**Assignment 1: Prop it Out**
**15-414/15-424 Bug Catching: Automated Program Verification**

Due: **11:59pm**, Thursday 9/6/18
Total Points: 50

1. **Table counting (4 points)** How many rows and columns does the full truth table for the following formula have? Why? Note that you do not need to write the truth table down to answer this question, and should assume that the truth table is constructed as the example given in Lecture 2.

$$\neg s \land ((r \to q \to p) \leftrightarrow (t \to u \to v \to p))$$

2. **Proof practice (6 points)** Conduct a proof in sequent calculus of the following formula. Be sure to say which proof rule you apply at each step.

$$((p \to q) \to p) \to p$$

3. **Biimplication (10 points)** The syntax of propositional logic provided the biimplication/bisubjunction operator $A \leftrightarrow B$ which is true iff both $A$ and $B$ have the same truth-value, so both are true or both are false. But this operator is missing sequent calculus proof rules.

   Your task is to design proof rules for when the equivalence operator is used in the succedent (right rule $\leftrightarrow$R) and when it is used in the antecedent (left rule $\leftrightarrow$L):

$$(\leftrightarrow\mathrm{R})\ \frac{\dots}{\Gamma \vdash F \leftrightarrow G, \Delta} \qquad (\leftrightarrow\mathrm{L})\ \frac{\dots}{\Gamma, F \leftrightarrow G \vdash \Delta}$$

4. **Soundness of $\leftrightarrow$ (20 points)** Proof rules cannot be used unless they are accompanied by a soundness proof. Use the semantics of the biimplication operator given in Lecture 2 to prove soundness of your proof rules $\leftrightarrow$R and $\leftrightarrow$L. That is, for each of the rules, prove that validity of all its premises implies validity of its conclusion.

5. **Use it! (10 points)** Use your proof rules $\leftrightarrow$R and $\leftrightarrow$L to conduct a sequent calculus proof for the formula:

$$(p \to q) \leftrightarrow \neg(p \land \neg q)$$