

Warm-up: What to eat?

We are trying healthy by finding the optimal amount of food to purchase.

We can choose the amount of **stir-fry** (ounce) and **boba** (fluid ounces).

Healthy Squad Goals

- $2000 \leq \text{Calories} \leq 2500$
- $\text{Sugar} \leq 100 \text{ g}$
- $\text{Calcium} \geq 700 \text{ mg}$

Food	Cost	Calories	Sugar	Calcium
Stir-fry (per oz)	1	100	3	20
Boba (per fl oz)	0.5	50	4	70

What is the cheapest way to stay “healthy” with this menu?

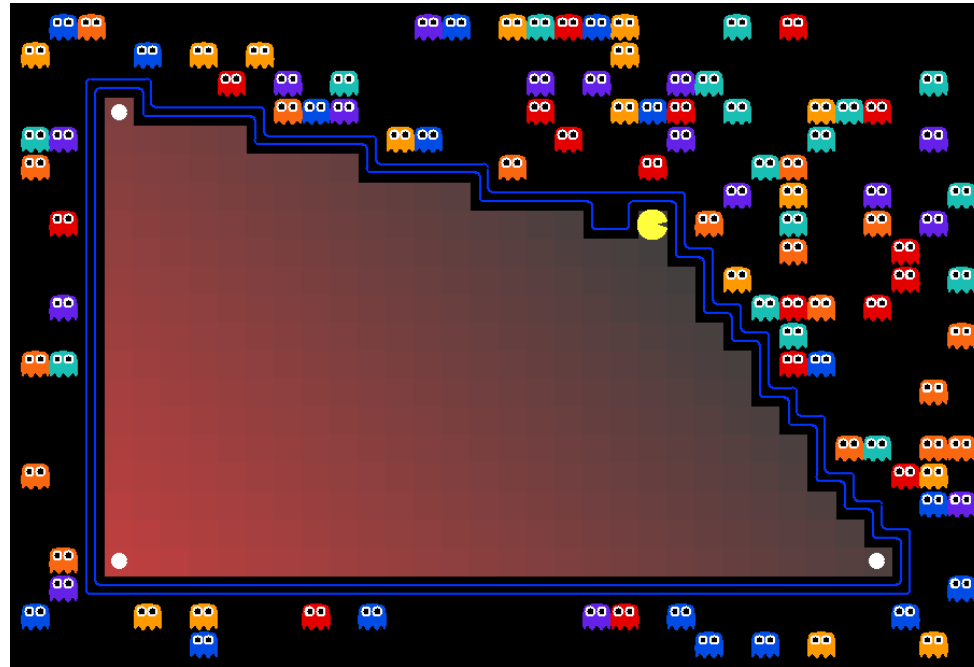
How much **stir-fry** (ounce) and **boba** (fluid ounces) should we buy?

Questions about schedule update?

9/5 Thu	Informed Search	AIMA Ch. 3.5-6	pptx (inked) pdf (inked)
9/10 Tue	Adversarial Search	AIMA Ch. 5.1-2, 5.5	pptx (inked) pdf (inked)
9/12 Thu	Constraint Satisfaction Problems	AIMA Ch. 6.1-3, 6.5 CSP Demo	pptx (inked) pdf (inked) Video: Forward Checking Video: AC-3 Video: Ordering: MRV and LCV
9/17 Tue	Optimization & Linear Programming	Boyd and Vandenberghe Ch. 2.2.1, 2.2.4, 4.3-4.3.1 Desmos Demos: <ul style="list-style-type: none">• Prereq: Dot Product• LP: Cost at points• LP: Zero cost• LP: Cost contours• LP: Constraint• LP: Cost with one constraint• LP	
9/19 Thu	Solving LPs & Integer Programming		
9/24 Tue	<i>Tentative:</i> Local Search & Ethics	AIMA Ch. 4.1, 6.4	
9/26 Thu	Logical Agents	AIMA Ch. 7.1-7	
10/1 Tue	MIDTERM 1 EXAM	In class	

AI: Representation and Problem Solving

Linear Programming



Instructor: Pat Virtue

Slide credits: CMU AI with drawings from <http://ai.berkeley.edu>

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Optimization

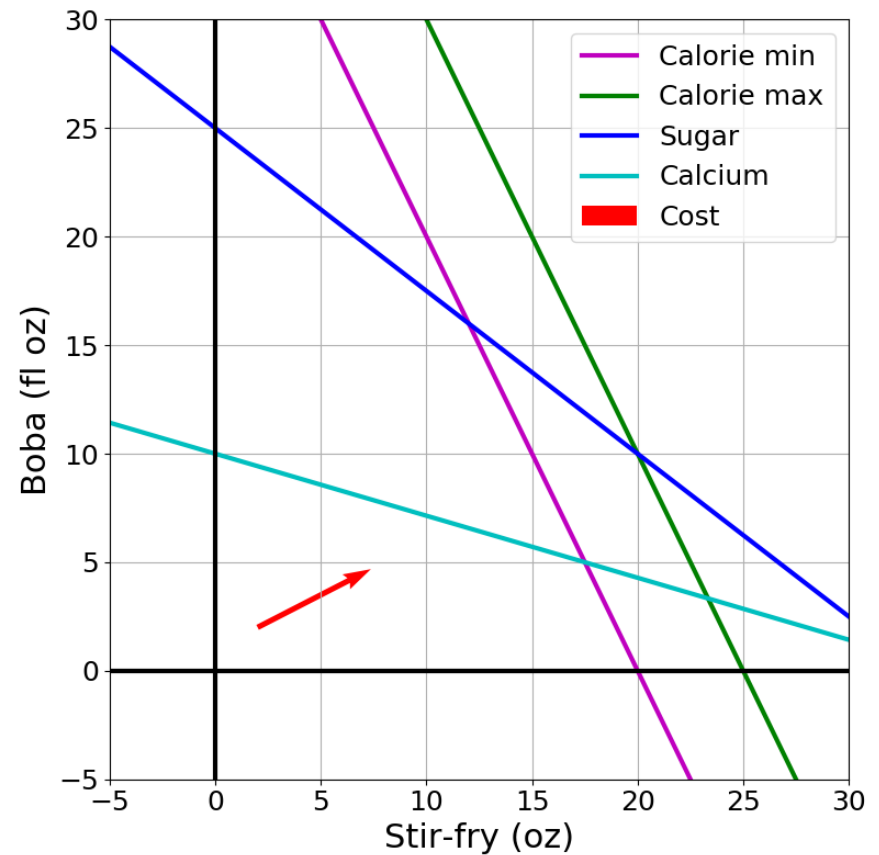
Problem
Description

Optimization
Representation

$$\min_{\mathbf{x}} \quad \mathbf{c}^T \mathbf{x}$$

$$\text{s.t.} \quad A\mathbf{x} \leq \mathbf{b}$$

Graphical Representation



Constraint Satisfaction Problems

Map coloring

Any \mathbf{x}
s.t. \mathbf{x} satisfies constraints
"such that"



