Bug Catching: Automated Program Verification
15414/15614 Spring 2021
Lecture 1: Introduction

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Course staff

Instructor

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Teaching Assistants

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Learning objectives

For this lecture

- What is this course about?
- What are the learning objectives for the course?
- How does it fit into the curriculum?
- How does the course work?
- Remember . . .

Bad code

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- "The Heartbleed bug allows anyone on the Internet to read the memory of the systems protected by the vulnerable versions of the OpenSSL software."
- "...this allows attackers to eavesdrop on communications, steal data directly from the services and users and to impersonate services and users."

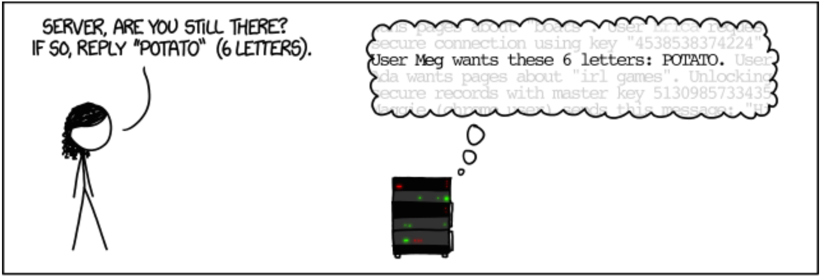
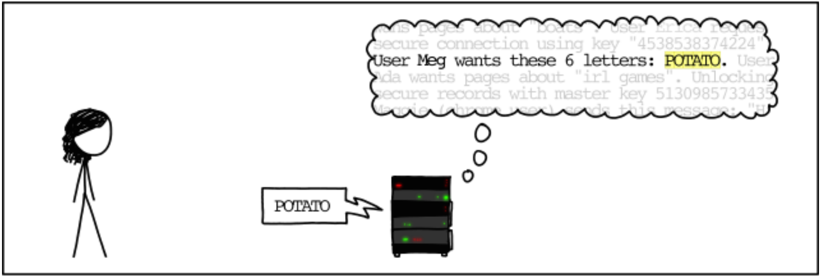


Image source: Randall Munroe, xkcd.com

Heartbleed, explained



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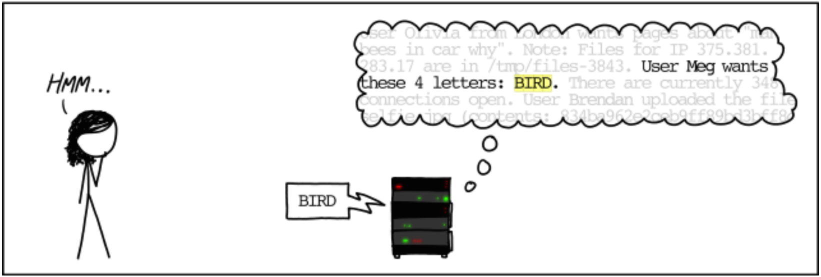

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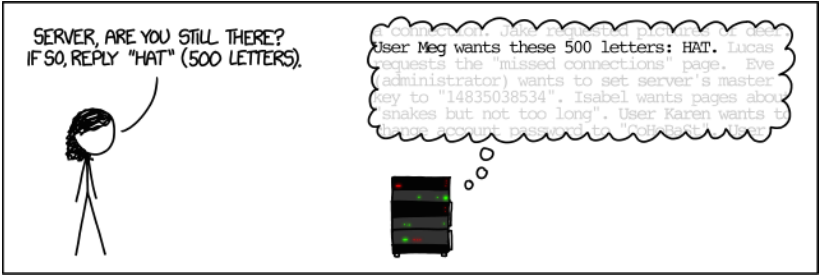

