SSA (1 of 2)

15-411/15-611 Compiler Design

Ben L. Titzer and Seth Goldstein

January 28, 2025

Today

- Trivial SSA
- φ-functions
- Dominance
- Placement & Renaming
- Bonus SSA in practice?

SSA

- Static single assignment is an **intermediate representation (IR)** where every variable has only *one* definition
    - Single **static** definition
    - (Could be in a loop which is executed dynamically many times.)
- φ-functions used at CFG join points
- All definitions dominate uses
- Variable names don't matter; IR implementation is literally nodes in a graph that point to each other

Advantages of SSA

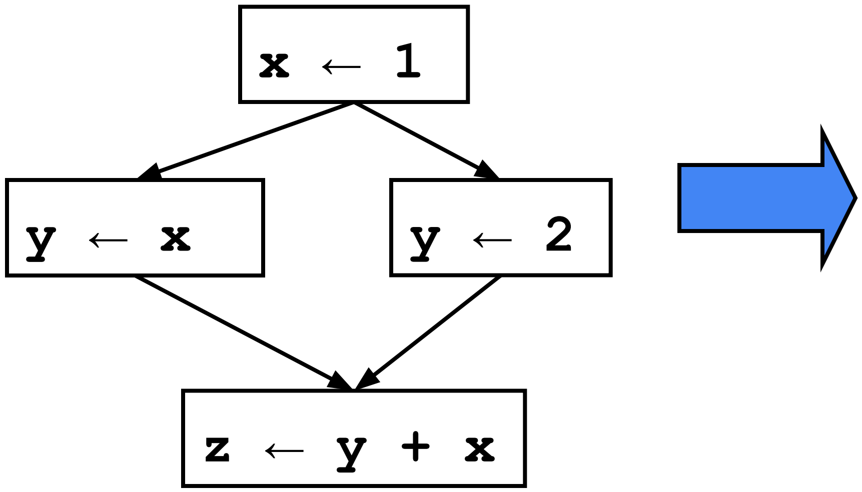
- Makes def-use-chains explicit
- Makes dataflow optimizations more *robust*
  - Easier to get right
  - Multiple optimizations can compose
  - Applies to more places
- Improves register allocation
  - Makes building interference graphs easier
  - Easier register allocation algorithm
  - Decoupling of spill, color, and coalesce
- For most programs reduces space/time requirements
  - Smaller IR, faster optimizations

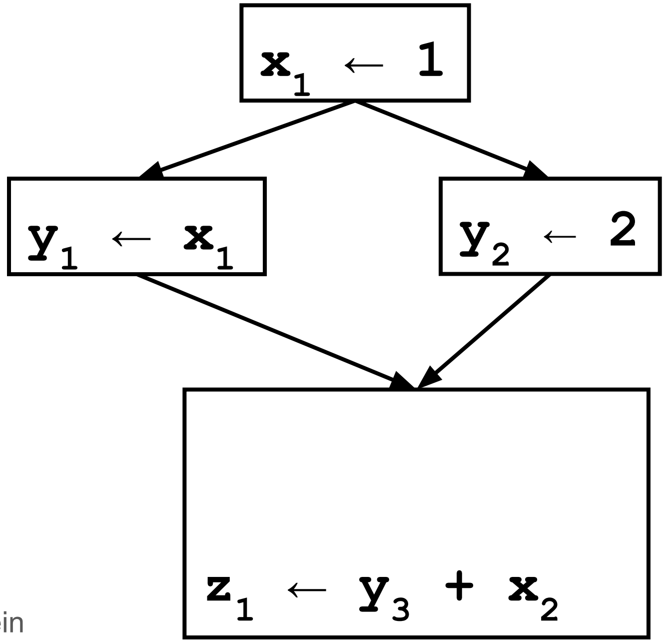
Implications of single definition

- Never have to worry about a variable being overwritten
  - Before SSA, compilers had to worry about variable names and redefinitions
  - A “node” in SSA IR represents a computation, rather than a storage location
- Improves pattern-matching optimizations
  - Constant propagation (y = 13; x + y ⇝ x + 13)
  - Constant folding (3 + 5 ⇝ 8)
  - Strength reduction (x + 0 ⇝ x)
  - Algebraic simplification (x + y - x ⇝ y)
- Improves reasoning across control flow
- Think of it as a “bulk solution” to many forward dataflow problems

Trivial SSA

- Each assignment generates a fresh variable.
- At each join point insert Φ functions for all live variables.





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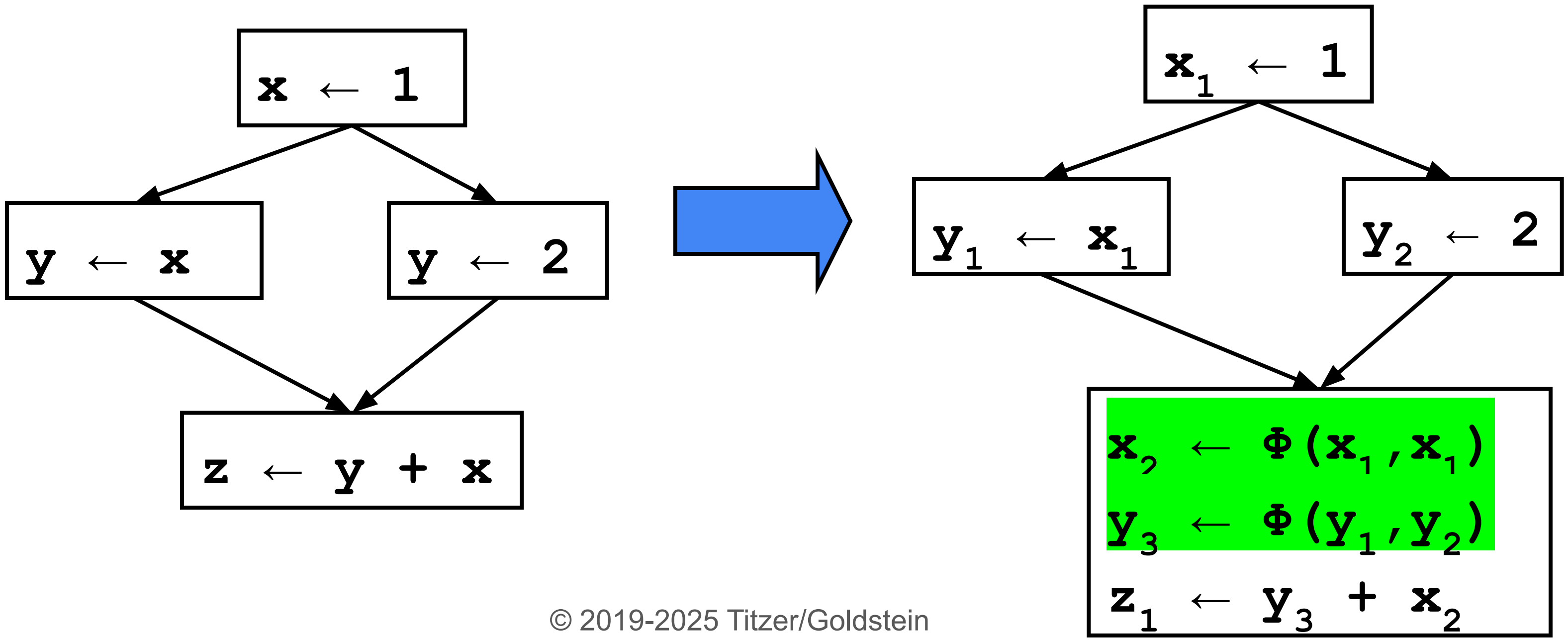

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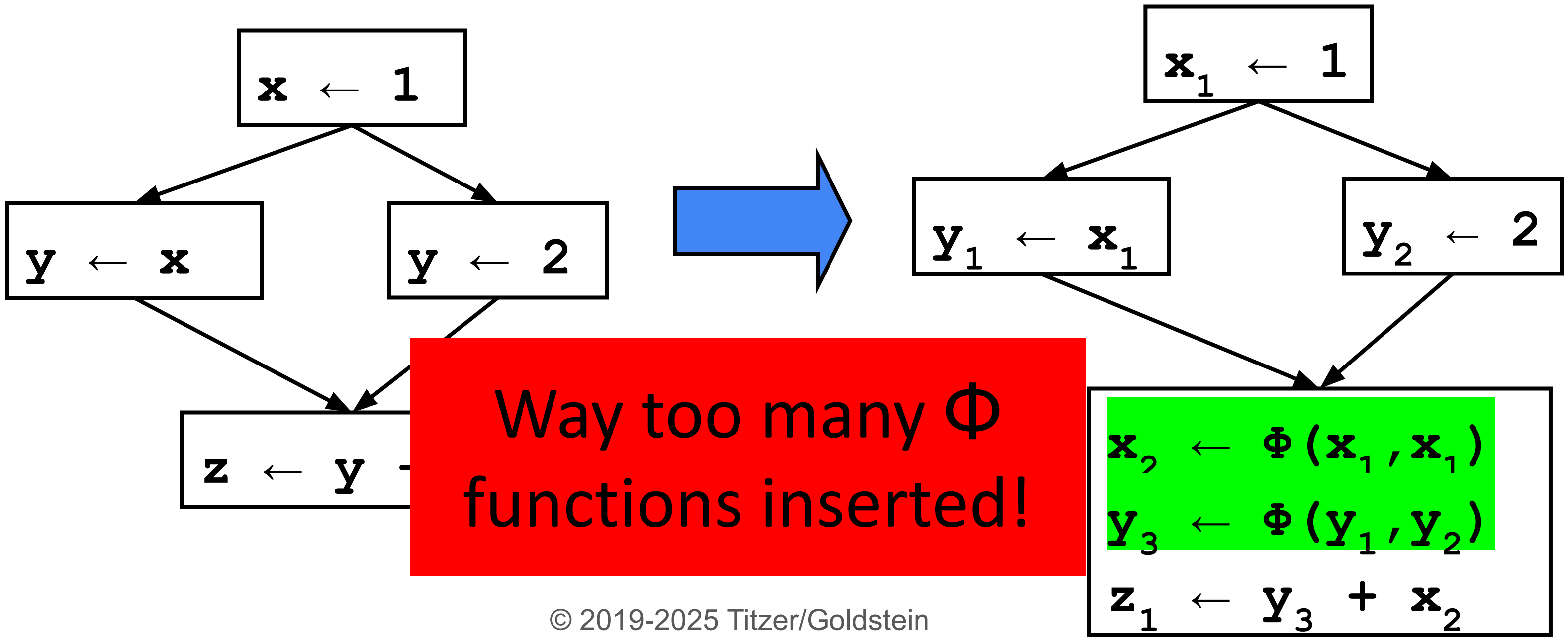

Minimal SSA

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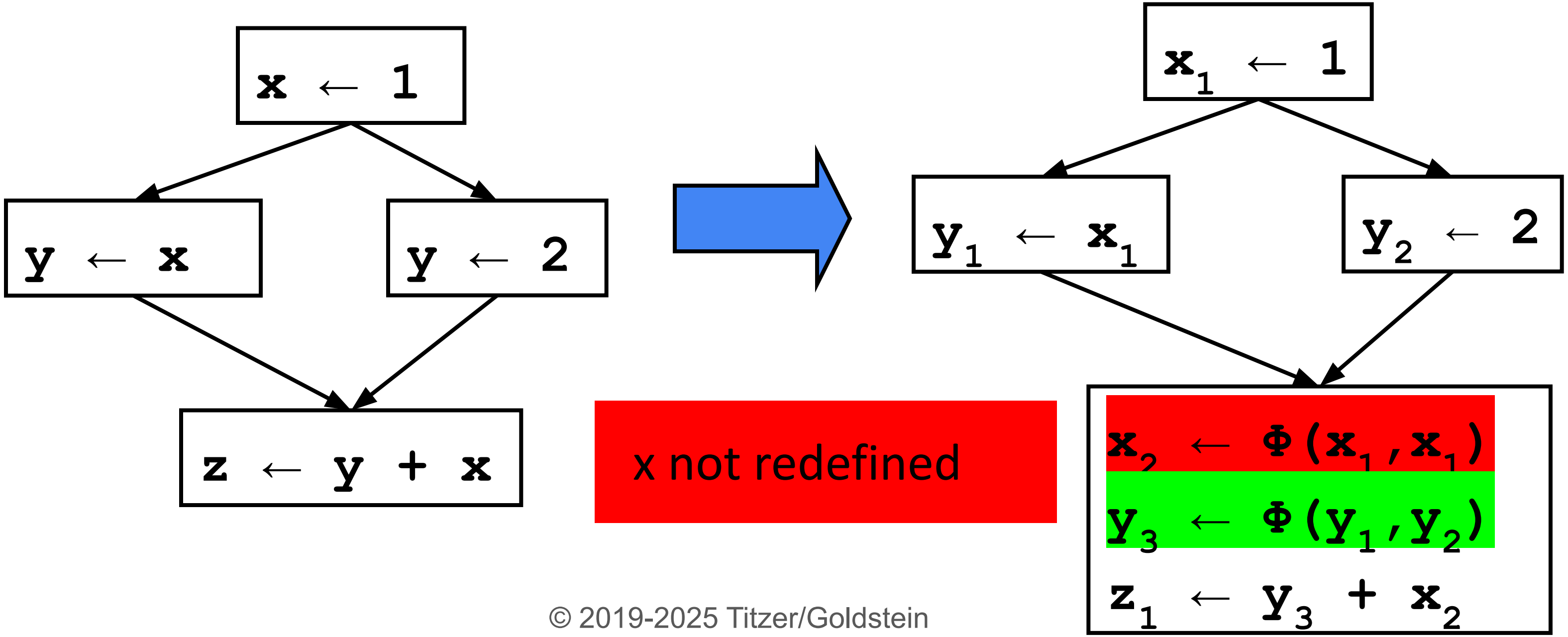
- At each join point insert Φ functions for all variables with multiple outstanding defs.



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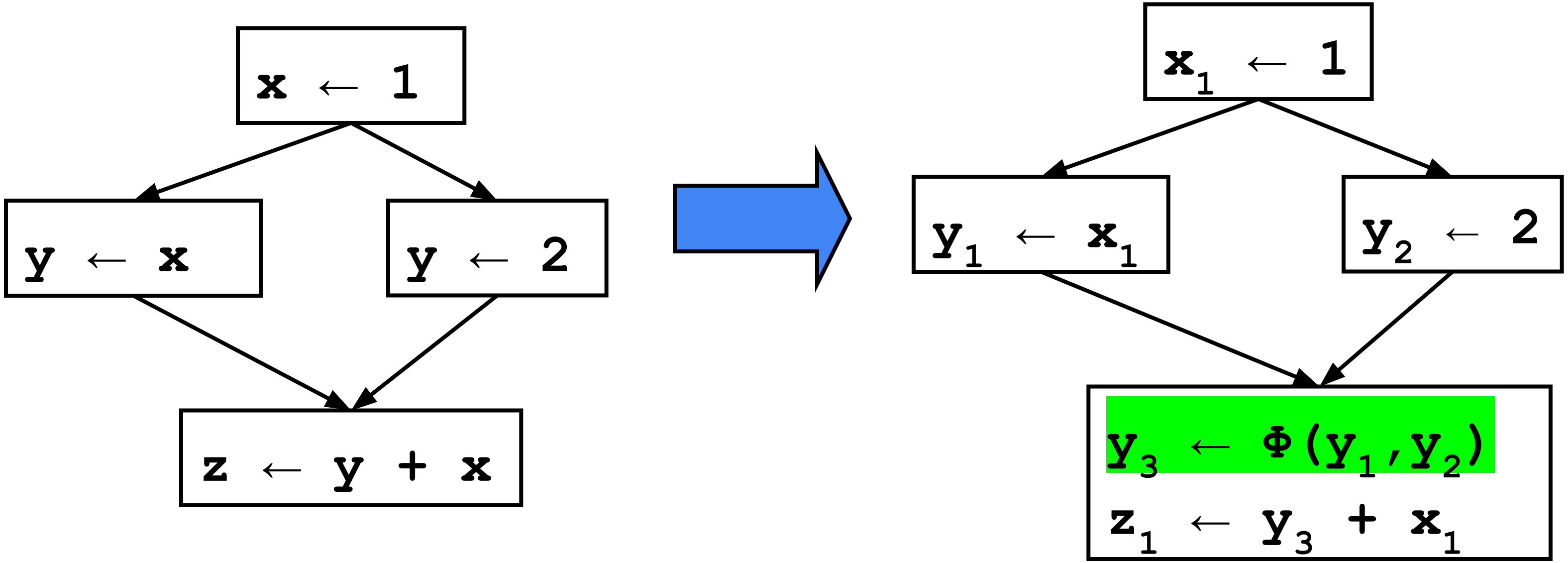

Handling cyclic control flow

- Introduce φ-functions to handle *joins* in CFG
- Loops have joins too!

```
x ← …
y ← …
while(x < 100){
   x ← x + 1
   y ← y + 1
}
```

```
   x ← …
   y ← …
   if (x >= 100) goto end
loop:
   x ← x + 1
   y ← y + 1
   if (x < 100) goto loop
end:
```

Handling cyclic control flow

- SSA requires single definition for each use
- Introduce φ-functions to handle joins at loop headers too

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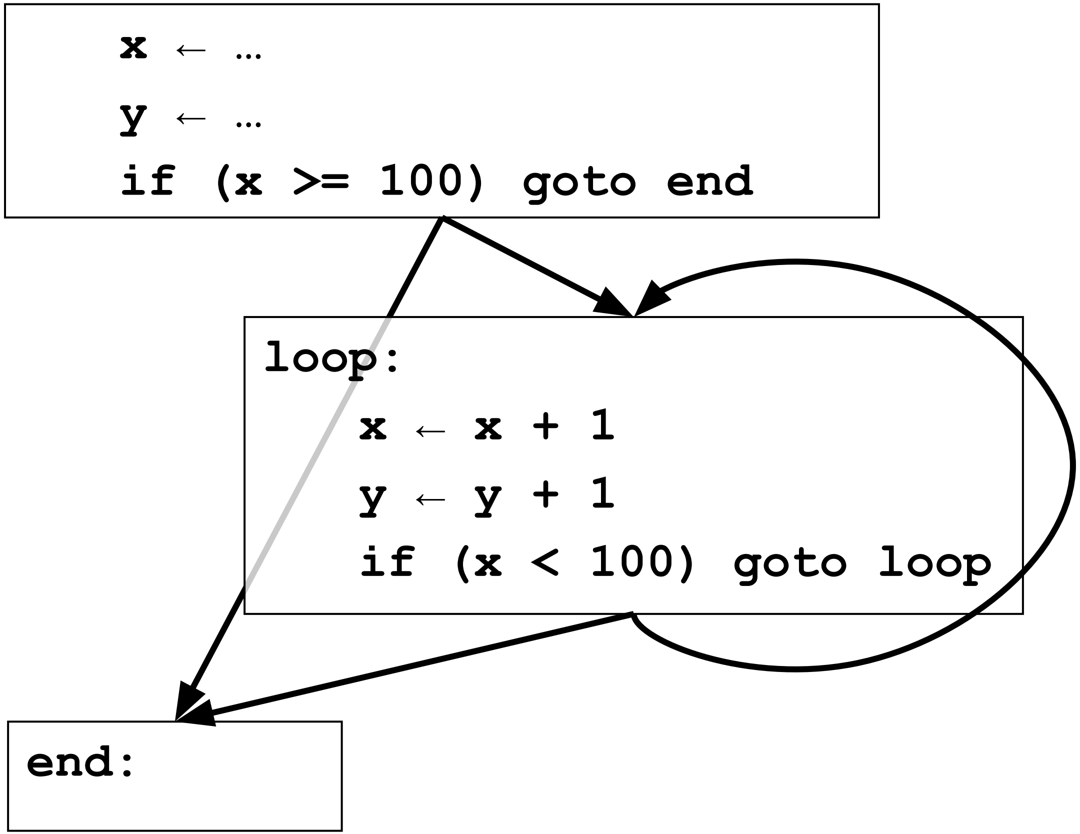


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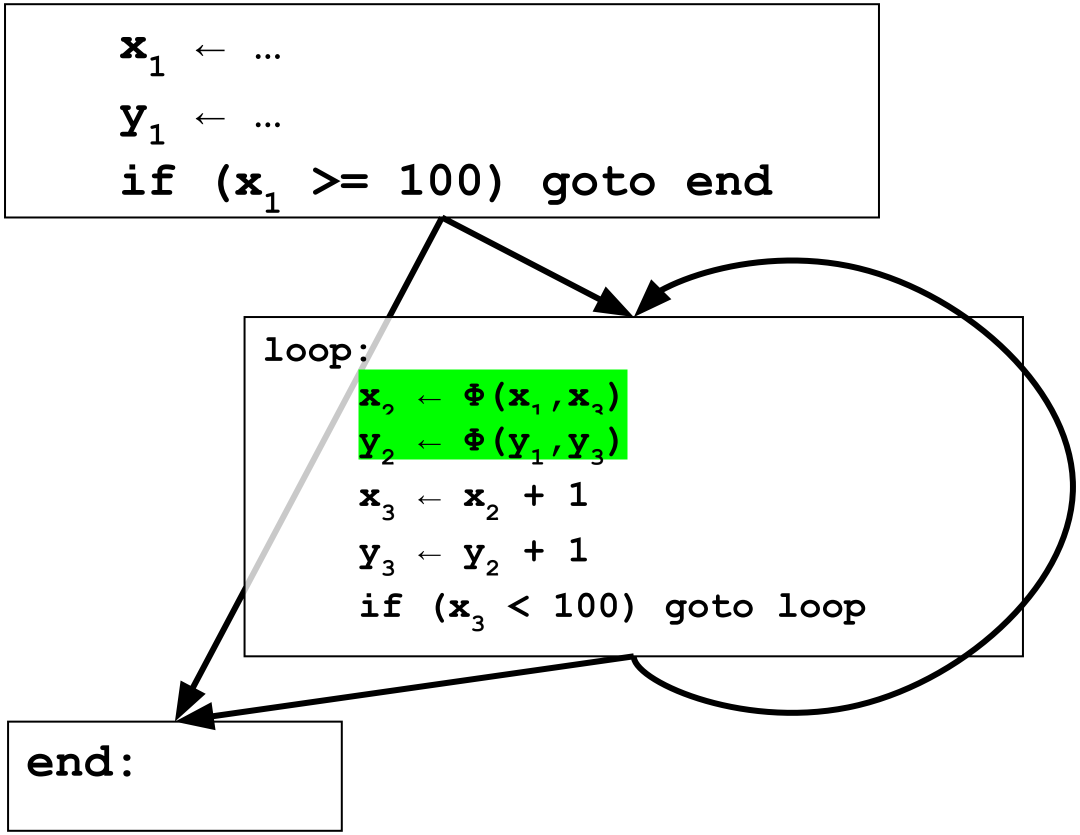


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end:
```

Handling cyclic control flow

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What's missing?

