**15-451/651 Algorithm Design & Analysis, Spring 2024**

# Extra Review Problems

## Computational Geometry II: Incremental Algorithms

1. **(Non-degenerate Triangles)** You are given a set $S$ of $n$ points in the plane. Give an expected or worst case $O(n^2)$ algorithm to count the number of non-degenerate triangles (i.e. ones with non-zero area) whose vertices are taken from $S$. In this problem the points of $S$ have integer coordinates, and arithmetic on these coordinates can be assumed to take $O(1)$ time, including computing the greatest common divisor.

2. **(Tracking the minimum)** You have a list of $n$ numbers and you wish to compute the minimum among them. You write a simple algorithm that maintains a variable holding the minimum element seen so far, and then process each element in the input, replacing the stored element if a new minimum is found.

   However, suppose that for some reason, replacing the minimum element seen so far is very expensive, so we're concerned with how many times that happens. (Processing the first element counts as changing the minimum.) For each part below write the tightest correct upper bound on the quantity in question. This is not a big-oh problem.

   (a) What is the worst-case number of times that the minimum element seen so far might change?

   (b) Suppose you randomly permute the input and then run the algorithm. Give an upper bound on the expected number of times the minimum element seen so far changes.