Assume vectors are column vectors

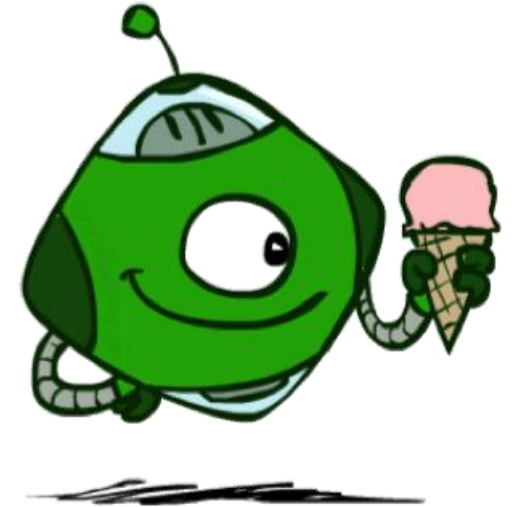
Notation Alert!

Optimization Formulation

Diet Problem

Any \mathbf{x}

s.t. \mathbf{x} satisfies constraints



Healthy Squad Goals

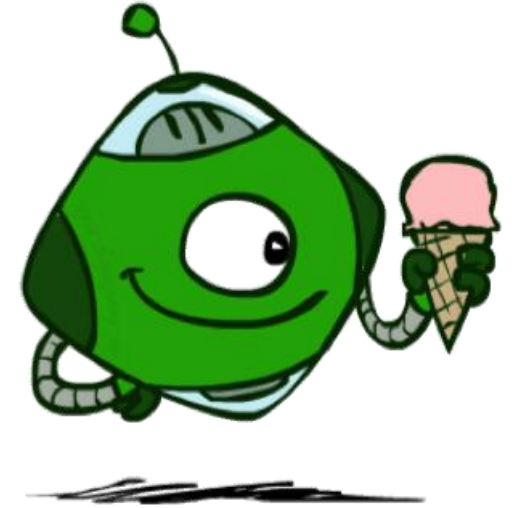
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Optimization Formulation

Diet Problem

$$\begin{array}{ll} \min_{\mathbf{x}} & \text{cost}(\mathbf{x}) \quad \text{Objective function} \\ \text{s.t.} & \mathbf{x} \text{ satisfies constraints} \end{array}$$



Healthy Squad Goals

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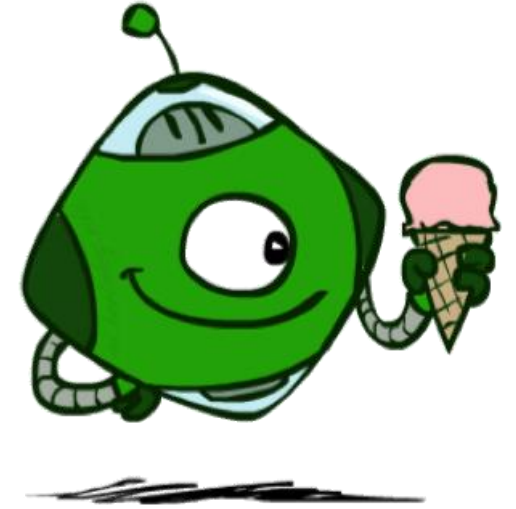
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Optimization Formulation

Diet Problem

$$\begin{array}{ll} \min_{\mathbf{x}} & \text{cost}(\mathbf{x}) \\ \text{s.t.} & \text{calories}(\mathbf{x}) \text{ contained} \\ & \text{sugar}(\mathbf{x}) \leq \text{limit} \\ & \text{calcium}(\mathbf{x}) \geq \text{limit} \end{array}$$



Healthy Squad Goals

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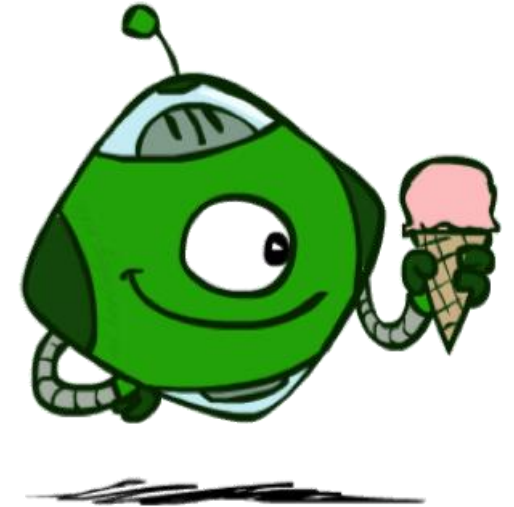
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Optimization Formulation

Diet Problem

$$\begin{array}{ll} \min_{x_1, x_2} & 1 x_1 + 0.5 x_2 \\ \text{s.t.} & 100 x_1 + 50 x_2 \geq 2000 \\ & 100 x_1 + 50 x_2 \leq 2500 \\ & 3 x_1 + 4 x_2 \leq 100 \\ & 20 x_1 + 70 x_2 \geq 700 \end{array}$$

Notation Alert!