```
   x ← …
   y ← …
   if (x >= 100) goto end
loop:
   x ← x + 1
   y ← y + 1
   if (x < 100) goto loop
end:
```

$x_1 \leftarrow \ldots$

$y_1 \leftarrow \ldots$

if (x_1 >= 100) goto end

loop:

$x_2 \leftarrow \Phi(x_1, x_3)$

$y_2 \leftarrow \Phi(y_1, y_3)$

$x_3 \leftarrow x_2 + 1$

$y_3 \leftarrow y_2 + 1$

if (x_3 < 100) goto loop

end:

Handling cyclic control flow

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   y ← …
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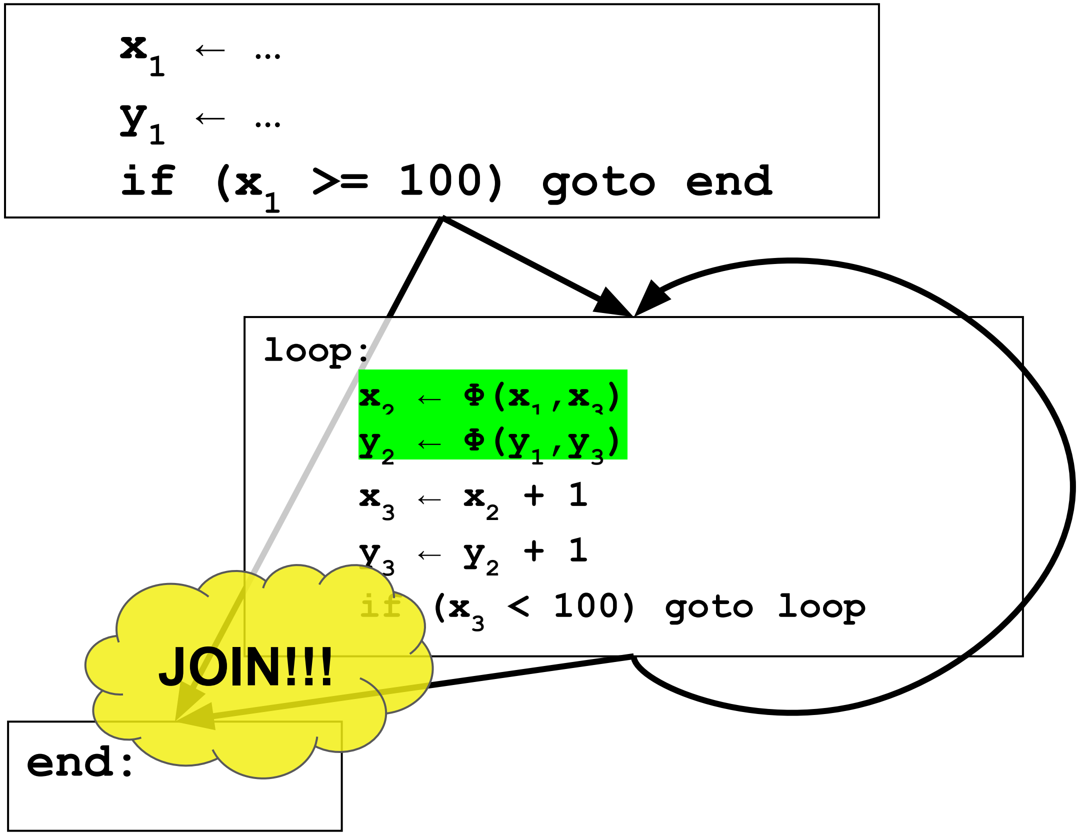

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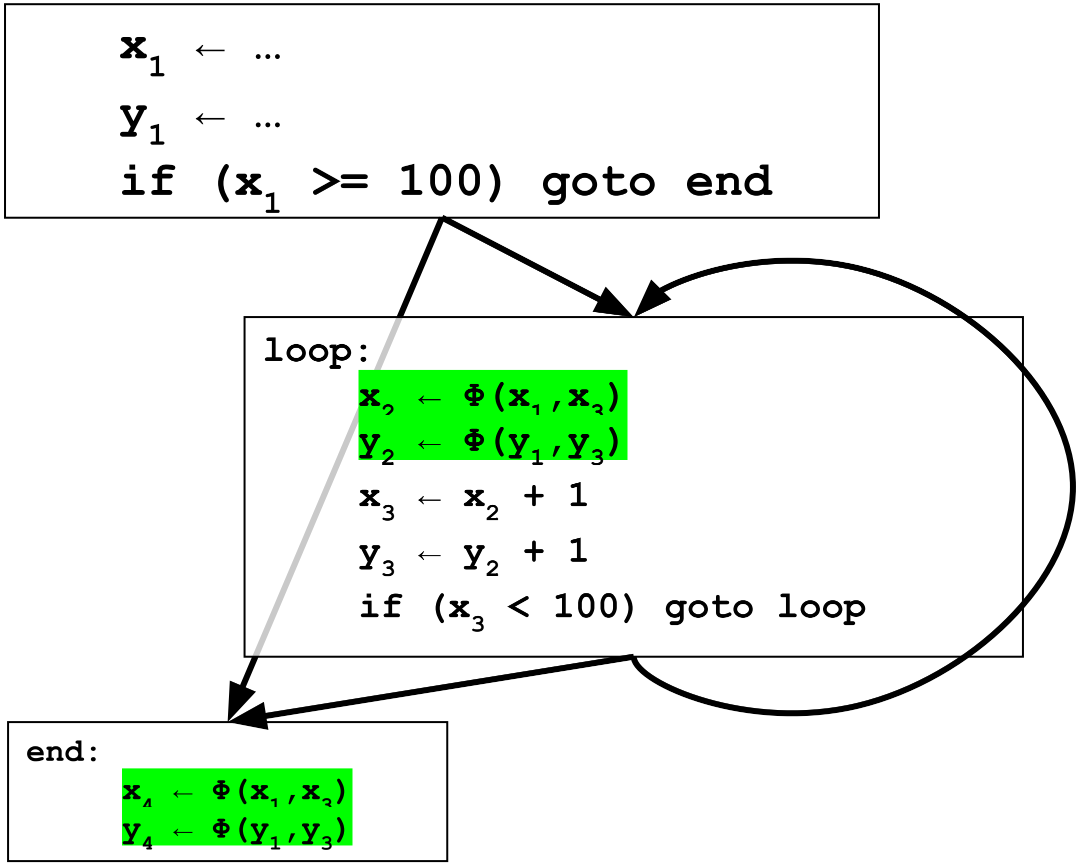


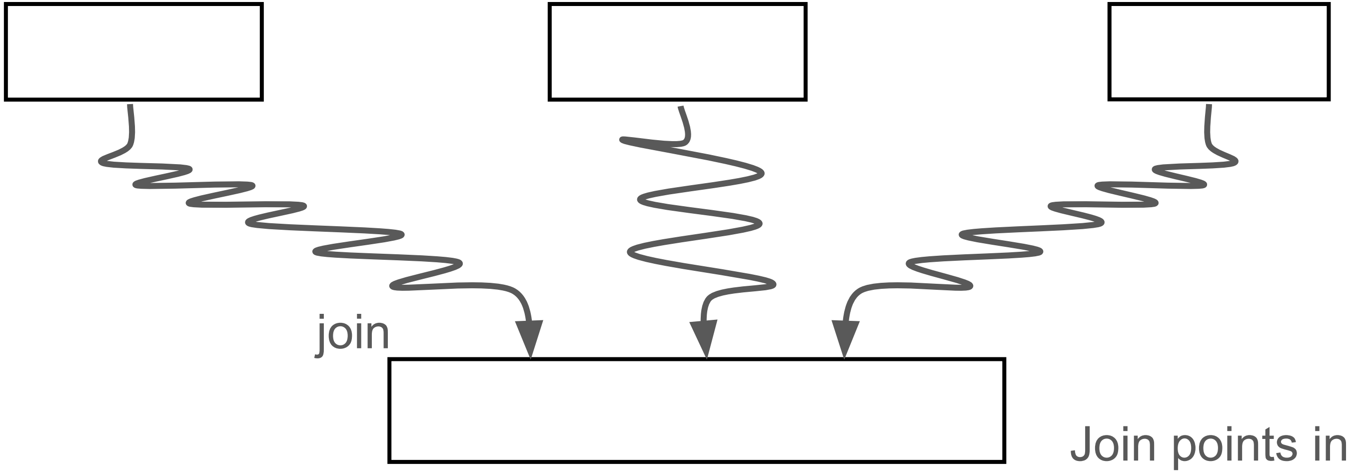
What is a Φ anyway?

- Φ is a fictional operator; it merges multiple definitions into a single definition at a join in the control flow graph.
- At a BB with *p* predecessors, there are *p* inputs to the Φ.

  $x_{new} \leftarrow \Phi(\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3, \ldots, \mathbf{x}_p)$

- What do the inputs to a Φ mean?
  - The inputs to φ-functions *positionally correspond* to the incoming control-flow edges.
  - They relate control flow merging and data flow merging.

What is a Φ anyway?



Join points in the control flow graph may require insertion of Φ functions.

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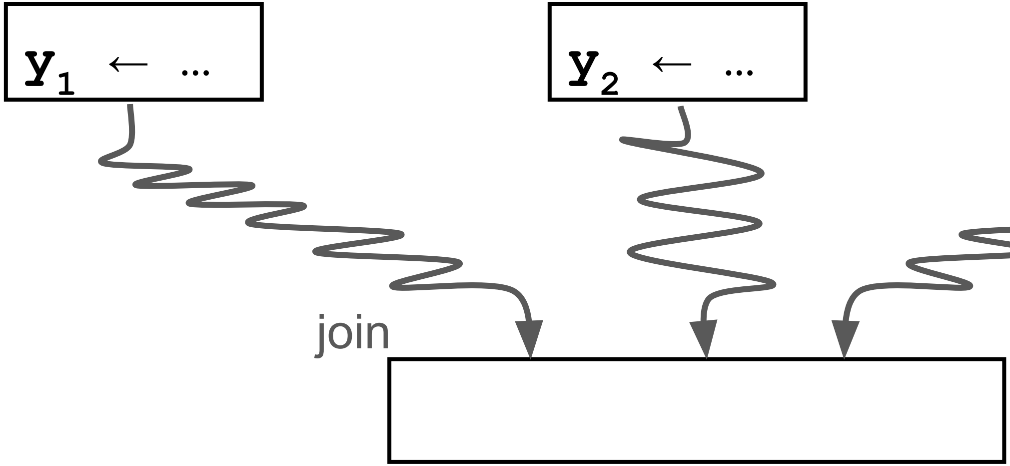
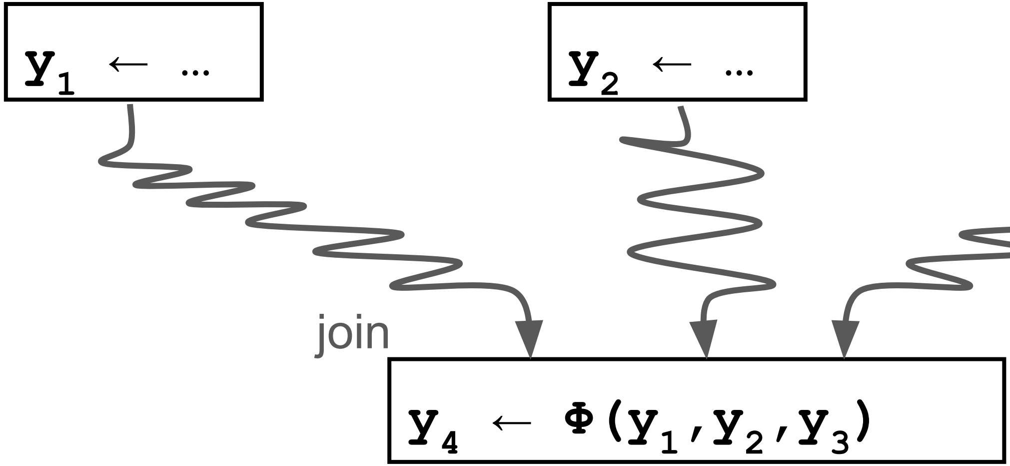


Join points in the control flow graph may require insertion of Φ functions, ***if there are different versions of the variable arriving.***

What is a Φ anyway?



Each incoming control edge supplies a corresponding data value for the Φ from the predecessor.

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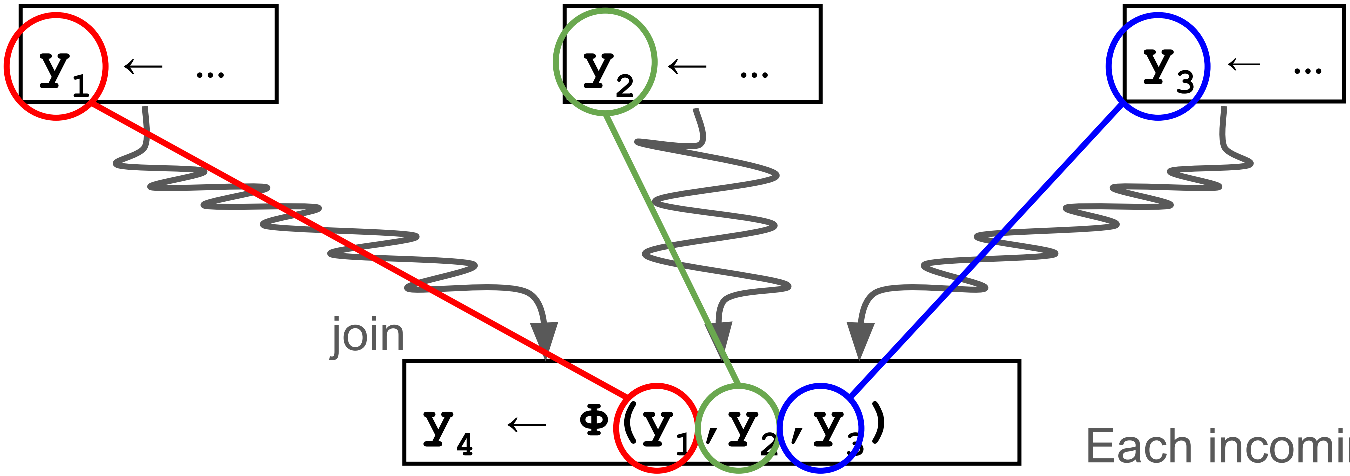

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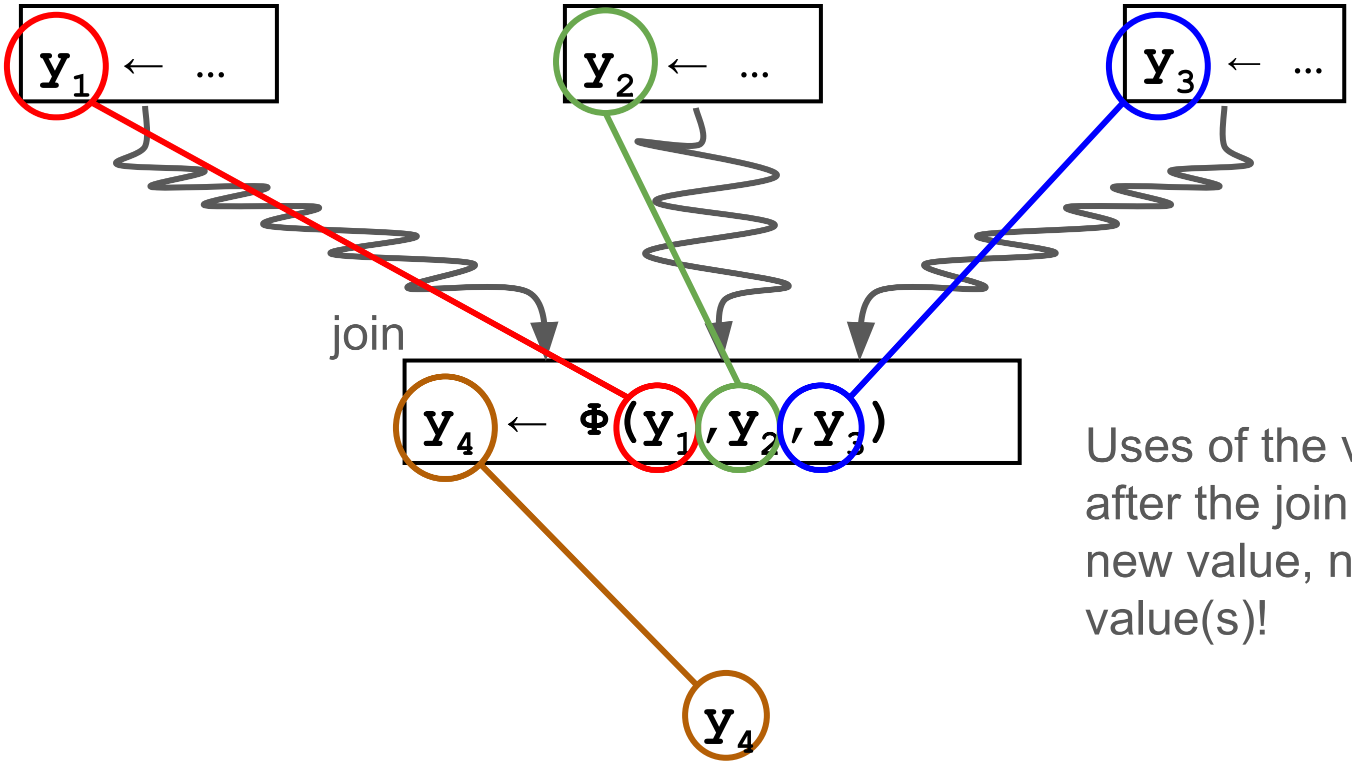
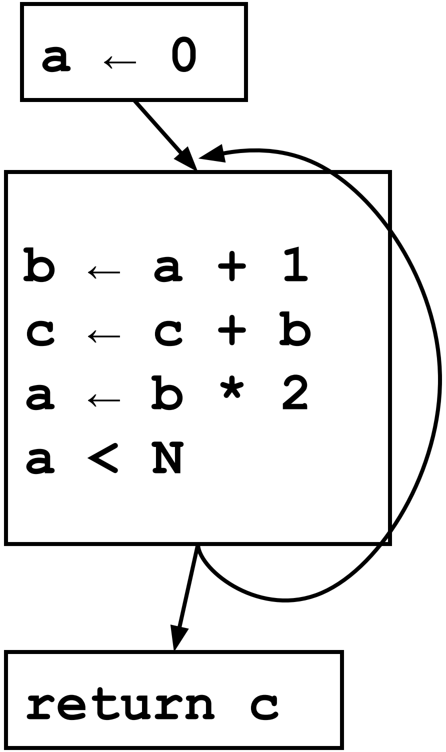


Uses of the variable after the join get the new value, not the old value(s)!

Another Loop Example



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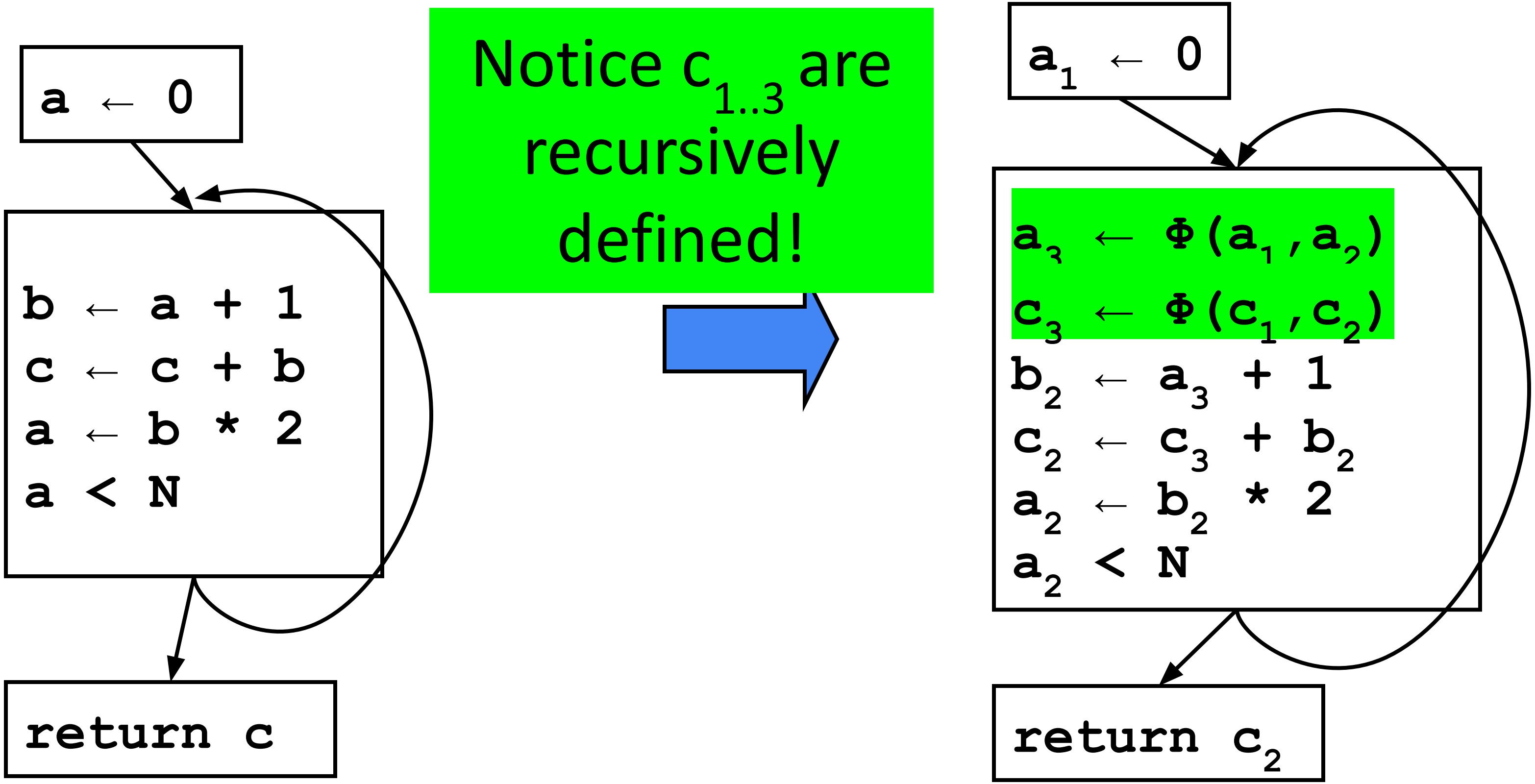
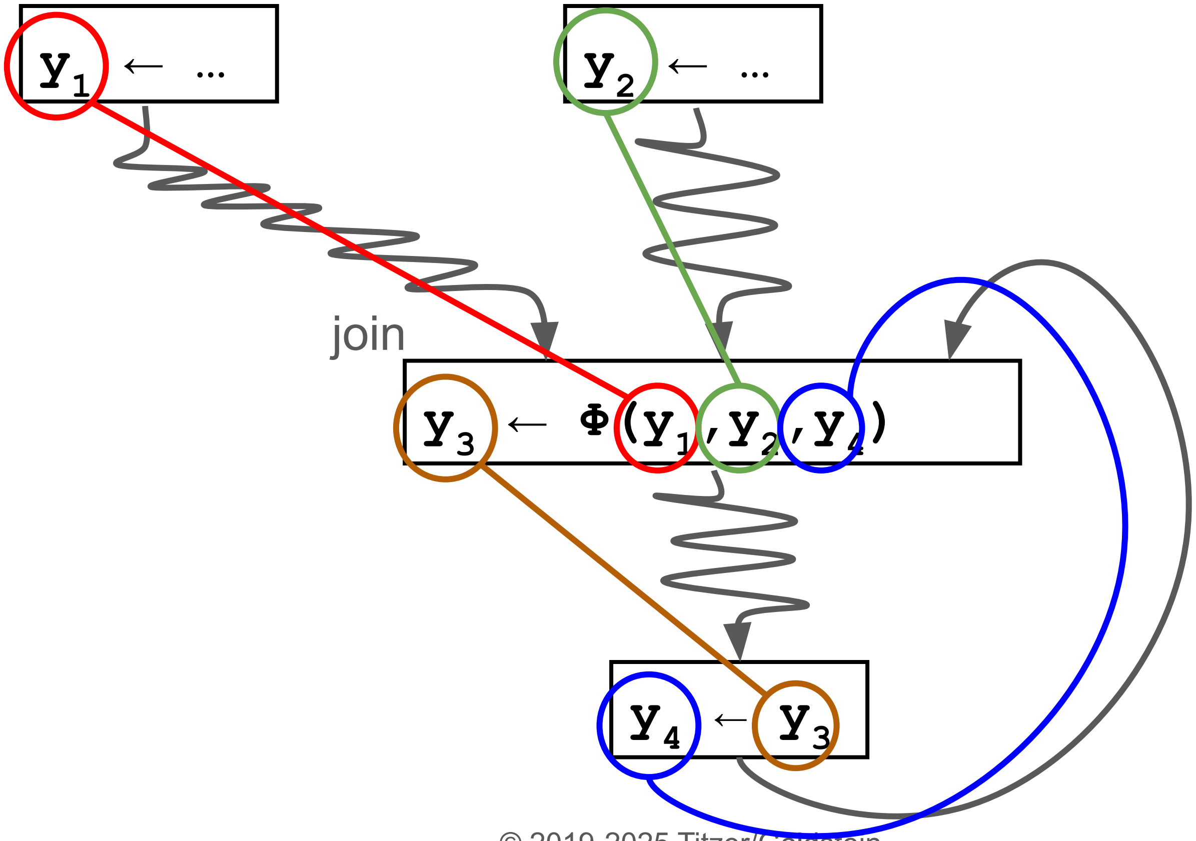


What is a Φ (for a loop) anyway?



Φs at loop headers relate the dataflow on a loop backedge with the control flow.

Allows finding induction variables really easily.

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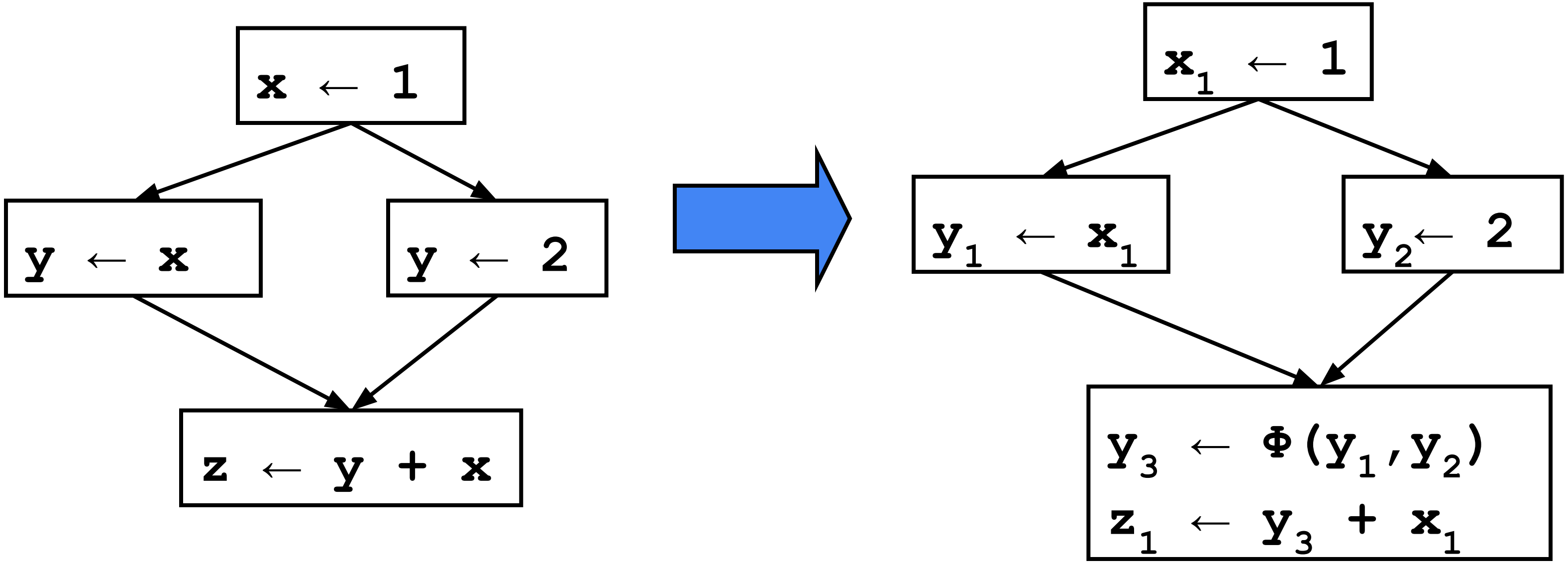
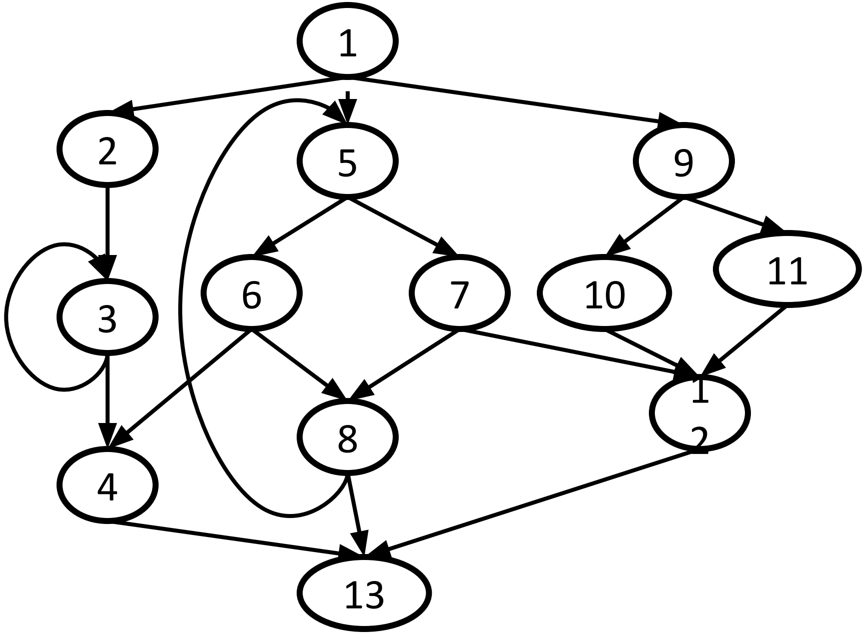


When do we insert Φ?



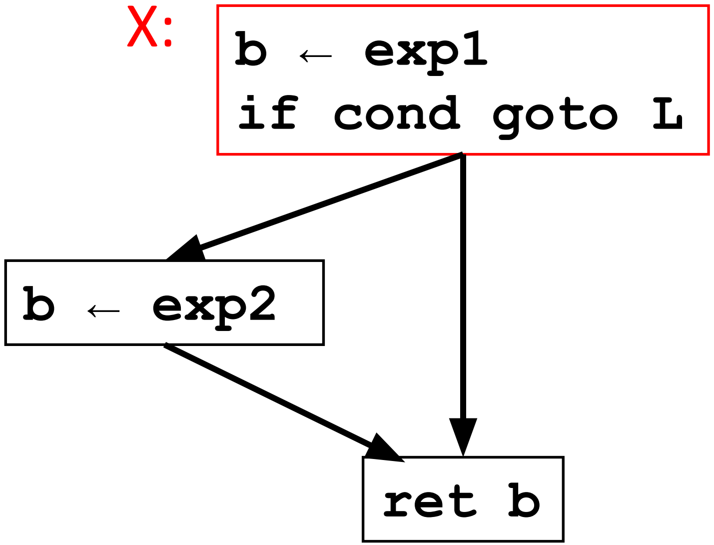
CFG

If there is a def of **a** in block 5, which nodes need a Φ()?

When do we insert Φ?

Require a Φ-function for variable b at node z of the flow graph:

- There is a block x containing a def of b
- There is a block y ≠ x containing a def of b
- There is a nonempty path P_{xz} of edges from x to z
- There is a nonempty path P_{yz} of edges from y to z
- Paths P_{xz} and P_{yz} do not have any node in common other than z, and…
- The node z does not appear within both P_{xz} and P_{yz} prior to the end, though it may appear in one or the other.



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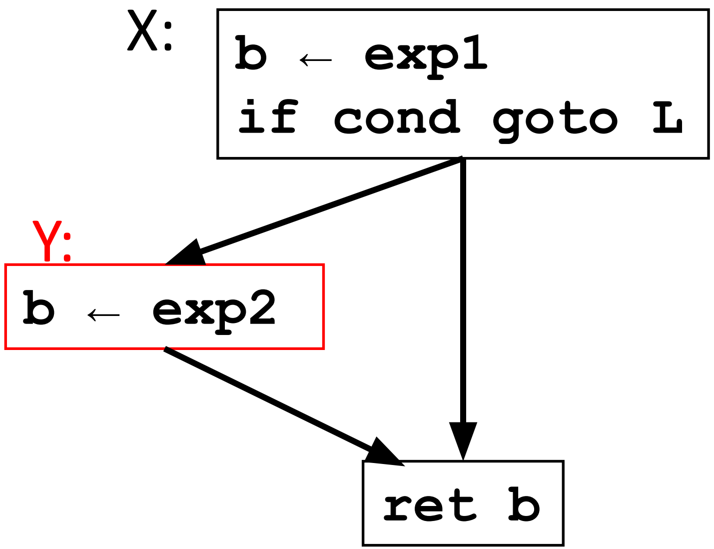

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Require a Φ-function for variable <u>b</u> at node <u>z</u> of the flow graph:

- There is a block x containing a def of b
- There is a block $y \neq x$ containing a def of b
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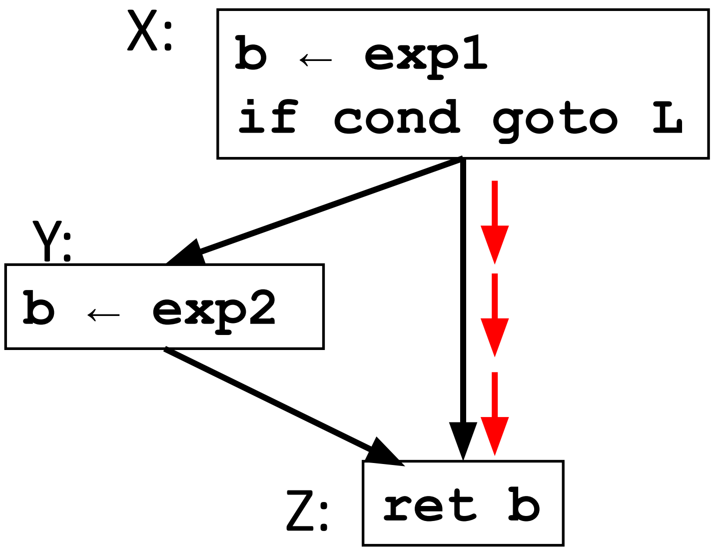

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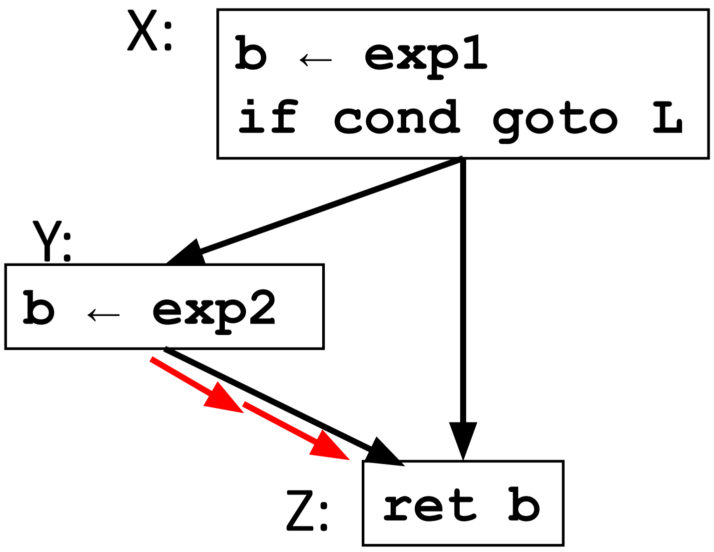

