15-451/651 Algorithm Design & Analysis, Spring 2024

Extra Review Problems

Computational Geometry I: Fundamentals

1. **(Point-in-convex-polygon)** Given a point q and a convex polygon P represented by points $P[1], P[2], \ldots, P[n]$ in counter-clockwise order

   (a) Determine whether q is in the polygon P in time $O(n)$.

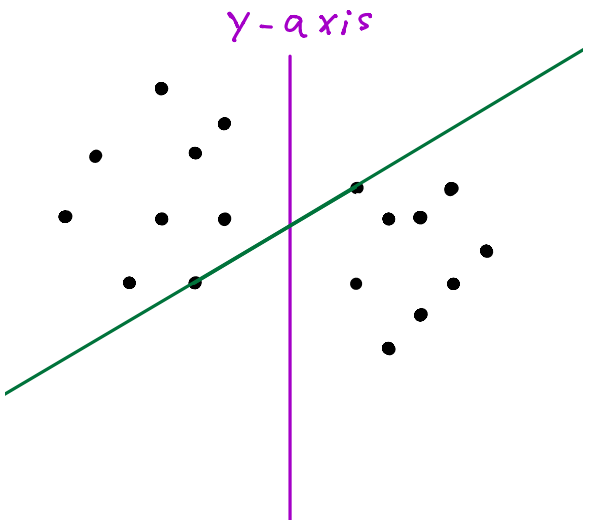
   (b) Speed up your algorithm to $O(\log n)$

2. **(Counting Triangles)** You're given a convex polygon P. The points of the polygon are called $A[0], A[1], \ldots, A[n-1]$ in counter-clockwise order, where $A[\,]$ is an array of points. You're also given a point q that is strictly inside of P. The point q does not lie on any of the lines passing through any pair of points in $A[\,]$.

   Of the $\binom{n}{3}$ triangles formed using the vertices of P, some of them contain q and some do not. Give an $O(n)$-time algorithm to count the number of such triangles that contain q. (Hint: it might be easier to count the number of triangles that *do not* contain q.)

3. **(Highest slope)** Let $p_1, p_2, \ldots, p_n$ be a set of points to the left of y-axis. (I.e. their x coordinates are all < 0.) You're also given $q_1, q_2, \ldots, q_n$ that are to the right of the y axis.

   The problem is to find the pair (i, j) such that the line from p_i to q_j has the highest slope of any such line.



   (a) Give a deterministic algorithm that solves this problem in $O(n \log n)$ time.

   (b) Show how to set the problem up as a 2-dimensional linear program.