Assignment 1: Prop it Out
15-414/15-424 Bug Catching: Automated Program Verification

Due: **11:59pm**, Thursday 9/6/18
Total Points: 50

1. **Table counting (3 points)** How many rows and columns does the truth-table for the following formula have? Why? Note that you do not need to write the truth table down to answer this question.

$$\neg s \land ((\neg r \to \neg q \to \neg p) \to p \to q \to r \to s)$$

2. **Proof practice (6 points)** Conduct a proof in sequent calculus of the following formula. Be sure to say which proof rule you apply at each step.

$$(p \to q) \land (q \to r) \to (p \to r)$$

3. **Soundness of →R (10 points)** Prove that →R rule is sound. That is, using the semantics of →, prove that validity of all its premises implies validity of its conclusion.

$$(\to\text{R}) \quad \frac{\Gamma, F \vdash G, \Delta}{\Gamma \vdash F \to G, \Delta}$$

4. **Biimplication (10 points)** The syntax of propositional logic provided the biimplication/bisubjunction operator $A \leftrightarrow B$ which is true iff both A and B have the same truth-value, so both are true or both are false. But this operator is missing sequent calculus proof rules.

   Your task is to design proof rules for when the equivalence operator is used in the succedent (right rule ↔R) and when it is used in the antecedent (left rule ↔L):

$$(\leftrightarrow\text{R}) \quad \frac{\dots}{\Gamma \vdash F \leftrightarrow G, \Delta} \qquad (\leftrightarrow\text{L}) \quad \frac{\dots}{\Gamma, F \leftrightarrow G \vdash \Delta}$$

5. **Use it! (6 points)** Use your proof rules ↔R and ↔L to conduct a sequent calculus proof for the formula:

$$(p \leftrightarrow q) \to \neg p \to \neg q$$

6. **Soundness of ↔ (10 points)** Proof rules cannot be used unless they are accompanied by a soundness proof. Quickly before anybody notices your answer to the previous two tasks, use the semantics of the biimplication operator to prove soundness of your proof rules ↔R and ↔L. That is, for each of the rules, prove that validity of all its premises implies validity of its conclusion.

7. **Revisiting soundness of propositional logic (5 points)** What do you need to change in the proof of the soundness theorem for propositional logic (Theorem 7 in the lecture notes) now that you have added proof rules ↔L and ↔R?