15-451/651 Algorithm Design & Analysis, Spring 2024

Recitation #12

Objectives

- Problem-Solving with Computational Geometry
- Using tools such as Convex Hull Algorithms, Randomized Incremental Algorithms, etc.

Recitation Problems

1. **(Circle with Most Points)** Given a set of points $S = \{p_1, \ldots, p_n\}$, and a radius $r > 0$, the goal is to find a circle of radius r that contains the maximum number of points from S. Give an $O(n^3)$ algorithm for this problem.

2. **(Width of a Set of Points)** You're given a set $S = \{p_1, \ldots, p_n\}$ of n points in the plane. A strip of width w is the region between two parallel lines, where the distance between the two lines is w. The goals is to find the strip of minimum width that contains all the points.

   (a) Give an $O(n^2)$ algorithm for this problem.

   (b) Give an $O(n \log n)$ algorithm for this problem.

3. **(Randomized Dart Throwing)** Consider the following randomized dart game and its associated cost function:

   Procedure `DartGame`:

   i Initially. there is a dart board of n consecutive, empty squares, arranged in a row.

   ii For n iterations, throw a dart at a uniformly random empty square, and pay cost equal to the number of consecutive empty squares to the left and right of the dart.

   The goal is to analyze the expected cost of this procedure.

   (a) Give a recurrence for the expected cost $C(n)$ of DartGame with n empty squares.

   (b) Consider reversing the dart game as follows: start with a full board of marked squares, and unmark a random square each iteration, again paying cost equal to the number of consecutive empty squares to the left and right of the unmarked square.

   Prove that the expected cost is the same for the original dart game and the reversed dart game.

(c) Prove an asymptotic bound on expected cost of DartGame using the reversed game.