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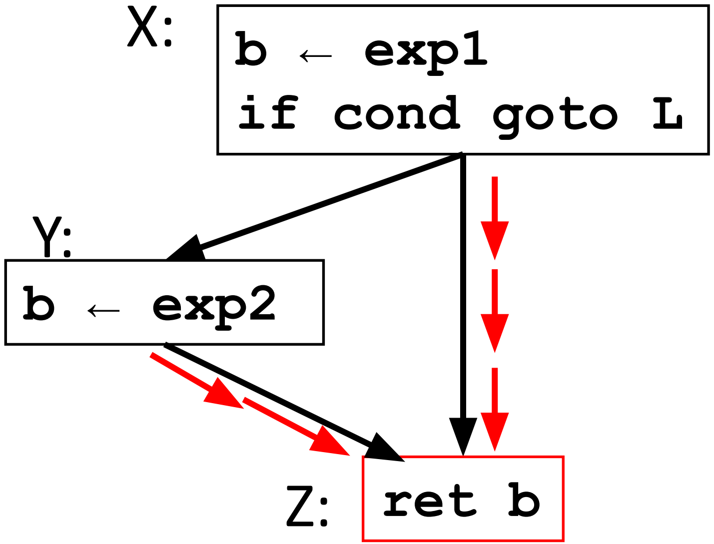

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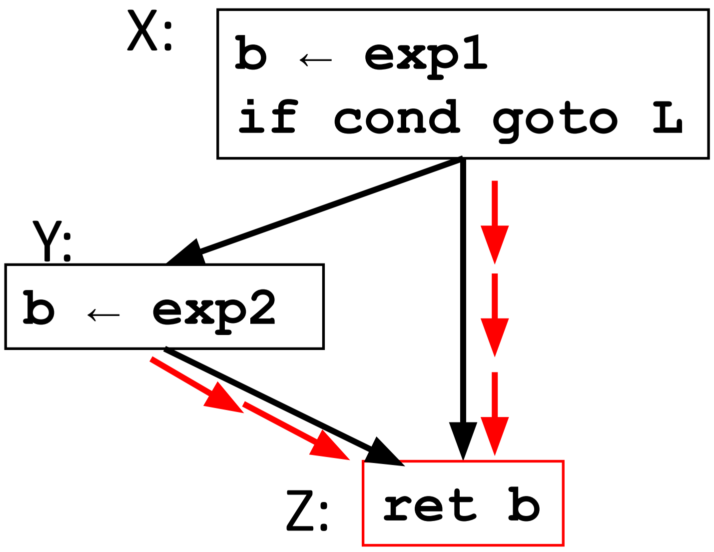

Iterative Insertion

- Implicit def of every variable in start node
- Inserting Φ-function creates new definition
- While there ∃ x,y,z that
  - satisfy path-convergence criteria
  - and z does not contain Φ-function for b
- do
  - insert $b \leftarrow \Phi(b,b,b,\ldots,b_n)$ at node z, z having n predecessors.

Dominance Property of SSA

- In SSA **definitions dominate uses***.
  - If x_i is used in $x \leftarrow \Phi(\ldots, x_i, \ldots)$, then BB($x_i$) dominates i^{th} predecessor of BB(Φ)
  - If x is used in $y \leftarrow \ldots\ x\ \ldots$, then BB(x) dominates BB(y)
- We can use this for an efficient algorithm to convert to SSA

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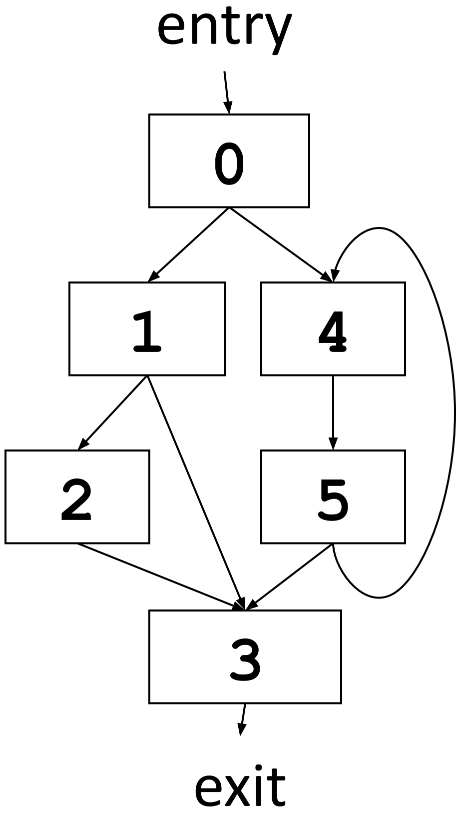
****well *double akshully***, we can insert assignments to convert any program to strict SSA

Side trip: Dominators

Dominators

- *a dom b*
  - block *a* dominates block *b* if every possible execution path from *entry* to *b* includes *a*



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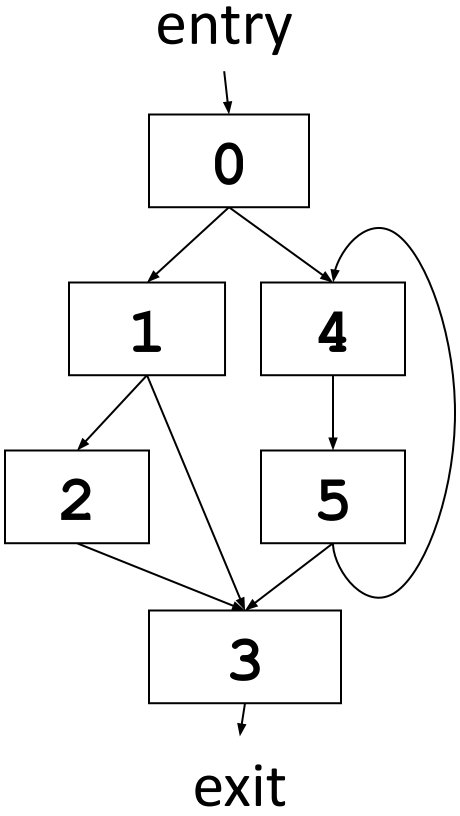

Dominators

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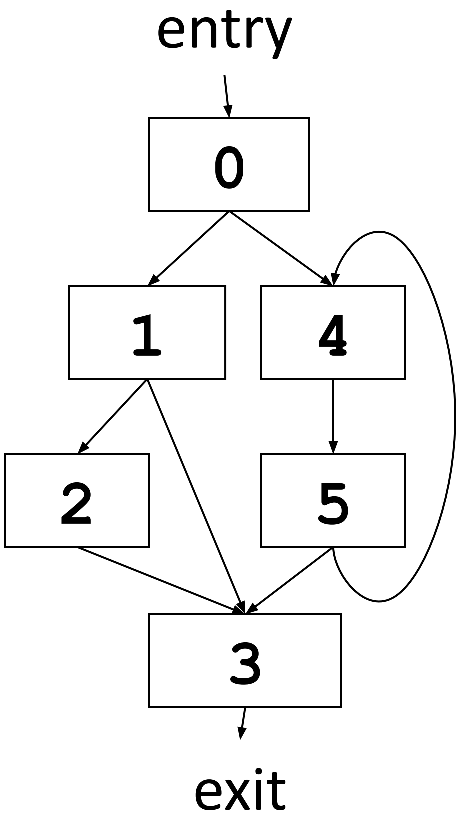
    - **entry** dominates everything
    - **0** dominates everything but entry
    - **1** dominates **2** and **1**



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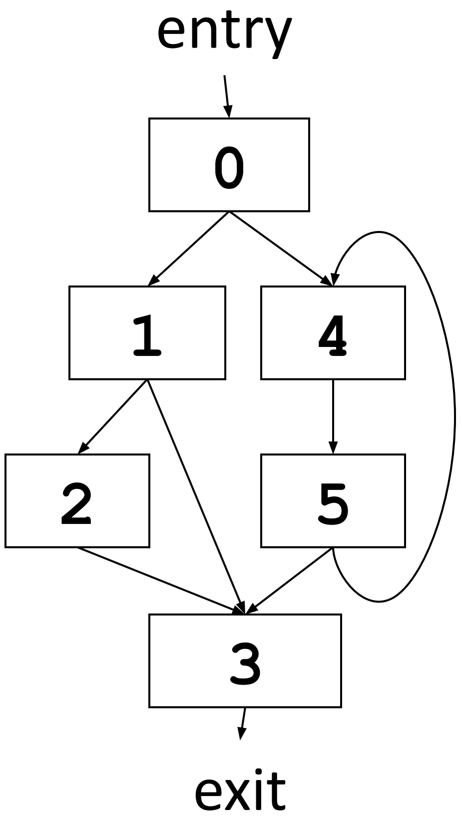
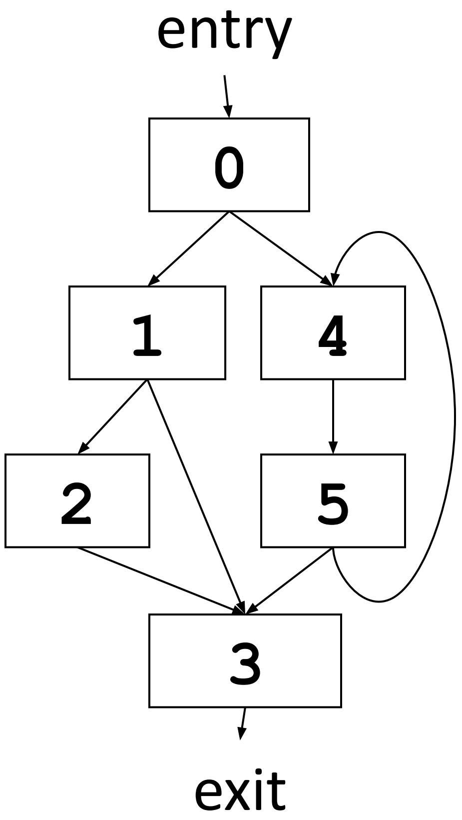
Dominators are useful in:

- *Dataflow analysis*
- *Constructing SSA*
- *Identifying "natural" loops*
- *Code motion*
- *…*



Definitions

- *a sdom b*
  - If *a* and *b* are different blocks and *a dom b*, we say that *a* *strictly dominates* *b*
- *a idom b*
  - If *a sdom b*, and there is no *c* such that *a sdom c* and *c sdom b*, we say that *a* is the *immediate dominator* of *b*



Properties of Dom

- Dominance is a partial order on the blocks of the flow graph, i.e.,
  - 1. Reflexivity: a dom a for all a
  - 2. Anti-symmetry: a dom b and b dom a implies a = b
  - 3. Transitivity: a dom b and b dom c implies a dom c

- NOTE: there may be blocks a and b such that neither a dom b or b dom a holds.

- The dominators of each node n are linearly ordered by the dom relation. The dominators of n appear in this linear order on any path from the initial node to n.

Computing dominators

- We want to compute *D*[*n*], the set of blocks that dominate *n*

Initialize each *D*[*n*] (except *D*[entry]) to be the set of all blocks, and then iterate until no *D*[*n*] changes:

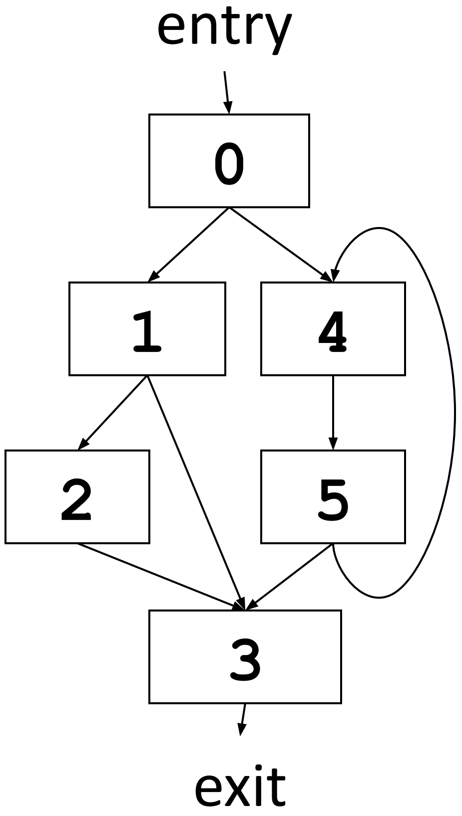
$$D[\text{entry}] = \{\text{entry}\}$$

$$D[n] = \{n\} \cup \left(\bigcap_{p \in \text{pred}(n)} D[p] \right), \quad \text{for } n \neq \text{entry}$$

Example

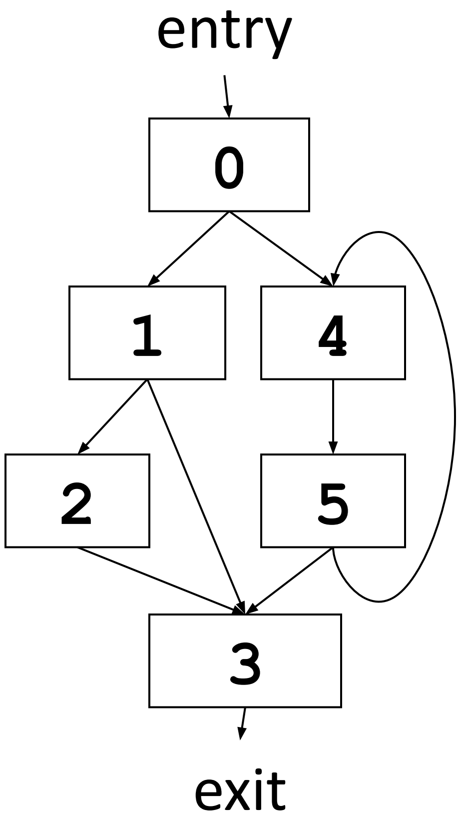
Initialization

block	D[n]
entry	{entry}
0	{entry,0,1,2,3,4,5,exit}
1	{entry,0,1,2,3,4,5,exit}
2	{entry,0,1,2,3,4,5,exit}
3	{entry,0,1,2,3,4,5,exit}
4	{entry,0,1,2,3,4,5,exit}
5	{entry,0,1,2,3,4,5,exit}
exit	{entry,0,1,2,3,4,5,exit}



Example

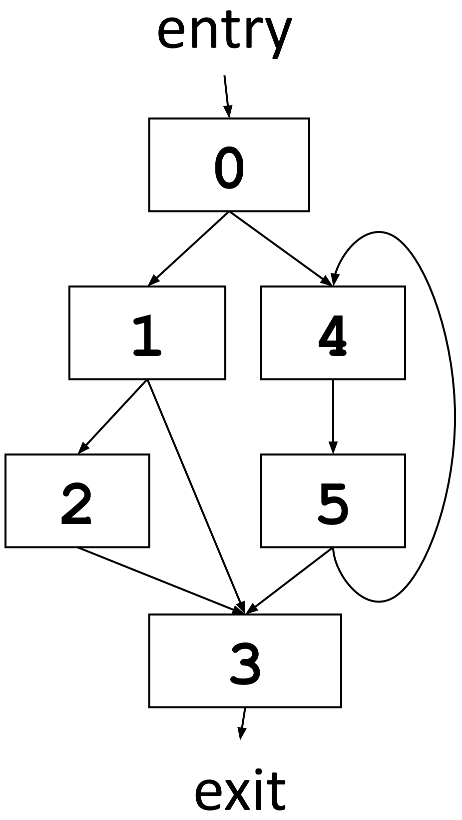
block	Initialization D[n]	First Pass D[n]
entry	{entry}	{entry}
0	{entry,0,1,2,3,4,5,exit}	{0,entry}
1	{entry,0,1,2,3,4,5,exit}	{1,0,entry}
2	{entry,0,1,2,3,4,5,exit}	{2,1,0,entry}
3	{entry,0,1,2,3,4,5,exit}	{3,1,0,entry}
4	{entry,0,1,2,3,4,5,exit}	{4,0,entry}
5	{entry,0,1,2,3,4,5,exit}	{5,4,0,entry}
exit	{entry,0,1,2,3,4,5,exit}	{exit,3,1,0,entry}



