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History of Computing

Growth of Analog Computing
& the Birth of Computing Theory

Analog Computers

- Instead of computing with numbers, one builds a physical model (an analog) of the system to be investigated
- Used when a system could not be readily investigated mathematically
- Special purpose instruments
- Their heyday was between WW I & WW II
  - Scaled models of dam projects, electrical grids, the Zuider Zee (Netherlands), California irrigation projects, British weather

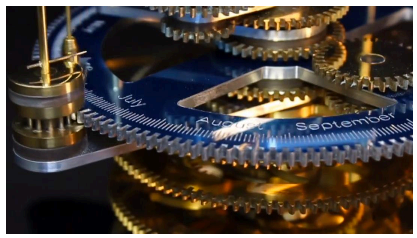
Antikythera Mechanism

Antikythera Mechanism

- In 1900, a Greek diver spotted a shipwreck that contained this mechanism.
- Earliest example of an analog computer ever found to calculate motion of the sun and moon against a fixed background of stars (for navigation presumably).
- Dated to around 100 BC.
- No one knows who built it or why it was lost.

Analog Computers


Antikythera Mechanism
Predicts astronomical positions and eclipses

Orrery
Used to determine relative positions of the planets and the sun

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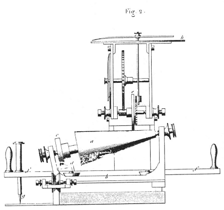
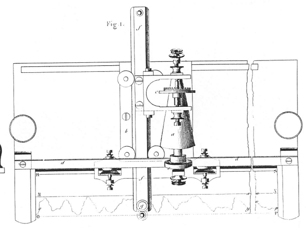
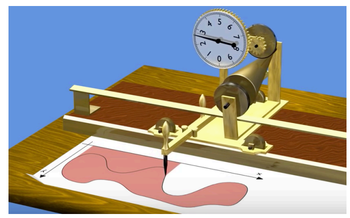
Orrery

Planimeter

- Ernst Planimeter (1834) - traces outline mechanically to compute area of region

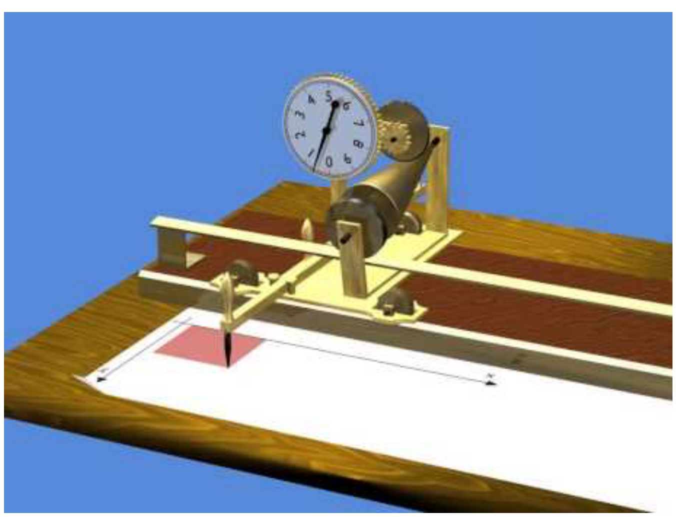






Polar planimeter

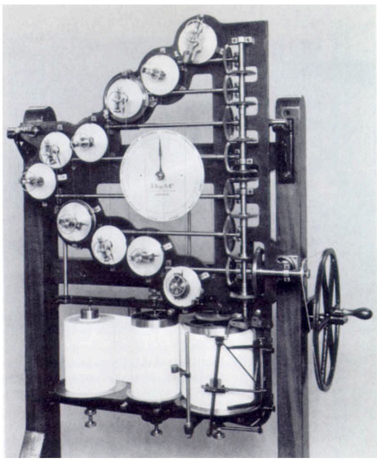
Ernst Planimeter

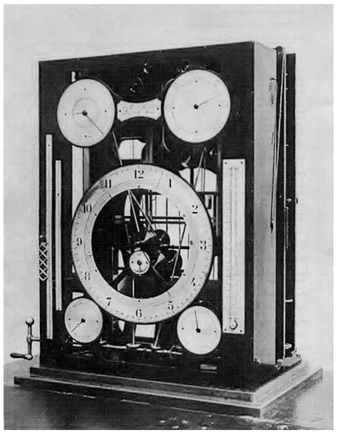


Analog Computers

- Lord Kelvin (1824-1907) (Sir William Thomson)
  - Father of Analog Computing
  - Invented analog tide-predicting machine (1876)
    - Used in thousands of ports throughout the world
  - Many other inventions