Healthy Squad Goals

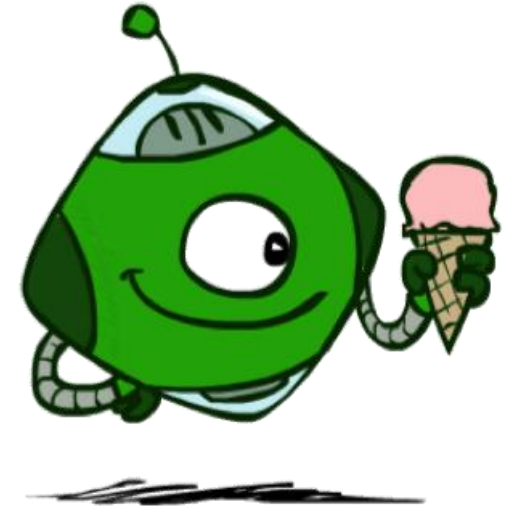
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Optimization Formulation

Diet Problem

$$\begin{aligned} \min_{x_1, x_2} \quad & c_1 x_1 + c_2 x_2 \\ \text{s.t.} \quad & a_{1,1} x_1 + a_{1,2} x_2 \geq b_1 \\ & a_{2,1} x_1 + a_{2,2} x_2 \leq b_2 \\ & a_{3,1} x_1 + a_{3,2} x_2 \leq b_3 \\ & a_{4,1} x_1 + a_{4,2} x_2 \geq b_4 \end{aligned}$$



Cost

$$\mathbf{c} = \begin{bmatrix} 1 \\ 0.5 \end{bmatrix}$$

Limit

$$\mathbf{b} = \begin{bmatrix} 2000 \\ 2500 \\ 100 \\ 700 \end{bmatrix} \begin{array}{l} \text{Calorie min} \\ \text{Calorie max} \\ \text{Sugar} \\ \text{Calcium} \end{array}$$

$$A = \begin{array}{cc} & \begin{array}{l} \text{Stir-fry} \\ \text{Boba} \end{array} \\ \begin{bmatrix} 100 & 50 \\ 100 & 50 \\ 3 & 4 \\ 20 & 70 \end{bmatrix} \end{array}$$

Notation Alert!

Optimization Formulation

Diet Problem

$$\begin{aligned} \min_{\mathbf{x}} \quad & \mathbf{c}^T \mathbf{x} \\ \text{s.t.} \quad & a_{1,1} x_1 + a_{1,2} x_2 \geq b_1 \\ & a_{2,1} x_1 + a_{2,2} x_2 \leq b_2 \\ & a_{3,1} x_1 + a_{3,2} x_2 \leq b_3 \\ & a_{4,1} x_1 + a_{4,2} x_2 \geq b_4 \end{aligned}$$

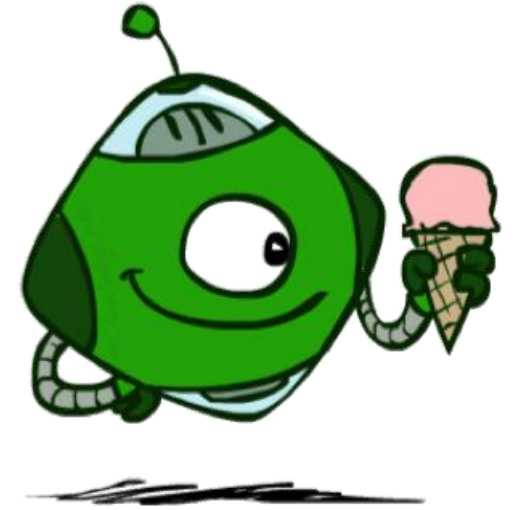
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Notation Alert!

Optimization Formulation

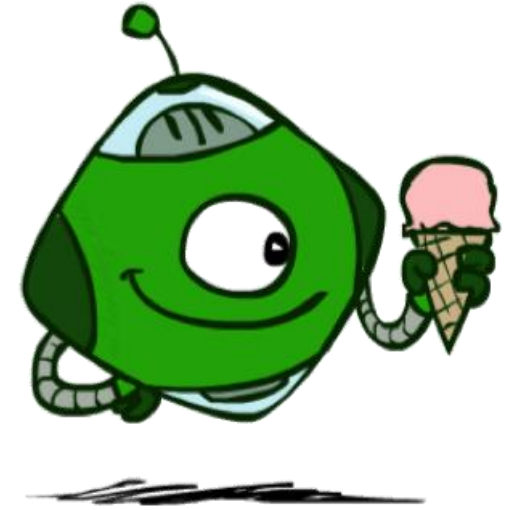
Diet Problem

$$\min_{\mathbf{x}} \quad \mathbf{c}^T \mathbf{x}$$

$$\begin{aligned} \text{s.t.} \quad & -a_{1,1} x_1 - a_{1,2} x_2 \leq -b_1 \\ & a_{2,1} x_1 + a_{2,2} x_2 \leq b_2 \\ & a_{3,1} x_1 + a_{3,2} x_2 \leq b_3 \\ & -a_{4,1} x_1 - a_{4,2} x_2 \leq -b_4 \end{aligned}$$

$$A = \begin{array}{cc} & \begin{array}{l} \text{Stir-fry} \\ \text{Boba} \end{array} \\ \begin{bmatrix} 100 & 50 \\ 100 & 50 \\ 3 & 4 \\ 20 & 70 \end{bmatrix} \end{array}$$

$$\mathbf{b} = \begin{array}{c} \text{Limit} \\ \begin{bmatrix} 2000 \\ 2500 \\ 100 \\ 700 \end{bmatrix} \end{array} \begin{array}{l} \text{Calorie min} \\ \text{Calorie max} \\ \text{Sugar} \\ \text{Calcium} \end{array}$$



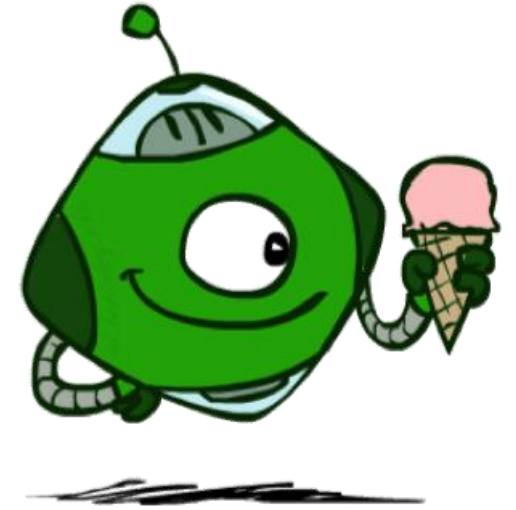
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Optimization Formulation

