15-451/651 Algorithm Design & Analysis, Spring 2024

Recitation #7

Objectives

- Understand the faster flow algorithms: Edmonds-Karp and Dinic's.
- Understand and apply Minimum-cost flows.

Recitation Problems

1. **(Super fast matching)** In lecture, we saw that Dinic's algorithm runs in time $O(n^2 m)$ on any graph, but on some graphs it runs even faster! For instance, in a *unit-capacity* network, where every edge has capacity one, we proved that Dinic's algorithm runs in time $O(m\sqrt{m})$. In this problem we will take this one step further.

   Suppose our graph has one additional restriction (we still keep the unit-capacity restriction): The net flow across every vertex (except s and t) can be at most one. This is equivalent to saying that every vertex other than s and t has either indegree one or outdegree one (but not necessarily both). Such a network is called a "unit network".

   (a) Prove that in a unit network, the number of blocking flows required to find a max flow is at most $O(\sqrt{n})$. (Hint: use a similar argument to the one in lecture for unit-capacity graphs)

   (b) Prove that we can solve the Bipartite Matching problem in $O(m\sqrt{n})$ time.

2. **(Oral homework scheduling)** You've been hired to help the 451 TAs schedule their oral sessions. There are n TAs, and TA i has s_i slots that they need to book a room for. There are m available room bookings, and each TA i has a list L_i of which room bookings $\{1, 2, \ldots, m\}$ would be suitable for them. Since there is a shortage of room bookings, however, the department has started to sell the bookings for money! The j^{th} room booking costs c_j dollars. Your job is to find a way to schedule all of the oral sessions for the minimum amount of money, or report that it is not possible.