15784 Midterm practice

Your name:

Please read instructions carefully. Do not worry if you cannot finish everything. Do not write down disorganized answers in the hope of getting partial credit; it's better to do a few questions completely right. Please write your answers down clearly (think before you write). You can use extra pages. Good luck!

Problem 1: Modified Rock-Paper-Scissors.

Consider the following modified version of Rock-Paper-Scissors, where losing with Paper to Scissors is considered doubly humiliating:

	Rock	Paper	Scissors
Rock	0,0	-1,1	1,-1
Paper	1,-1	0,0	-2,2
Scissors	-1,1	2,-2	0,0

a. Wright argues that in every equilibrium of this game, every pure strategy must receive positive probability from both players. Is Wright right or wrong? Explain why.

b. Based on your answer in **a**, compute a Nash equilibrium of this game. Is it the unique equilibrium? Why (not)?

Problem 2: A game with a hidden coin flip.

In this problem, you will solve a simple game (reminiscent of "Liar's Dice" if you happen to know it). Player 1 flips a coin and sees the result; player 2 does not see the result. Heads is a "winning" coin flip, Tails is a "losing" coin flip. Player 1 makes a claim about the coin flip to player 2, either claiming to have flipped Heads, or claiming to have flipped Tails. Player 2 can choose to Dispute the claim, or to Accept it.

If player 2 chooses to Dispute, player 1 must show the coin. (Of course, player 1 cannot change the result of the coin flip.) If player 1 lied, player 2 wins; if player 1 told the truth, player 1 wins. If player 2 chooses to Accept, then whatever player 1 claimed stands (regardless of what she actually flipped), and player 2 must flip the coin to compete with that claim.

For example, suppose player 1 flips Tails, but then claims to have flipped Heads. If player 2 Disputes, player 2 wins, because player 1 lied. If player 2 Accepts, then player 1's claim of Heads stands (and the fact that she actually flipped Tails becomes irrelevant), and player 2 must flip the coin to compete with Heads. If player 2 flips Tails, then he loses, because Tails is worse than Heads. If player 2 flips Heads, we have a tie.

Suppose the utility for winning is 1, the utility for losing is -1, and the utility for a tie is 0 (it's a zero-sum game). Give the extensive form of the game, convert it to normal (matrix) form (explaining what the strategies mean and calculating the expected utilities), and solve for the equilibrium of this game. (*Hint:* the normal form should be 4×4.)

Problem 3: The distancing dilemma.

During a pandemic, two people are approaching each other on a narrow trail. They can't come close to each other. On both sides of the trail there is mud. If one person goes and stands in the mud, they can pass each other. (If they both go into the mud on their respective sides, they can also pass each other.) So, initially, both players have a choice between Mud (M) and Trail (T). If you choose T and the other M, you'll get a utility of 2. If you choose M (in which case you'll definitely get to pass), you'll get a utility of 1 – you'll pass but have muddy shoes.

However, if *both* choose T, the situation hasn't resolved yet – they'll still be stuck. Realizing that this has happened, they will get another chance where, again, they can choose M or T. The payoffs will be the same as above, except if they *again* both choose T, they'll have to give up and turn around, for a utility of 0. (Let's say otherwise an officer will come and tell both of them to go home, for causing trouble.) So there are 2 rounds of the game in this case, and never more.

a. Draw the above game in extensive form. Hint: While this is (at most) a 2-round game in which in both rounds the players choose simultaneously, only one person can move at a node in an extensive-form game. Thus, for each of the rounds, you'll have to sequentialize the moves by the players, but you can make them effectively simultaneous within each round by not letting the other person learn what the first person did in the same round (but after the first round completes, they'll know everything that happened up to that point). So the tree will be four moves deep on the side where in the first round they both choose T.

b. Give the normal form of this game. Note that it should be a 4×4 game because the strategy must specify an action at every information set (even if that information set is not reachable given the other part of the strategy). There should be a fair amount of repetition in the matrix.

c. Give a pure-strategy subgame-perfect Nash equilibrium in which player 1 obtains utility 2. The strategies should be ones from your game in **b**.

d. Solve for a symmetric subgame-perfect Nash equilibrium of this game (which will involve randomization). (An equilibrium is symmetric if the row and column player use the same strategy.) Hint: you can do this by a sort of backward induction: first solve for an equilibrium of the second round (after both choose T), and then replace this subgame by the values of that equilibrium, and solve the first round.