Diet Problem

$$\begin{aligned} \min_{\mathbf{x}} \quad & \mathbf{c}^T \mathbf{x} \\ \text{s.t.} \quad & a_{1,1} x_1 + a_{1,2} x_2 \leq b_1 \\ & a_{2,1} x_1 + a_{2,2} x_2 \leq b_2 \\ & a_{3,1} x_1 + a_{3,2} x_2 \leq b_3 \\ & a_{4,1} x_1 + a_{4,2} x_2 \leq b_4 \end{aligned}$$



Cost

$$\mathbf{c} = \begin{bmatrix} 1 \\ 0.5 \end{bmatrix}$$

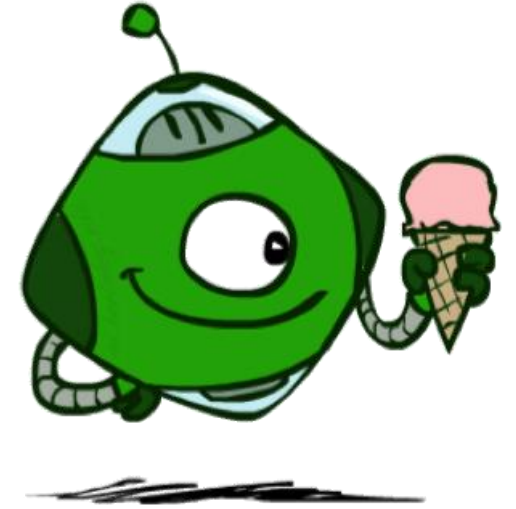
Limit

$$\mathbf{A} = \begin{array}{cc} \text{Stir-fry} & \text{Boba} \\ \begin{bmatrix} -100 & -50 \\ 100 & 50 \\ 3 & 4 \\ -20 & -70 \end{bmatrix} & \end{array} \quad \mathbf{b} = \begin{array}{c} \begin{bmatrix} -2000 \\ 2500 \\ 100 \\ -700 \end{bmatrix} \\ \begin{array}{l} \text{Calorie min} \\ \text{Calorie max} \\ \text{Sugar} \\ \text{Calcium} \end{array} \end{array}$$

Optimization Formulation

Diet Problem

$$\begin{array}{ll} \min_{\mathbf{x}} & \mathbf{c}^T \mathbf{x} \\ \text{s.t.} & A\mathbf{x} \leq \mathbf{b} \end{array}$$



Cost

$$\mathbf{c} = \begin{bmatrix} 1 \\ 0.5 \end{bmatrix}$$

Limit

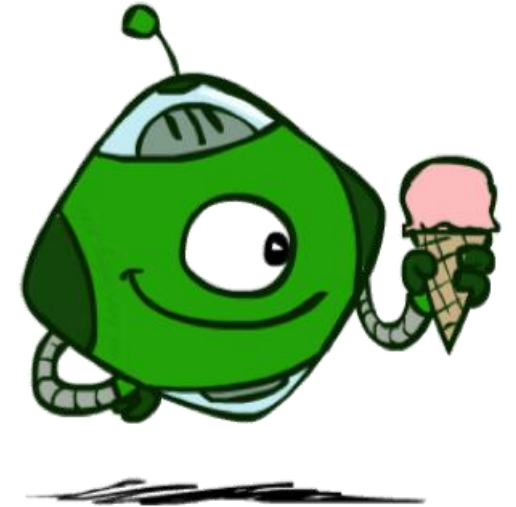
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Notation Alert!

Poll 1

What has to increase to add more nutrition constraints?

$$\begin{array}{ll} \min_{\mathbf{x}} & \mathbf{c}^T \mathbf{x} \\ \text{s.t.} & A\mathbf{x} \leq \mathbf{b} \end{array}$$



Select all that apply

- A) length \mathbf{x}
- B) length \mathbf{c}
- C) height A
- D) width A
- E) length \mathbf{b}

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Poll 1

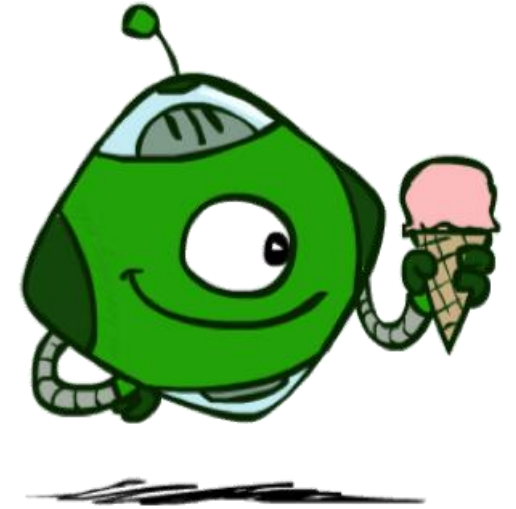
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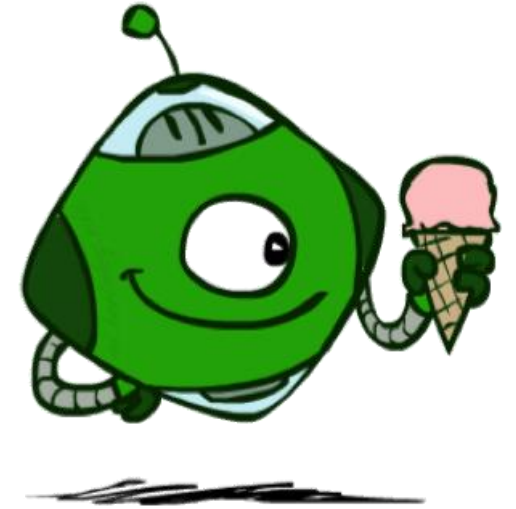
Poll 2

What has to increase to add more menu items?

$$\begin{array}{ll} \min_{\mathbf{x}} & \mathbf{c}^T \mathbf{x} \\ \text{s.t.} & A\mathbf{x} \leq \mathbf{b} \end{array}$$