Update rule: $$D[n] = \{n\} \cup \left(\bigcap_{p \in \text{pred}(n)} D[p] \right);$$

Example

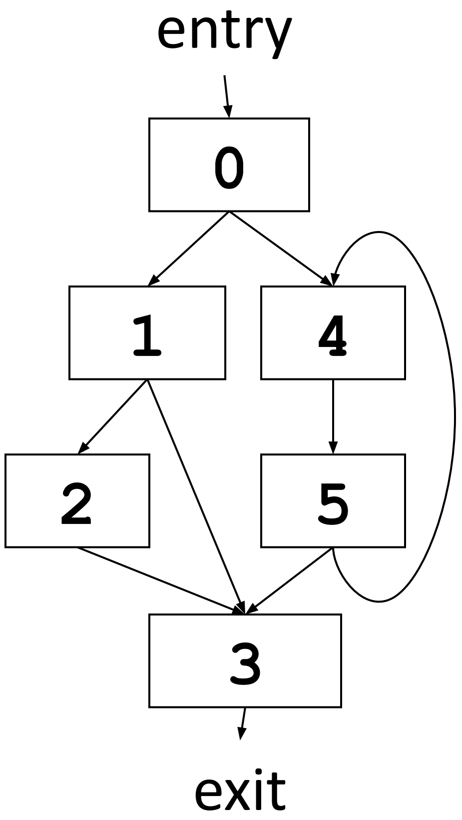
block	First Pass D[n]	Second Pass D[n]
entry	{entry}	{entry}
0	{0,entry}	{0,entry}
1	{1,0,entry}	{1,0,entry}
2	{2,1,0,entry}	{2,1,0,entry}
3	{3,1,0,entry}	{3,0,entry}
4	{4,0,entry}	{4,0,entry}
5	{5,4,0,entry}	{5,4,0,entry}
exit	{exit,3,1,0,entry}	{exit,3,0,entry}



Update rule: $$D[n] = \{n\} \cup \left(\bigcap_{p \in \mathrm{pred}(n)} D[p] \right)$$

Example

block	Second Pass D[n]	Third Pass D[n]
entry	{entry}	{entry}
0	{0,entry}	{0,entry}
1	{1,0,entry}	{1,0,entry}
2	{2,1,0,entry}	{2,1,0,entry}
3	{3,0,entry}	{3,0,entry}
4	{4,0,entry}	{4,0,entry}
5	{5,4,0,entry}	{5,4,0,entry}
exit	{exit,3,0,entry}	{exit,3,0,entry}



Update rule: $$D[n] = \{n\} \cup \left(\bigcap_{p \in \mathrm{pred}(n)} D[p] \right),$$

Computing dominators

- Iterative algorithm is $O(n^2 e)$
  - assuming bit vector set
  - choosing a good iteration order matters
- More efficient algorithm due to Lengauer and Tarjan
  - $O(e \cdot \alpha(e,n))$
  - much more complicated
  - Books provide simple algorithms that are fast in practice(faster than Tarjan algorithm for realistic CFGs)
  - For a clever algorithm see: "A Simple, Fast Dominance Algorithm" by Cooper, Harvey, and Kennedy

$\alpha(e,n)$ is *inverse Ackermann*

Immediate dominators

- Let *sD*[*n*] be the set of blocks that strictly dominate *n*, then

$$\mathbf{sD[n] = D[n] - \{n\}}$$

- To compute *iD*[*n*], the set of blocks (size <= 1) that immediately dominate *n*
- Set $\mathbf{iD[n] = sD[n]}$
- Repeat until no iD[n] changes:

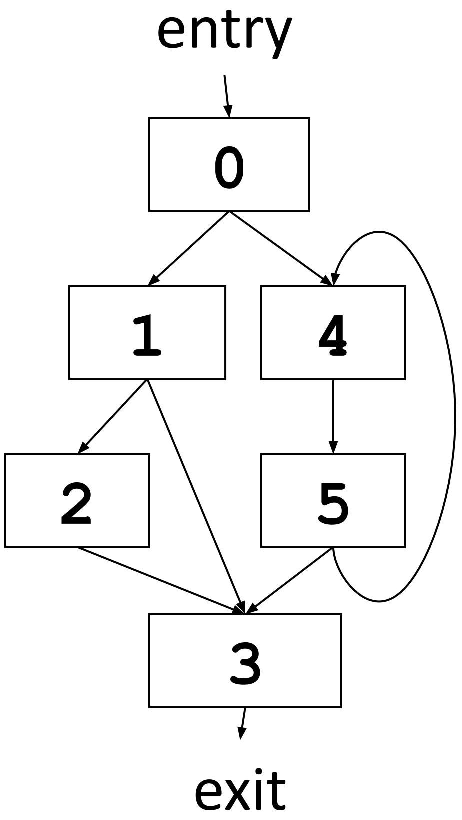
$$\mathbf{iD[n] = iD[n]} - \bigcup_{\mathbf{d \in iD[n]}} \mathbf{(sD[d])}$$

Example

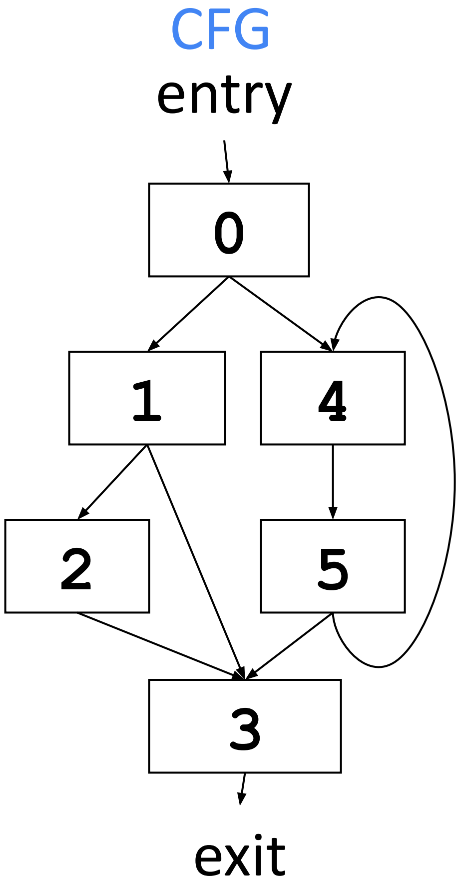
block	Initialization iD[n]=sD[n]	First Pass iD[n]
entry	{}	{}
0	{entry}	{entry}
1	{0,entry}	{0}
2	{1,0,entry}	{1}
3	{0,entry}	{0}
4	{0,entry}	{0}
5	{4,0,entry}	{4}
exit	{3,0,entry}	{3}

Update rule: $$iD[n] = iD[n] - \bigcup_{d \in iD[n]} (sD[d])$$

CFG

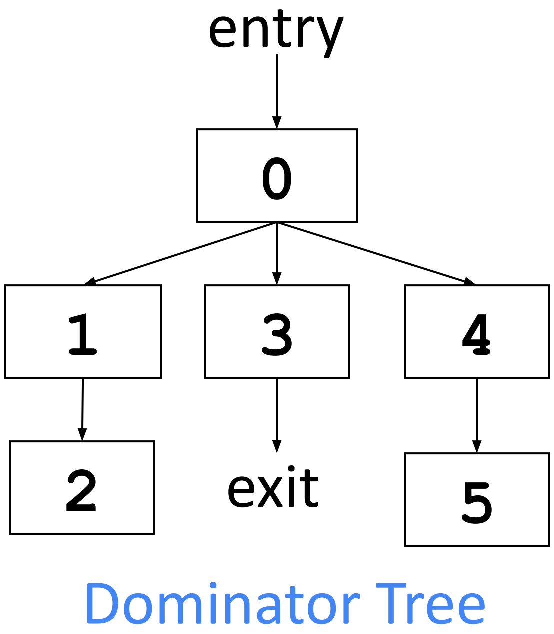


Dominator Tree



In the *dominator tree* the initial node is the entry block, and the parent of each other node is its immediate dominator.

block	iD[n]
entry	{}
0	{entry}
1	{0}
2	{1}
3	{0}
4	{0}
5	{4}
exit	{3}

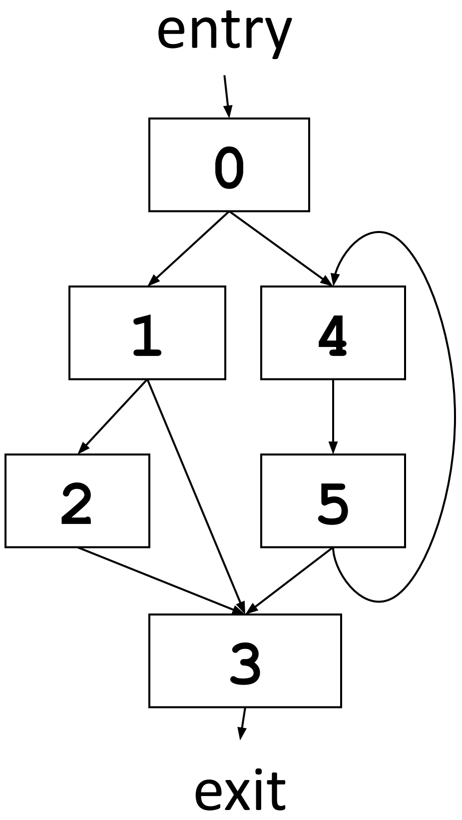


Post-Dominance

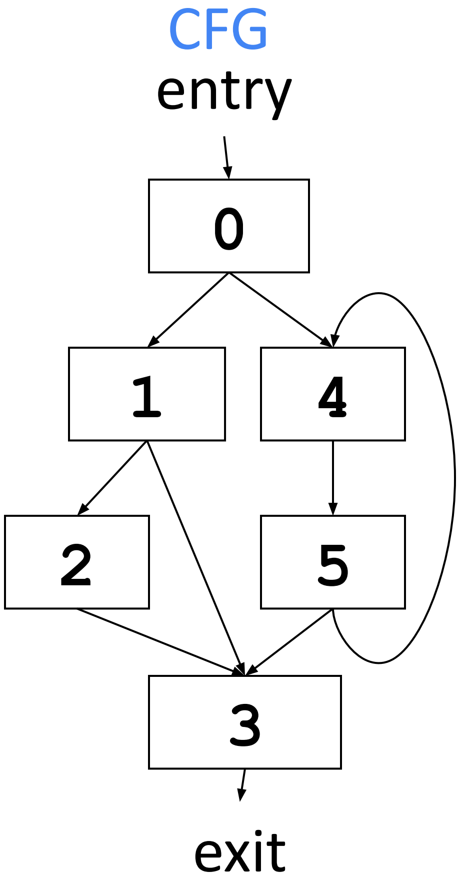
- Block *a* post-dominates *b* (a pdom b) if every path from *a* to the exit block includes *b*
- pdom on CFG is the same as dom on the reverse (all edges reversed) CFG
- **0** post-dominates ?

  1 post-dominates ?

  4 post-dominates ?

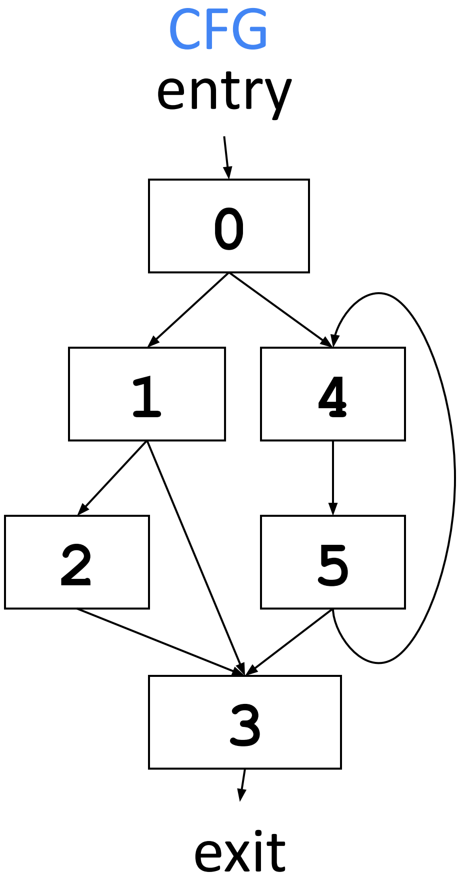


Dominance Frontier



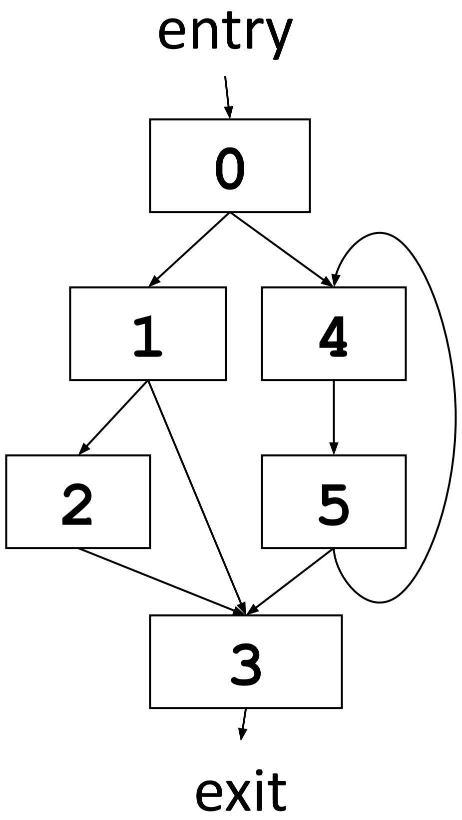
- *z* is in the dominance frontier of *x* If *z* is the first node we encounter on the path from *x* which *x* does not *strictly* dominate.
- For some path from node *x* to *z*,
  $x \rightarrow \ldots \rightarrow y \rightarrow z$
  where x dom y but not x sdom z.
- Intuitively, the dominance frontier consists of nodes "just outside the dominator tree"

Dominance Frontier

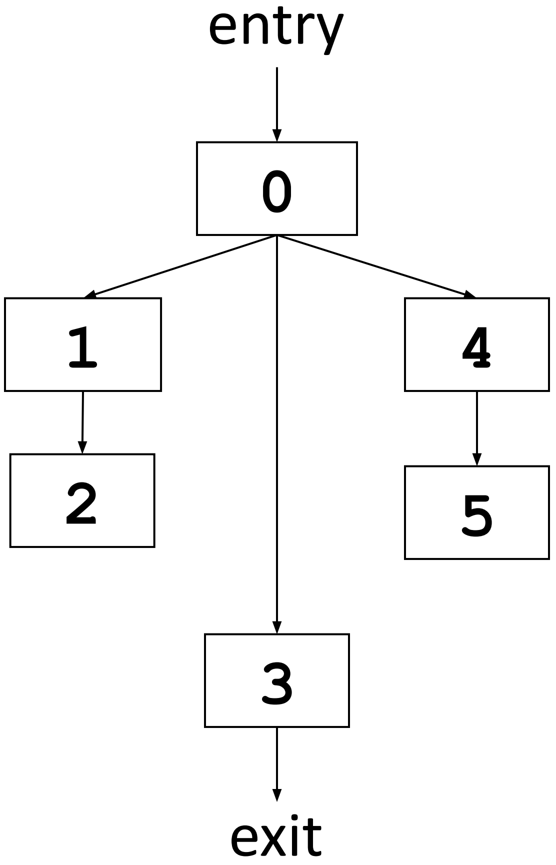


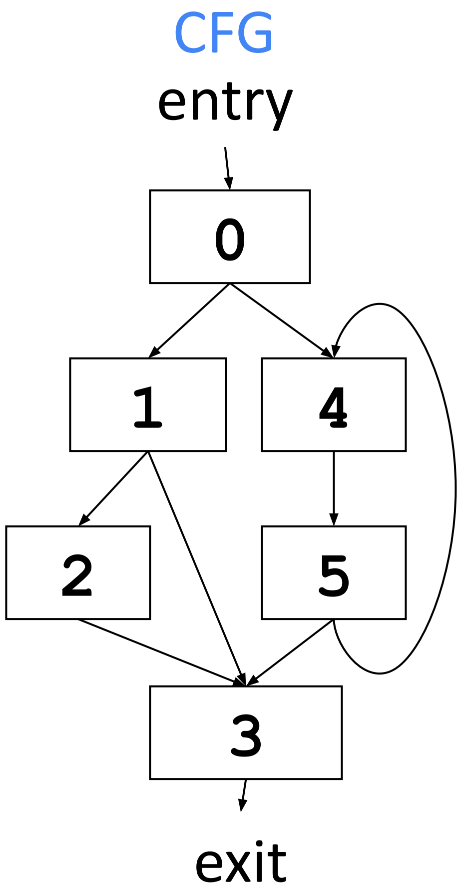
- z is in the dominance frontier of x If z is the first node we encounter on the path from x which x does not *strictly* dominate.
- For some path from node x to z,
  $x \rightarrow \ldots \rightarrow y \rightarrow z$
  where x dom y but not x sdom z.
- Dominance frontier of **1**? {3}
- Dominance frontier of **2**? {3}
- Dominance frontier of **4**? {3,4}

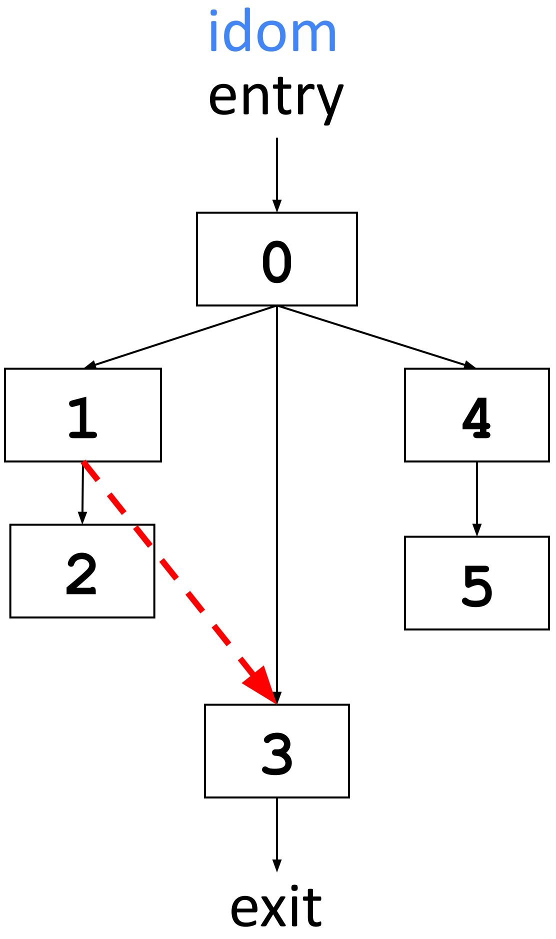
CFG



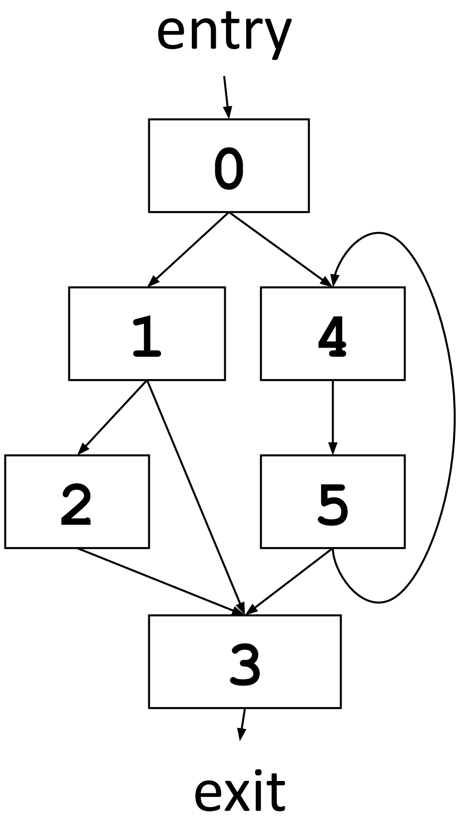