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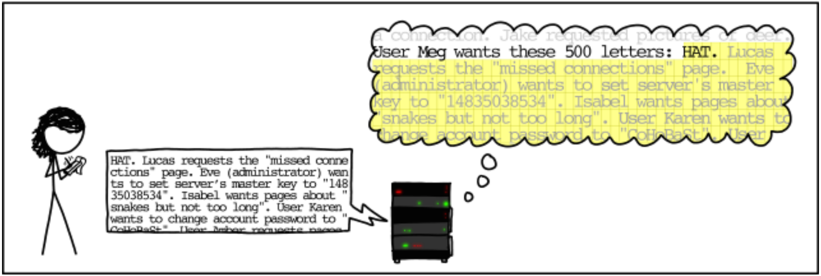

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25 April 2014

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25 April 2014

EXPLOITS AND VULNERABILITIES | NEWS

Five years later, Heartbleed vulnerability still unpatched

Posted: September 12, 2019 by Gilad Maayan

Algorithms vs. code

```
int binarySearch(int key, int[] a, int n) {
    int low = 0;
    int high = n;

    while (low < high) {
        int mid = (low + high) / 2;

        if(a[mid] == key) return mid; // key found
        else if(a[mid] < key) {
            low = mid + 1;
        } else {
            high = mid;
        }
    }
    return -1;  // key not found.
}
```

Code matters

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But what if `low` $+$ `high` $> 2^{31} - 1$?

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Then `mid = (low + high) / 2` becomes negative

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Algorithm may be correct—but we run code, not algorithms.

How do we fix it?

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Solution: `mid = low + (high - low)/2`

The fix

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int binarySearch(int key, int[] a, int n)
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    int low = 0;
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The fix

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int binarySearch(int key, int[] a, int n)
//@requires 0 <= n && n <= \length(a);
/*@ensures (\result == -1 && !is_in(key, A, 0, n))
  @      || (0 <= \result && \result < n
  @             && A[\result] == key); @*/
{
    int low = 0;
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    while (low < high) {
        int mid = low + (high - low) / 2;