Tide Predicting Machines



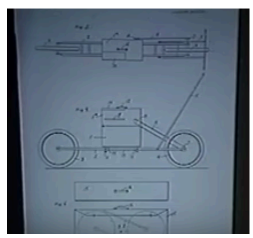


by Lord Kelvin (1876, left) and William Ferrel (1882, right)

Tide Prediction Machine

The National Oceanography Centre in Liverpool

Vannevar Bush



- Developed the profile tracer
  - a bicycle wheel with gadgetry for measurement
  - a one-problem analog computer
    - used to plot ground contours
- During WW II, Bush became chief scientific adviser to Roosevelt
- Another analog computer he developed was the differential analyzer

Vannevar Bush

Vannevar Bush Symposium, MIT 1995


THE EPISTLE OF THE
APOSTLE PAVL TO
THE ROMANES

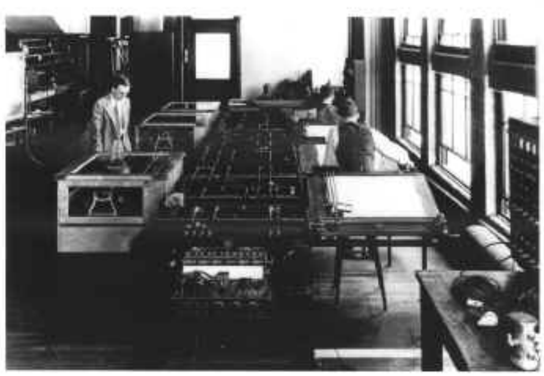
Differential Analyzer

- Designed by Vannevar Bush at MIT
  - starting in the 1920s and completed in the early 1930s
- More of a general purpose computer (still limited)
  - Useful for differential equations
    - Describe many aspects of the physical environment involving rates of change
      - Accelerating projectiles
      - Oscillating electric currents
  - Differential Equation Example [Khan Academy]:
  - Solve for y(x): $y'' + 2y' = 3y$
  - The solution is a <u>function</u> y of x (rather than a single numerical value). e.g. $y(x) = e^{-3x}$

Differential Analyzer

- Rockefeller Differential Analyzer completed in 1942 at MIT
  - Massive machine
    - 100-tons
    - 2000 vacuum tubes
    - 150 motors
  - Fell into secrecy during World War II
  - Emerging after WWII, the Differential Analyzer was already obsolete, being replaced by digital computers like ENIAC

Differential Analyzer



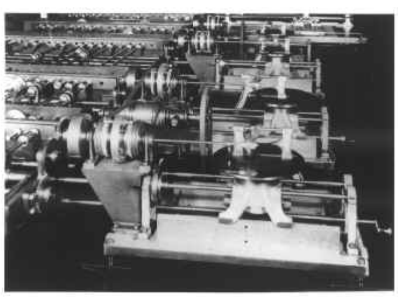
The Differential Analyzer (MIT Museum)

Differential Analyzer

Vannevar Bush

Operator’s console of the Differential Analyzer (MIT Museum)

