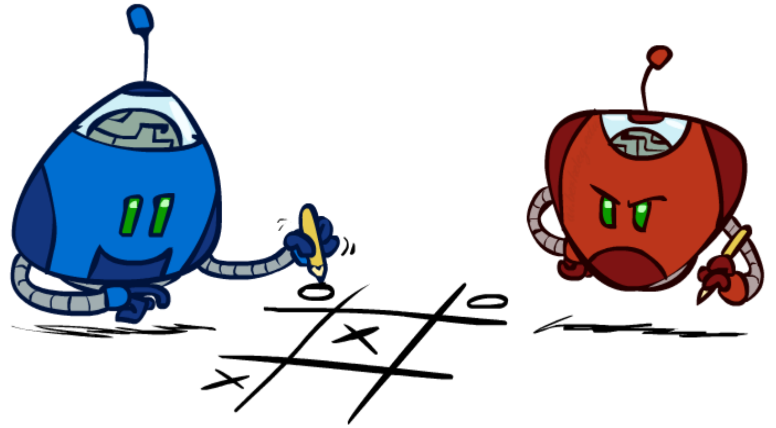
Minimax

States

Actions

Values





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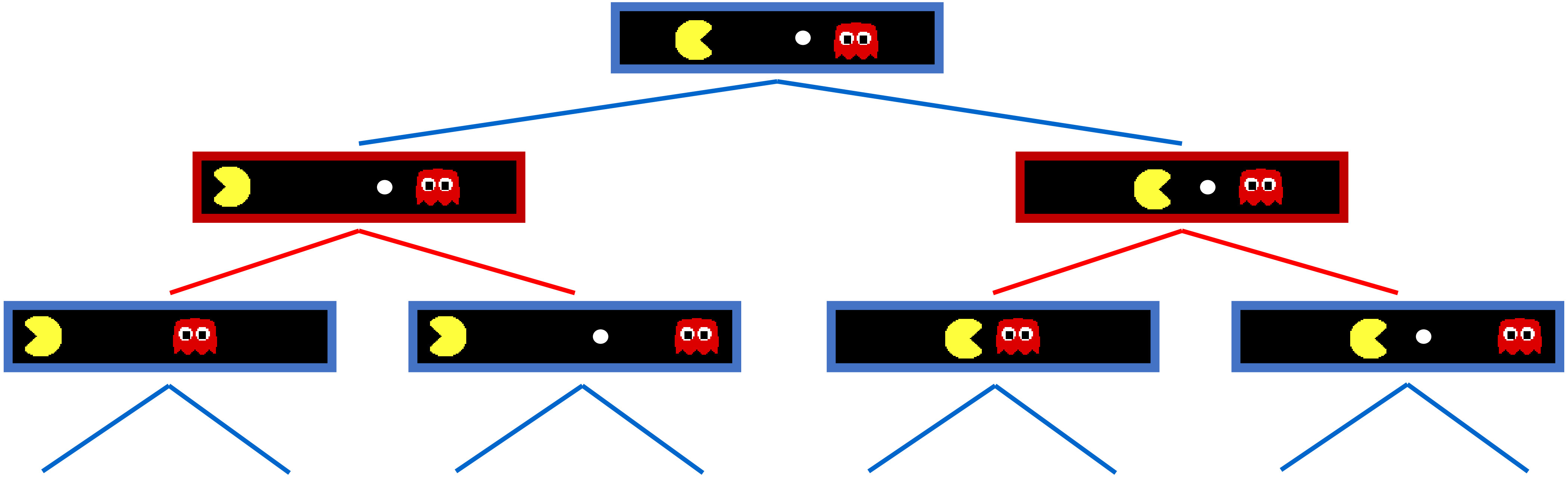

Minimax

States

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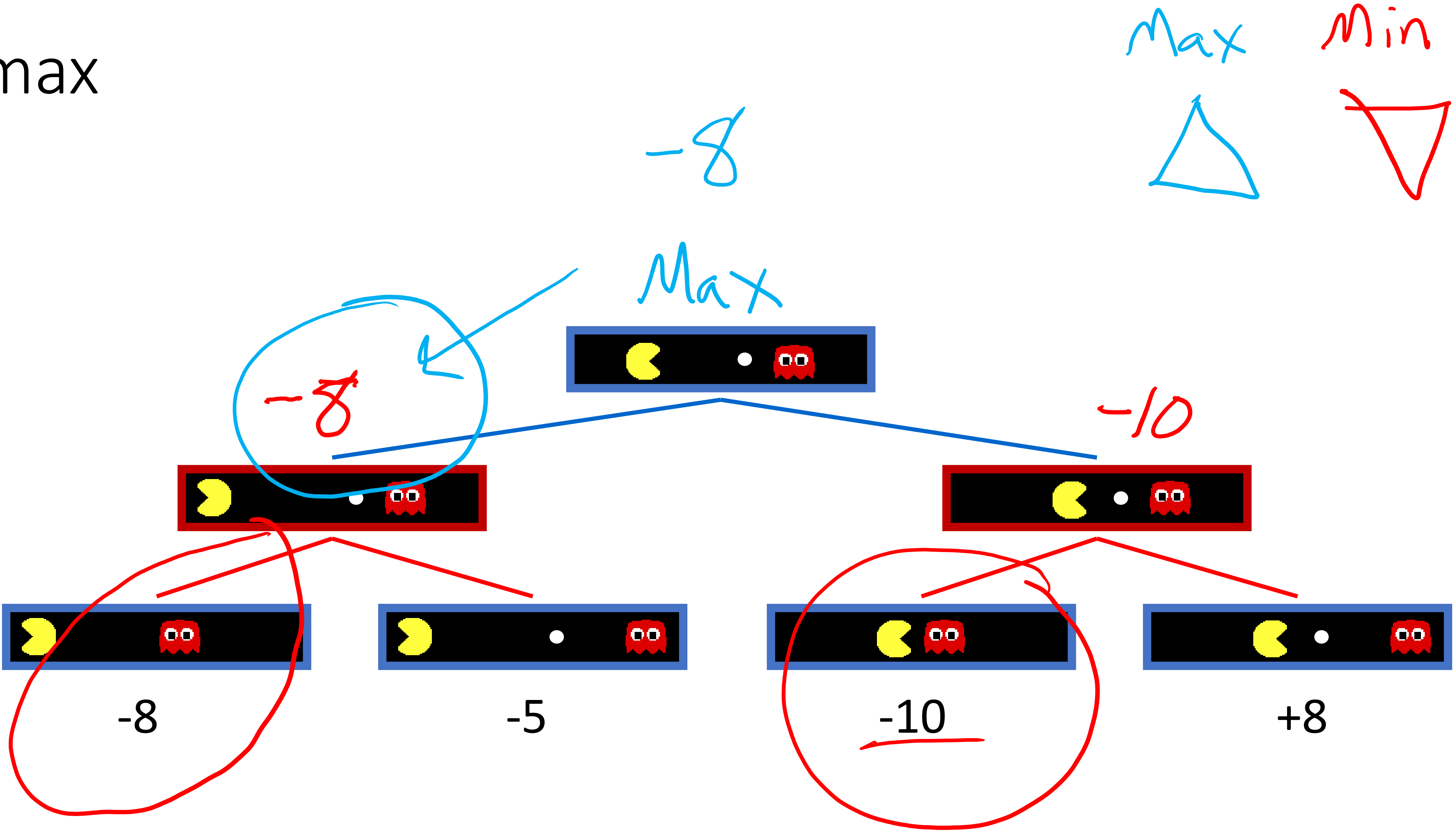
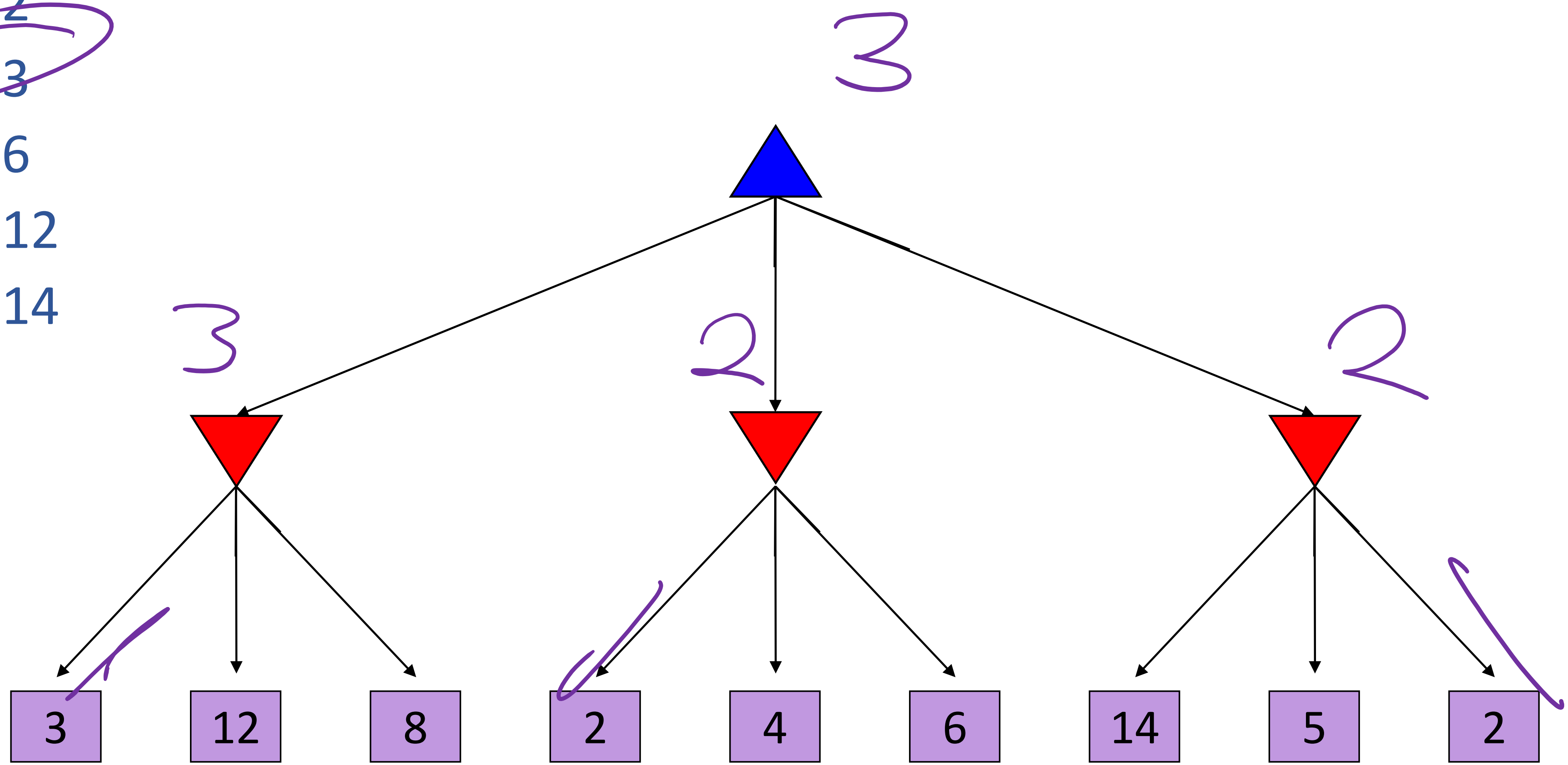


Poll 1

What is the minimax value at the root?

A) 2
B) 3
C) 6
D) 12
E) 14



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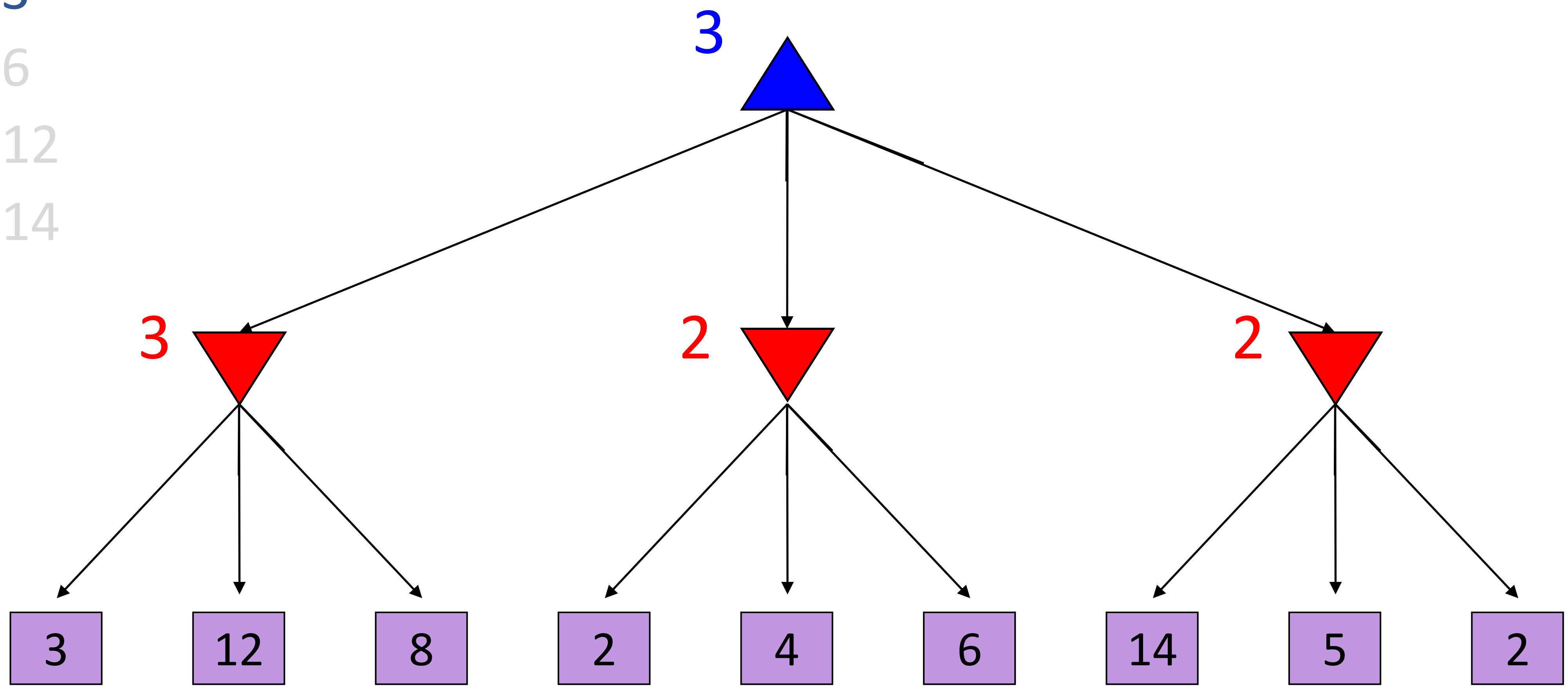
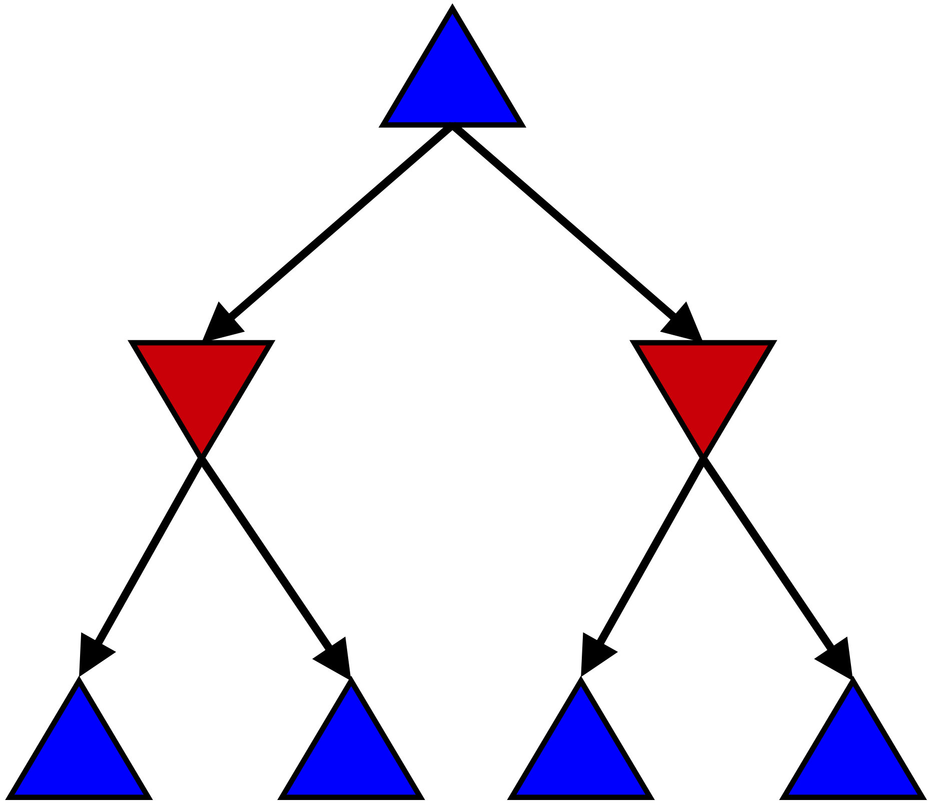
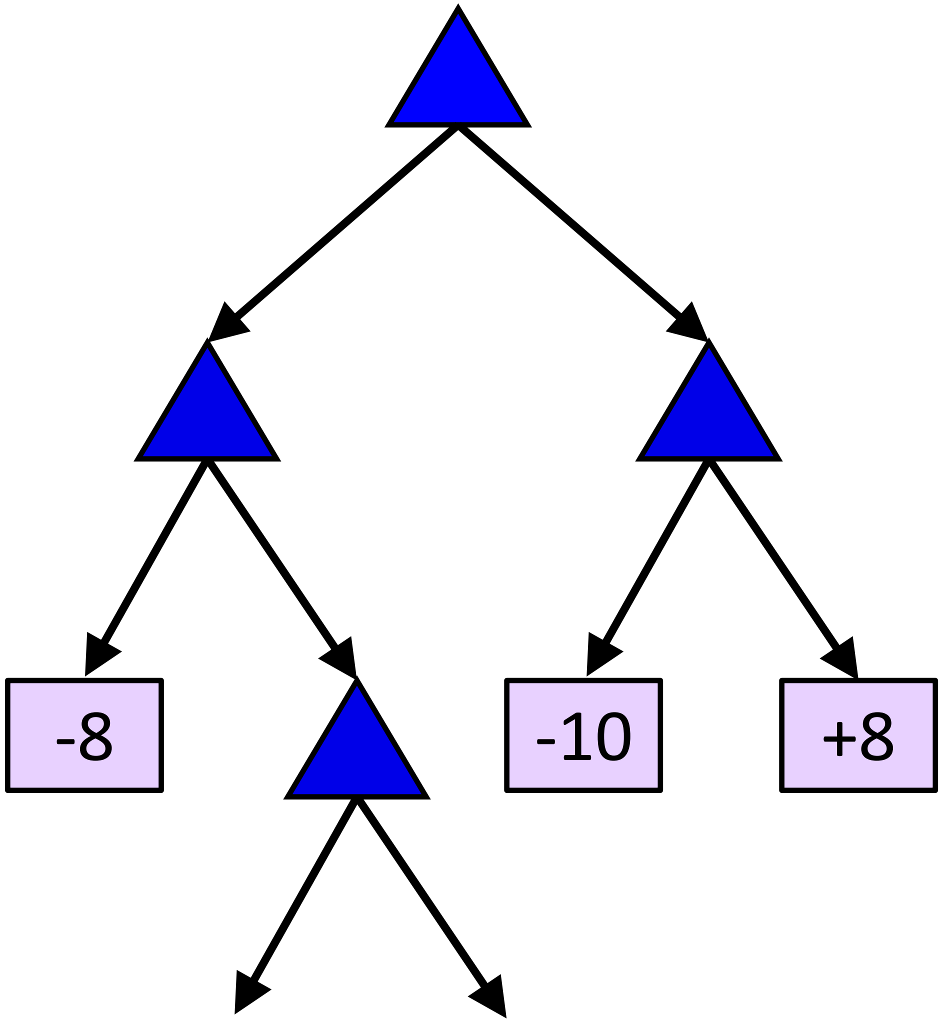