idom



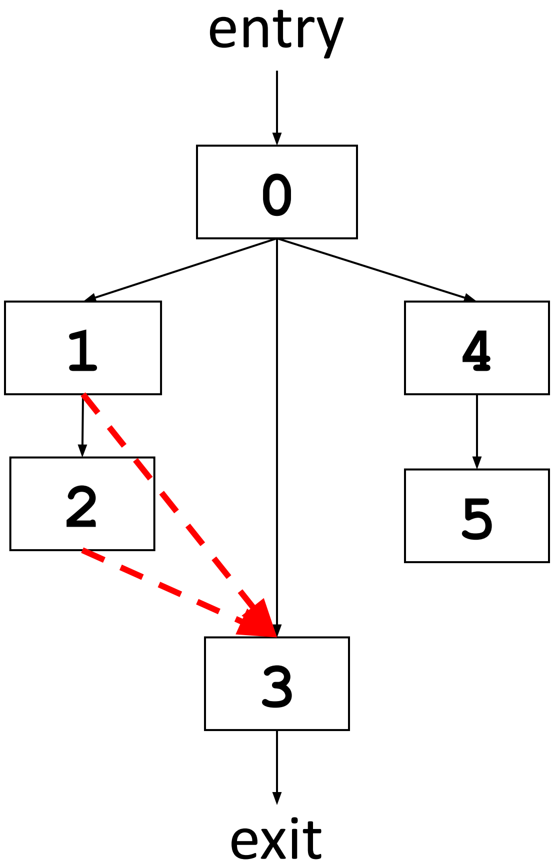

CFG
entry
0
1
4
2
5
3
exit

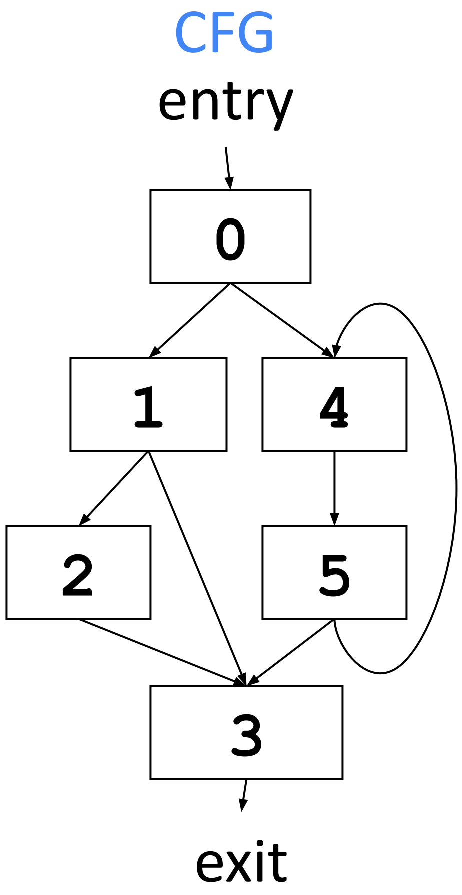

idom
entry
0
1
4
2
5
3
exit

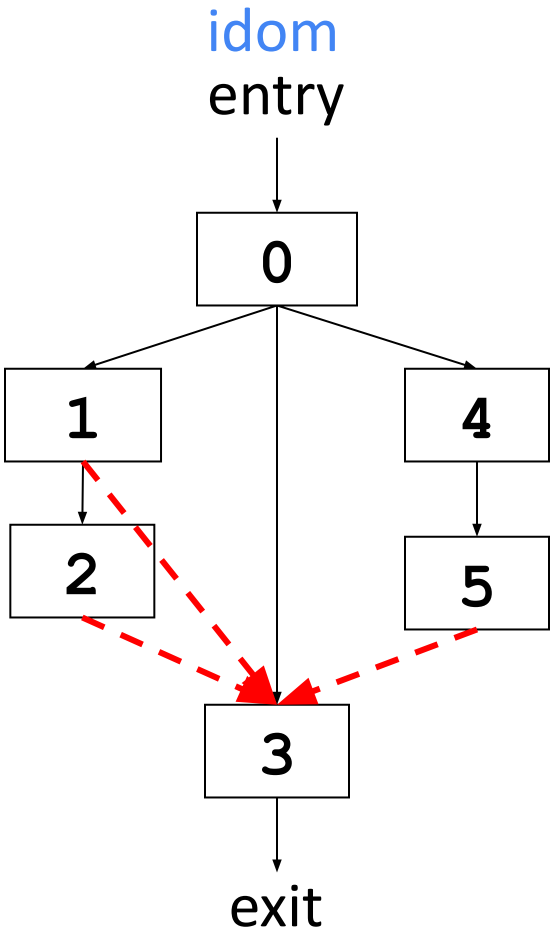
CFG

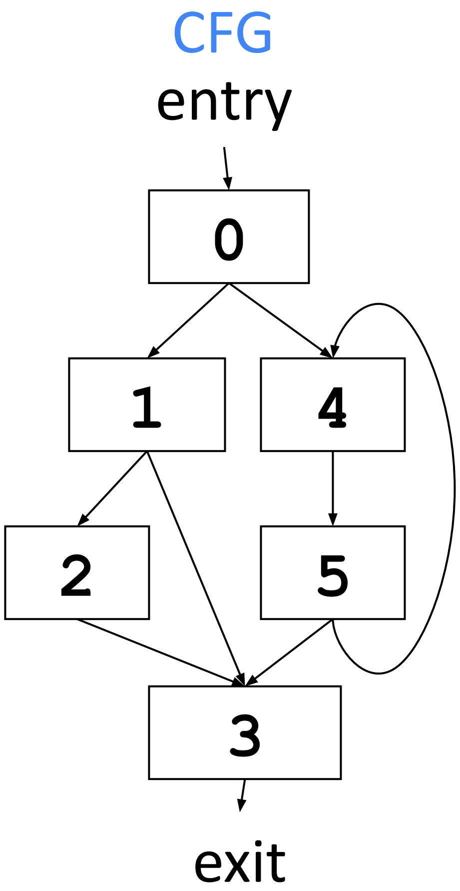


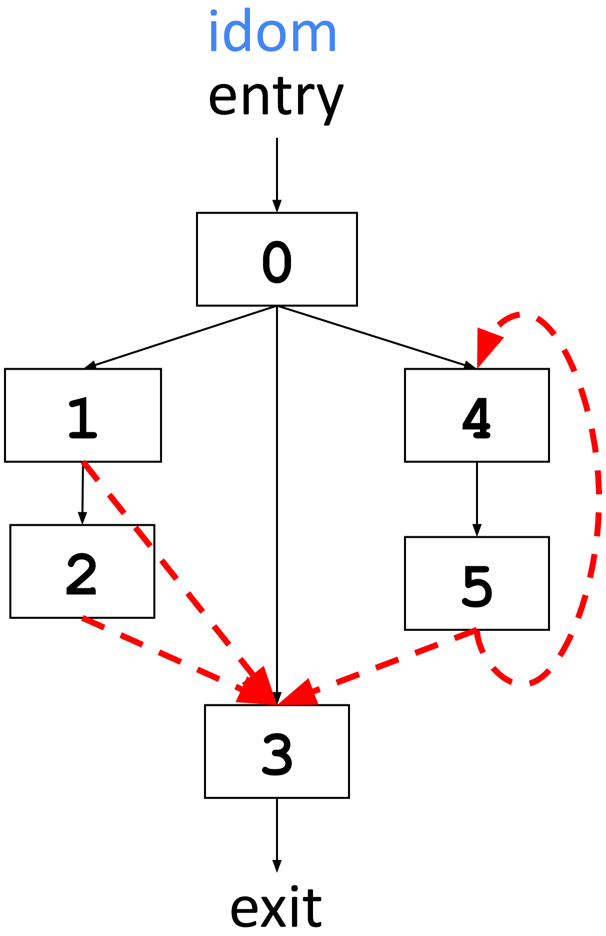
idom




CFG
entry
0
1
4
2
5
3
exit


idom
entry
0
1
4
2
5
3
exit


CFG
entry
0
1
4
2
5
3
exit


idom
entry
0
1
4
2
5
3
exit

Calculating the Dominance Frontier

- Let *dominates*[n] be the set of all blocks which block n dominates
  - subtree of dominator tree with n as the root
- The dominance frontier of n, *DF[n]* is

$$DF[n] = \bigcup_{s \in dominates[n]} succ(s) - dominates[n] - \{n\}$$

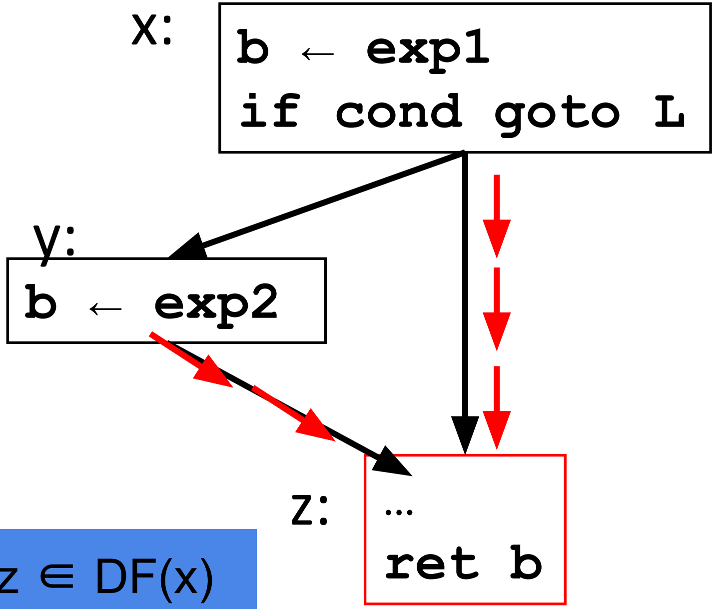
Recap

- *a dom b*
  - every possible execution path from *entry* to *b* includes *a*
- *a sdom b*
  - *a dom b* and *a* != *b*
- *a idom b*
  - *a* is “closest” dominator of *b*
- *a pdom b*
  - every path from *a* to the exit block includes *b*
- Dominator trees
- Dominance frontier

Back to inserting Φs

Require a Φ-function for variable b at node z of the flow graph:

- There is a block x containing a def of b
- There is a block $y \neq x$ containing a def of b
- There is a nonempty path P_{xz} of edges from x to z
- There is a nonempty path P_{yz} of edges from y to z
- Paths P_{xz} and P_{yz} do not have any node in common other than z, and…
- The node z does not appear within both P_{xz} and P_{yz} prior to the end, though it may appear in one or the other.



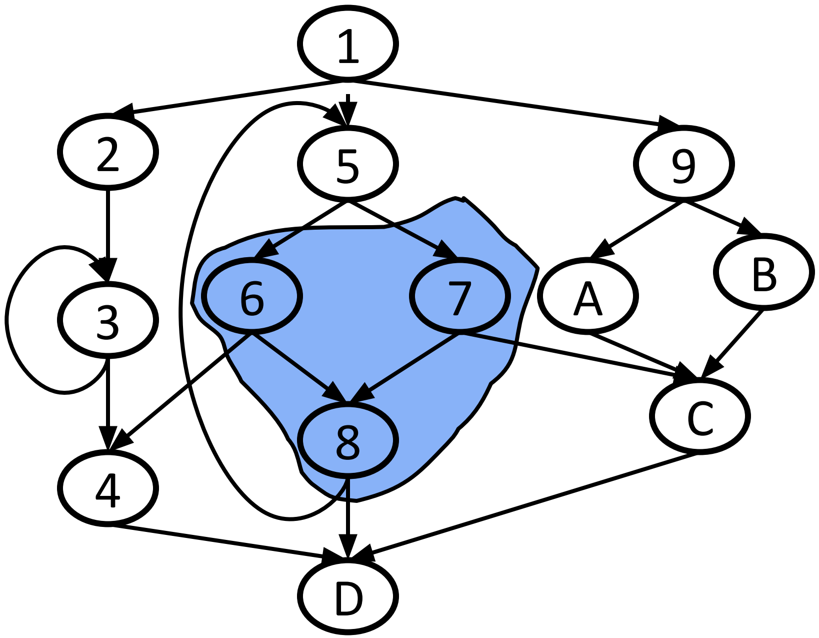
In other words, $z \in DF(x)$

Using Dominance for SSA Construction

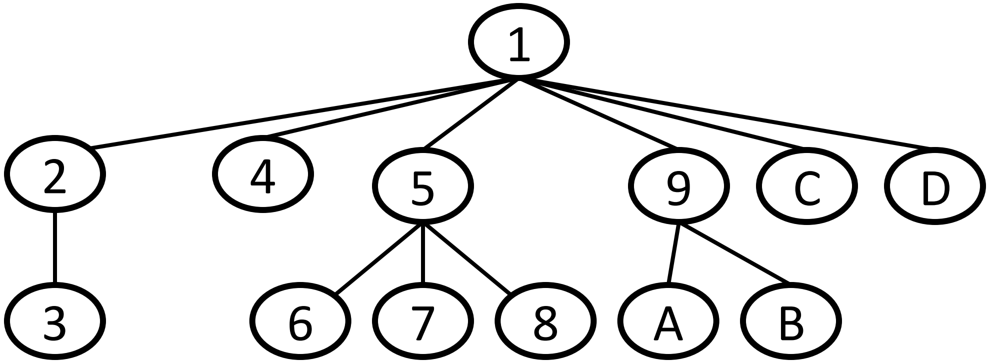
- **Dominance-Frontier Criterion**: Whenever node x contains a definition of some variable a, then any node z ∈ DF(x), z needs a Φ-function for a.

- **Iterated dominance frontier**: since a Φ-function itself is a definition, we must iterate the dominance-frontier criterion until there are no nodes that need Φ-functions.

Dominance



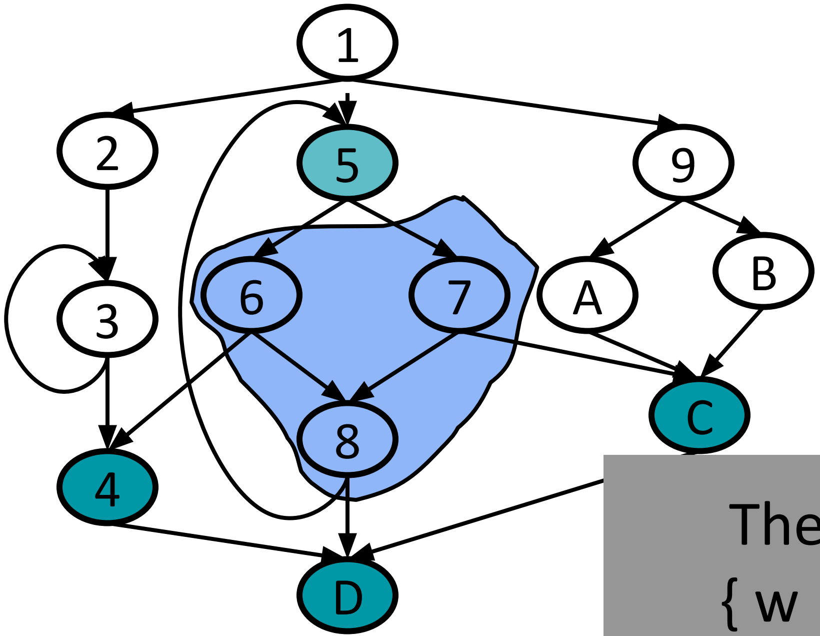
CFG

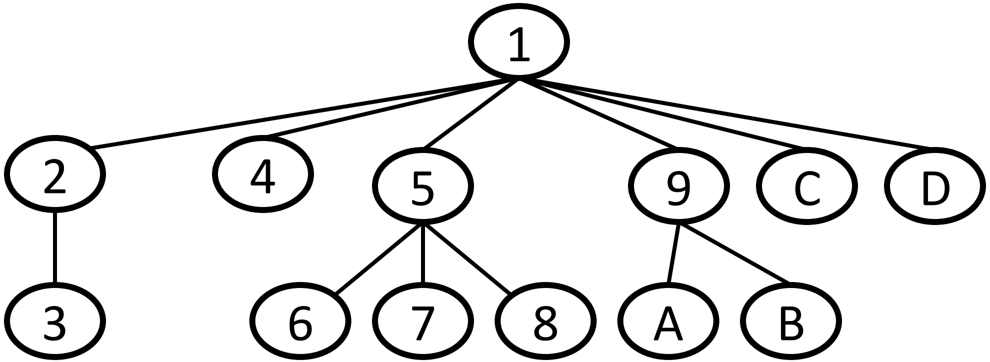


D-Tree

If there is a def of a in block 5, which nodes need a $\Phi()$?

Dominance Frontier





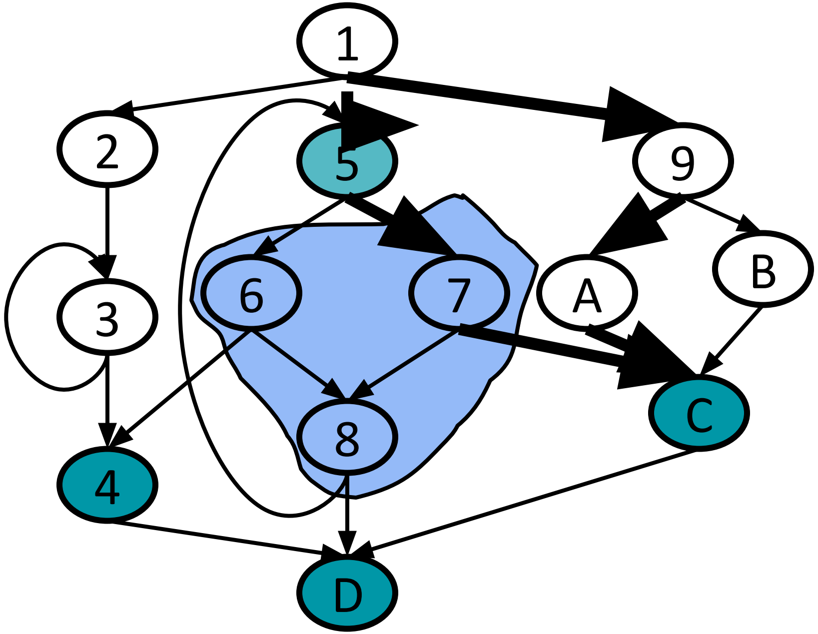
The dominance Frontier of a node x =
{ w | x dom pred(w) AND !(x sdom w)}

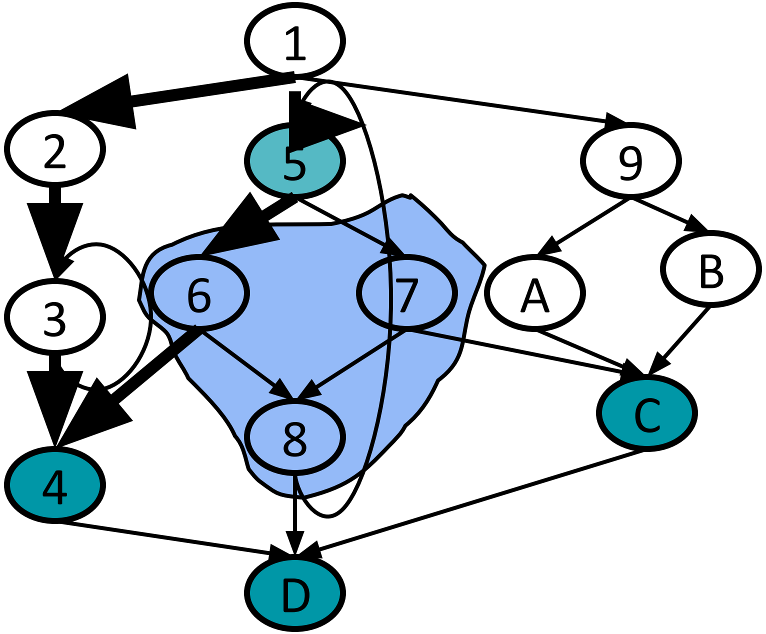
CFG

D-Tree

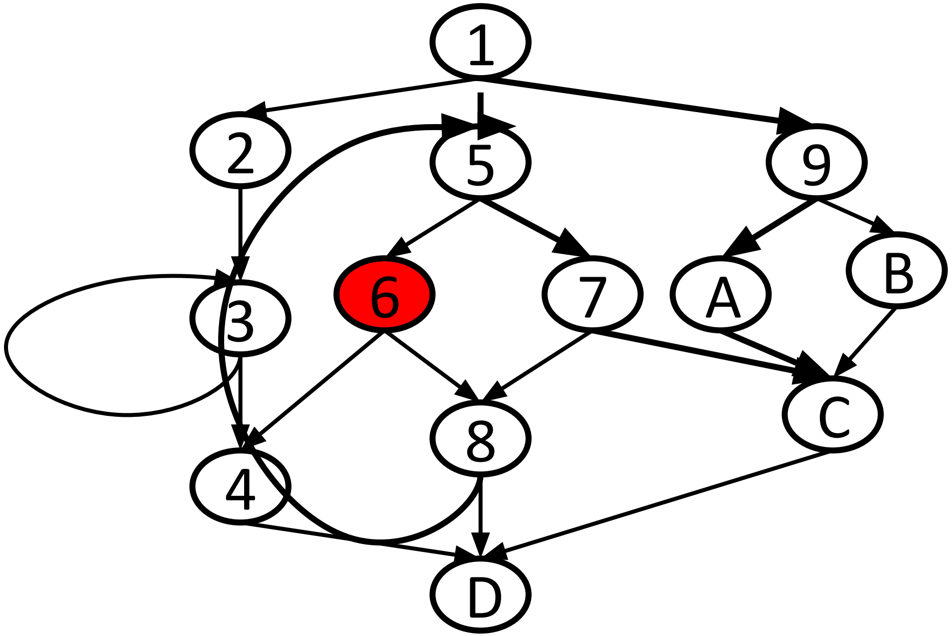
15-411/611

Dominance Frontier & path-convergence

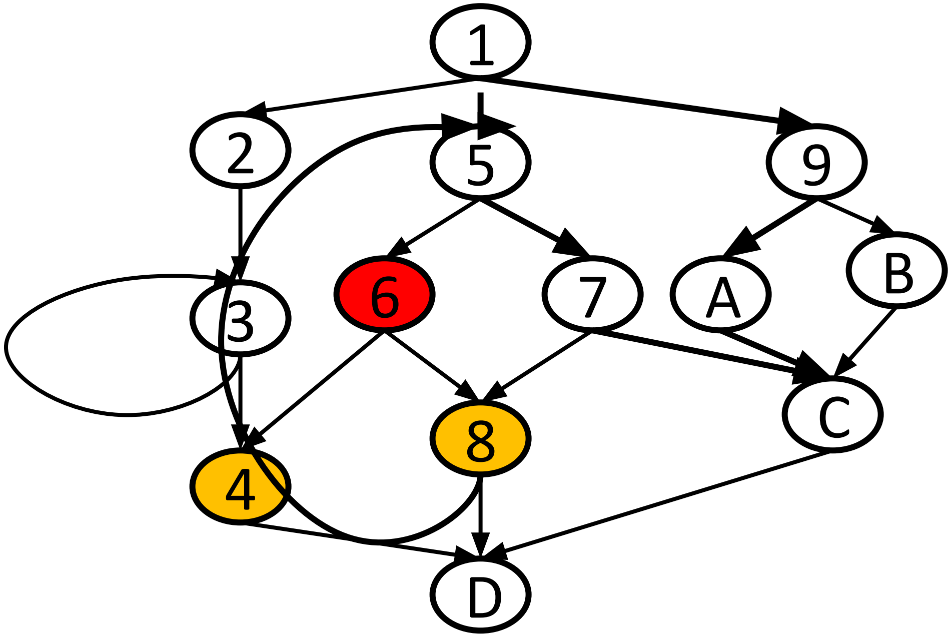




Dominance Frontier Criterion

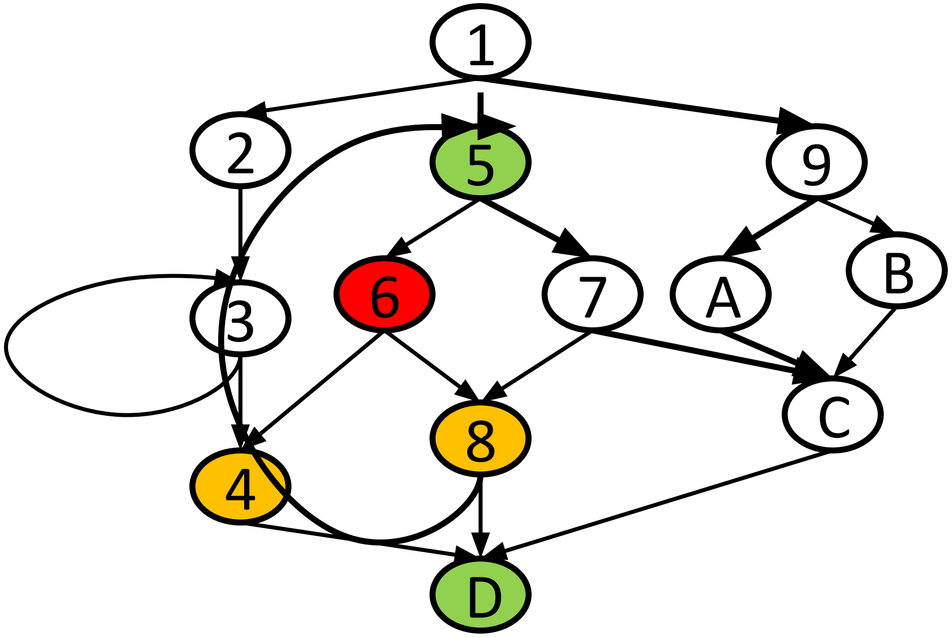


Dominance Frontier Criterion



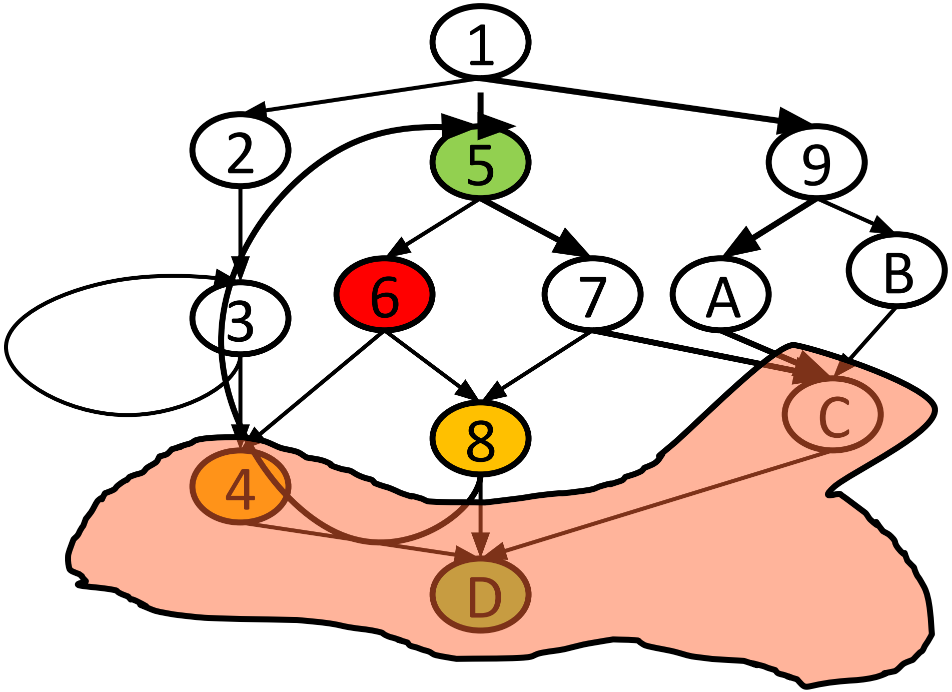
And, Iterating

Dominance Frontier Criterion



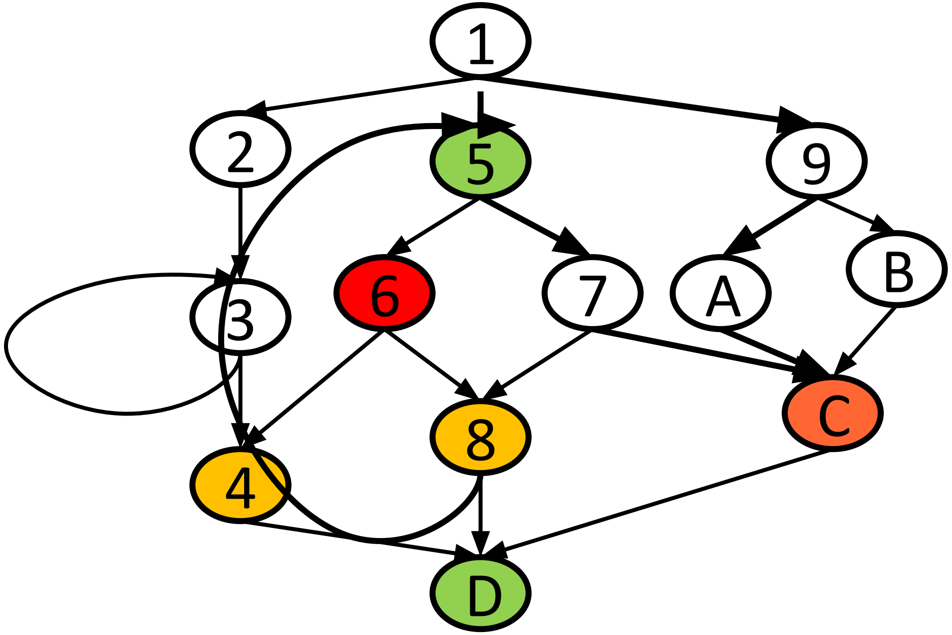
And, Iterating

Dominance Frontier Criterion



And, Iterating

Dominance Frontier Criterion



Done

Computing Dominance Frontier

- We just covered a $O(n^3)$ iterative algorithm – ***embarrassing!***

- There's also a near linear time algorithm due to Tarjan and Lengauer (Chap 19.2)
  - SSA construction therefore near linear
  - SSA form makes many optimizations linear (no need for iterative data flow)

Using DF to Place $\Phi()$

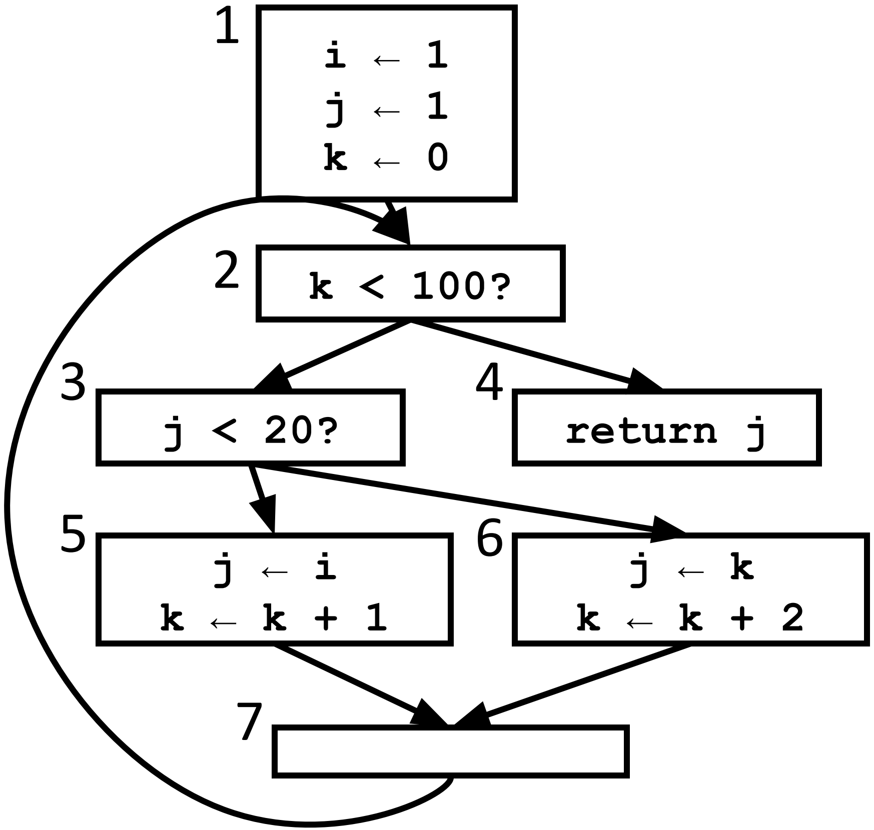
- Gather all the defsites of every variable
- Then, for every variable
  - foreach defsite
    - foreach node in DF(defsite)
      - if we haven't put $\Phi()$ in node put one in
      - If this node didn't define the variable before: add this node to the defsites

- This essentially computes the Iterated Dominance Frontier on the fly, creating minimal SSA

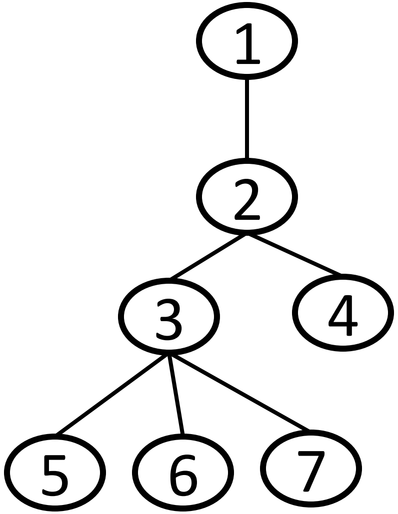
Using DF to Place Φ()