Select all that apply

- A) length \mathbf{x}
- B) length \mathbf{c}
- C) height A
- D) width A
- E) length \mathbf{b}



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Poll 2

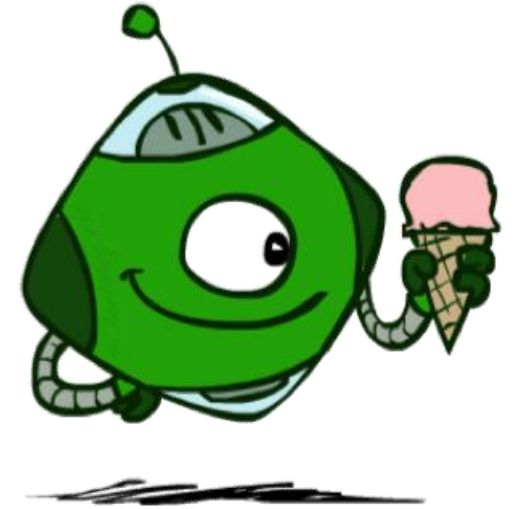
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$$\mathbf{b} = \begin{bmatrix} -2000 \\ 2500 \\ 100 \\ -700 \end{bmatrix}$$



Question

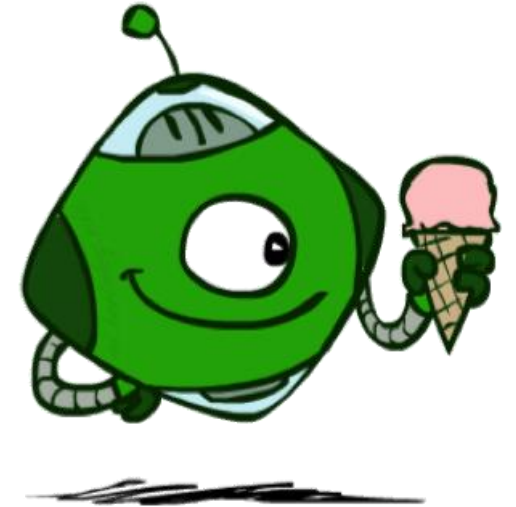
If $A \in \mathbb{R}^{M \times N}$, which of the following also equals N ?

$$\begin{array}{ll} \min_{\mathbf{x}} & \mathbf{c}^T \mathbf{x} \\ \text{s.t.} & A\mathbf{x} \leq \mathbf{b} \end{array}$$

Select all that apply

- A) length \mathbf{x}
- B) length \mathbf{c}
- C) length \mathbf{b}

Notation Alert!



Linear Programming

Linear objective with linear constraints

$$\begin{array}{ll} \min_{\mathbf{x}} & \mathbf{c}^T \mathbf{x} \\ \text{s.t.} & A\mathbf{x} \leq \mathbf{b} \end{array}$$

As opposed to general optimization

$$\begin{array}{ll} \min_{\mathbf{x}} & f_0(\mathbf{x}) \\ \text{s.t.} & f_i(\mathbf{x}) \leq 0, \quad i = 1 \dots M \\ & \mathbf{a}_i^T \mathbf{x} = \mathbf{b}_i, \quad i = 1 \dots P \end{array}$$

Linear Programming

Different formulations

Inequality form

$$\begin{array}{ll} \min. & \mathbf{c}^T \mathbf{x} \\ \text{s.t.} & \mathbf{Ax} \leq \mathbf{b} \end{array}$$

General form

$$\begin{array}{ll} \min. & \mathbf{c}^T \mathbf{x} + \mathbf{d} \\ \text{s.t.} & \mathbf{Gx} \leq \mathbf{h} \\ & \mathbf{Ax} = \mathbf{b} \end{array}$$

Standard form

$$\begin{array}{ll} \min. & \mathbf{c}^T \mathbf{x} \\ \text{s.t.} & \mathbf{Ax} = \mathbf{b} \\ & \mathbf{x} \geq \mathbf{0} \end{array}$$

Important to pay attention to form!

Can switch between formulations!