Minimax Code



Max Code



Max Code

```python
def max_value(state):

    if state.is_leaf:
        return state.value
    # TODO Also handle depth limit

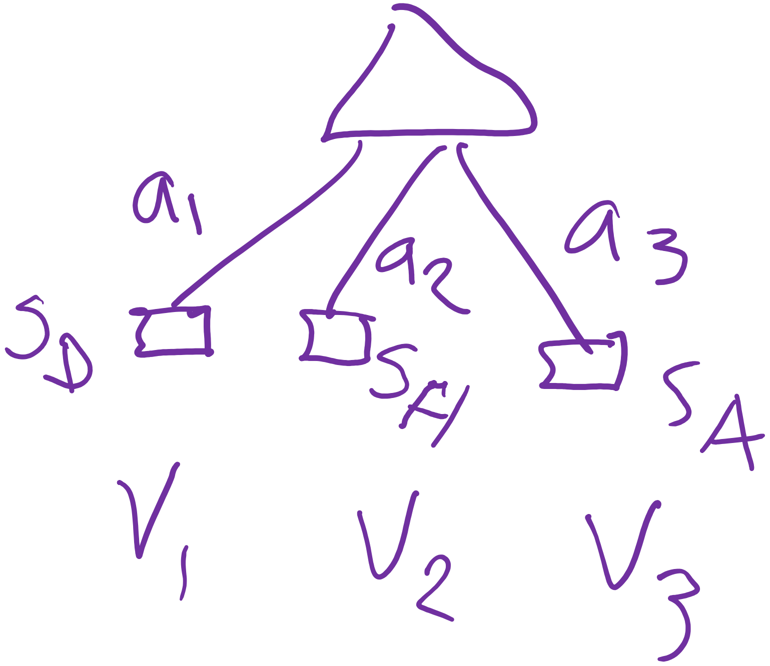
    best_value = -10000000

    for action in state.actions:
        next_state = state.result(action)

        next_value = max_value(next_state)

        if next_value > best_value:
            best_value = next_value

    return best_value
```



Minimax Code

```
def max_value(state):

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    # TODO Also handle depth limit

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    for action in state.actions:
        next_state = state.result(action)

        next_value = min_value(next_state)

        if next_value > best_value:
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    return best_value

def min_value(state):
```

Minimax Notation

$a \in \{ action(s) \}$

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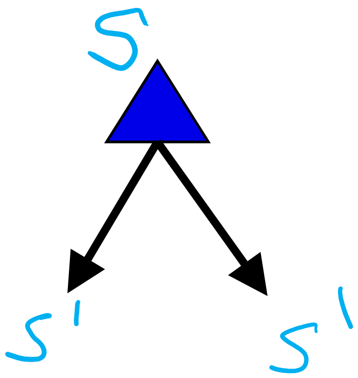
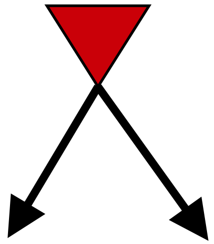


$$V(s) = \max_{a} V(s'),$$

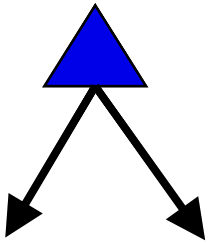
where $s' = result(s, a)$



$$V(s) = \min_{a} V(s')$$

$a = result(s, a)$

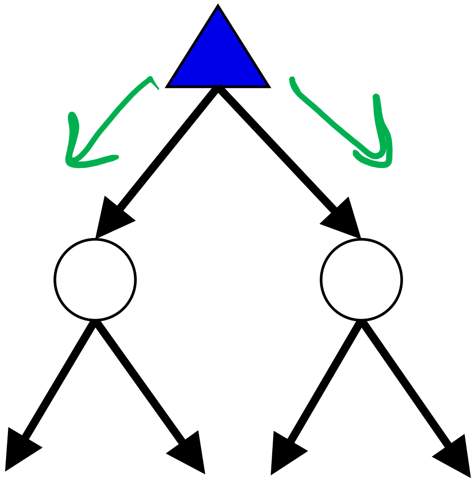
Minimax Notation



$$V(s) = \max_a V(s'),$$

where $s' = result(s, a)$

best_action



$$\hat{a} = \operatorname*{argmax}_a V(s'),$$

action

where $s' = result(s, a)$

Generic Game Tree Pseudocode

```
function  minimax_decision( state )
    return  argmax a in state.actions  value( state.result(a) )

function  value( state )
    if  state.is_leaf
        return state.value

    if  state.player  is  MAX
        return  max a in state.actions  value( state.result(a) )

    if  state.player  is  MIN
        return  min a in state.actions  value( state.result(a) )
```

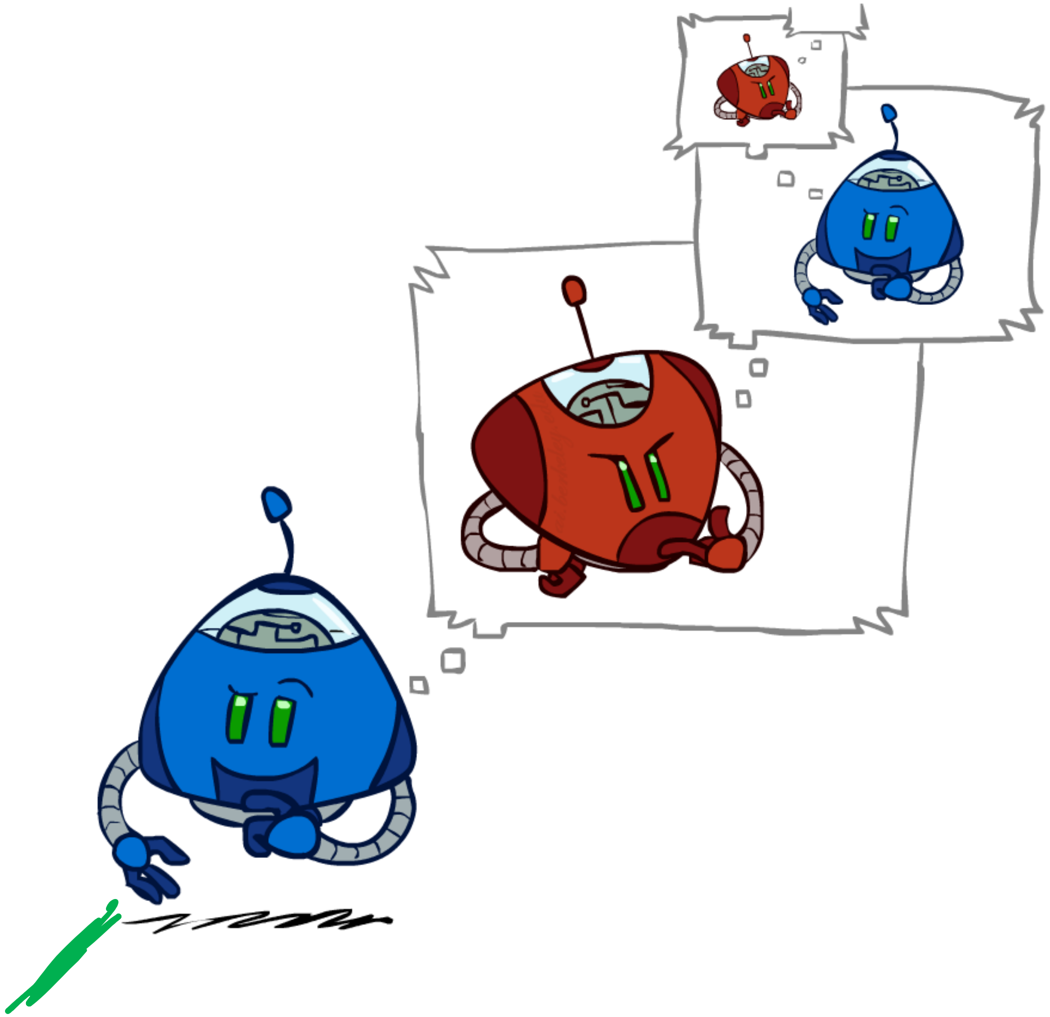
Minimax Efficiency

How efficient is minimax?

- Just like (exhaustive) DFS
- Time: $O(b^m)$
- Space: $O(bm)$

Example: For chess, $b \approx 35$, $m \approx 100$

- Exact solution is completely infeasible
- Humans can't do this either, so how do we play chess?
- Bounded rationality – Herbert Simon



Resource Limits

Resource Limits

Problem: In realistic games, cannot search to leaves!

Solution 1: Bounded lookahead

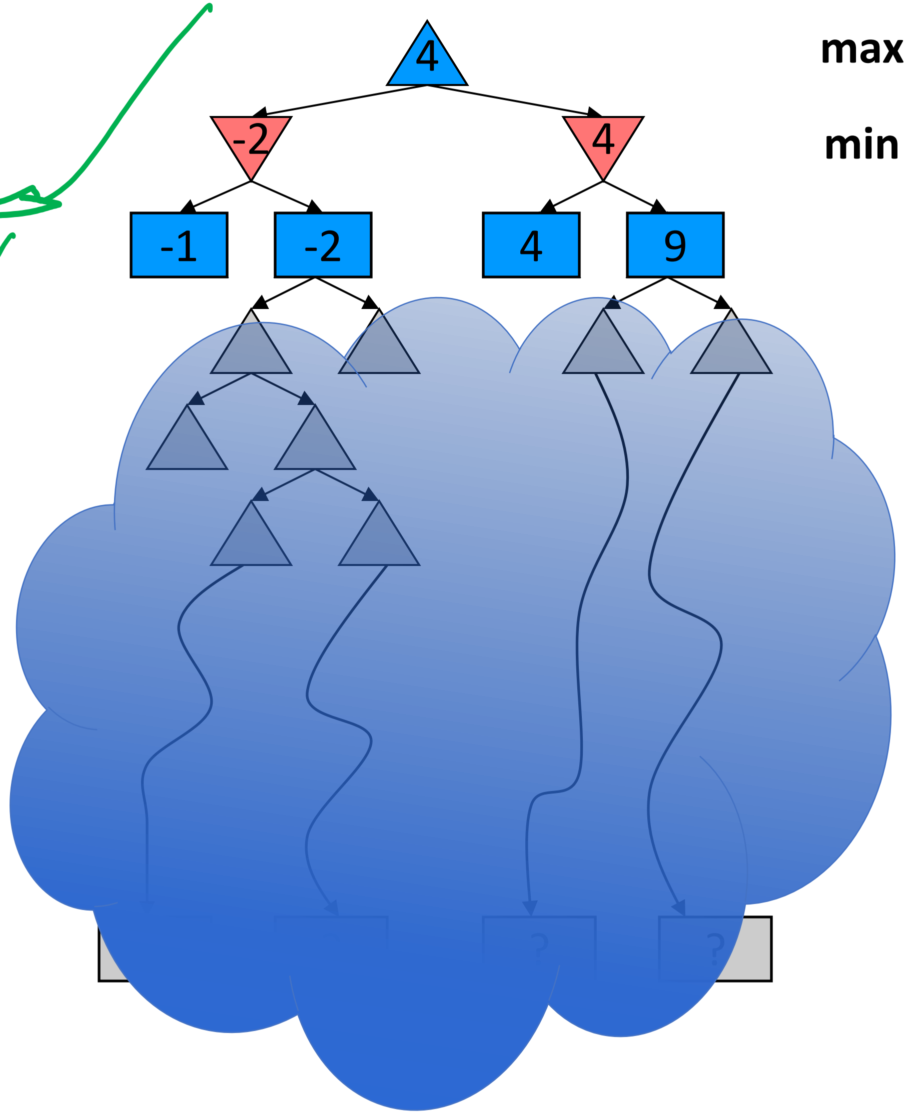
- Search only to a preset ***depth limit*** or ***horizon***
- Use an ***evaluation function*** for non-terminal positions

Guarantee of optimal play is gone

More plies make a BIG difference

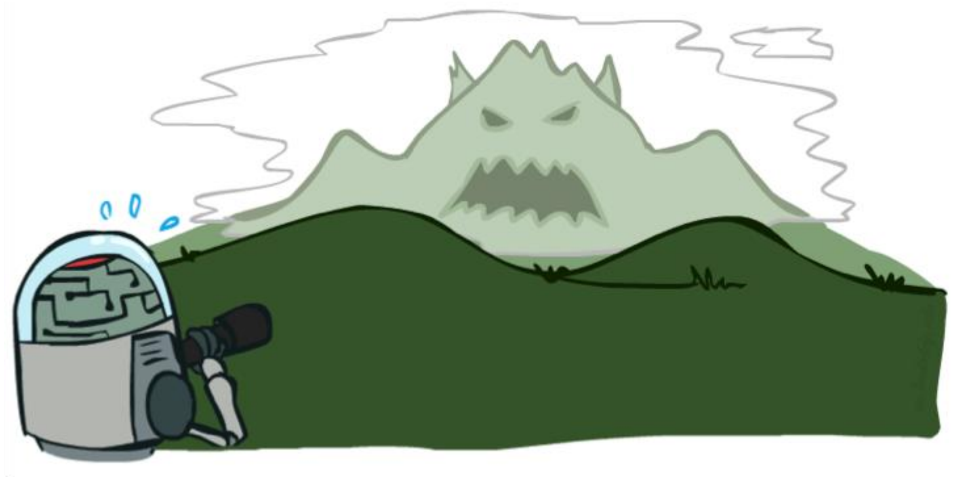
Example:

- Suppose we have 100 seconds, can explore 10K nodes / sec
- So can check 1M nodes per move
- For chess, b=~35 so reaches about depth 4 – not so good



Depth Matters

- Evaluation functions are always imperfect
- Deeper search => better play (usually)
- Or, deeper search gives same quality of play with a less accurate evaluation function
- An important example of the tradeoff between complexity of features and complexity of computation

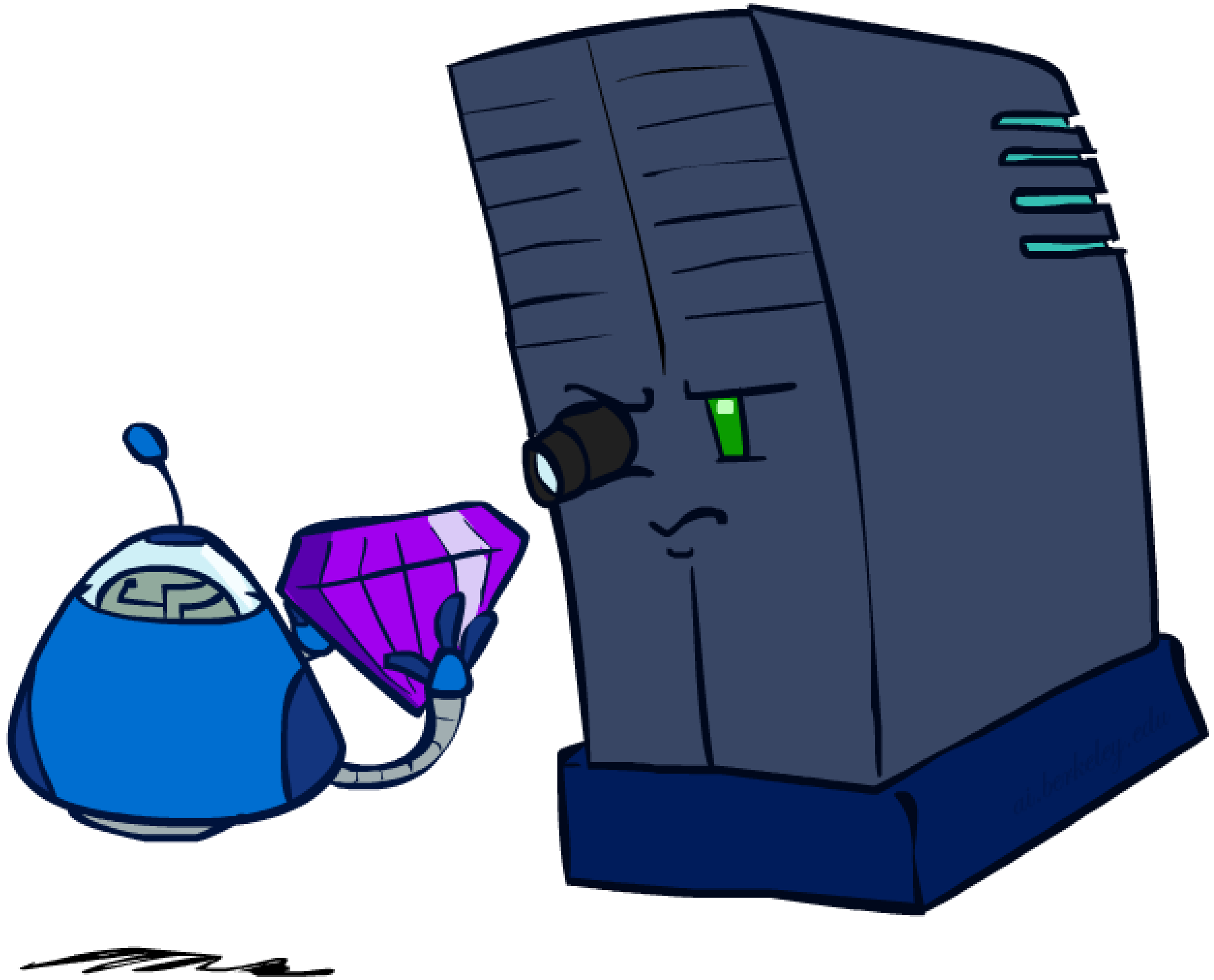


[Demo: depth limited (L6D4, L6D5)]

Demo Limited Depth (2)

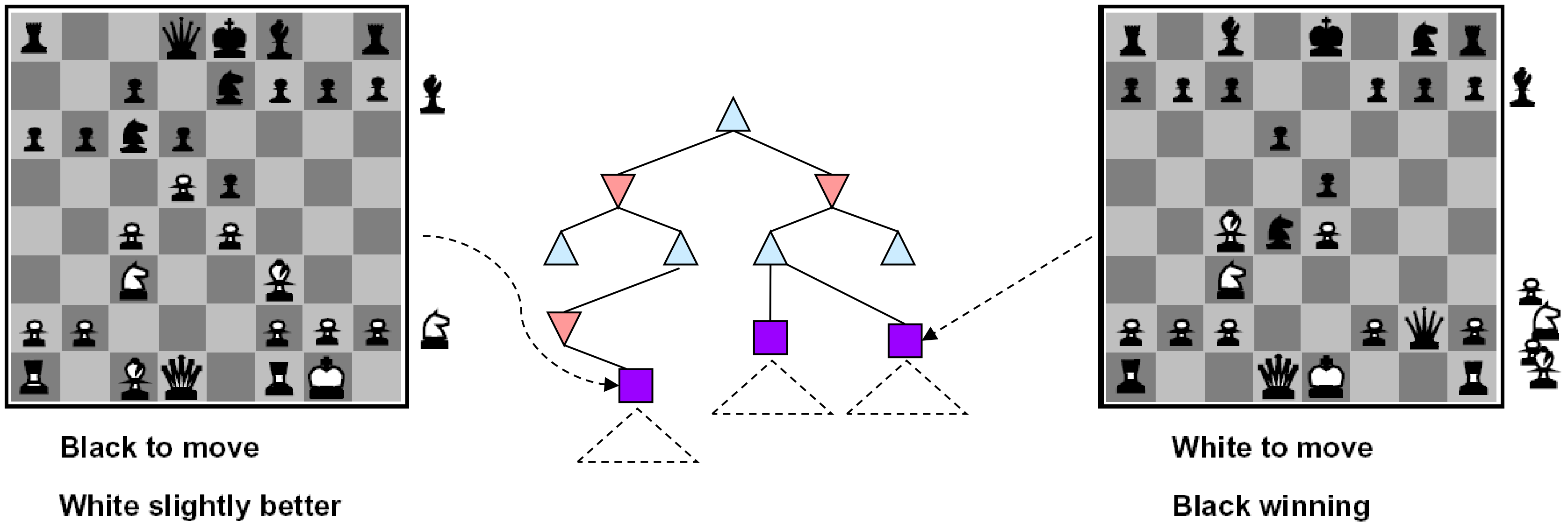
Demo Limited Depth (10)

Evaluation Functions



Evaluation Functions

Evaluation functions score non-terminals in depth-limited search

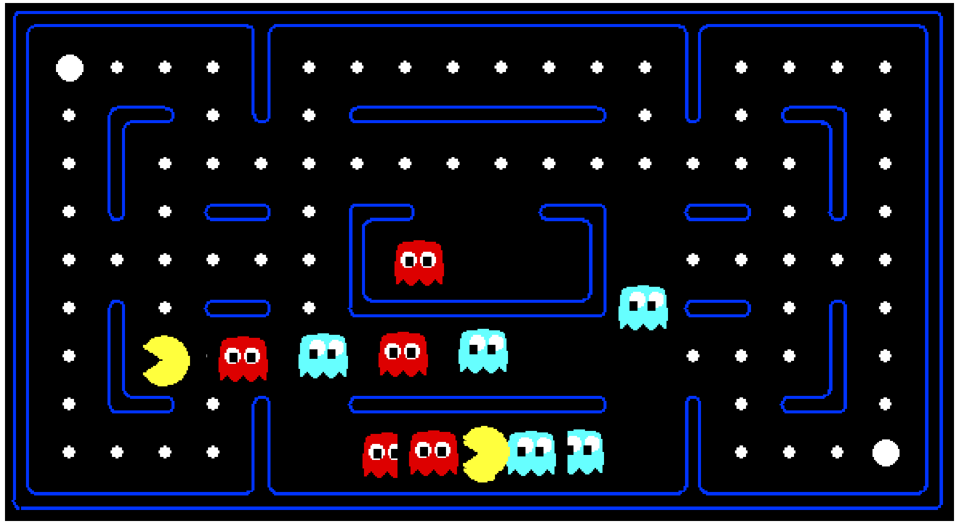


Ideal function: returns the actual minimax value of the position

In practice: typically weighted linear sum of features:

- EVAL(s) = $w_1 f_1(s) + w_2 f_2(s) + \ldots. + w_n f_n(s)$
- E.g., $w_1 = 9$, $f_1(s)$ = (num white queens – num black queens), etc.

Evaluation for Pacman

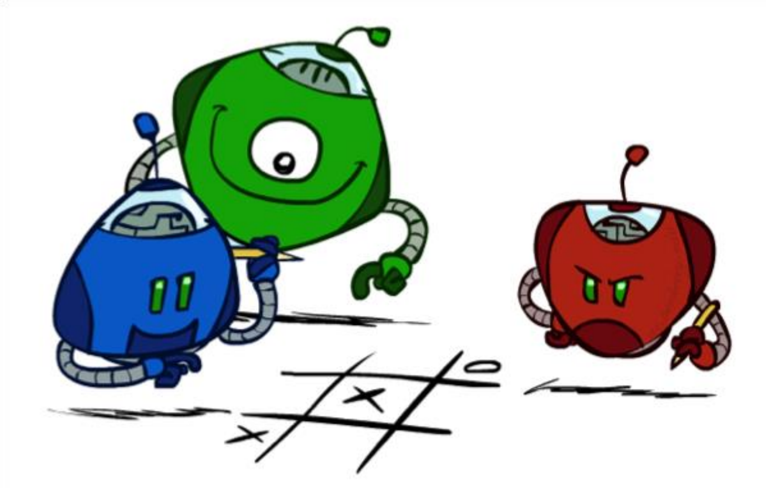


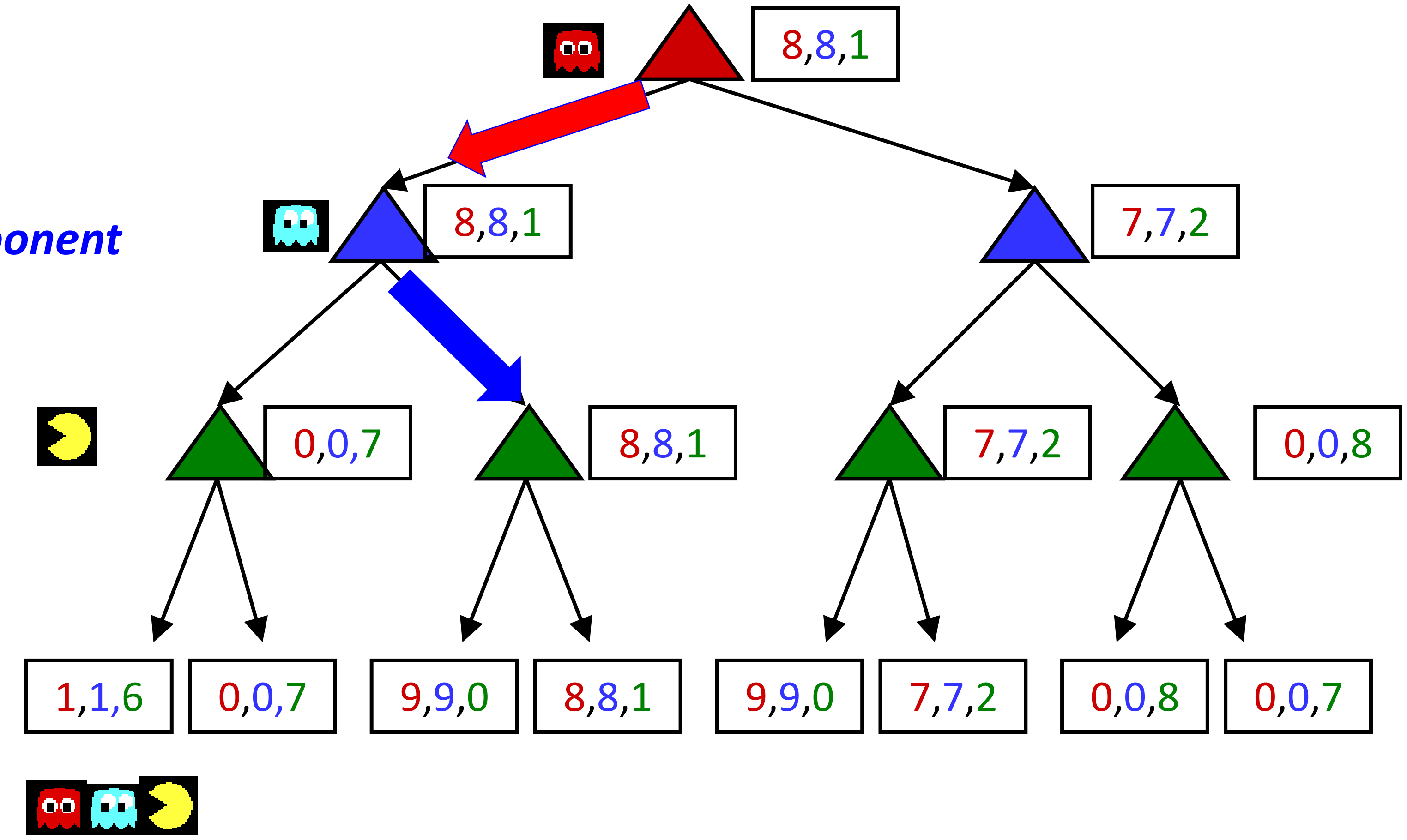
Generalized minimax

What if the game is not zero-sum, or has multiple players?

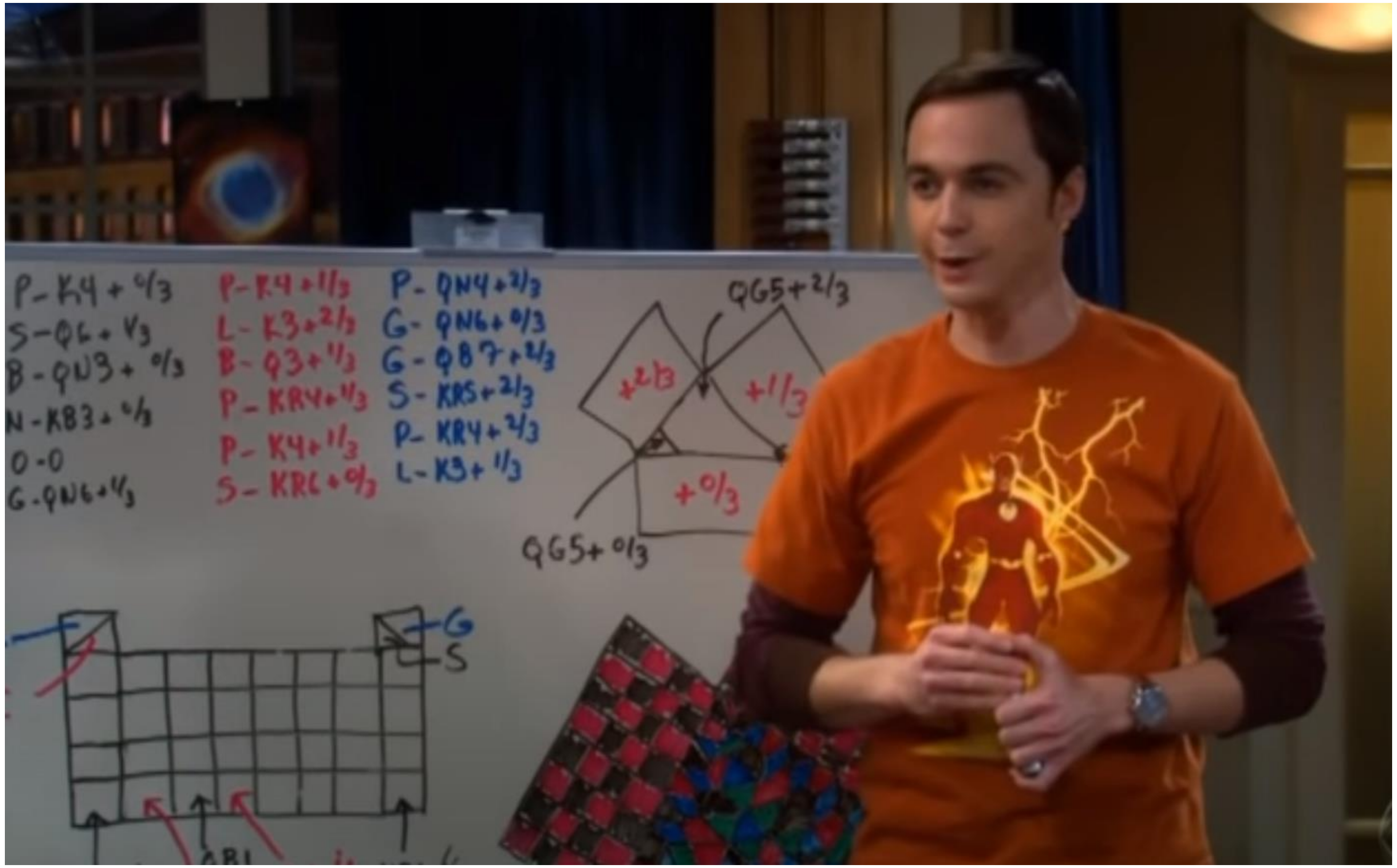
Generalization of minimax:

- Terminals have ***utility tuples***
- Node values are also utility tuples
- ***Each player maximizes its own component***
- Can give rise to cooperation and competition dynamically…





Generalized minimax



[Three Person Chess](https://www.youtube.com/watch?v=HHVPutfveVs)

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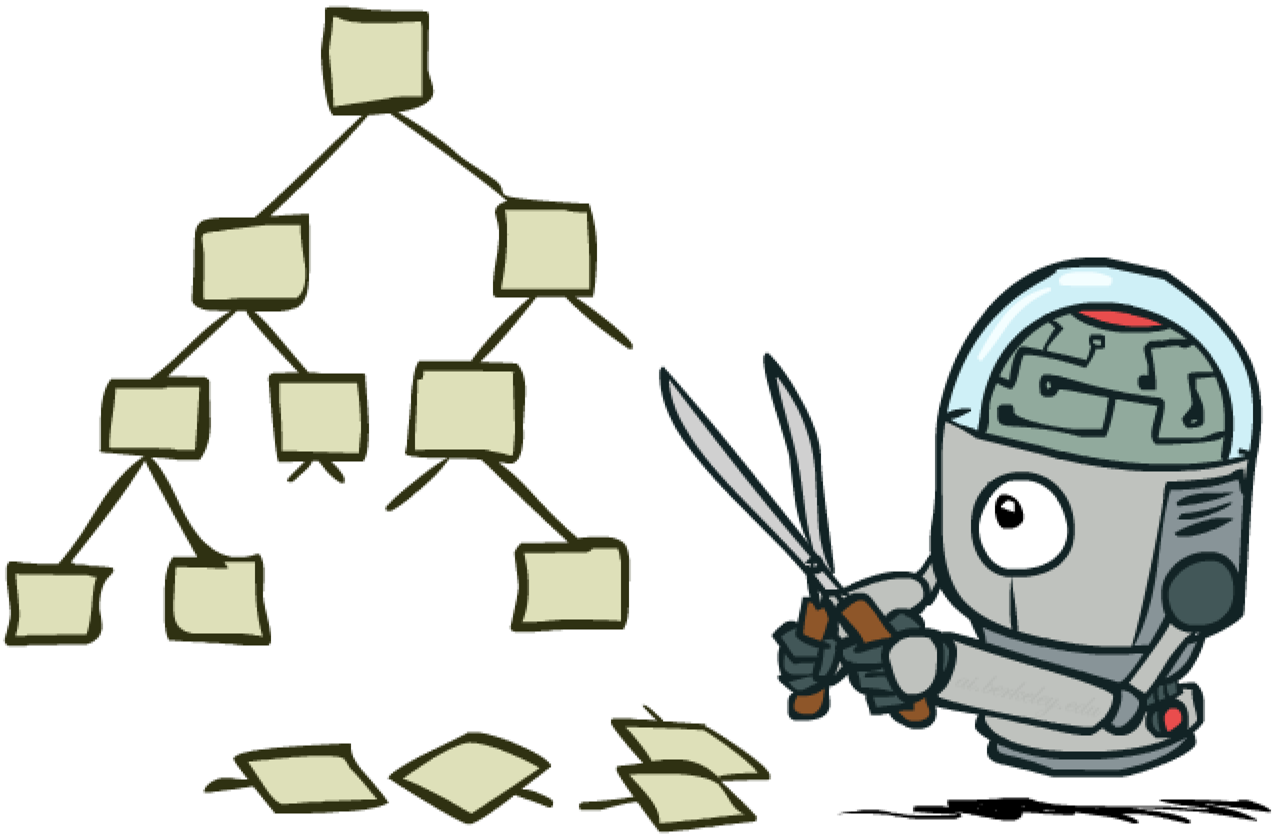
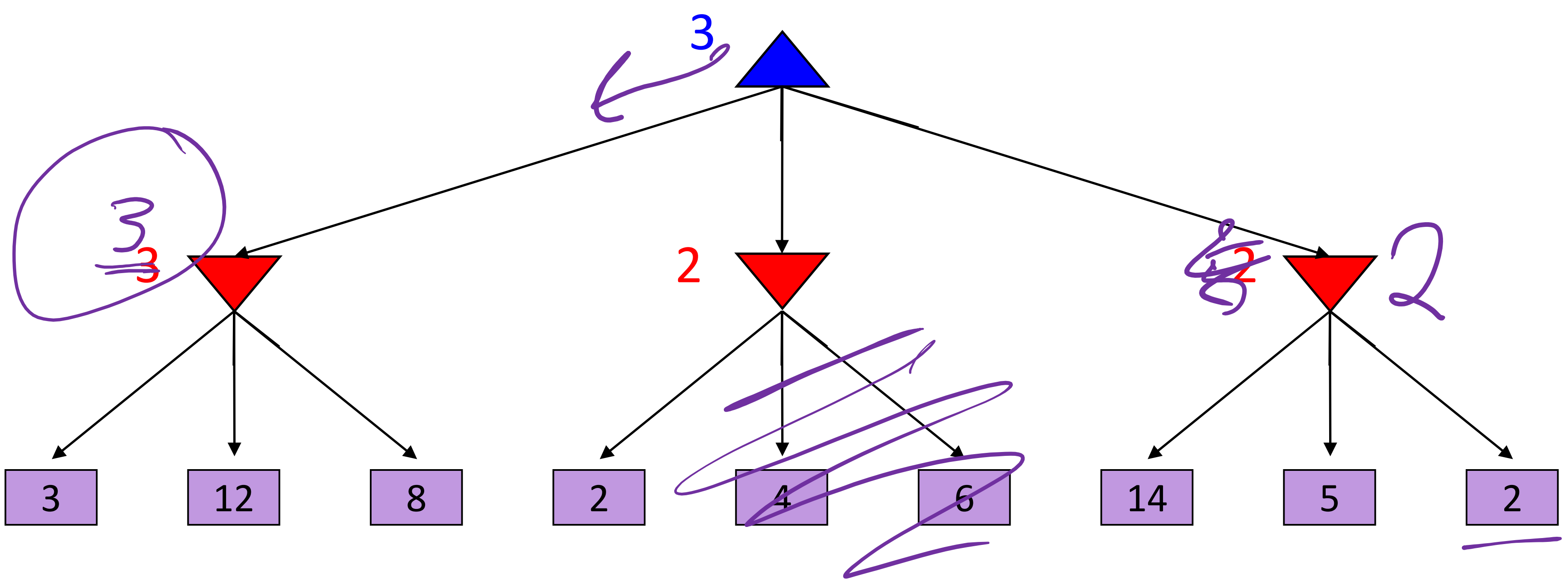
Game Tree Pruning

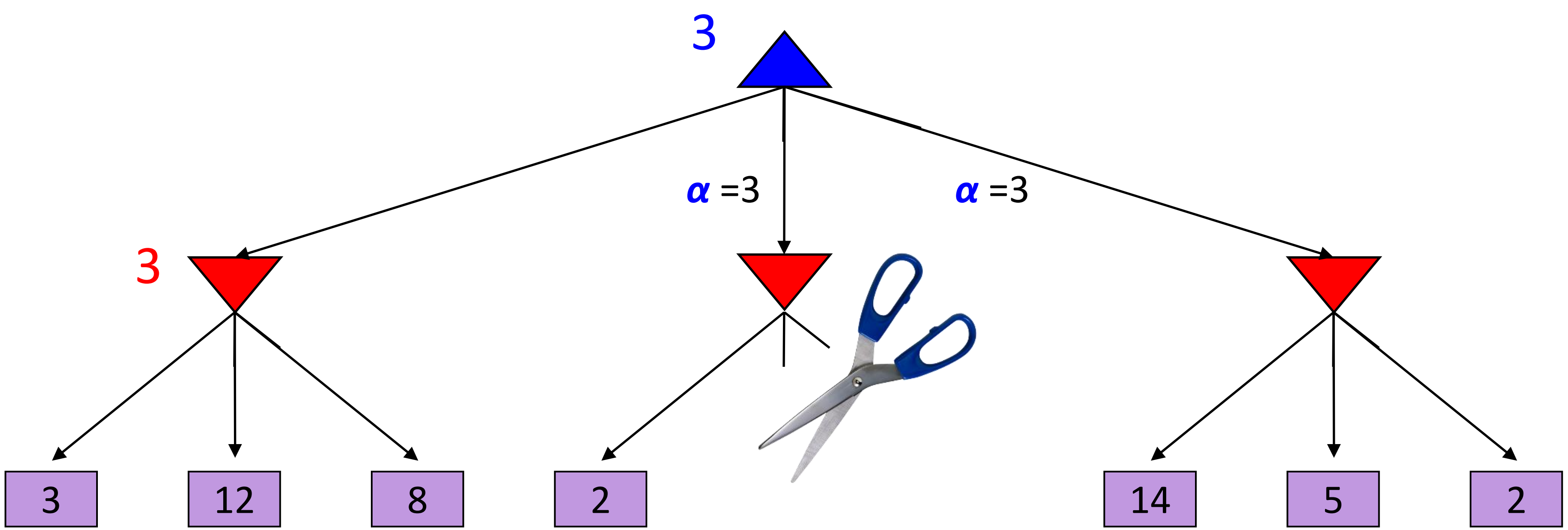


Minimax Example



Alpha-Beta Example

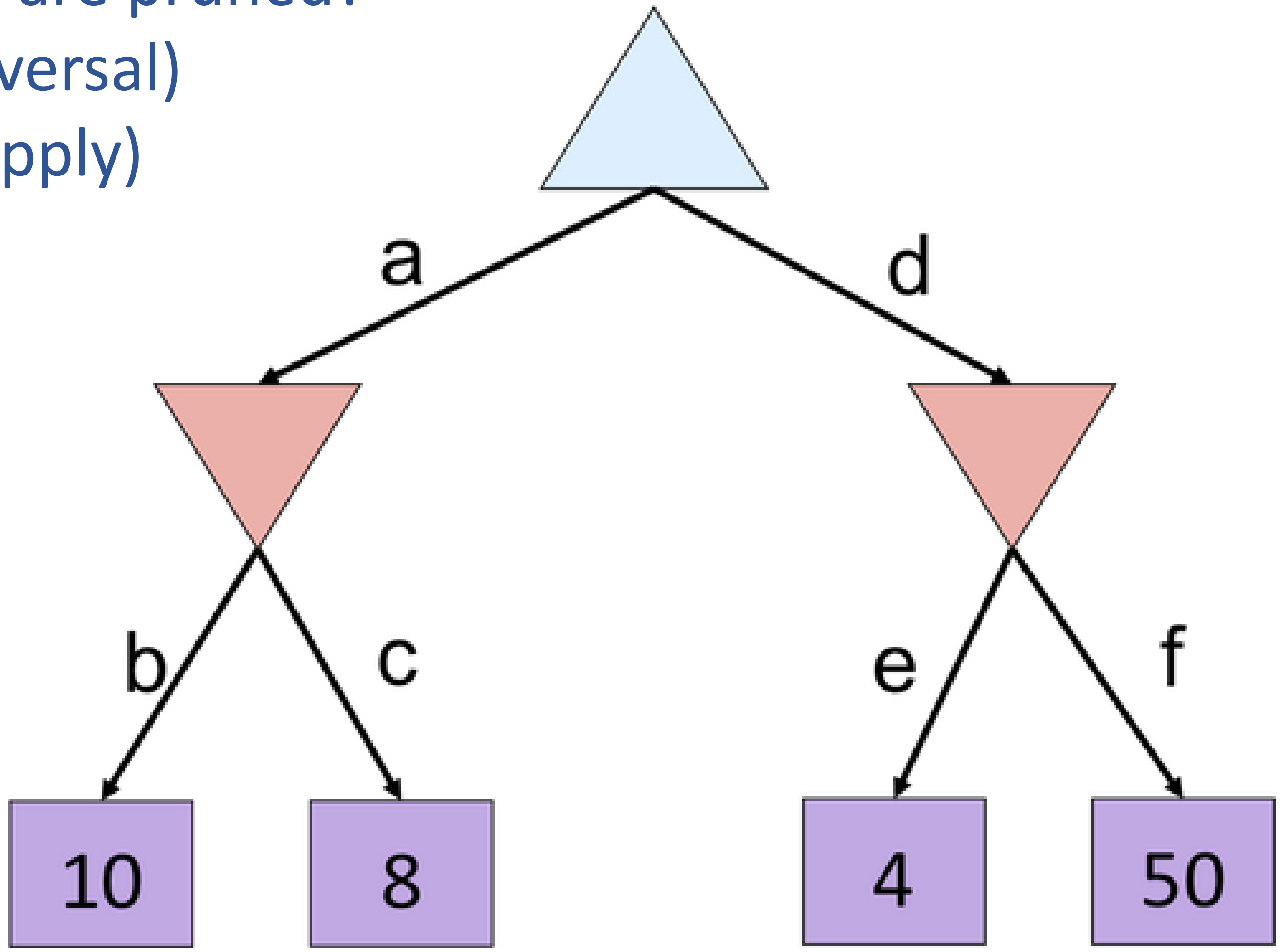
α = best option so far from any MAX node on this path



The order of generation matters: more pruning is possible if good moves come first

Poll 2

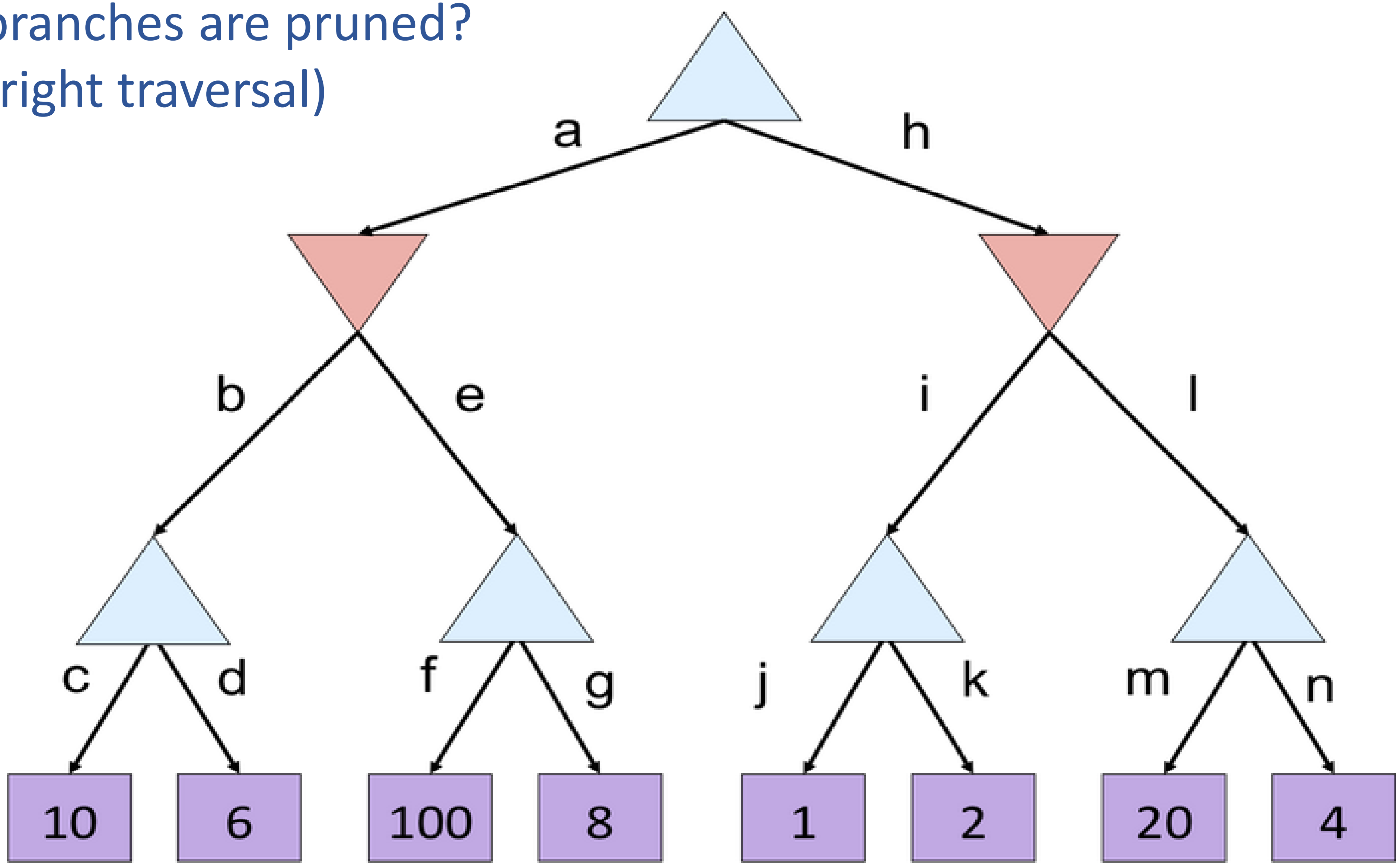
Which branches are pruned?
(Left to right traversal)
(Select all that apply)



