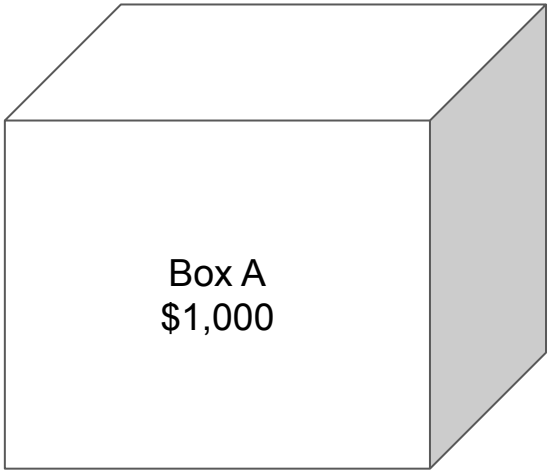
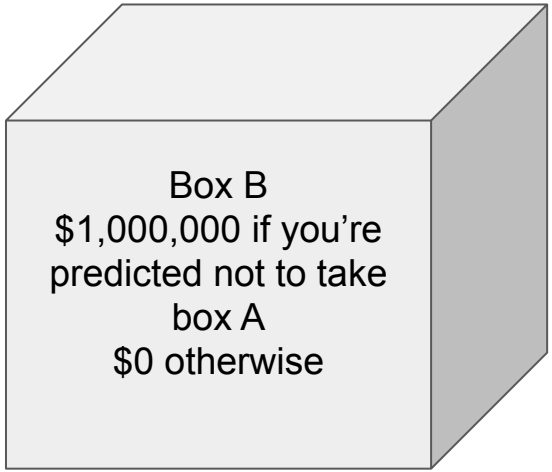
CS 15-784 Cooperative AI Decision Theory

Newcomb's problem (Nozick 1969)





Causal Decision Theory: I can't causally affect the content of Box B. No matter the content of Box B, it's better to take Box A.

Evidential Decision Theory: Rejecting Box A gives me evidence that Box B contains $1,000,000.

Agenda

- Why does this matter in the context of this course?
- Four theoretical ideas (EDT, CDT, ratificationism, policy choice)
- More arguments
- Learning

Decision theory as a foundation for game theory

It would be good to recover game theory from principles of single-agent decision making (decision theory).

(Example: regret learning → Nash equilibrium.)

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If is_game(current_situation):
        Return game_theory(current_situation)
Else:
        Return decision_theory(current_situation)
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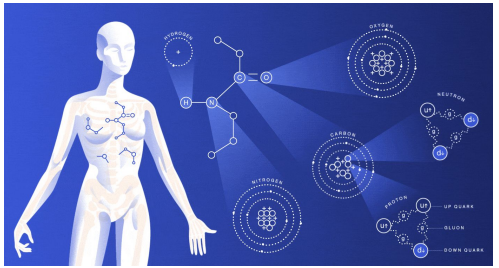

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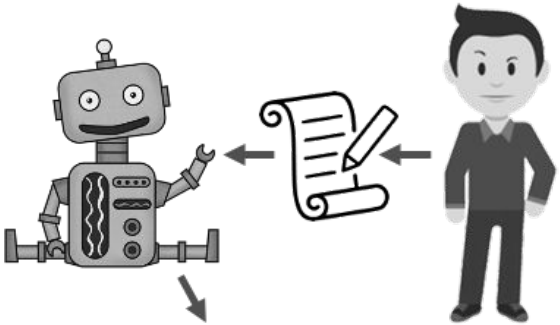
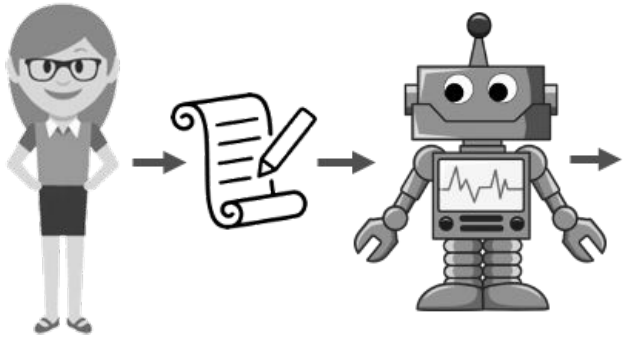
Newcomb’s problem is a single-agent scenario exhibiting a key feature of multi-agent strategic interactions.

