15-451/651 Algorithm Design & Analysis, Fall 2024

Recitation #13

Objectives

- Provide practice reducing problems to properties and applications of polynomials.

Recitation Problems

1. **(Polynomials and fields review)**

   (a) Write down a polynomial that interpolates the points $(1,1),(4,1),(6,2)$. You do not have to simplify your polynomial.

   (b) Let ω be a primitive eighth root of unity. What are the cardinalities of the following sets?

   - $\{\omega^0, \omega^1, \omega^2, \omega^3, \omega^4, \omega^5, \omega^6, \omega^7\}$
   - $\{\omega^1, \omega^3, \omega^9, \omega^{16}, \omega^{27}, \omega^{45}\}$
   - $\{\omega^{-8}, \omega^0, \omega^8, \omega^{8888}\}$

2. **(2-SUM and 3-SUM)**

   You are given an array of n integers A, each element of which is at most $O(n)$ in size. Your goal is to determine the possible sums that you can make by taking integers from this array.

   (a) Solve the 2-SUM problem. That is, output a list of all of the integers that you can possibly make by summing any pair of elements from A. You are allowed to use the same element twice.

(b) Now solve 2-SUM but without allowing the same element to be used twice.

(c) Solve the 3-SUM problem, That is, output a list of all of the integers that you can possibly make by summing any triple of elements from A. You are allowed to use the same element multiple times.

(d) Finally, solve 3-SUM but without allowing the same element to be used multiple times.

3. **(Evenly Spaced Ones)**

Given a binary string S of length n, we wish to determine whether there exists three evenly spaced ones within S. For example, 11100000, 110110010 both have three evenly spaced 1s, while 1011 does not.

(a) Derive a brute-force algorithm solving this problem with $O(n^2)$ complexity.

(b) Derive an algorithm with $O(n \log n)$ complexity that uses polynomial multiplication and convolutions.