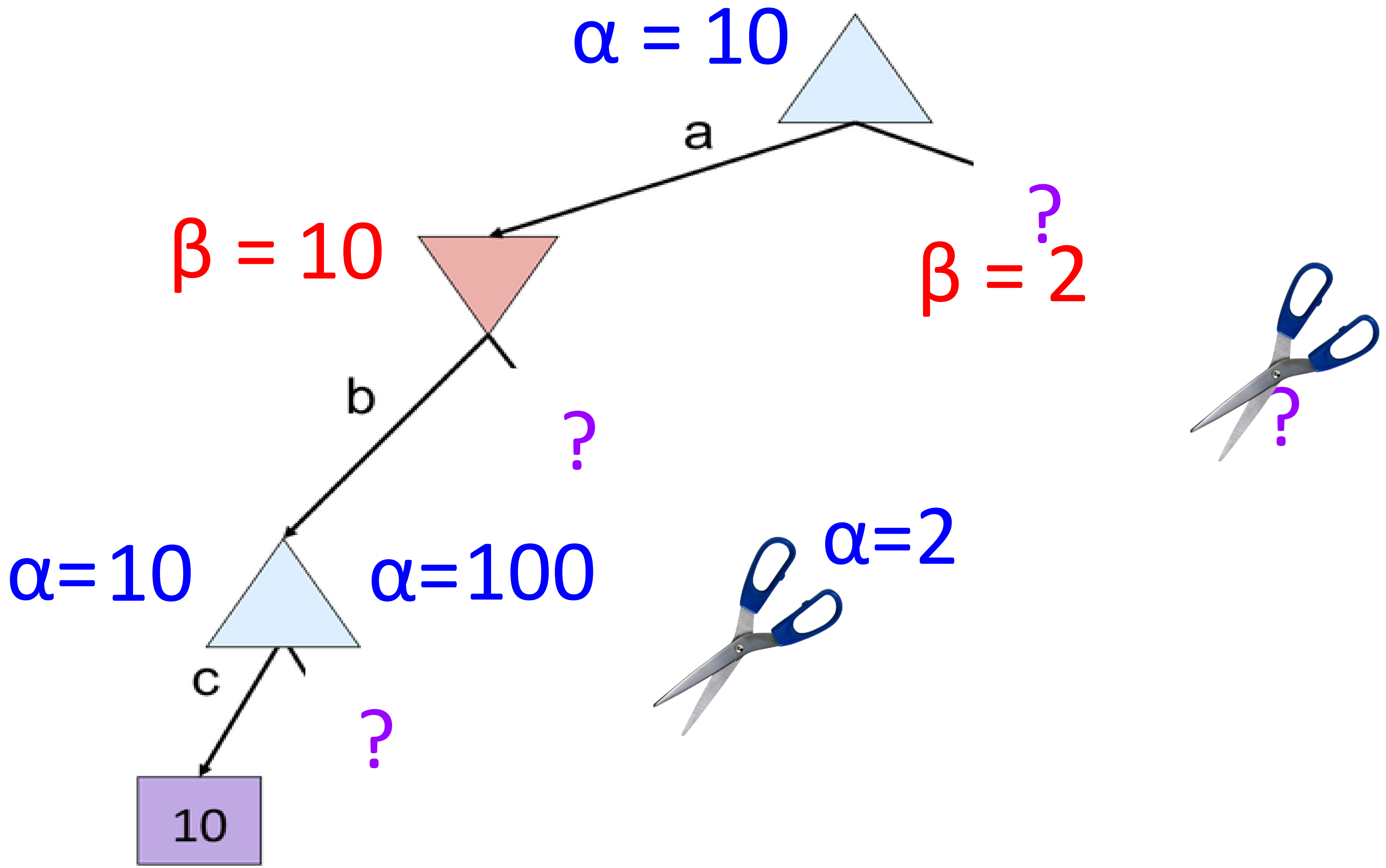
Poll 3

Which branches are pruned?
(Left to right traversal)

A) e, l
B) g, l
C) g, k, l
D) g, n



Poll 3



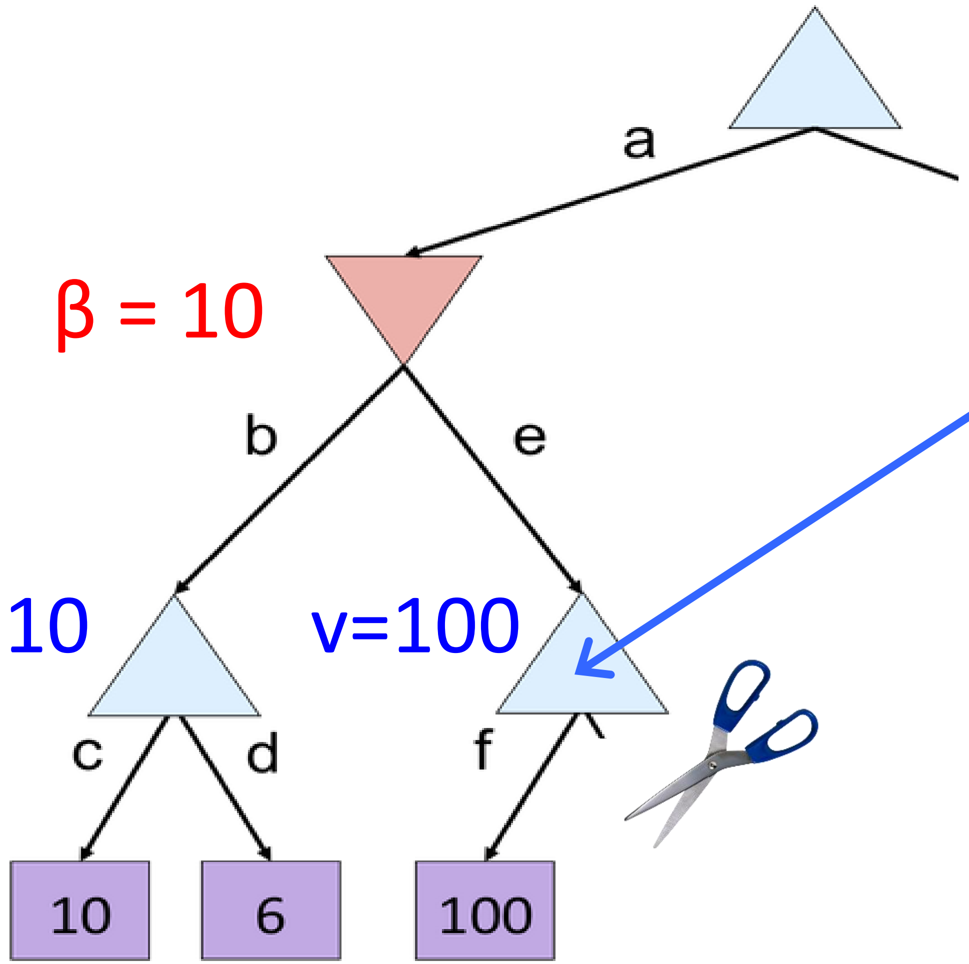
Alpha-Beta Implementation

α: MAX's best option on path to root
β: MIN's best option on path to root

```
def max-value(state, α, β):
    initialize v = -∞
    for each successor of state:
        v = max(v, value(successor, α, β))
        if v ≥ β
            return v
        α = max(α, v)
    return v
```

```
def min-value(state , α, β):
    initialize v = +∞
    for each successor of state:
        v = min(v, value(successor, α, β))
        if v ≤ α
            return v
        β = min(β, v)
    return v
```

Alpha-Beta Poll 3



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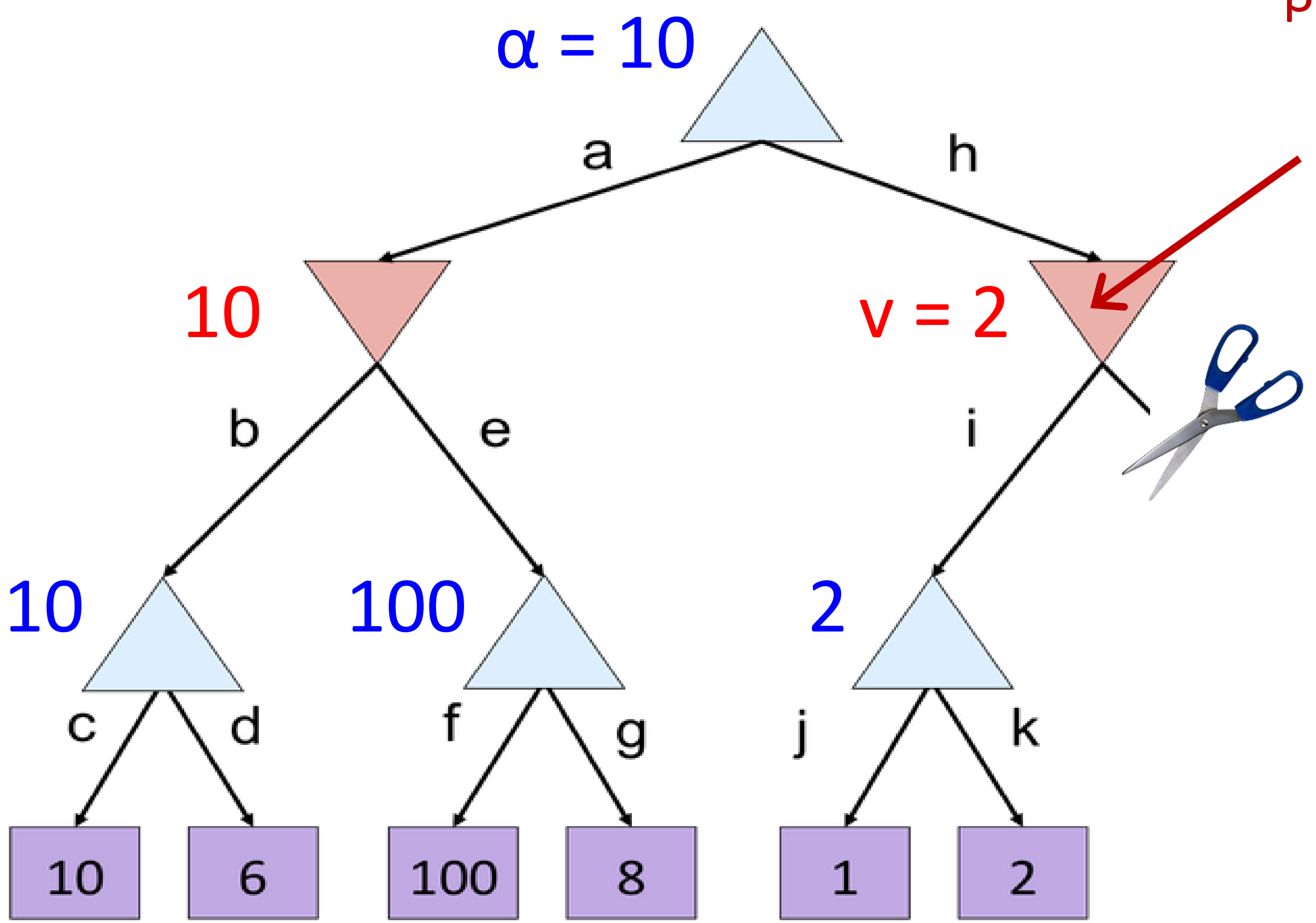
Alpha-Beta Poll 3



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```

Alpha-Beta Pruning Properties

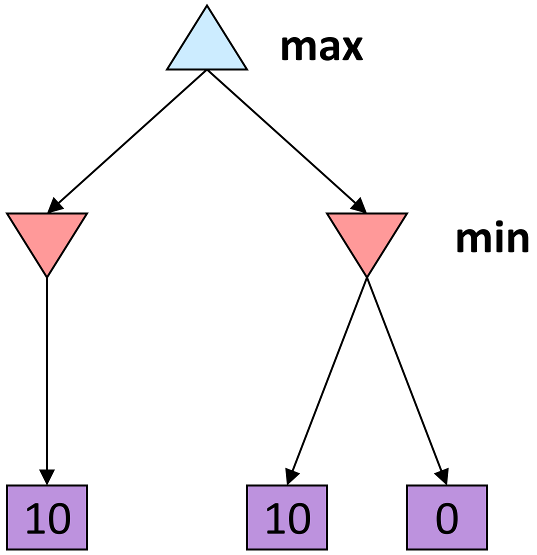
Theorem: This pruning has ***no effect*** on minimax value computed for the root!

Good child ordering improves effectiveness of pruning

- Iterative deepening helps with this

With "perfect ordering":

- Time complexity drops to $O(b^{m/2})$
- Doubles solvable depth!
- 1M nodes/move => depth=8, respectable



This is a simple example of metareasoning (computing about what to compute)

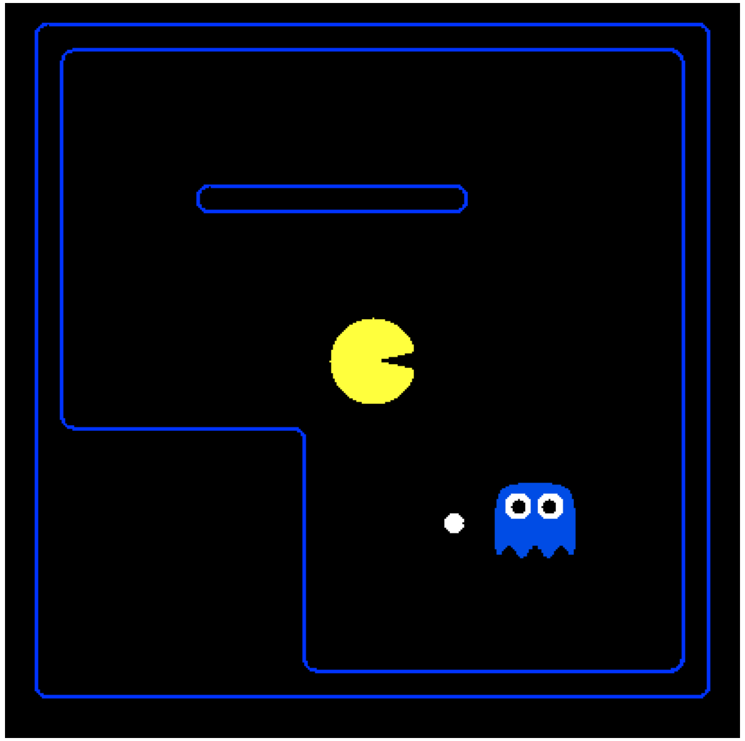
Minimax Demo

Points

+500 win

-500 lose

-1 each move



Fine print

- Pacman: uses depth 4 minimax
- Ghost: uses depth 2 minimax

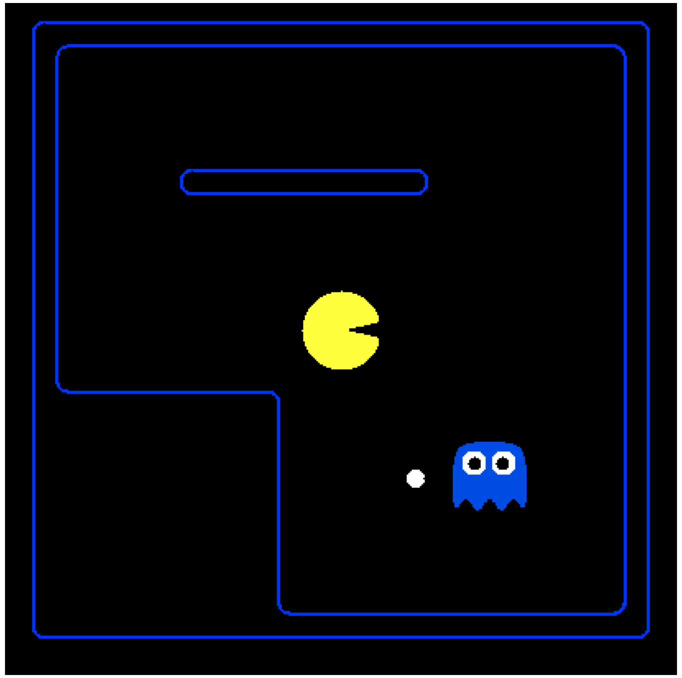
Poll 4

How well would a minimax Pacman perform against a ghost that moves randomly?