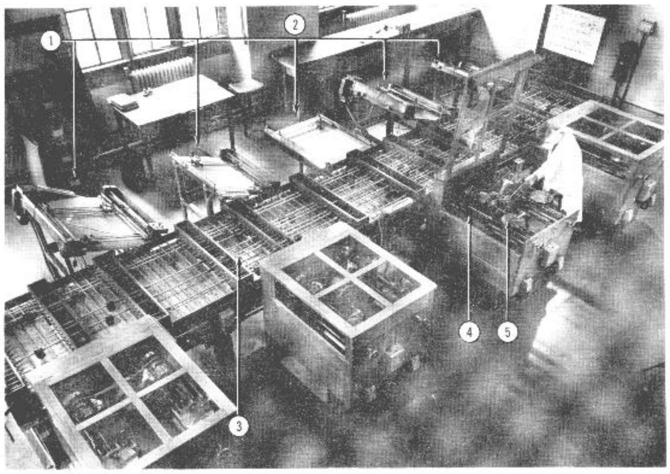
Differential Analyzer



Close-up of wheel and disk integrators on the machine (MIT Museum)

Close up of bus rods which carry variables between different calculating units (MIT Museum)

Differential Analyzer



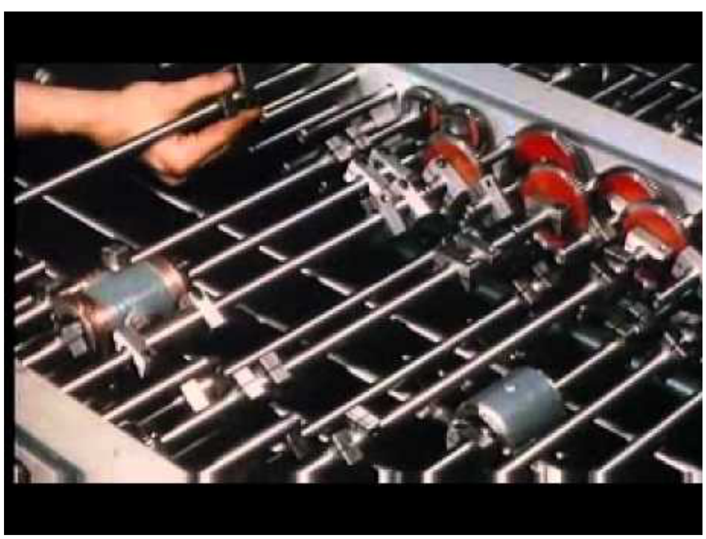
1 Input table
2 Output table
3 Shafts and gears used for interconnection
4 Torque amplifier
5 Integrator disk

FIG. 4. The differential analyzer system, showing integrators, torque amplifiers, and shafting.

Another view

Differential Analyzer at UCLA

1948



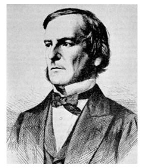
Advantages of Analog Calculation

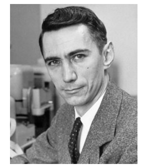
- Ability to solve a given problem numerically even without the ability to find a formal mathematical solution
- Ability to solve even a very complex problem in a relatively short time
- Ability to explore the consequences of a wide range of hypothetical different configurations of the problem being simulated in a short period of time
- Ability to transmit information between components at very high rates

Disadvantages of Analog Calculation

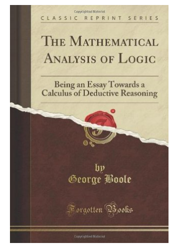
- An analog device is not universal.
  - not sufficiently general to solve an arbitrary category of problems
- It is difficult if not impossible to store information and results.
- It does not give exact results.
  - Accuracy can vary between 0.02% and 3%
- The components of an analog computer will function as required only when the magnitudes of their voltages or motions lie within certain limits.

Logic





- George Boole
  - Boole represented logical expressions in a mathematical form (*Boolean Algebra*) – 1847
    e.g. A = person’s age is at least 18.
  - In conjunction with Boole, Augustus DeMorgan formalized a set of logical operations (DeMorgan transformations).