        if(a[mid] == key) return mid; // key found
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The fix

```
int binarySearch(int key, int[] a, int n)
//@requires 0 <= n && n <= \length(a);
//@requires is_sorted(a, 0, n);
/*@ensures (\result == -1 && !is_in(key, A, 0, n))
  @       || (0 <= \result && \result < n
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- Humans are fallible, bugs are subtle
- What's the specification?

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$$\mathit{Specification} \Longleftrightarrow \mathit{Implementation}$$

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$$\textit{Specification} \iff \textit{Implementation}$$

- Specification must be **precise**
- Meaning of code must be **comprehensive**
- Reasoning must be **sound**

Course objectives

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- Identify and formalize program correctness
- Understand language semantics
- Apply mathematical reasoning to program correctness
- Learn how to write correct software, from beginning to end
- Use automated tools that assist verifying your code
- Understand how verification tools work

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- Make you better programmers

Course outline

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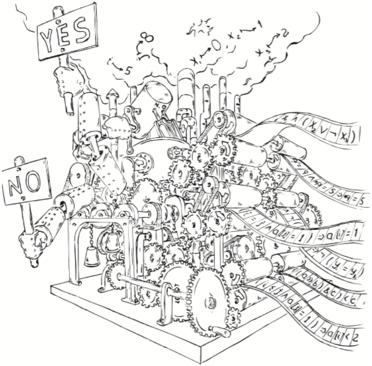
Part III: Mechanized reasoning

- Techniques for automated proving

Algorithmic approaches

Formal proofs are tedious

Automatic methods can:



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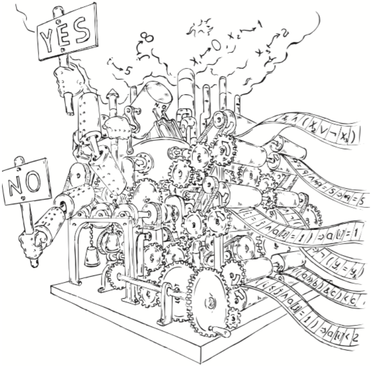

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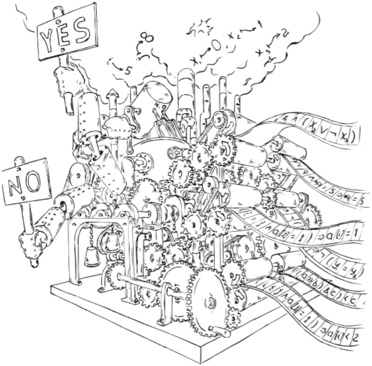

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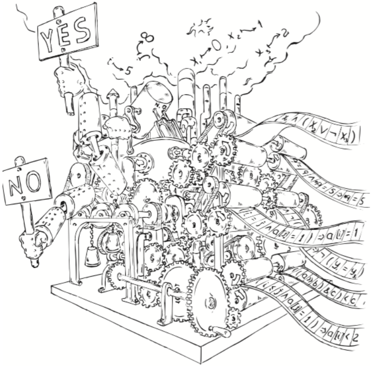

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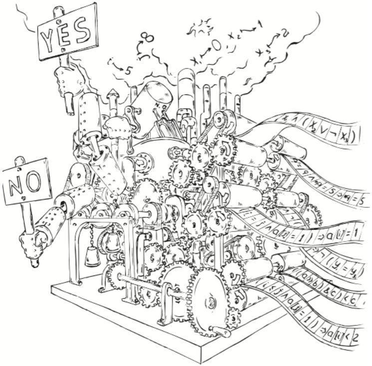
This is what you will learn!

- Make use of these methods
- How (and when) they work



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Different traditions and techniques

Functional programming: dependent types

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- Interactive provers for VC (Coq)

We focus on automated proving

Reasoning about correctness

Functional Correctness

- Specification
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- Derived from semantics
- Exhaustive proof rules
- Automatable*

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int[] array_copy(int[] A, int n)
//@requires 0 <= n && n <= \length(A);
//@ensures \length(\result) == n;
{
  int[] B = alloc_array(int, n);

  for (int i = 0; i < n; i++)
  //@loop_invariant 0 <= i;
  {
    B[i] = A[i];
  }

  return B;
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When it works! (It's not quite like a type-checker . . .)

Automated verifiers

Systems that prove that programs match their specifications

Basic idea:

1. Translate programs into *proof obligations*
2. Encode proof obligations as satisfiability
3. Solve using a decision procedure

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Verifiers are complex systems

- We'll deep-dive into selected components
- Understand "big picture" for the rest

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Recent developments

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```
Implement a ‘sat‘ function in Why3 that matches this signature:

    sat (cnf : cnf) : option valuation

If ‘cnf‘ is satisfiable, then ‘sat‘ should return ‘Some rho‘,
where ‘rho‘ is a satisfying assignment for ‘cnf‘.
Otherwise, it should return ‘None‘.
```

Recent developments

```
let rec sat (cnf : cnf) : option valuation =
  ensures  {
    match result with
    | None -> unsat cnf
    | Some rho -> sat_with rho cnf
  }
  let rho = Array.make cnf.nvars False in
  let rec dfs (n : int) : bool =
    requires { valid_valuation rho cnf }
    requires { 0 <= n <= cnf.nvars }
    variant  { cnf.nvars - n }
    ensures  {
      if result == true then
        sat_with rho cnf
      else
        forall rho':valuation. array_eq_sub rho' rho 0 n -> not (sat_with rho' cnf)
    }
```

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MF

The `dfs` function needs additional specifications to prove that the array accesses are in bounds.

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The following is an updated version of the `dfs` function with added specifications:

Copy code

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let rec dfs (rho : valuation) (n : int) : bool =
  requires { valid_valuation rho cnf }
  requires { 0 <= n < cnf.nvars }
  variant  { n }
  ensures  {
    if result == true then
      sat_with rho cnf
    else
      forall i:int. 0 <= i <= n -> not (sat_with (set rho i true) cnf)
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Recent developments

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The `ensures` specification on `dfs` is not correct. Specifying `sat_with (set rho i true) cnf` for all `i` in the range from `0` to `n` is not relevant; what is relevant is that if `result <> true`, then `unsat cnf`.