A. Better than against a minimax ghost

B. Worse than against a minimax ghost

C. Same as against a minimax ghost

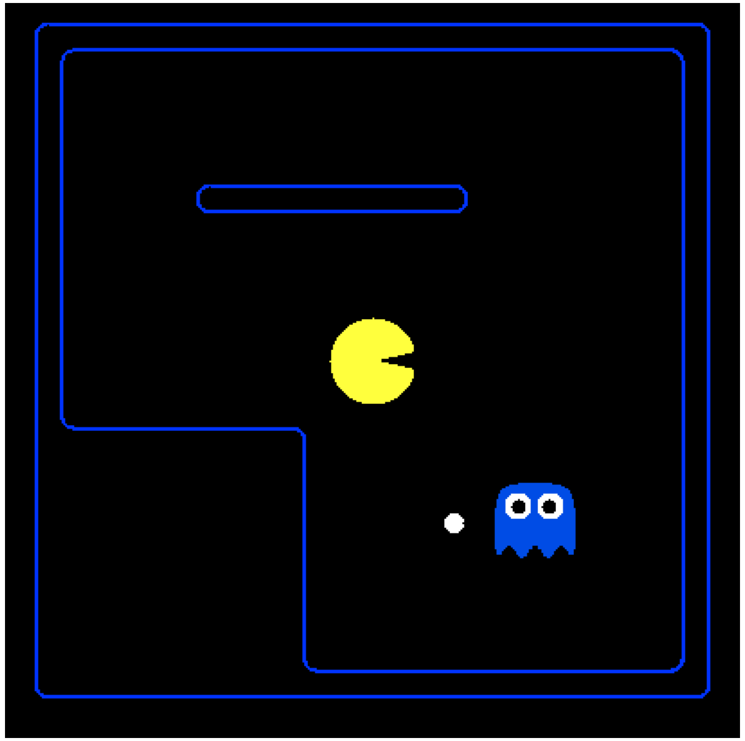


Fine print

- Pacman: uses depth 4 minimax as before
- Ghost: moves randomly

Demo

Assumptions vs. Reality

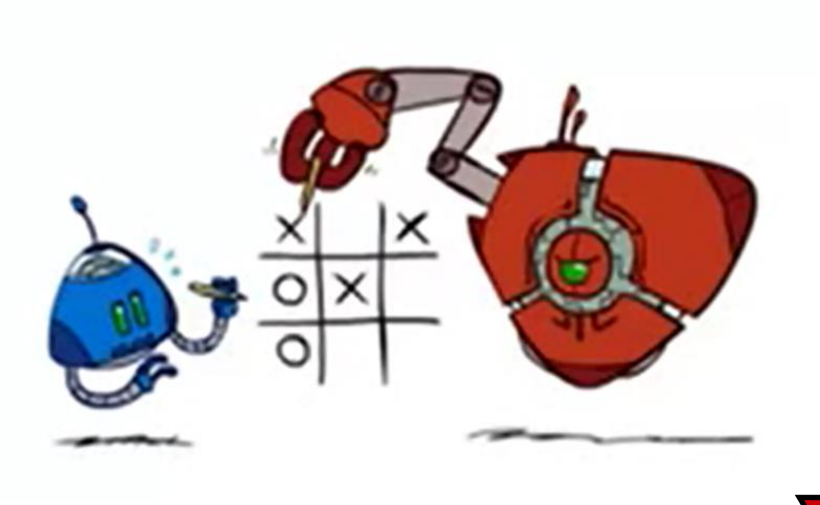


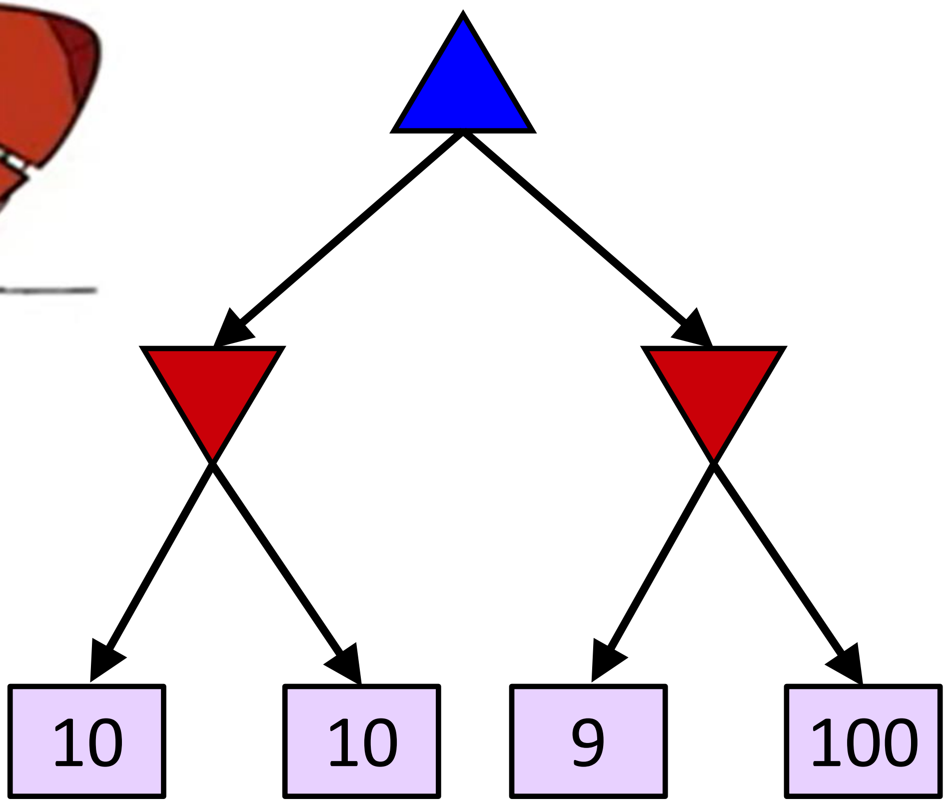
	Minimax Ghost	Random Ghost
Minimax Pacman		

Results from playing 5 games

Modeling Assumptions

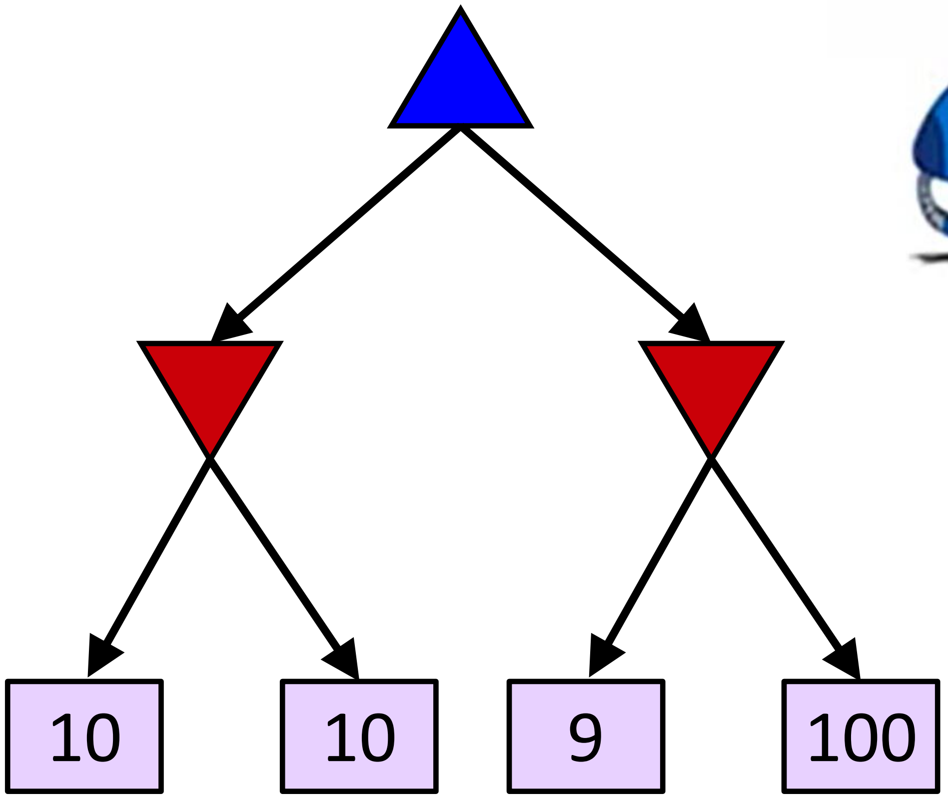
Know your opponent

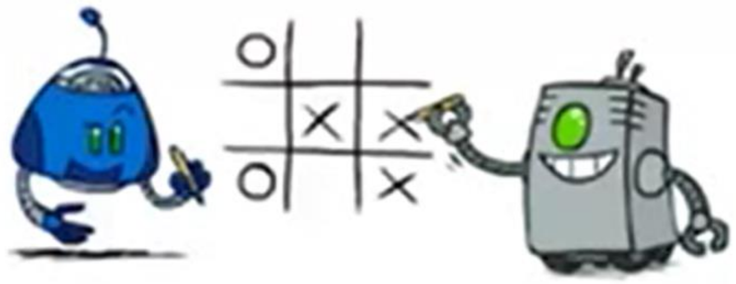




Modeling Assumptions

Know your opponent





Modeling Assumptions

Minimax autonomous vehicle?

Image: https://corporate.ford.com/innovation/autonomous-2021.html

Minimax Driver?

