15-451/651 Algorithm Design & Analysis

Spring 2023, Recitation #12

Objectives

- Problem-Solving with Computational Geometry
- Using tools such as Convex Hull Algorithms, Sweep Line/Angle Algorithms, etc.

Recitation Problems

1. **(Circle with Most Points)** Given a set of points $S = \{p_1, \ldots, p_n\}$, and a radius $r > 0$, the goal is to find a circle of radius r that contains the maximum number of points from S. Give an $O(n^3)$ algorithm for this problem.

2. **(Width of a Set of Points)** You're given a set $S = \{p_1, \ldots, p_n\}$ of n points in the plane. A strip of width w is the region between two parallel lines, where the distance between the two lines is w. The goals is to find the strip of minimum width that contains all the points. Give an $O(n^2)$ algorithm for this problem.

   (a) Give an $O(n^2)$ algorithm for this problem.

   (b) Give an $O(n \log n)$ algorithm for this problem.

3. **(Left-Turning Spiral)** You're given a set $S = \{p_1, \cdots, p_n\}$ of n points in the plane. Your goal is to draw a **left-turning spiral**, which is defined as follows:

   - It is a path that visits every node once,
   - It doesn't intersect itself,
   - Every time you make a turn, it is a leftward, or counterclockwise, turn.

   (a) Give an $O(n^2 \log n)$ algorithm to find a left-turning spiral. (Hint: Use the convex hull.)

   (b) Inspired by this solution, solve the problem in $O(n^2)$, without ever explicitly finding the convex hull.

Further Review

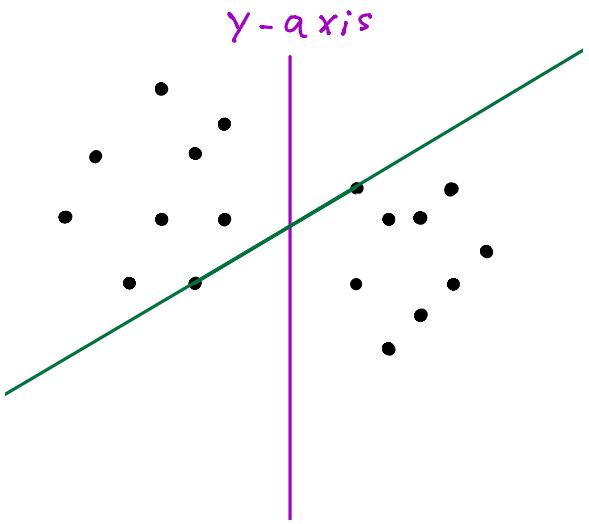
1. **(Circle with Most Points Again)** Give an $O(n^2 \log n)$ algorithm for problem 1. (Hint: sweep-*angle*.)

2. **(Non-degenerate Triangles)**

   You are given a set S of n points in the plane. Give an expected or worst case $O(n^2)$ algorithm to count the number of non-degenerate triangles (i.e. ones with non-zero area) whose vertices are taken from S. In this problem the points of S have integer coordinates, and arithmetic on these coordinates can be assumed to take $O(1)$ time, including computing the greatest common divisor.

3. **(Highest Slope)** Let $p_1, p_2, \ldots, p_n$ be a set of points to the left of y-axis. (I.e. their x coordinates are all < 0.) You're also given $q_1, q_2, \ldots, q_n$ that are to the right of the y axis.

   The problem is to find the pair (i, j) such that the line from p_i to q_j has the highest slope of any such line.



   (a) Give a deterministic algorithm that solves this problem in $O(n \log n)$ time.

   (b) Show how to set the problem up as a 2-dimensional linear program.

4. **(Winding Polygonal Line)** This is an extension to Problem 3 (Left-Turning Spiral). Instead of turning left every time, you're given a string of length $n-2$ consisting of only L and R. If the ith character is an L then you need to make a left (counterclockwise) turn between points i, $i+1$, and $i+2$ on your path, and if it's an R you need to make a right (clockwise) turn between these points. Find an algorithm in $O(n^2)$ to solve this version of the problem.