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The following is an updated version of the `dfs` function with a corrected `ensures` specification:

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(Limitations of) recent developments

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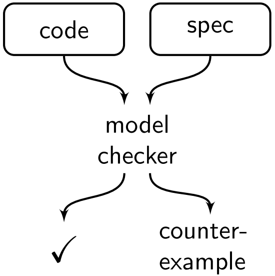
If you are interested in research related to this, let me know!

Model Checking

Fully-automatic techniques for finding bugs (or proving their absence)

- Specifications written in *propositional temporal logic*
- Verification by exhaustive state space search
- Diagnostic counterexamples



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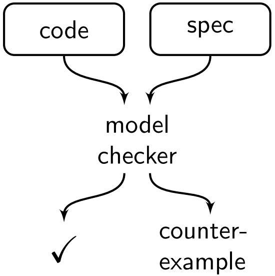

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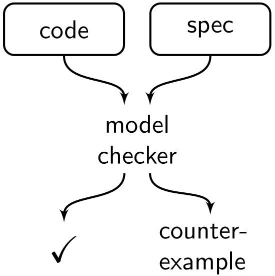

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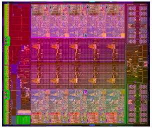
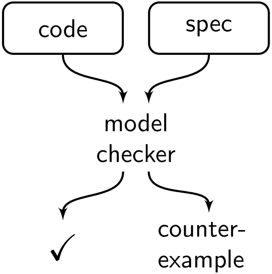
10^{70} atoms 10^{500000} states





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Now widely used for bug-finding:

- Hardware, software, protocols, ...
- Microsoft, Intel, Amazon, Google, NASA, ...

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- Abstraction & refinement

Now widely used for bug-finding:

- Hardware, software, protocols, ...
- Microsoft, Intel, Amazon, Google, NASA, ...

Ed Clarke, 1945–2020
Turing Award, 2007

Model Checking

Clever ways of dealing with state explosion:

- Partial order reduction
- Bounded model checking
- Symbolic representations
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First developed this course!

Grading

Breakdown:

- 50% assignments (written + programming)
- 15% mini-project 1
- 15% mini-project 2
- 20% final exam

6 assignments
done individually

2 mini-projects
pick from small menu
can work with a partner

Participation:

- Come to lecture
- Answer questions (in class and on Piazza!)
- Contribute to discussion

Written parts of assignments

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- Correctness of your answer
- How you present your reasoning

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Strive for **clarity & conciseness**

- Show each step of your reasoning
- State your assumptions
- Answers without these $\longrightarrow$ no points

Programming parts of assignments

For the programming, you will:

- Implement some functionality (data structure or algorithm)
- Specify correctness for that functionality
- Use Why3 to prove it correct

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- Correct specification
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Partial credit depending on how many of these you achieve

Clarity & conciseness is necessary for partial credit!

Mini-Projects

Mini-projects are intended to build proficiency in:

- Writing good specifications
- Applying course principles to practice
- Making effective use of automated tools
- **Writing useful & correct code**

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Gradual progression to sophistication:

1. Familiarize yourself with Why3
2. Implement and prove something
3. Work with more complex data structures
4. Implement and prove something really interesting
5. Optimize your implementation, still verified

Late Policy

Late days

- 5 late days to use throughout the semester
- No more than 2 late days on any assignment
- Late days do not apply to mini-projects!

Logistics

Website: `http://www.cs.cmu.edu/~15414`

Course staff contact: Piazza

Lecture: Tuesdays & Thursdays, 12:20-1:40pm

Office Hours: TBD, schedule on website and course calendar soon

Assignments: Gradescope