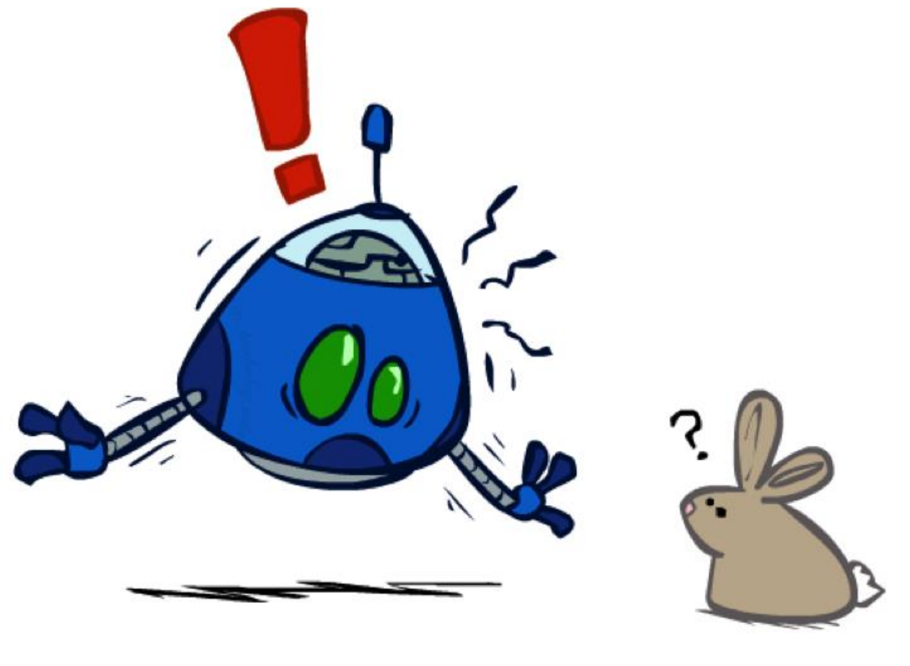
https://youtu.be/5PRrwlkPdNI?t=52

Clip: How I Met Your Mother, CBS

Modeling Assumptions

Dangerous Pessimism

Assuming the worst case when it's not likely

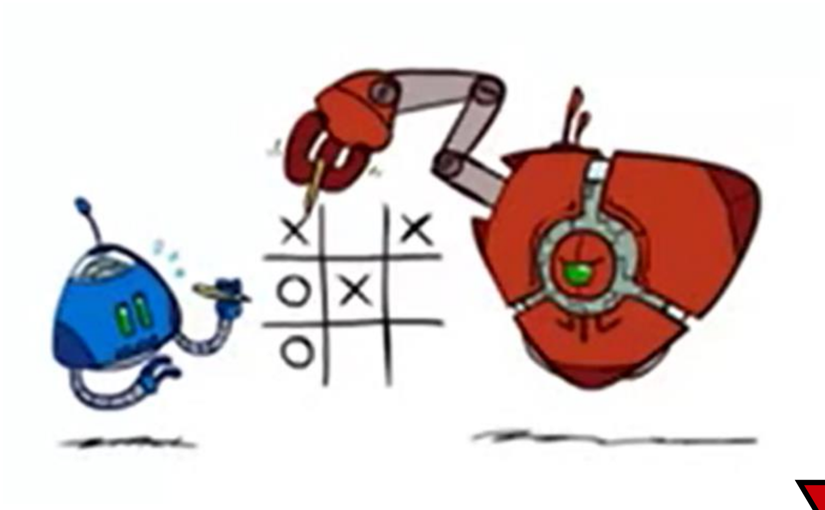


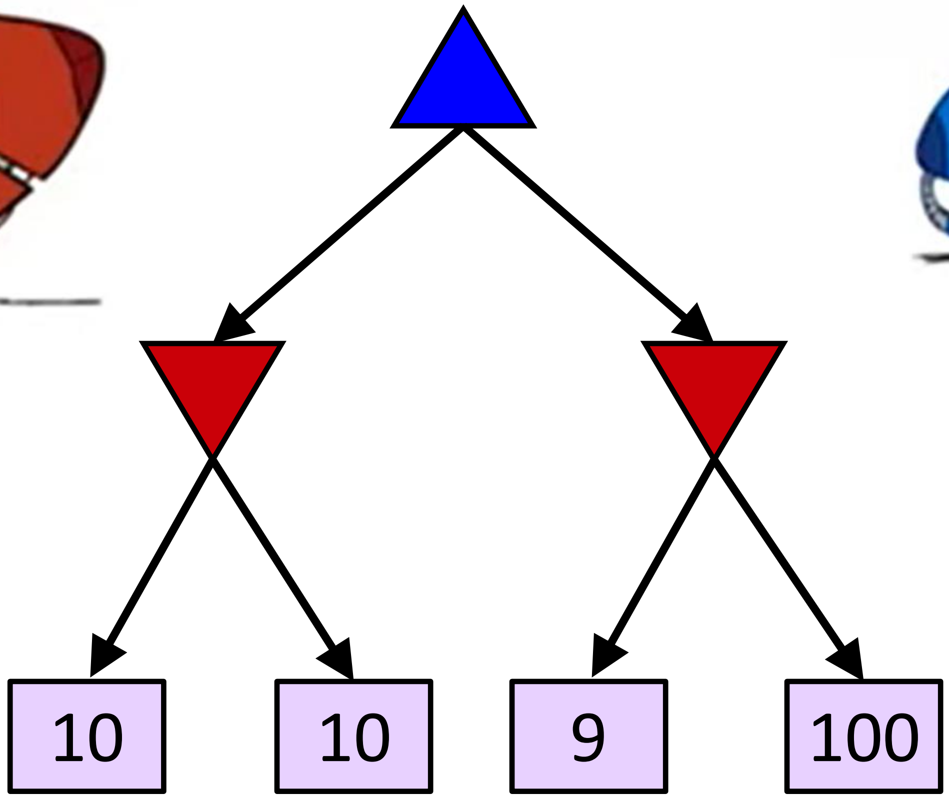
Dangerous Optimism

Assuming chance when the world is adversarial

Modeling Assumptions

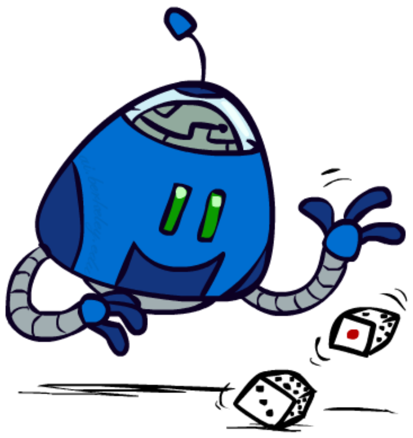
Know your opponent

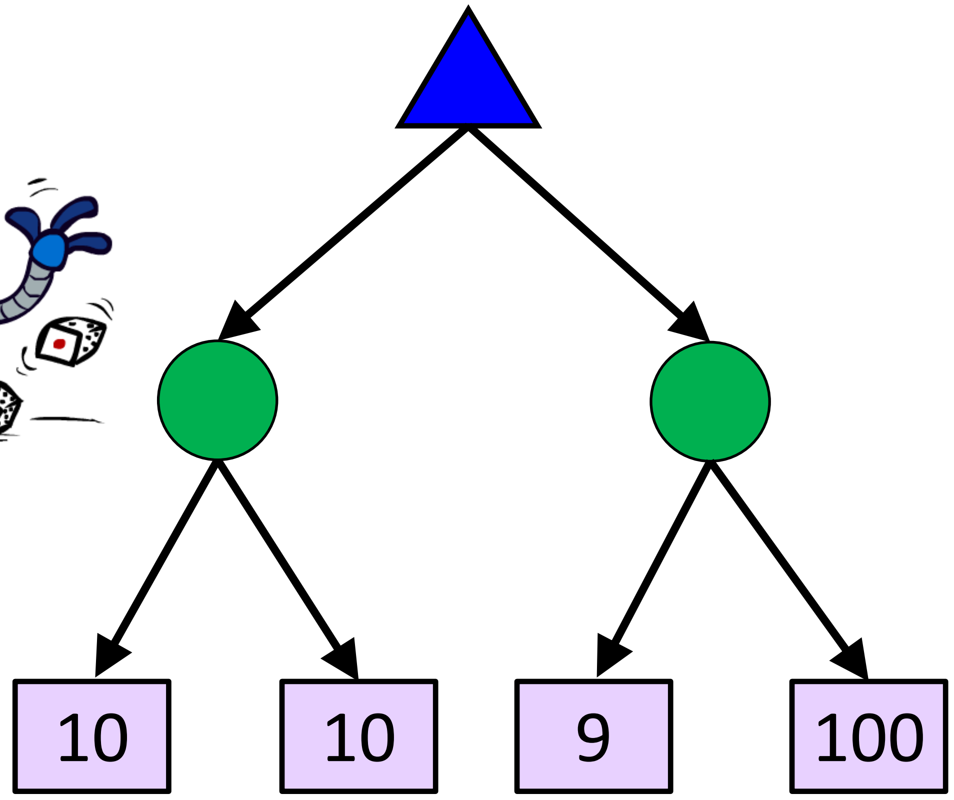




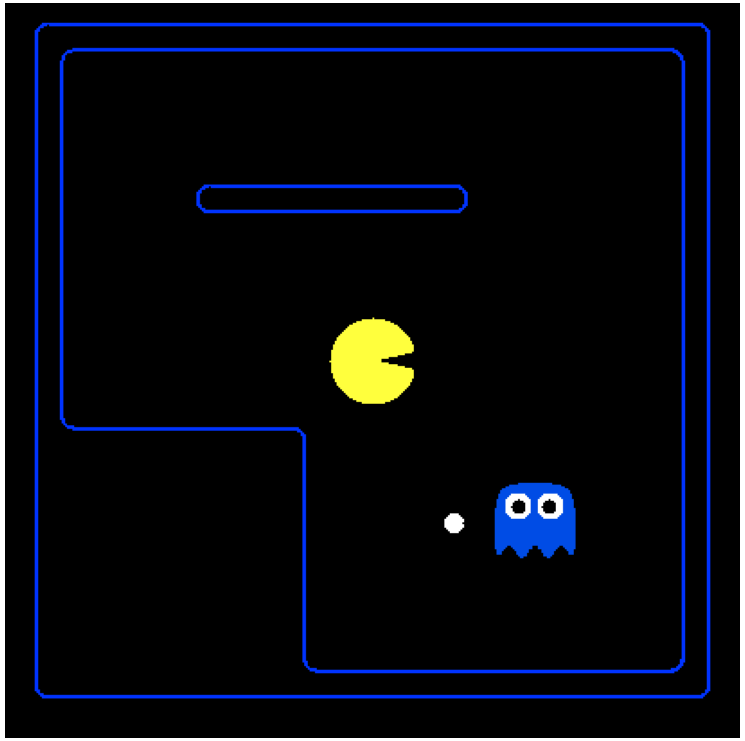
Modeling Assumptions

Chance nodes: Expectimax





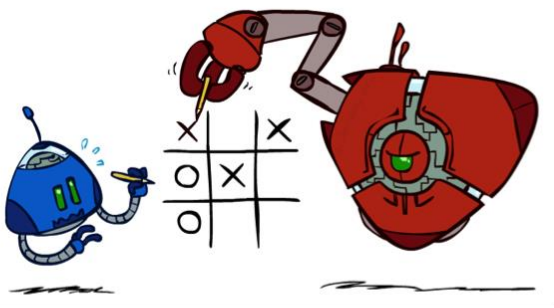
Assumptions vs. Reality

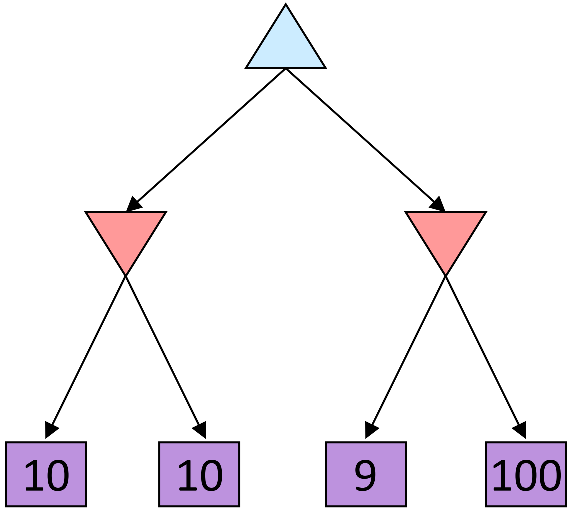


	Minimax Ghost	Random Ghost
Minimax Pacman	Won 5/5 Avg. Score: 493	Won 5/5 Avg. Score: 464
Expectimax Pacman		

Results from playing 5 games

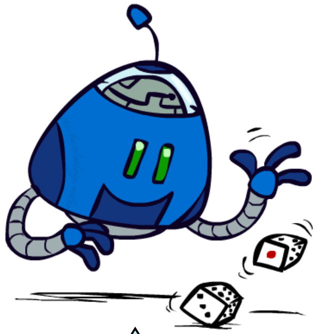
Chance outcomes in trees

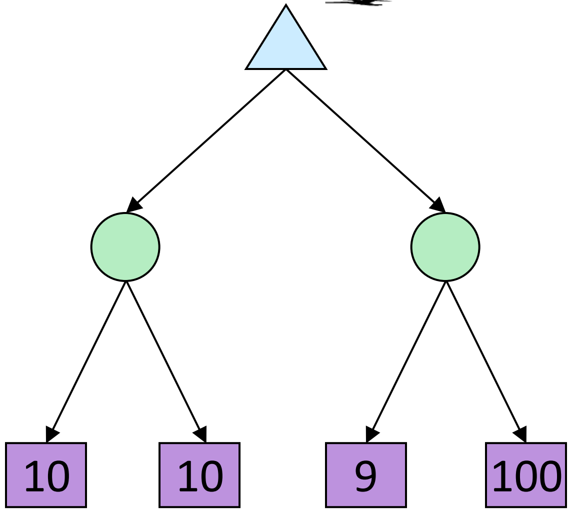




Tictactoe, chess

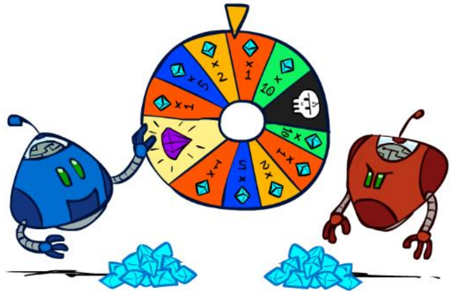
Minimax

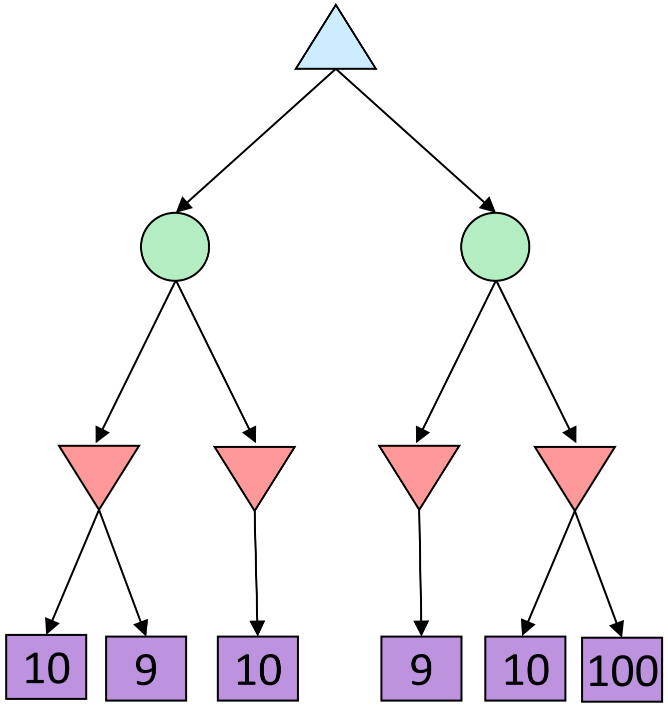




Tetris, investing

Expectimax

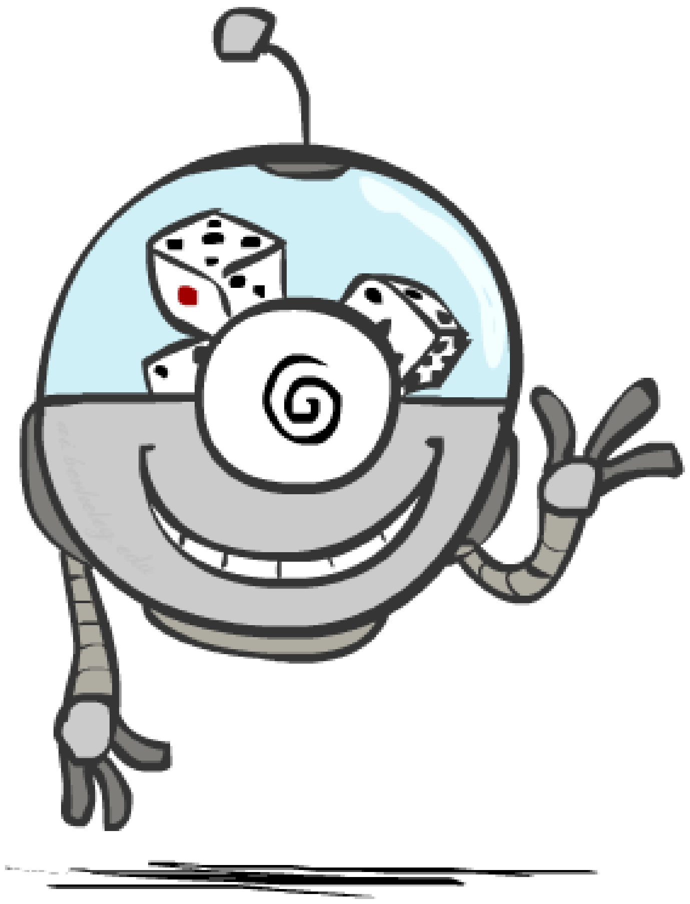




Backgammon, Monopoly

Expectiminimax

Probabilities



Probabilities

A random variable represents an event whose outcome is unknown

A probability distribution is an assignment of weights to outcomes

Example: Traffic on freeway

- Random variable: T = whether there's traffic
- Outcomes: T in {none, light, heavy}
- Distribution:

  P(T=none) = 0.25, P(T=light) = 0.50, P(T=heavy) = 0.25

Probabilities over all possible outcomes sum to one

0.25

0.50

0.25

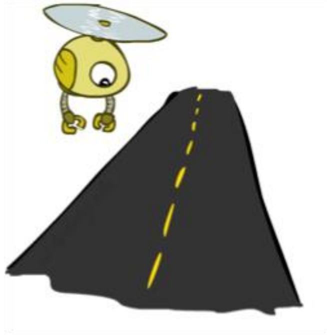
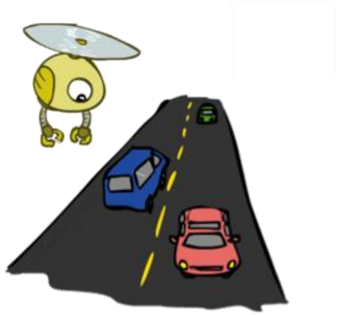
Expected Value

Expected value of a function of a random variable:

Average the values of each outcome,
weighted by the probability of that outcome

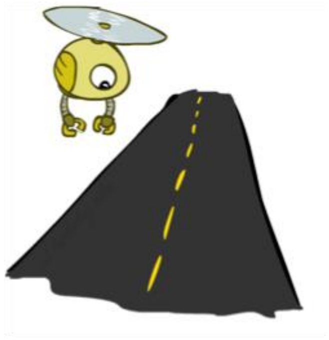
Example: How long to get to the airport?

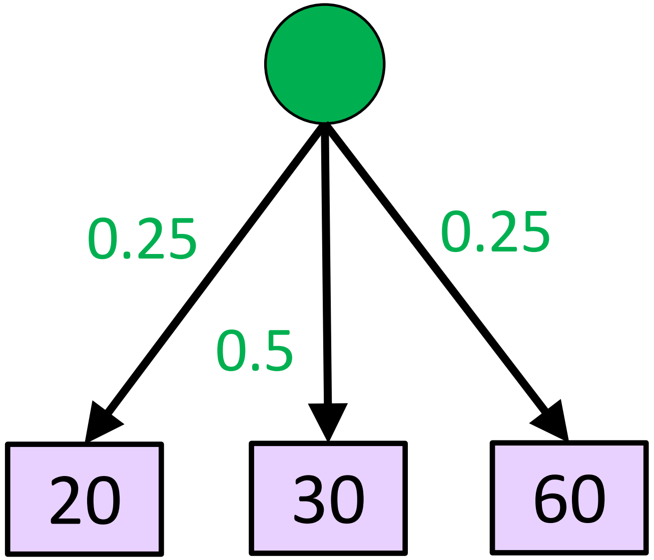
Time:	20 min		30 min		60 min		
	x	+	x	+	x	⇒	35 min
Probability:	0.25		0.50		0.25		


Expectations

Time:	20 min		30 min		60 min
	x	+	x	+	x
Probability:	0.25		0.50		0.25





Max node notation

$$V(s) = \max_{a} V(s'),$$

where $s' = result(s, a)$

Chance node notation

$$V(s) = \sum_{s'} P(s')\, V(s')$$

Poll 5

Expectimax tree search:

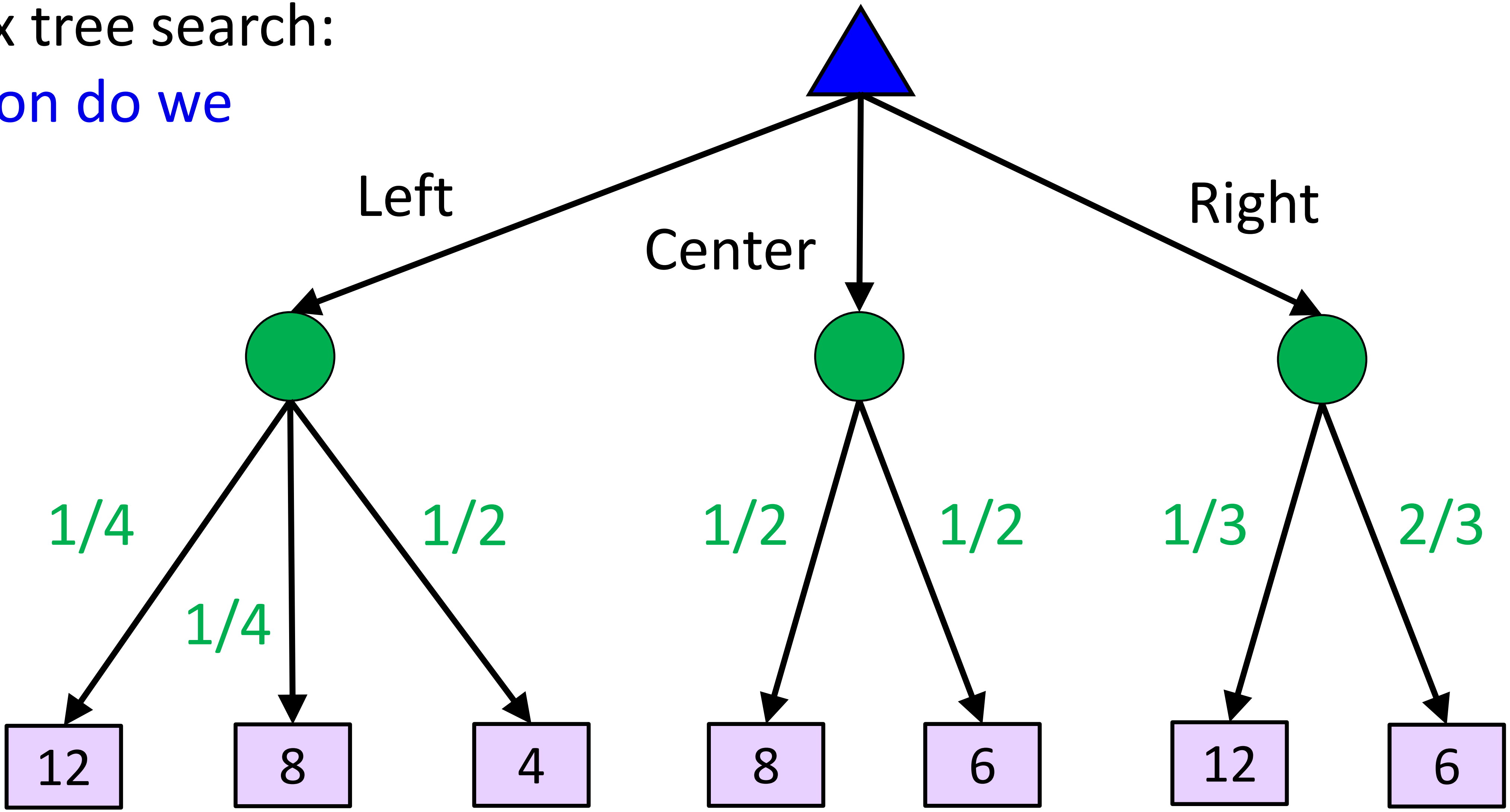
Which action do we choose?