```
foreach node n {
    foreach variable v defined in n {
        orig[n] ∪= {v}
        defsites[v] ∪= {n}
    }
    foreach variable v {
        W = defsites[v]
        while W not empty {
            foreach y in DF[n]
            if y ∉ PHI[v] {
                insert "v ← Φ(v,v,…)" at top of y
                PHI[v] = PHI[v] ∪ {y}
                if v ∉ orig[y]: W = W ∪ {y}
            }
        }
    }
}
```

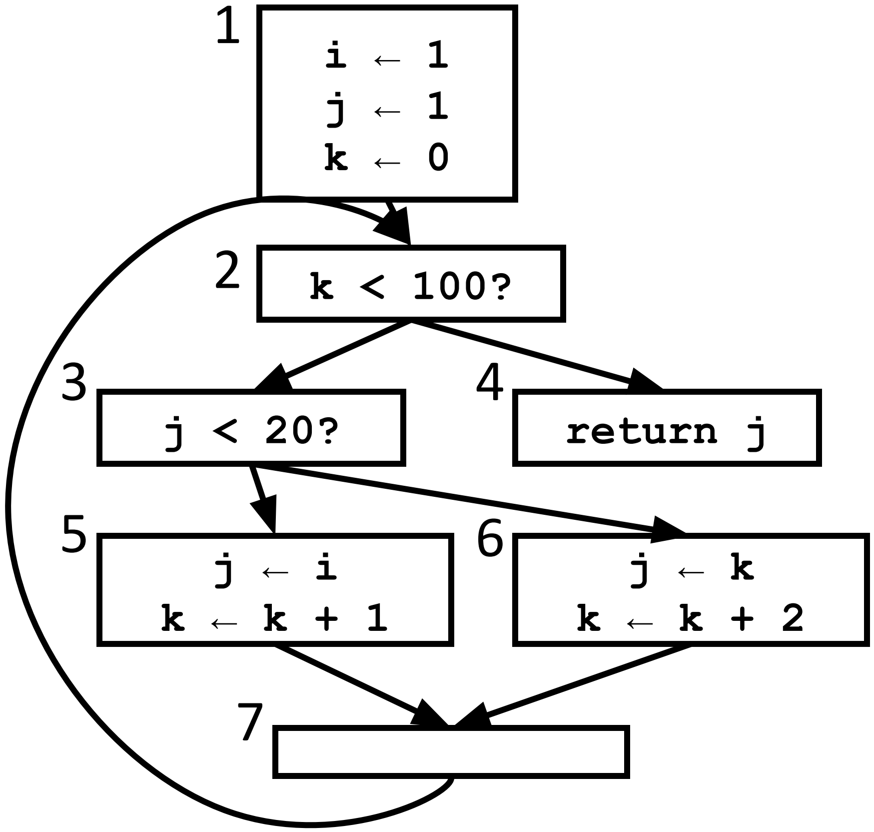
Computing SSA



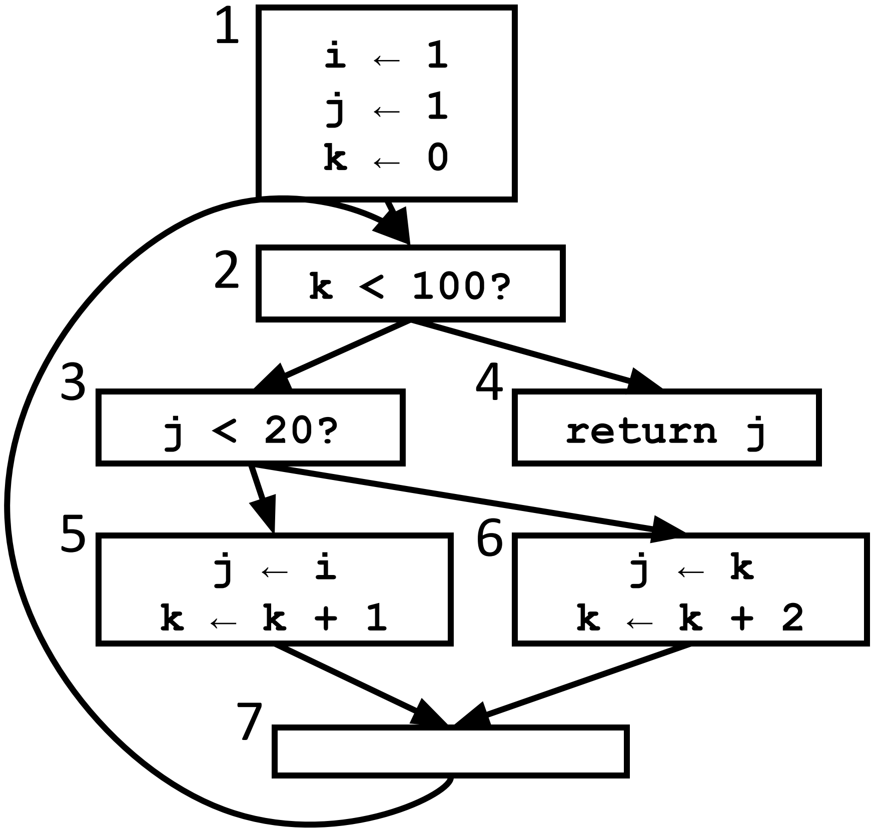
D-tree

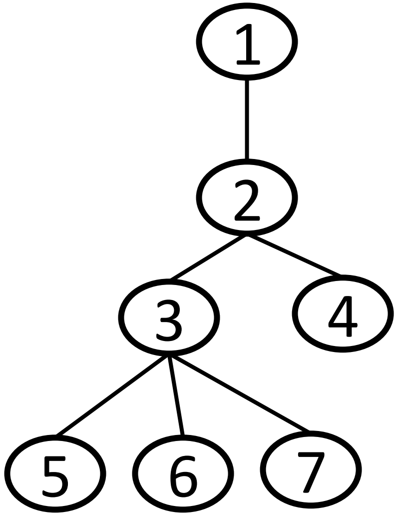


Compute D-tree



Compute Dominance Frontier (DFs)

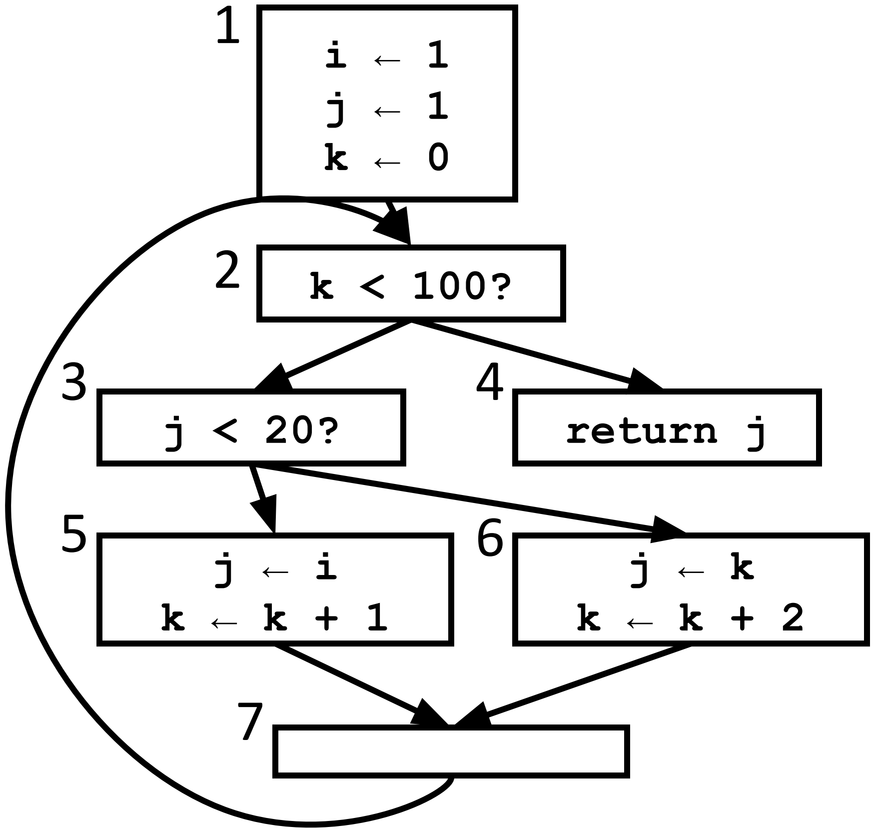




DFs

1	{}
2	{2}
3	{2}
4	{}
5	{7}
6	{7}
7	{2}

Compute defsites



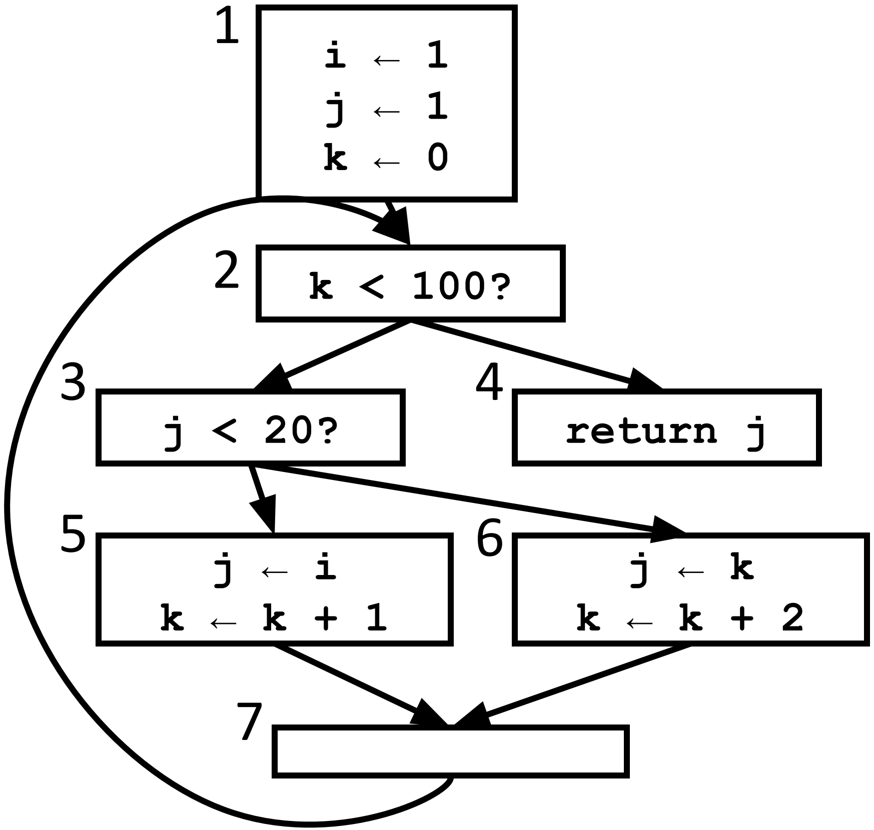
DFs	
1	{}
2	{2}
3	{2}
4	{}
5	{7}
6	{7}
7	{2}

orig[n]	
1	{ i,j,k}
2	{}
3	{}
4	{}
5	{j,k}
6	{j,k}
7	{}

defsites[v]	
i	{1}
j	{1,5,6}
k	{1,5,6}

var j: W={1,5,6}

Inspect variables



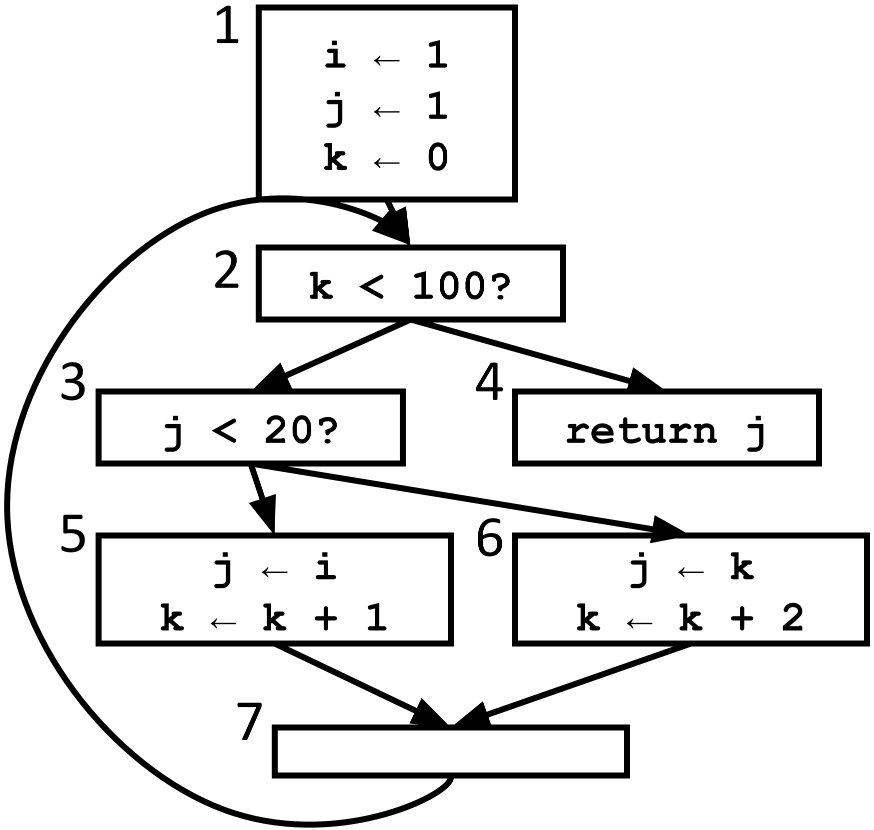
DFs	
1	{}
2	{2}
3	{2}
4	{}
5	{7}
6	{7}
7	{2}

orig[n]	
1	{ i,j,k}
2	{}
3	{}
4	{}
5	{j,k}
6	{j,k}
7	{}

defsites[v]	
i	{1}
j	{1,5,6}
k	{1,5,6}

var j: W={1,5,6}

Insert ϕ for j

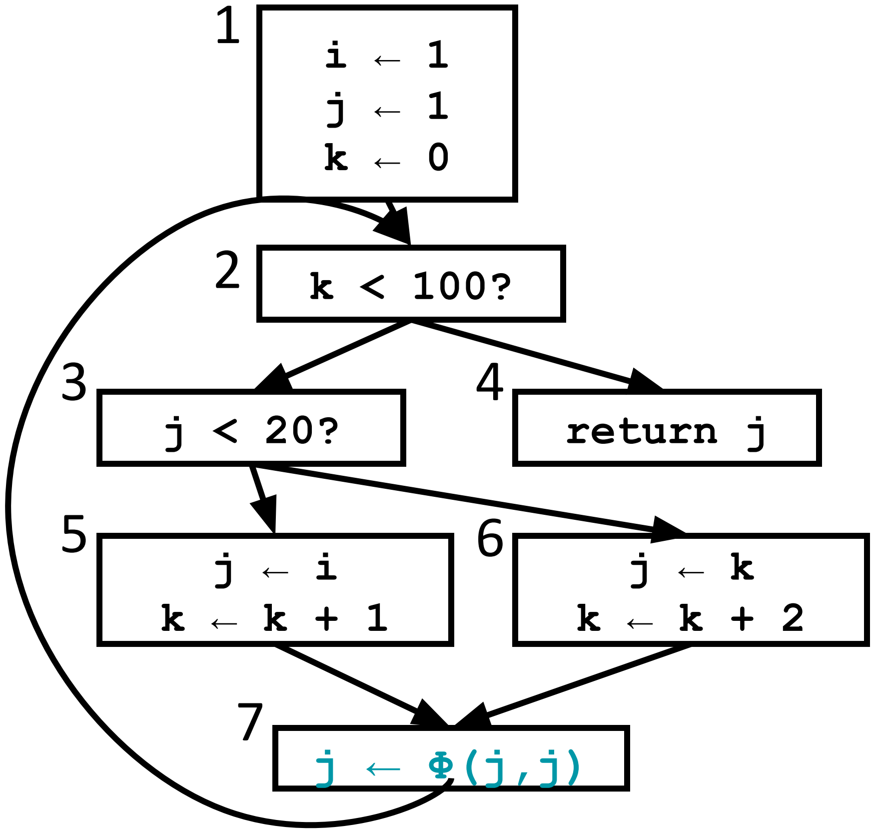


DFs		orig[n]		defsites[v]	
1	{}	1	{ i,j,k}	i	{1}
2	{2}	2	{}	j	{1,5,6}
3	{2}	3	{}	k	{1,5,6}
4	{}	4	{}		
5	{7}	5	{j,k}		
6	{7}	6	{j,k}		
7	{2}	7	{}		

var j: W={1,5,6}

DF[1] ∪ DF[5] ∪ DF[6] ={7}

Insert ϕ for j



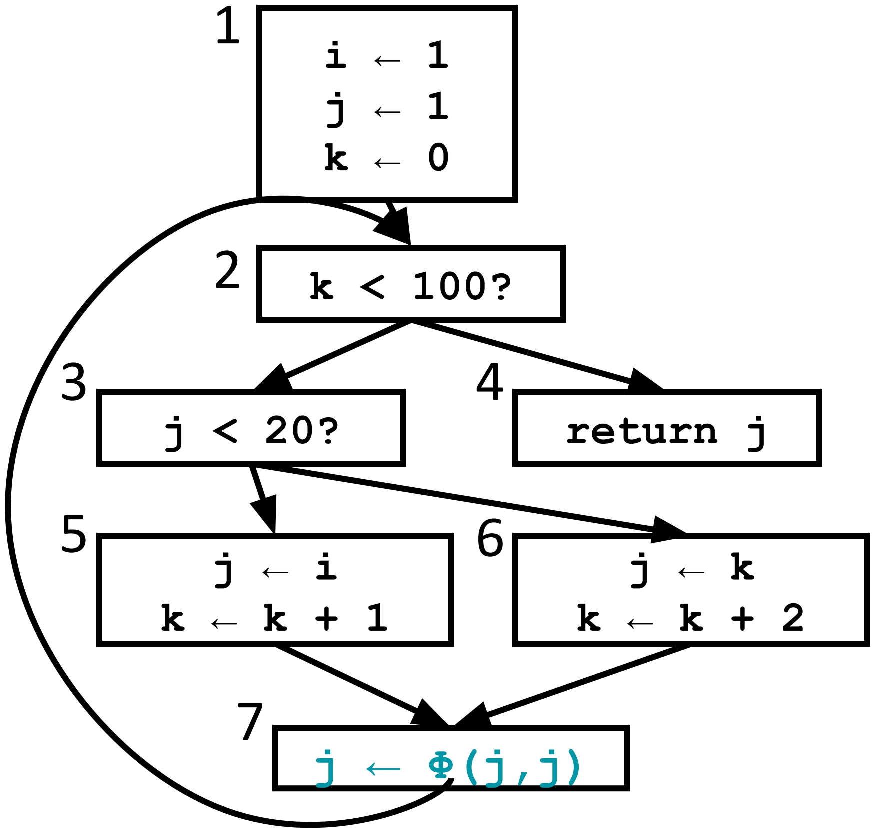
DFs	
1	{}
2	{2}
3	{2}
4	{}
5	{7}
6	{7}
7	{2}

orig[n]	
1	{ i,j,k}
2	{}
3	{}
4	{}
5	{j,k}
6	{j,k}
7	{}

defsites[v]	
i	{1}
j	{1,5,6}
k	{1,5,6}

var j: W={1,5,6}

Handle new write for j



DFs

1	{}
2	{2}
3	{2}
4	{}
5	{7}
6	{7}
7	{2}

orig[n]

1	{ i,j,k}
2	{}
3	{}
4	{}
5	{j,k}
6	{j,k}
7	{}

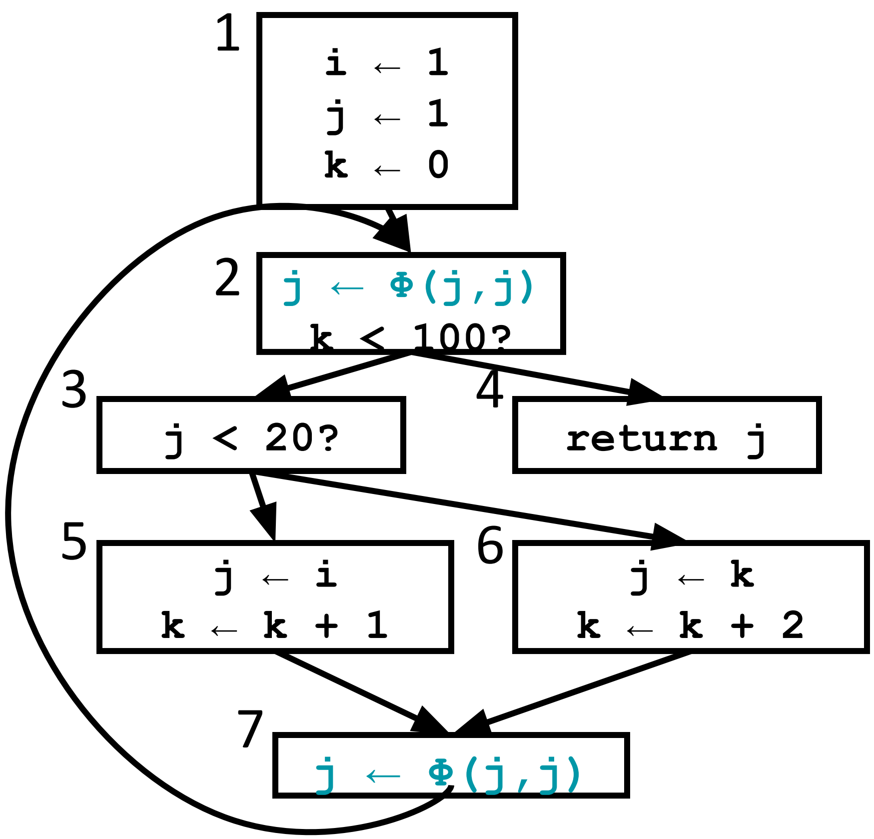
defsites[v]

i	{1}
j	{1,5,6}
k	{1,5,6}

var j: W={1,5,6,7}

DF[1] ∪ DF[5] ∪ DF[6] ∪ DF[7]
={7,2}

Insert more ϕ for j



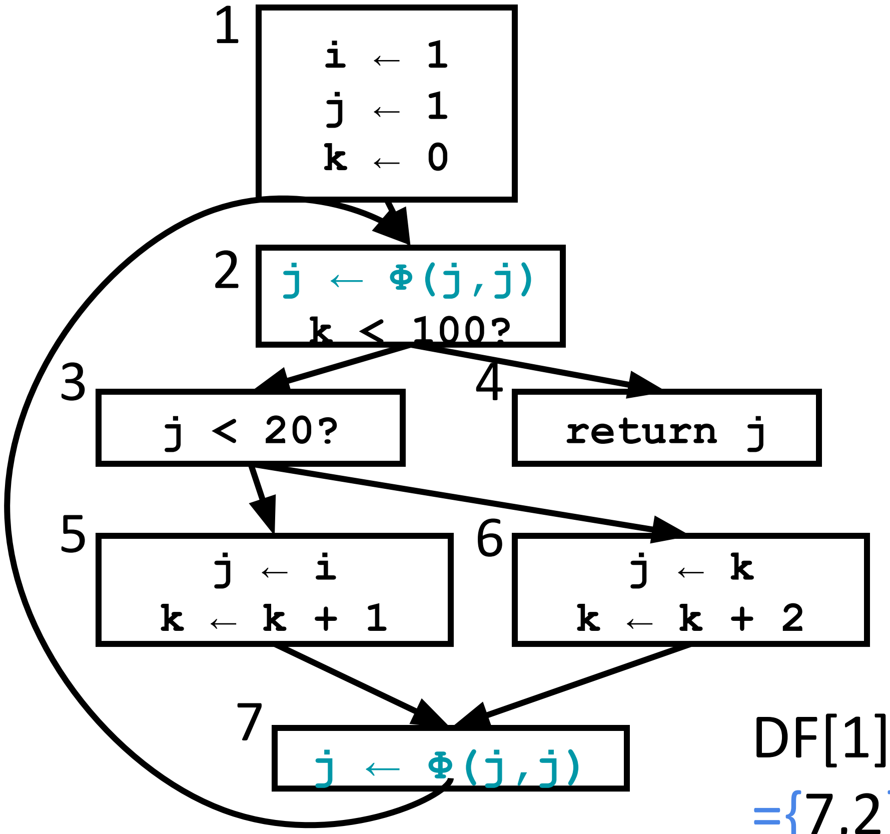
	DFs		orig[n]
1	{}	1	{ i,j,k}
2	{2}	2	{}
3	{2}	3	{}
4	{}	4	{}
5	{7}	5	{j,k}
6	{7}	6	{j,k}
7	{2}	7	{}

	defsites[v]
i	{1}
j	{1,5,6}
k	{1,5,6}

var j: W={1,5,6,7}

DF[1] ∪ DF[5] ∪ DF[6] ∪ DF[7]
={7,2}

Update writes for j



DFs	
1	{}
2	{2}
3	{2}
4	{}
5	{7}
6	{7}
7	{2}

orig[n]	
1	{ i,j,k}
2	{}
3	{}
4	{}
5	{j,k}
6	{j,k}
7	{}

defsites[v]	
i	{1}
j	{1,5,6}
k	{1,5,6}

var j: W={1,5,6,7,2}

DF[1] ∪ DF[5] ∪ DF[6] ∪ DF[7] ∪ DF[2]
={7,2}

Renaming Variables

- Placing ϕ is not enough, need to update names
- Walk down the dominator tree, renaming variables incrementally
- Replace uses with most recent renamed def
  - For straight-line code this is easy
  - If there are branches and joins?

Renaming for Straight-Line Code

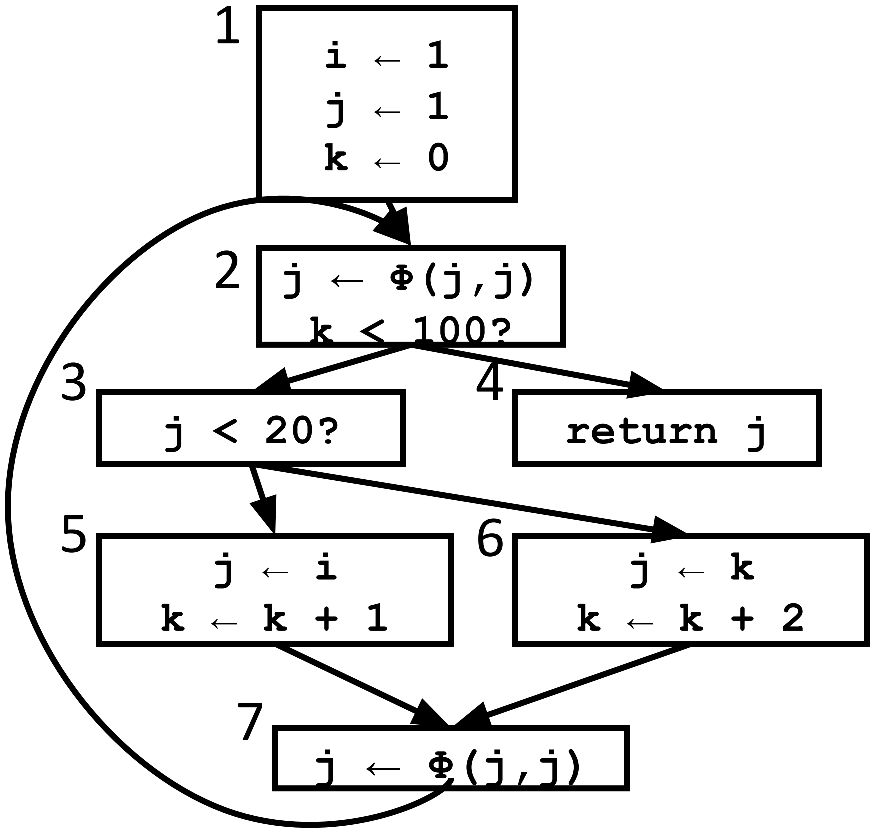
- Need to extend for ϕ-functions.
- Need to maintain property that definitions dominate uses.

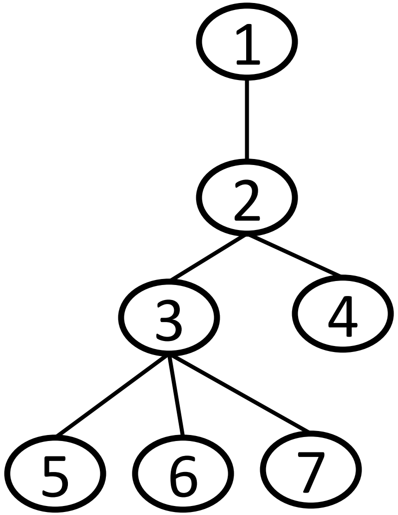
for each variable a:
    Count[a] = 0
    Stack[a] = [0]
renameBasicBlock(*B*):
    for each instruction *S* in block *B*:
        for each use of a variable *x* in *S*:
            i = top(Stack[*x*])
            replace the use of *x* with x_i
        for each variable *a* that *S* defines
            count[*a*] = Count[*a*] + 1
            i = Count[*a*]
            push *i* onto Stack[*a*]
            replace definition of *a* with a_i

Renaming in CFG