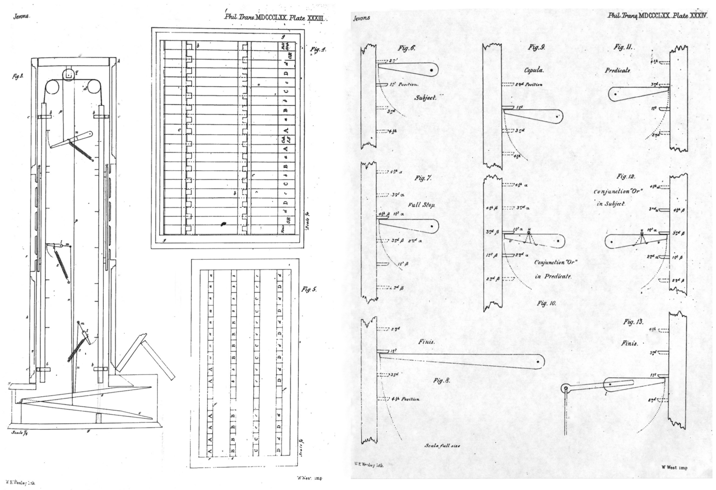
- Claude Shannon
  - Shannon showed how Boole's concepts of TRUE and FALSE could be used to represent the functions of switches in electronic circuits -1938.

George Boole

Logic Machines

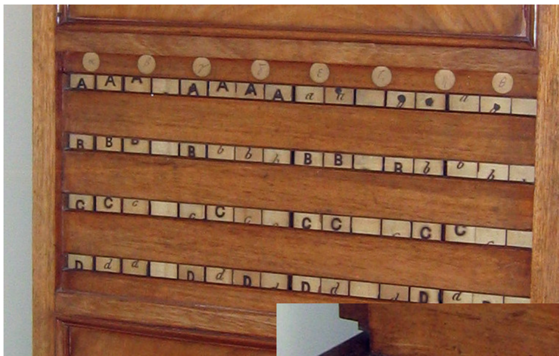
- Logic Piano (1869)
  - William Stanley Jevons
  - economist
  - Jevons' paradox
    - increased efficiency leads to increase resource use
  - invented a machine to solve 4-variable logic equations, eliminating logical impossibilities

Logic Machines



Source: Rutherford Journal

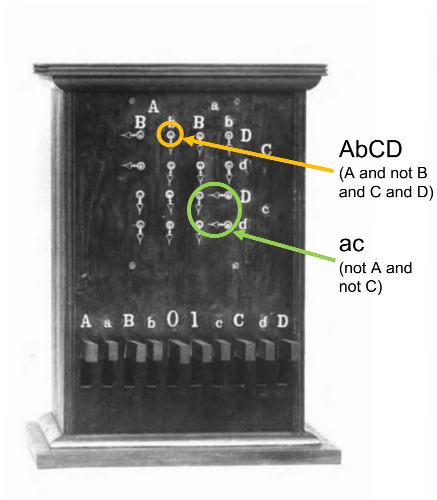
Logic Machines

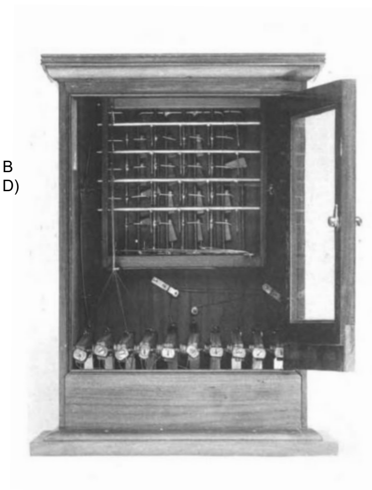


Logic Machines

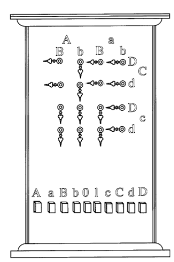
- Marquand Logic Machine (1881) - Allan Marquand
  - “The machine displayed all the valid implications of a simple logical proposition [of 4 variables.” (IBM Archives)
  - Created first circuit diagram of an electrical logic machine (1885)

Logic Machines





Logic Machines



- Example:
  A implies B and B implies C.
  - Press 1 key to reset machine (all horizontal).
  - A —> B is equivalent to A and not B implies false.
    - Press A and b and 0 keys. Ab pointers fall.
  - B —> C is equivalent to B and not C implies false.
    - Press B and c and 0 keys. Bc pointers fall.
  - Remaining horizontal pointers yield solution:
    ABC + aBC + abC + abc = true
    or simply BC + ab is true.