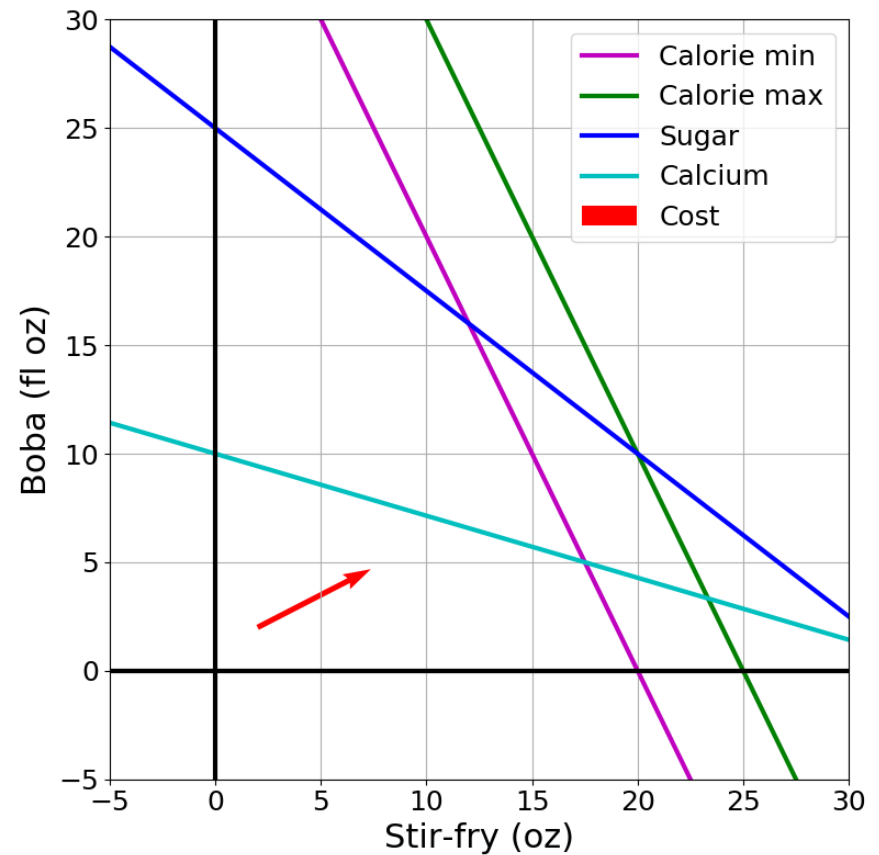
Optimization

Problem
Description

Optimization
Representation

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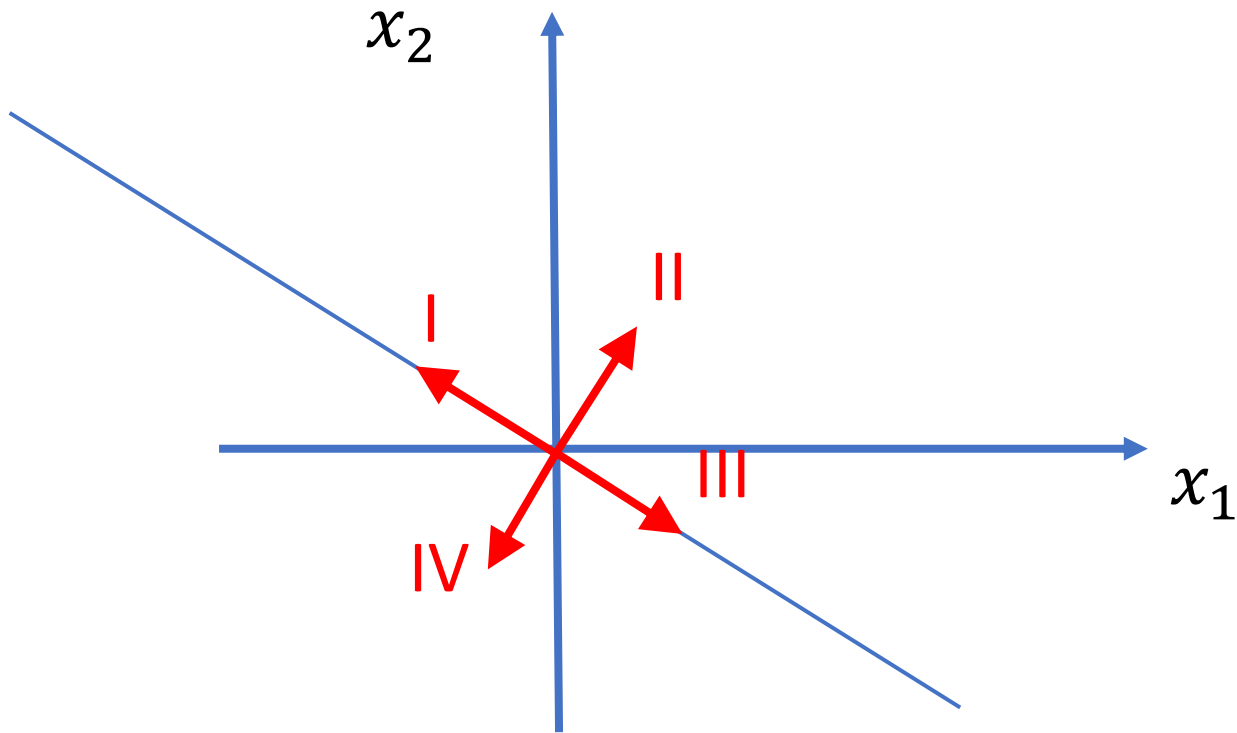
Graphical Representation



Poll 3

Which of these points have cost $\mathbf{c}^T \mathbf{x} = 0$?