```
rename(n):
    renameBasicBlock(n)
        for each successor Y of n, where n is the jth predecessor of Y:
        for each phi-function f in Y, where the operand of f is 'a'
        i = top(Stack[a])
        replace jth operand with a_i
        for each child of n in D-tree, X:
            rename(X)
    for each instruction S ∈ n:
            for each variable v that S defines:
                pop Stack[v]
```

Rename j variables

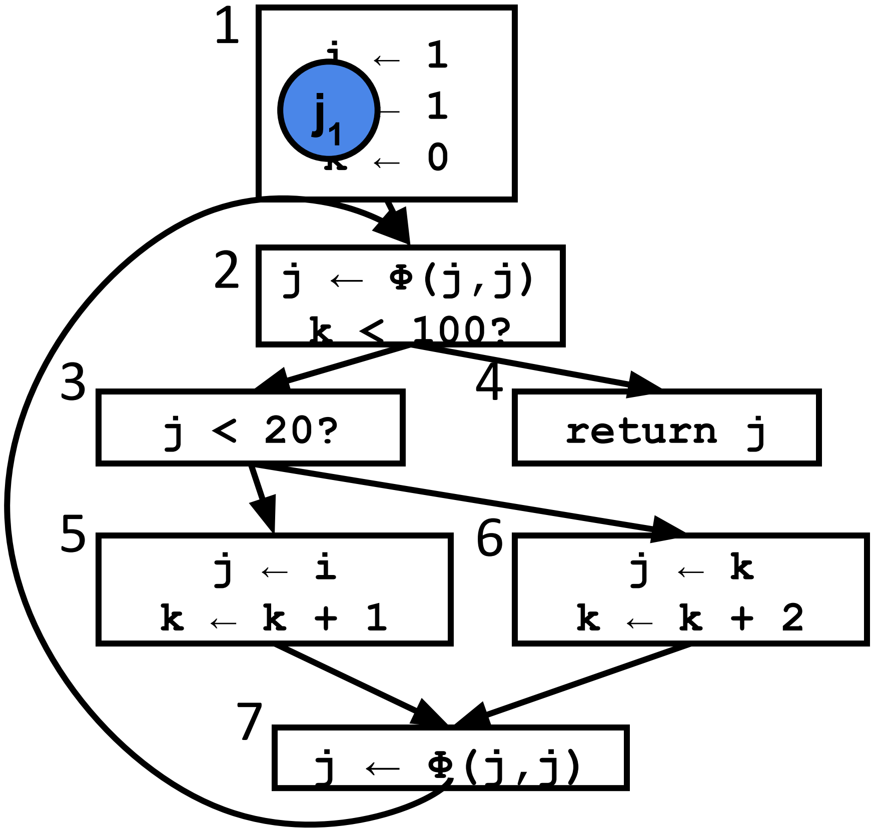


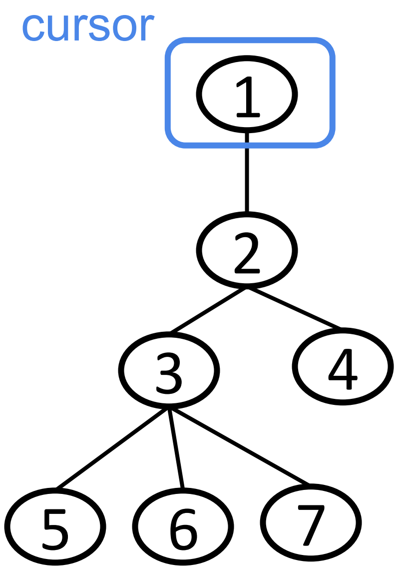


defsites[v]

i	{1}
j	{1,5,6,7,2}
k	{1,5,6}

Rename j variables





defsites[v]

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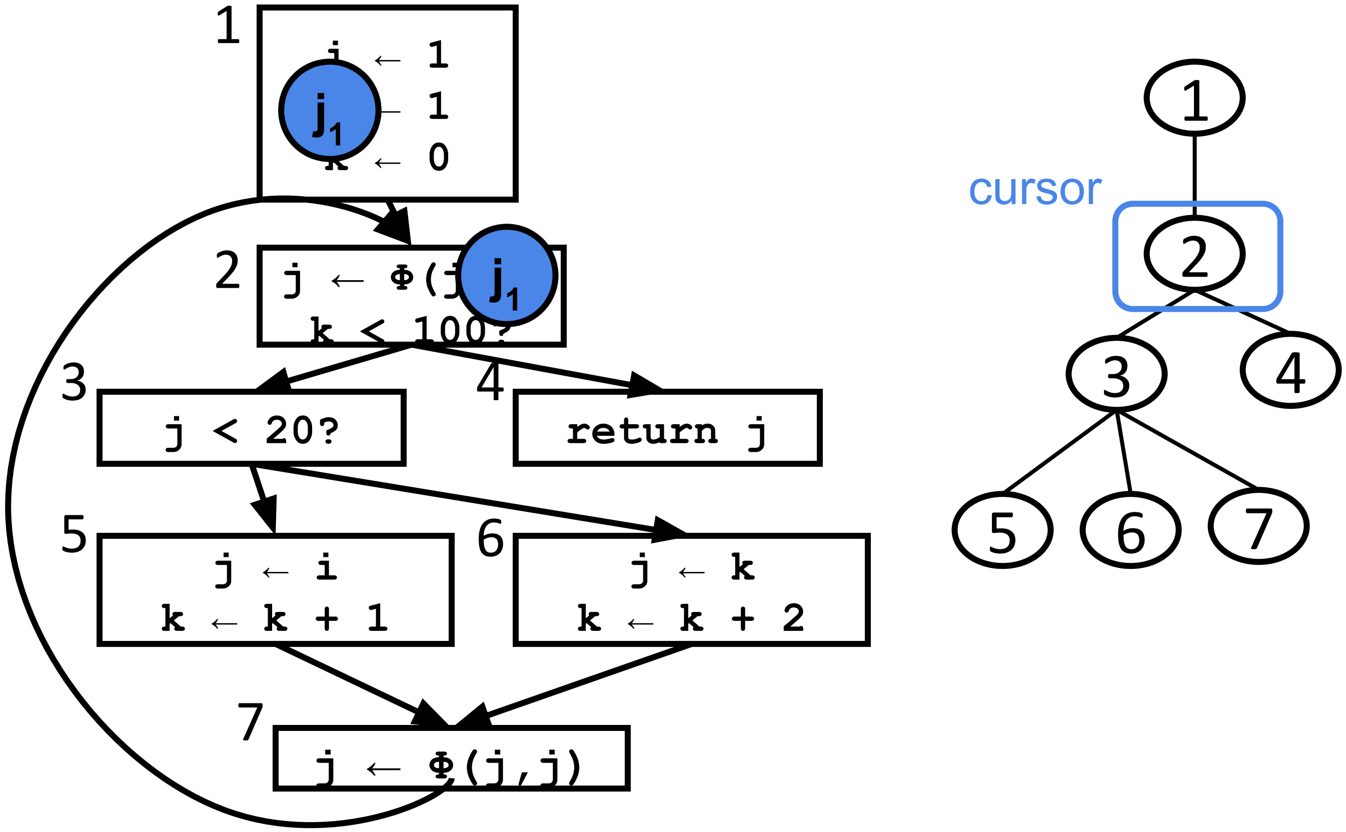
j	{1,5,6,7,2}
k	{1,5,6}

Rename j variables



defsites[v]

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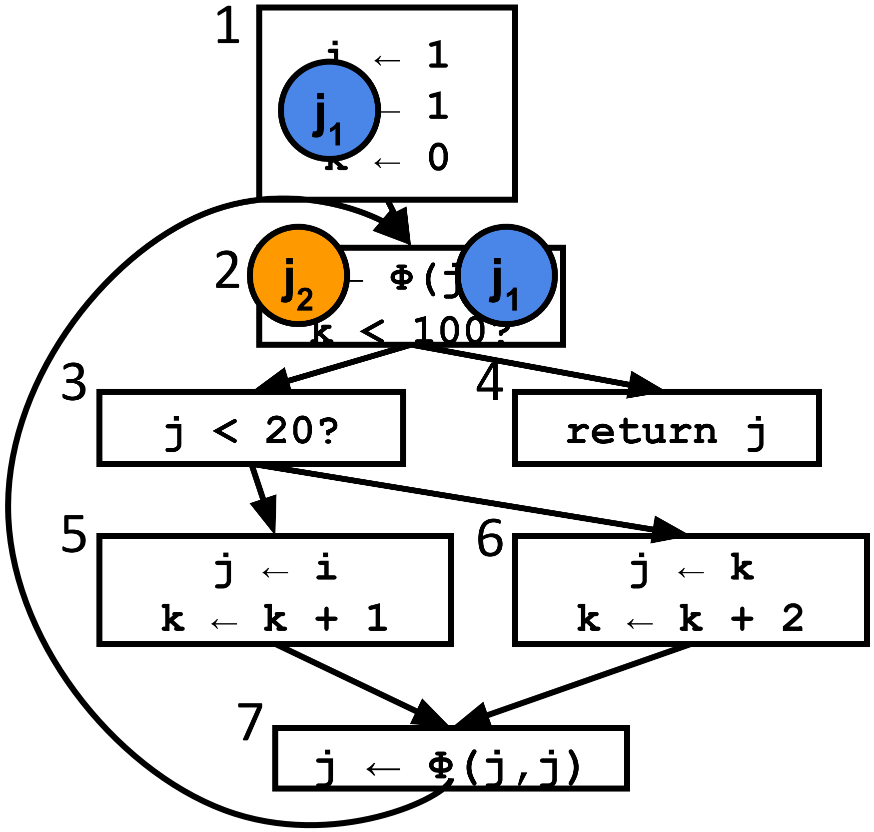
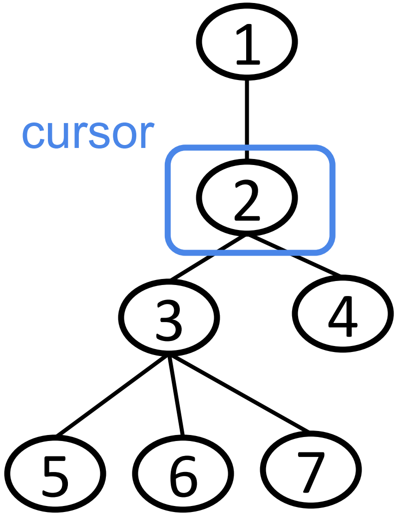
j	{1,5,6,7,2}
k	{1,5,6}

Rename j variables





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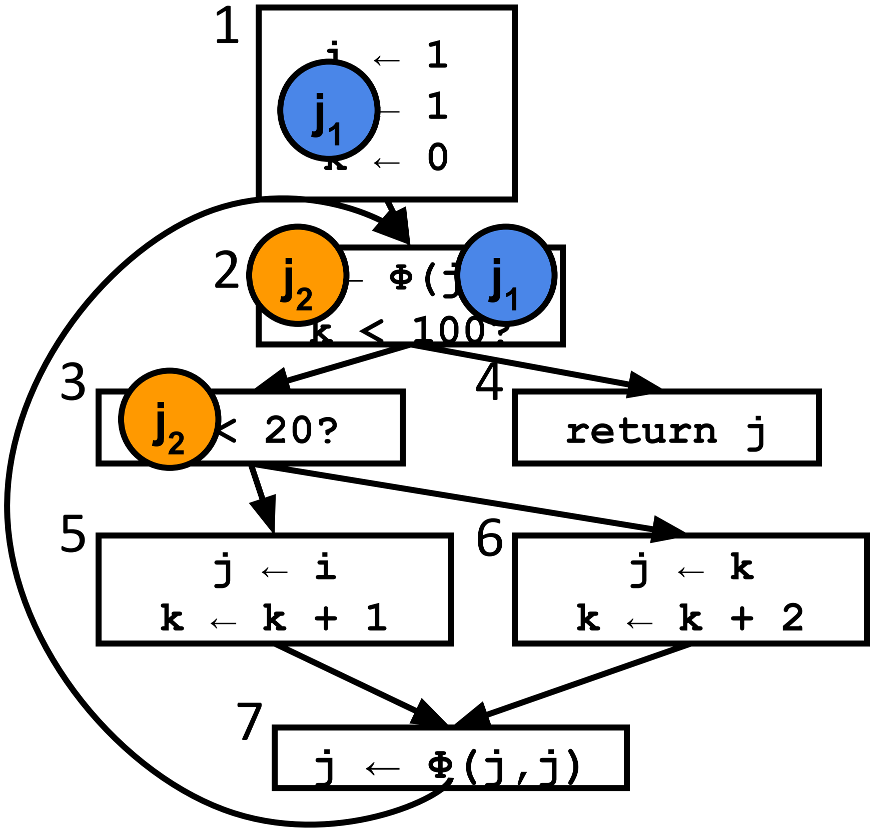
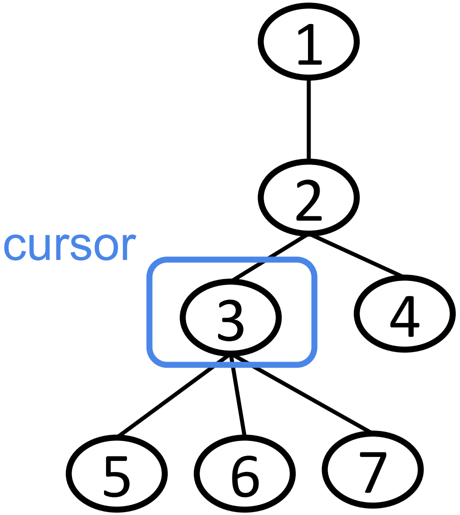
k	{1,5,6}

Rename j variables





defsites[v]

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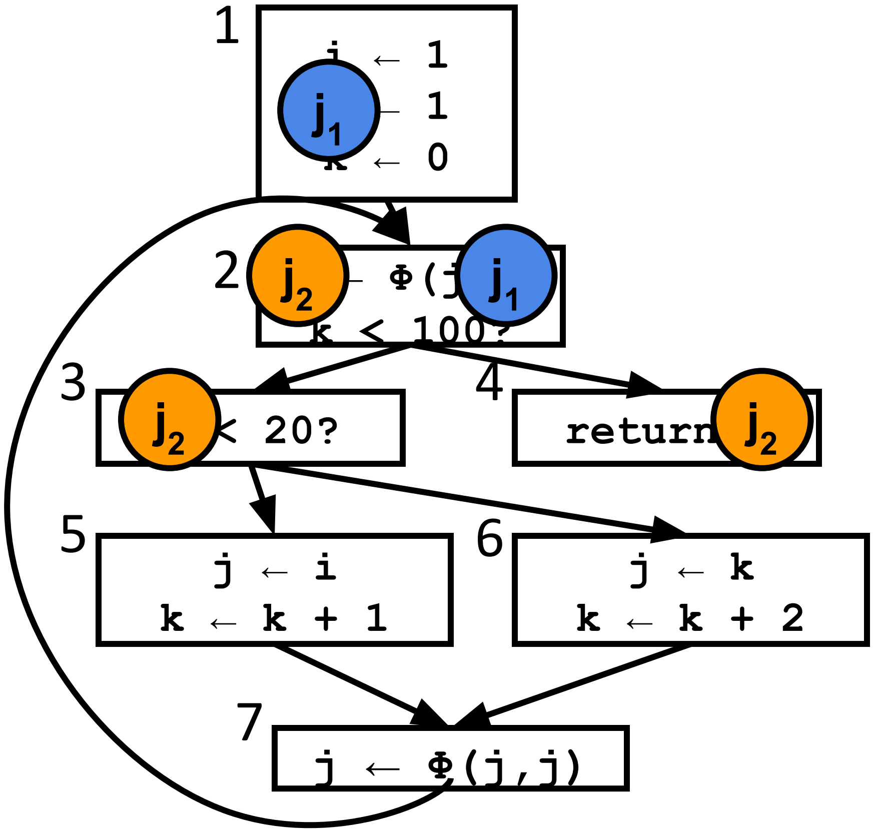
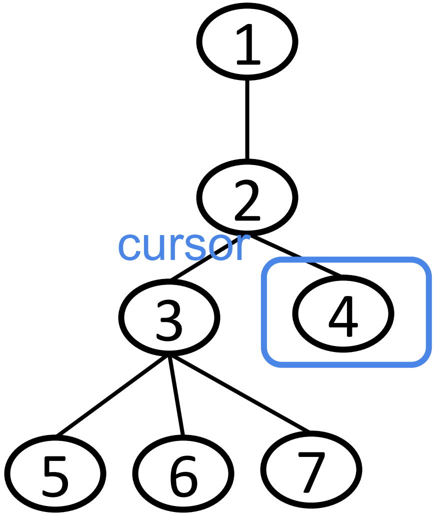
k	{1,5,6}

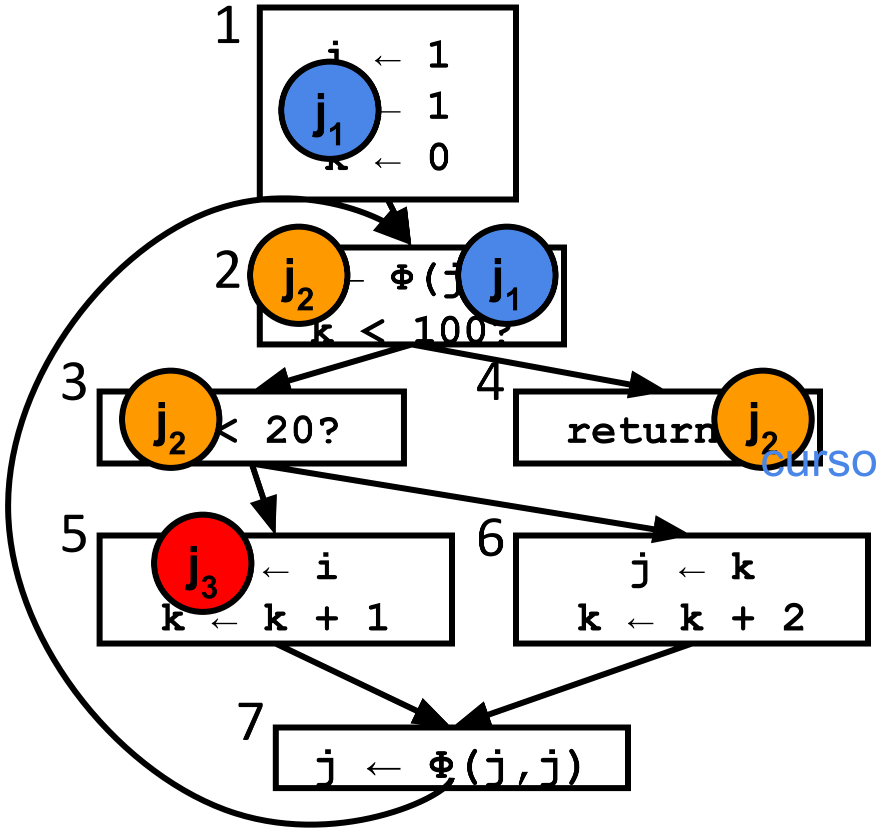
Rename j variables

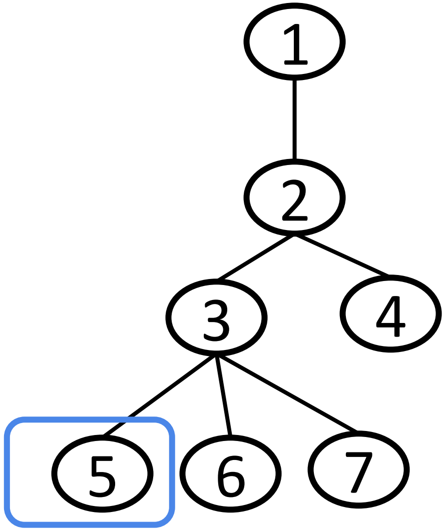




defsites[v]	
i	{1}
j	{1,5,6,7,2}
k	{1,5,6}

Rename j variables





defsites[v]