Theoretical Foundations of Computation

- David Hilbert – Decidability
  - Given a formal, abstract and symbolic system defining mathematics, does there exist a definite procedure guaranteed to be able to decide, in a finite number of steps, whether an arbitrary assertion stated in the system is true or is false?

Decidability

- "I am lying."
- "The following sentence is true.
  The previous sentence is false."

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Theoretical Foundations of Computation

- Kurt Gödel – Incompleteness Theorem
  - Provided arithmetic is consistent, mathematics is incomplete in that there exist propositions which cannot be proved.
  - Proved using Gödel Numbers
    <a,b,c,d,e,…> = $2^a 3^b 5^c 7^d 11^e$…
    where a,b,c,d,e,… are positive integers
    - Example: Each word in the dictionary can be mapped to a unique Gödel Number.
      cab: $2^3 3^1 5^2 = 600$
      deaf: $2^4 3^5 5^1 7^6 = 2287096560$

Kurt Gödel

Alan Mathison Turing

- Born in 1912 in London, England
- Sent away from his parents every autumn to the isolation of boarding school
- Studied chemistry and mathematics when it was out of fashion to do so
- Entered King's College, Cambridge in 1931
- Began studying the work of John von Neumann in quantum mechanics in 1932
- Believed in the "liberal-libertarian" school of thought championed by Keynes and Forster.

Alan Turing's Contribution

- Wrote *"On Computable Numbers, with an application to the Entscheidungsproblem"* in 1936
  - Defines the notion of an algorithm
  - Introduces the fundamental concept of a computing model commonly referred to as a "Turing machine"
  - Introduces the notion of a universal algorithmic automaton, more commonly referred to later as a universal Turing machine
  - Considered to be one of the major contributions to the logical foundations of Computer Science

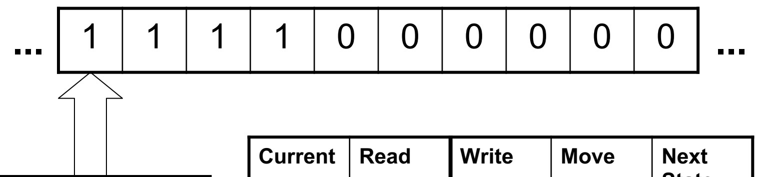
Turing Machine

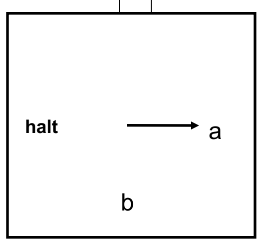
- An abstract mathematical device capable of reading and writing units of information on a tape that is partitioned into a succession of squares and potentially infinite in length.
  - A tape (of infinite length) divided into “cells” horizontally along its surface
  - A finite set of symbols that can be stored on the tape
  - A reading and writing device capable of reading data from a single cell, erasing data in a single cell, writing data to a single cell or moving left or right along the tape one cell at a time
  - A control unit that determines its operation

Turing Machine

- The machine has a number of states that it can be in.
- The behavior of the machine is determined by the current state of the machine and the symbol at the cell on the tape where the read/write device is situated.
- Given this information, the machine may
  - change to another state or remain in the same state
  - move left or right on the tape
  - write data to the tape or erase data on the tape

Turing Machine Components





Current State	Read	Write	Move	Next State
a	0	1	R	b
a	1	1	R	a
b	0	0	L	Halt

Universal Turing Machine

- A Turing machine that is capable of simulating any other Turing machine.
  - One tape holds the "program“ (machine to be simulated).
  - A second tape holds the current "state" of the Turing machine that we are emulating.
  - A third tape holds the "input" and will, upon completion, hold the "output."
- Turing was describing a modern computer in 1936 before it was realistically possible to construct one.
- No one has devised a more general model for computation to this day.

The Halting Problem

- Turing's work gave rise to the notion that computers could not solve every theoretically possible problem
  - A universal program U cannot exist that is capable of answering, within a finite number of steps, the following question relative to any program P:
    - *Will P terminate after a finite number of steps?*
- Many scientists claimed that a Turing machine could compute any intellectual process, and Turing proves them wrong.

Alan Turing