A: Left

B: Center

C: Right

D: Eight



Poll 5

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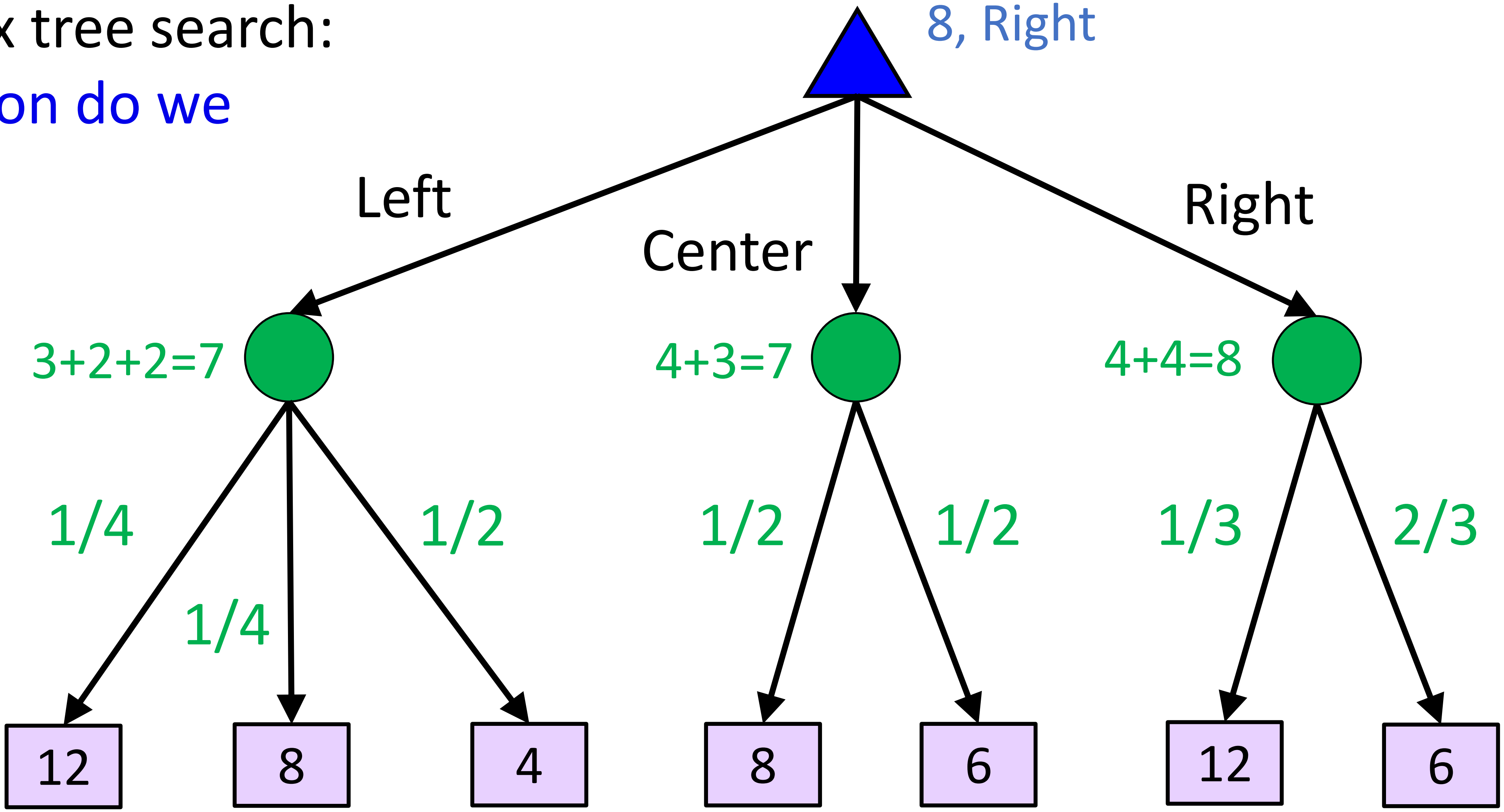
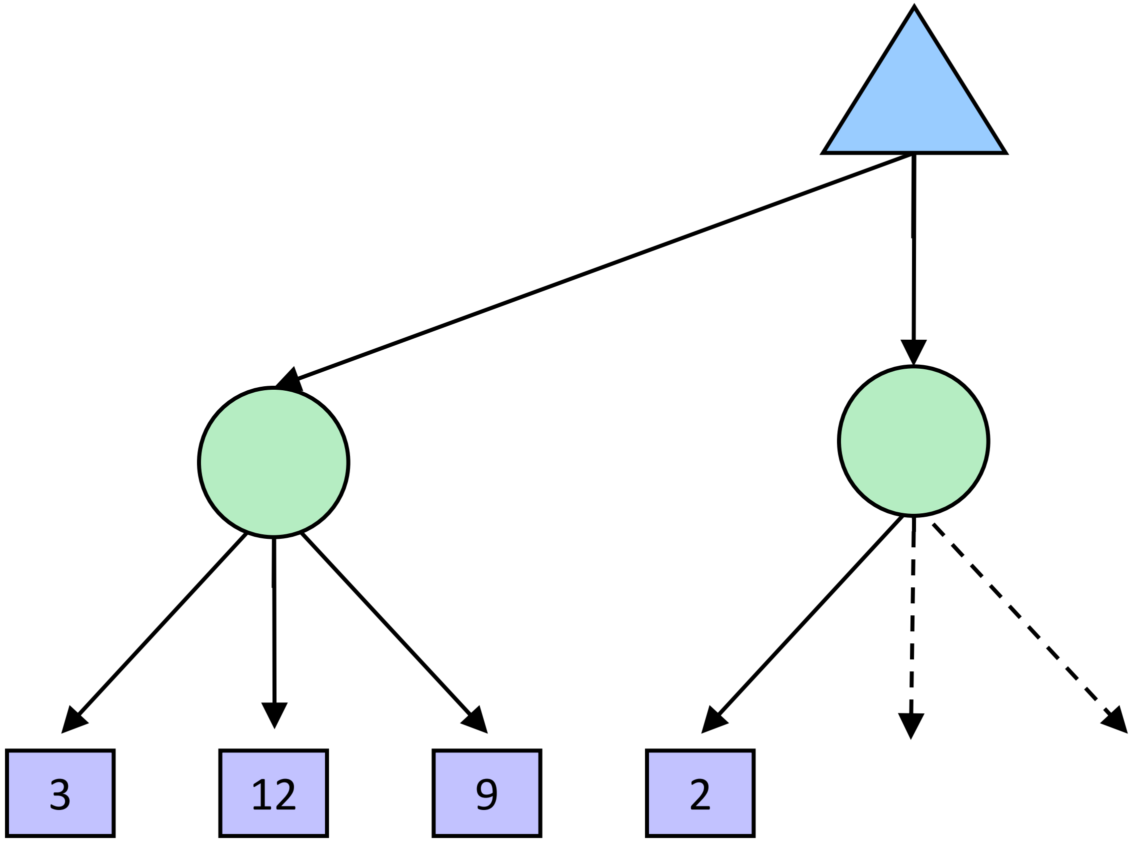
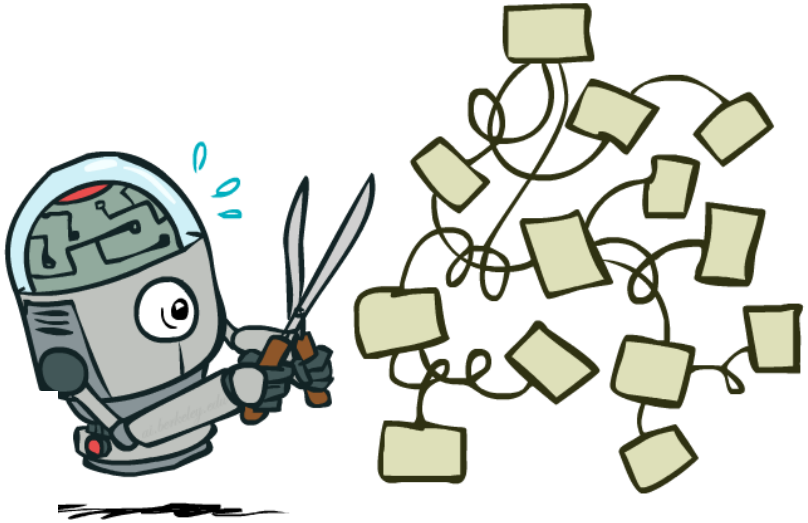


Expectimax Pruning?





Expectimax Code

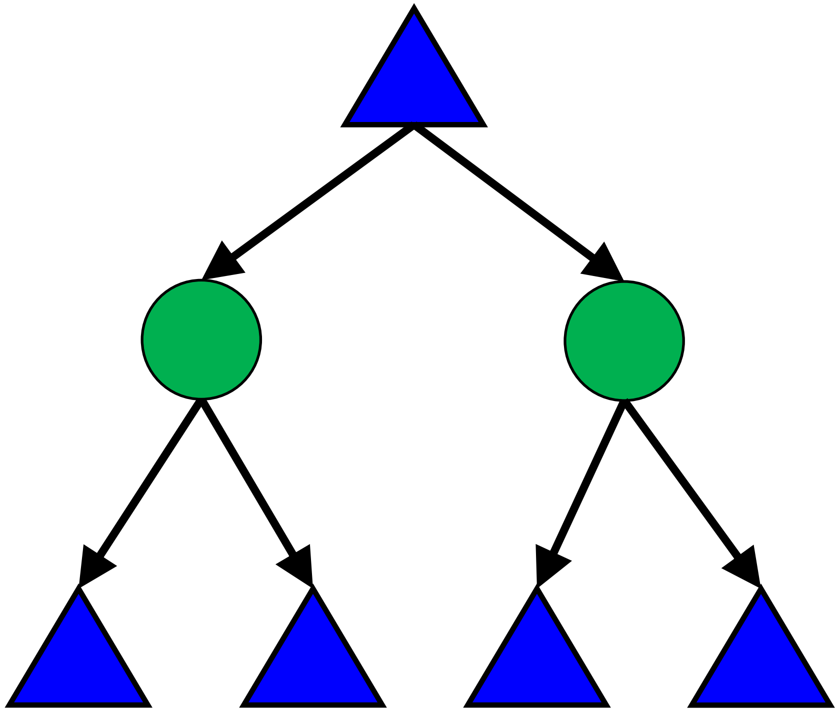
```
function  value( state )
    if  state.is_leaf
        return  state.value

    if  state.player  is  MAX
        return  max a in state.actions  value( state.result(a) )

    if  state.player  is  MIN
        return  min a in state.actions  value( state.result(a) )

    if  state.player  is  CHANCE
        return  sum s in state.next_states  P( s ) * value( s )
```

Preview: MDP/Reinforcement Learning Notation



$$V(s) = \max_a \sum_{s'} P(s')\, V(s')$$

Preview: MDP/Reinforcement Learning Notation

Standard expectimax: $V(s) = \max_a \sum_{s'} P(s'|s,a)V(s')$

Bellman equations: $V(s) = \max_a \sum_{s'} P(s'|s,a)[R(s,a,s') + \gamma V(s')]$

Value iteration: $V_{k+1}(s) = \max_a \sum_{s'} P(s'|s,a)[R(s,a,s') + \gamma V_k(s')], \quad \forall\, s$

Q-iteration: $Q_{k+1}(s,a) = \sum_{s'} P(s'|s,a)[R(s,a,s') + \gamma \max_{a'} Q_k(s',a')], \quad \forall\, s,a$

Policy extraction: $\pi_V(s) = \operatorname{argmax}_a \sum_{s'} P(s'|s,a)[R(s,a,s') + \gamma V(s')], \quad \forall\, s$

Policy evaluation: $V_{k+1}^{\pi}(s) = \sum_{s'} P(s'|s,\pi(s))[R(s,\pi(s),s') + \gamma V_k^{\pi}(s')], \quad \forall\, s$

Policy improvement: $\pi_{new}(s) = \operatorname{argmax}_a \sum_{s'} P(s'|s,a)[R(s,a,s') + \gamma V^{\pi_{old}}(s')], \quad \forall\, s$

Preview: MDP/Reinforcement Learning Notation

Standard expectimax: $V(s) = \max_a \sum_{s'} P(s'|s,a)V(s')$

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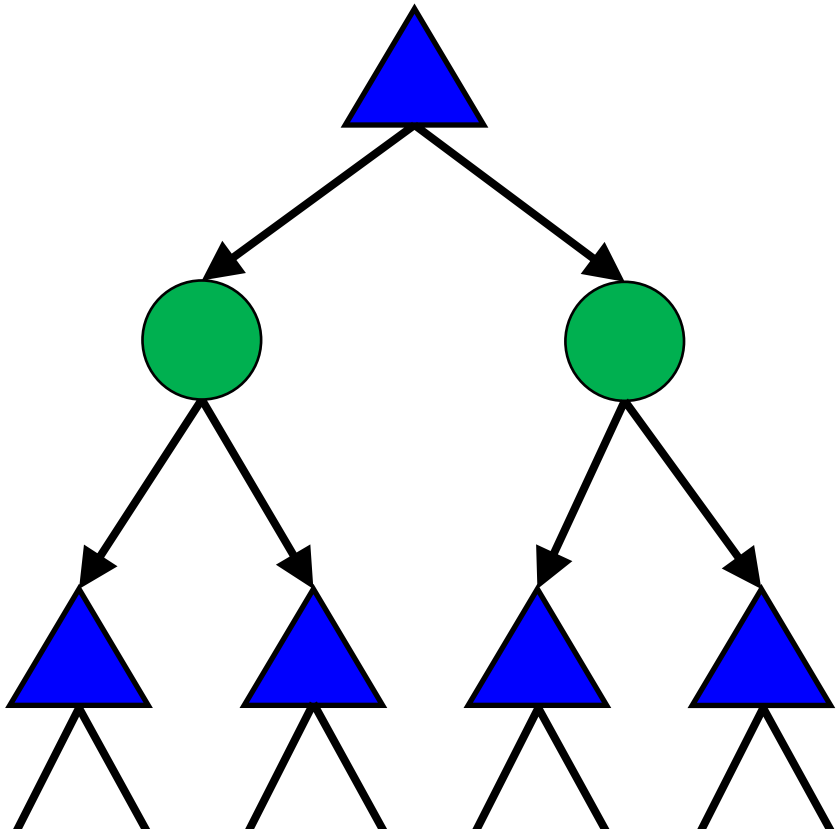
Why Expectimax?

Pretty great model for an agent in the world

Choose the action that has the: highest expected value



Bonus Question

Let’s say you know that your opponent is actually running a depth 1 minimax, using the result 80% of the time, and moving randomly otherwise

Question: What tree search should you use?

A: Minimax

B: Expectimax

C: Something completely different

Summary

Games require decisions when optimality is impossible

- Bounded-depth search and approximate evaluation functions

Games force efficient use of computation

- Alpha-beta pruning

Game playing has produced important research ideas

- Reinforcement learning (checkers)
- Iterative deepening (chess)
- Monte Carlo tree search (Go)
- Solution methods for partial-information games in economics (poker)

Video games present much greater challenges – lots to do!

- $b = 10^{500}$, $|S| = 10^{4000}$, $m = 10{,}000$