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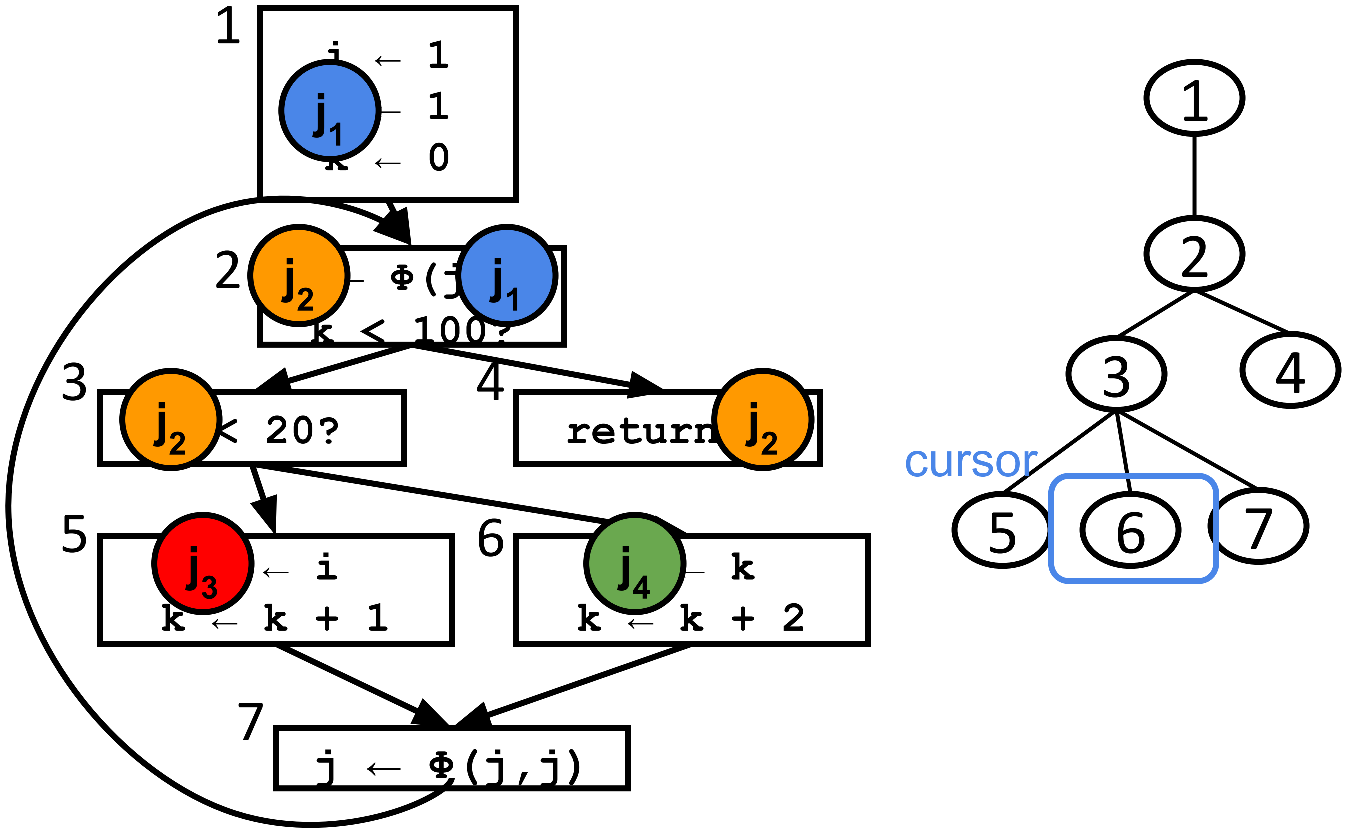
j	{1,5,6,7,2}
k	{1,5,6}

Rename j variables



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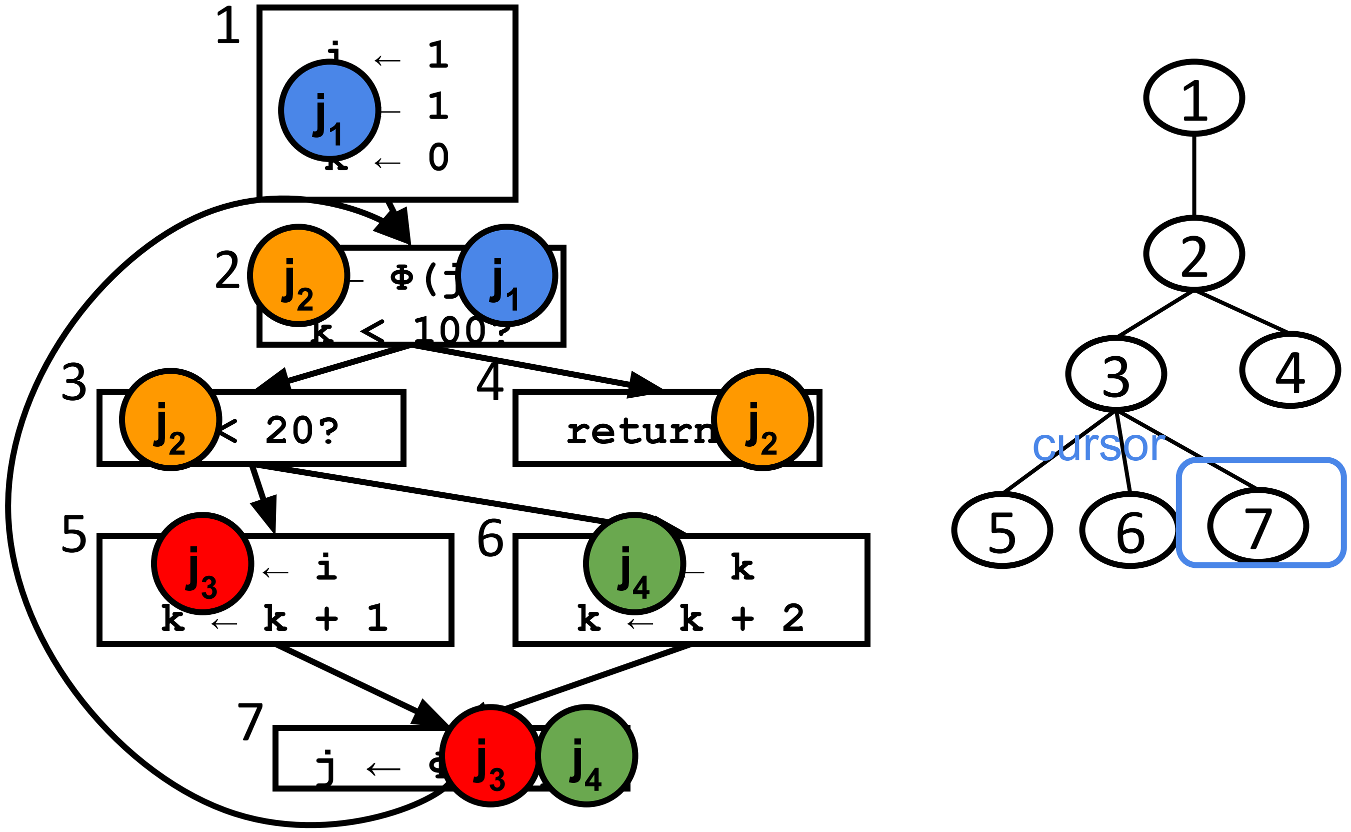
j	{1,5,6,7,2}
k	{1,5,6}

Rename j variables



defsites[v]

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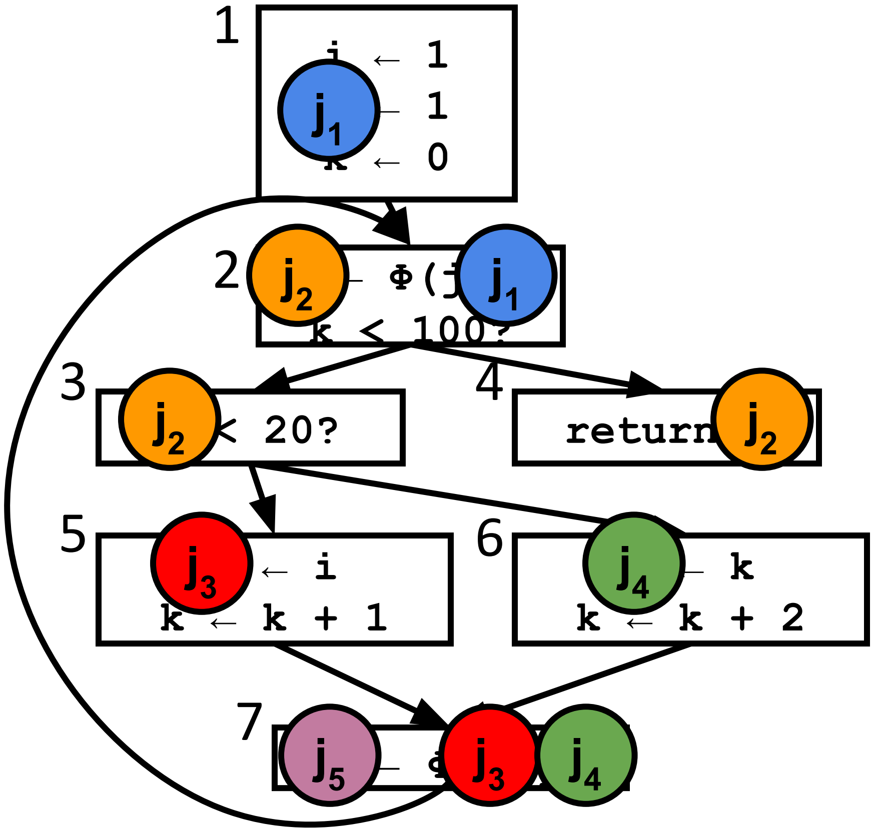
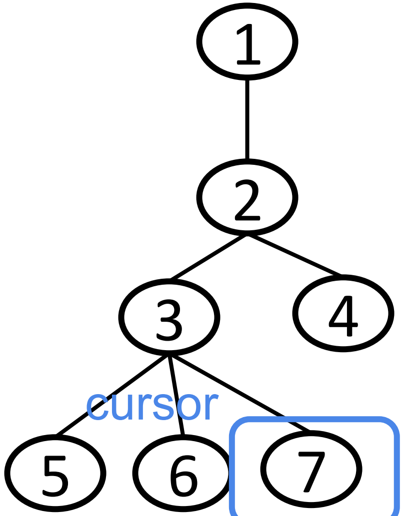
j	{1,5,6,7,2}
k	{1,5,6}

Rename j variables





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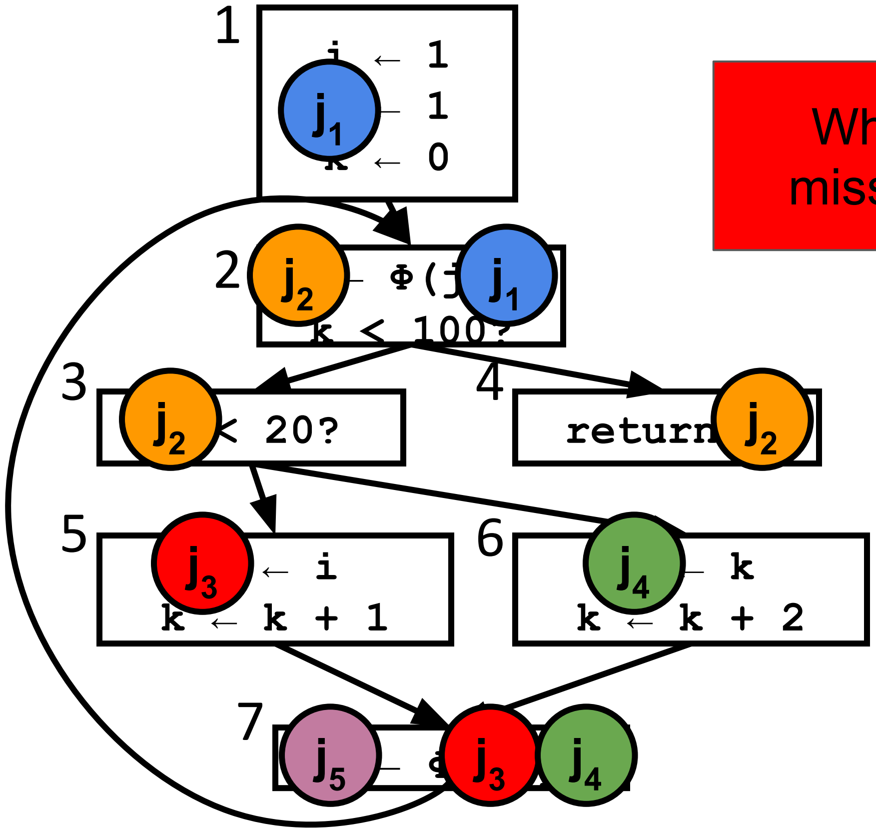
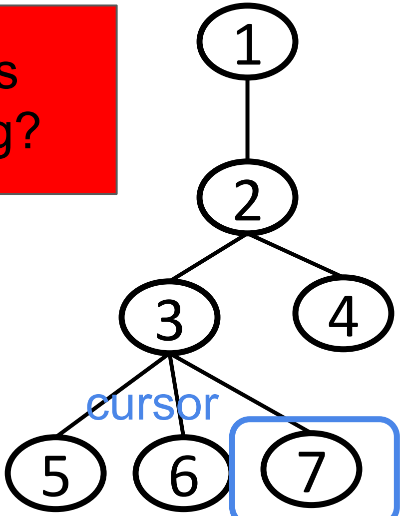
j	{1,5,6,7,2}
k	{1,5,6}

Rename j variables



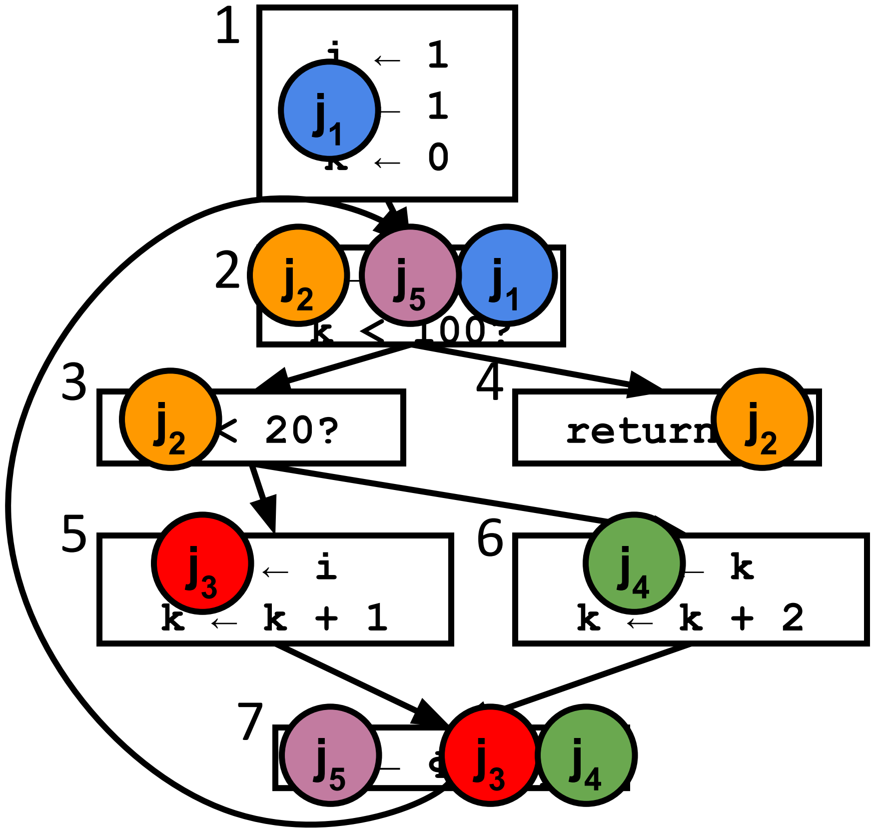
What's missing?

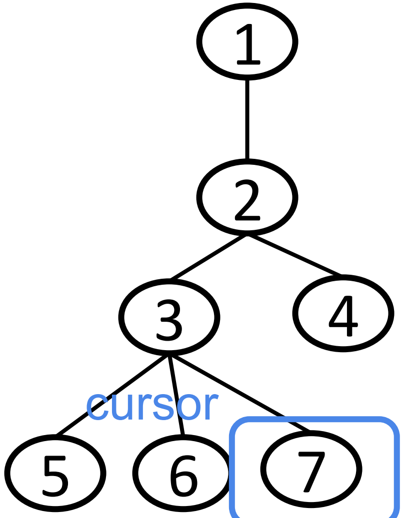


defsites[v]

i	{1}
j	{1,5,6,7,2}
k	{1,5,6}

Rename j variables





defsites[v]

i	{1}
j	{1,5,6,7,2}
k	{1,5,6}

Flavors of SSA

- Minimal SSA
  - at each join point with >1 outstanding definition insert a φ-function
  - Some may be dead
- Pruned SSA
  - only add live φ-functions
  - must compute LIVEOUT
- Semi-pruned SSA
  - Same as minimal SSA, but only on names live across more than 1 basic block

Summary

- SSA is a useful and efficient IR.
- Definitions dominate uses
- Constructing SSA can be efficient

  (No need to do Lengaur-Tarjan Algorithm, instead see A Simple, Fast Dominance Algorithm by Cooper, Harvey, and Kennedy)
- Don’t do any optimizations yet!

Next time

- More practice building SSA
- Constant propagation with SSA
- Deconstructing SSA
- SSA in practice