15-451/651 Algorithm Design & Analysis, Spring 2024

Extra Review Problems

Fingerprinting

1. **(Short answer / multiple choice)**

   (i) The Karp-Rabin algorithm may yield

   (a) A false positive

   (b) A false negative

   (c) Either of the above

   (ii) In the Karp-Rabin algorithm, suppose we have a text of length m and want to find a string of length n. After we pick a prime, what is the tightest bound of the runtime of this algorithm?

   (a) $O(m+n)$

   (b) $O(\log m + \log n)$

   (c) $O(n \log m)$

   (d) $O(m \log n)$

   (e) $O(mn)$

   (iii) Given an n-bit number x, the number of distinct prime divisors of x is at most (pick the smallest correct value from among the following):

   (a) $\log\log n$

   (b) $\log n$

   (c) n

   (d) n^2

   (iv) Given any x in the range $2, 3, ..., n$ the number of distinct prime divisors of x is at most (pick the smallest correct value from among the following):

   (a) $O(\log\log n)$

   (b) $O(\log n)$

   (c) $O(n)$

   (d) $O(n^2)$

   (e) $O(2^n)$

2. **(General analysis of Karp-Rabin)**

   (a) In lecture we analyzed the complexity of the Karp-Rabin algorithm for $\Sigma = \{0,1\}$, and showed that to achieve 1% error, this required a random $O(\log m + \log n)$-bit prime. Generalize this for any Σ. How many bits should p be to retain 1% error?

   (b) Now suppose we want an error rate of δ, how large should our prime be?