for cost vector: $\mathbf{c} = \begin{bmatrix} -2 \\ 1 \end{bmatrix}$



Cost Contours

Given the cost vector $[c_1, c_2]^T$ where will

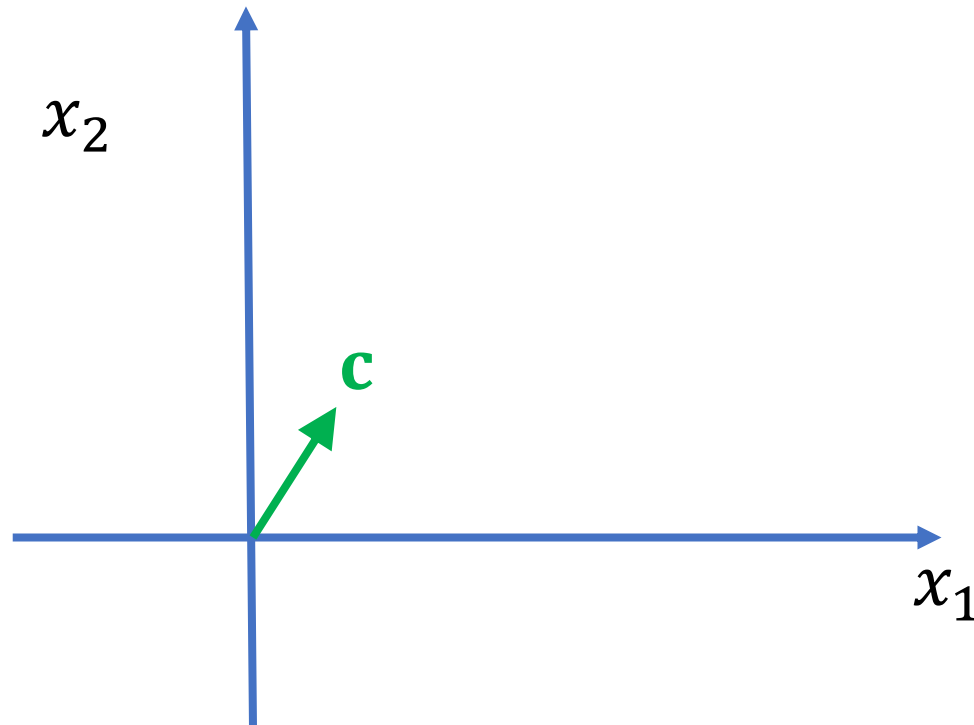
$$\mathbf{c}^T \mathbf{x} = 0 ?$$

$$\mathbf{c}^T \mathbf{x} = 1 ?$$

$$\mathbf{c}^T \mathbf{x} = 2 ?$$

$$\mathbf{c}^T \mathbf{x} = -1 ?$$

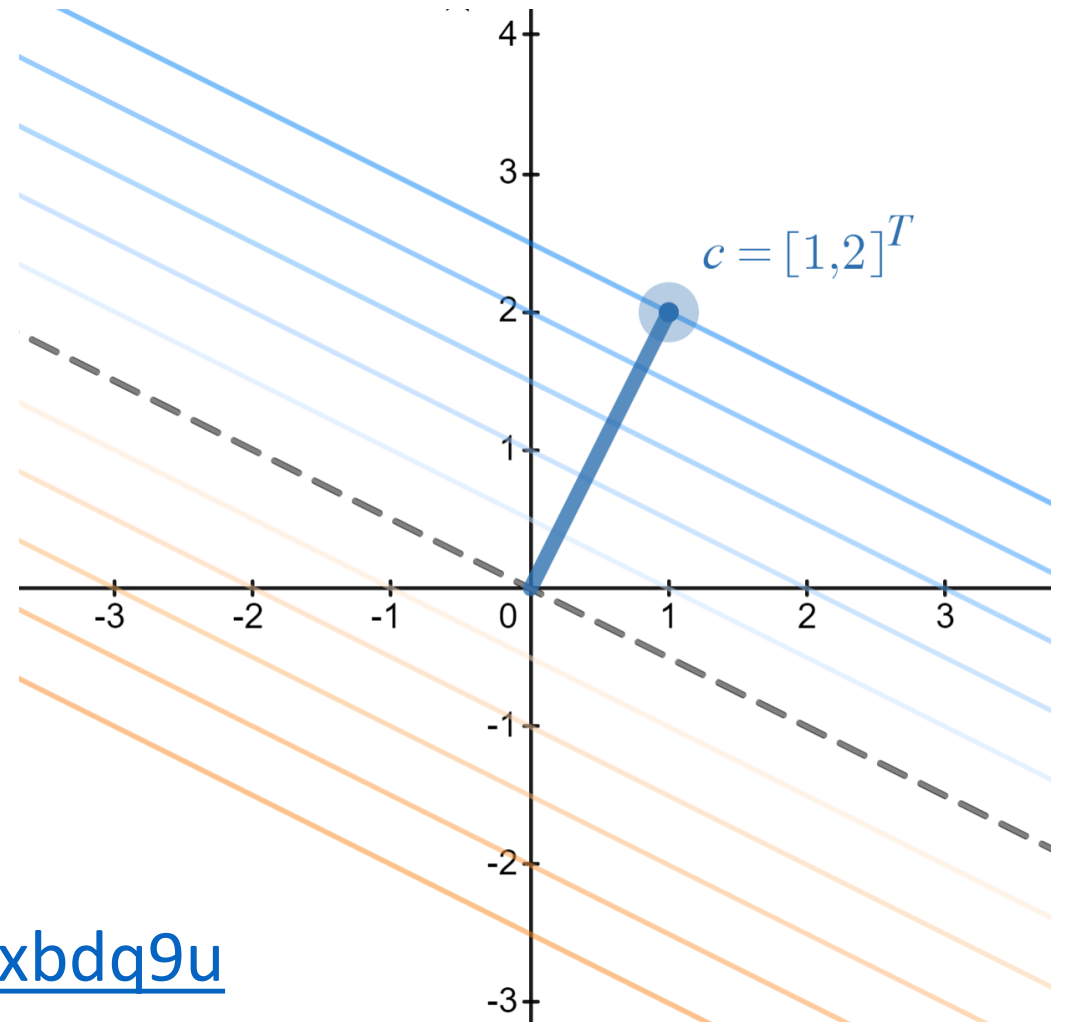
$$\mathbf{c}^T \mathbf{x} = -2 ?$$



Question

As the magnitude of \mathbf{c} increases, the distance between the contours lines of the objective $\mathbf{c}^T \mathbf{x}$:

- A) Increases
- B) Decreases



<https://www.desmos.com/calculator/8d9kxbdq9u>

Graphics Representation

Geometry / Algebra I Quiz

What shape does this inequality represent?

$$a_1 x_1 + a_2 x_2 \leq b_1$$

Graphics Representation

Geometry / Algebra I Quiz

What shape do these represent?

1. $a_1 x_1 + a_2 x_2 = b_1$

2. $a_1 x_1 + a_2 x_2 \leq b_1$

3. $a_{1,1} x_1 + a_{1,2} x_2 \leq b_1$
 $a_{2,1} x_1 + a_{2,2} x_2 \leq b_2$
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Feasible region:

All points x that satisfy the constraints

Graphics Representation

Geometry / Algebra I Quiz

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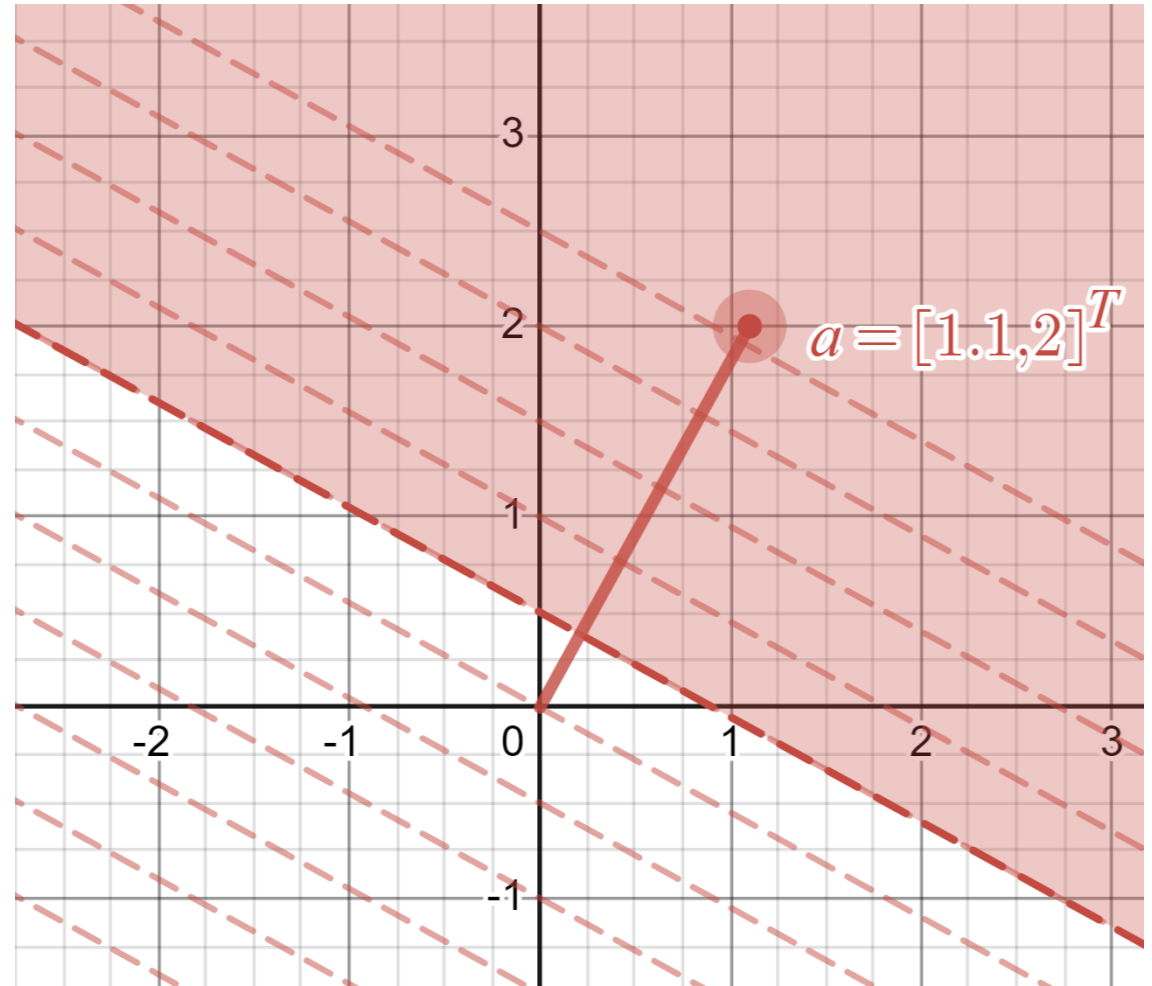
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3.



Graphics Representation

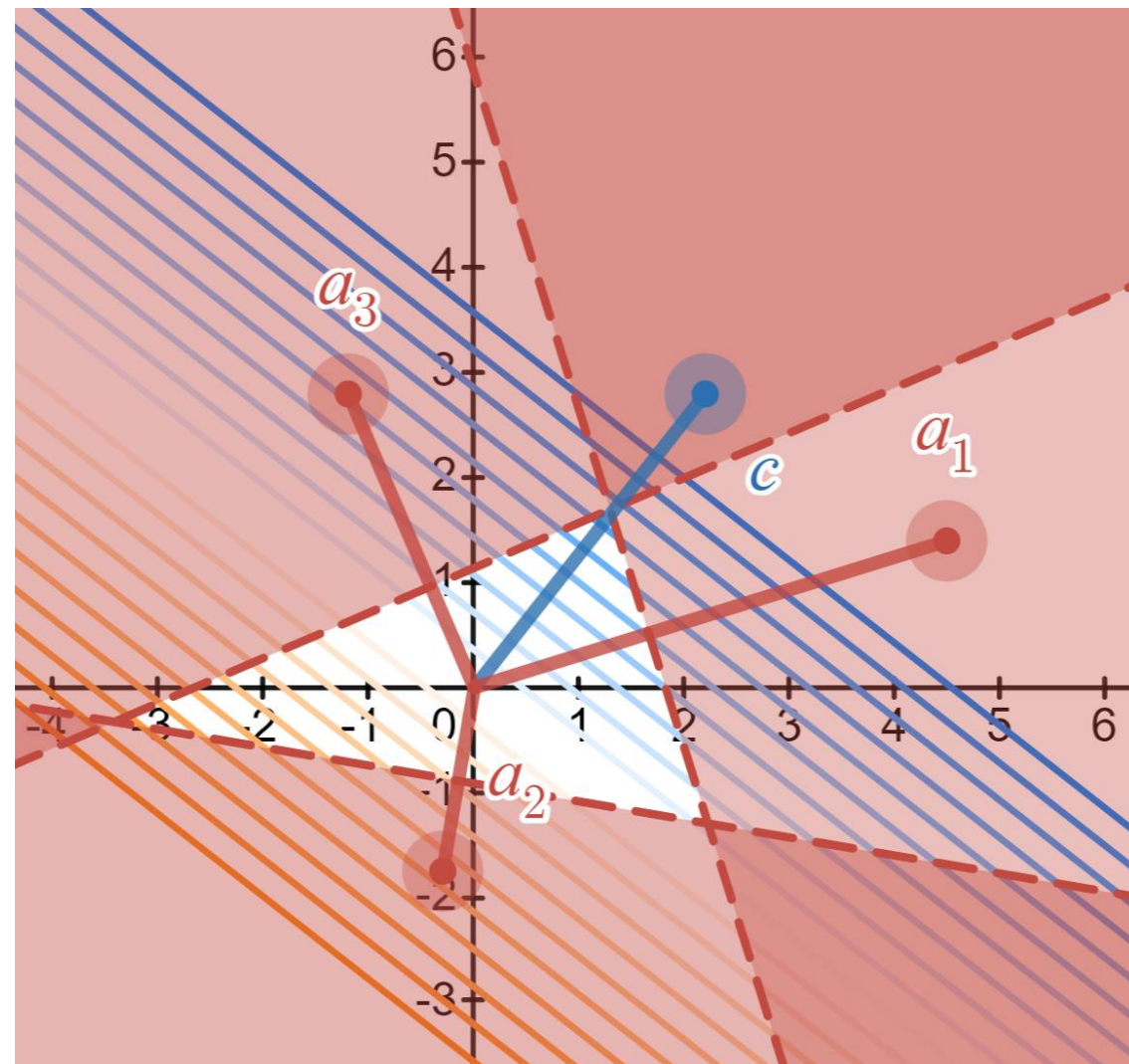
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<https://www.desmos.com/calculator/plp1thgsbh>