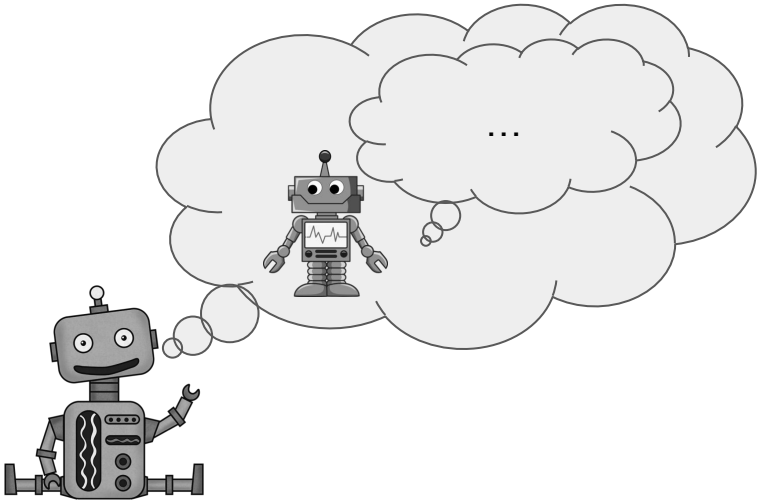
Recall program equilibrium

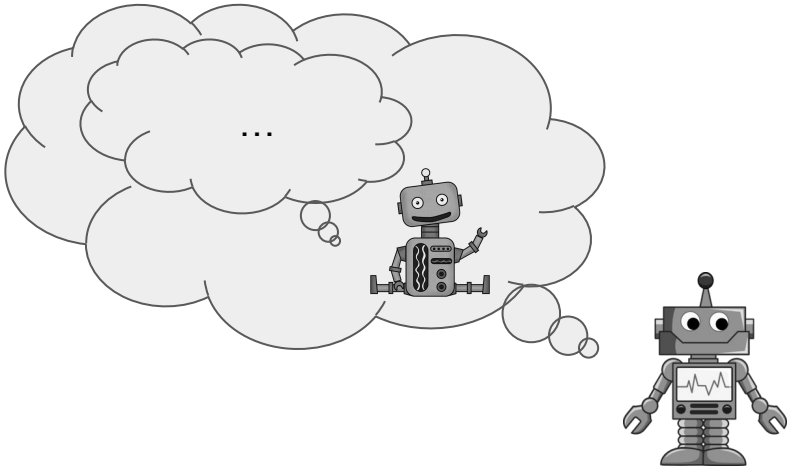




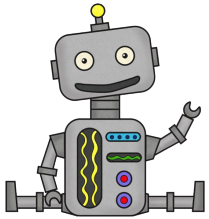
	Cooperate	Defect
Cooperate	6,6	0,10
Defect	10,0	4,4

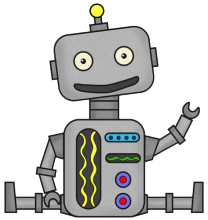
Recall program equilibrium





The Prisoner’s Dilemma against a similar opponent





	Cooperate	Defect
Cooperate	6,6	0,10
Defect	10,0	4,4

Recall program equilibrium (again)

Cooperate with Copies (CwC):
Input: opponent program prog_{-i}, this program CwC
Output: Cooperate or Defect

```
1: if prog_{-i} = CwC then
2:     return Cooperate
3: end if
4: return Defect
```

	"Return Cooperate"	"Return Defect"	CwC	…
"Return Cooperate"	6,6	0,10	0,10	
"Return Defect"	10,0	4,4	4,4	
CwC	10,0	4,4	6,6	
…				

(CwC,CwC) is a Nash equilibrium.

Causal and evidential decision theory

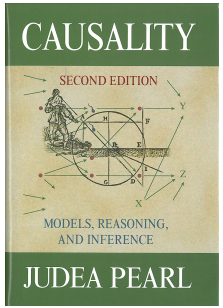
Evidential Decision Theory:

$$\arg\max_{a \in A} \sum_{\text{outcome } o} P(o \mid a) u(o)$$

Causal Decision Theory:

$$\arg\max_{a \in A} \sum_{\text{outcome } o} P(a \to o) u(o)$$

E.g., *do*-calculus



Piazza poll

Is the following statement true or false?

Newcomb’s problem is primarily about the conflict between two principles of rational choice: expected utility maximization and the dominance principle.

- True
- False

Piazza poll – Solution

Is the following statement true or false?

Newcomb's problem is primarily about the conflict between two principles of rational choice: expected utility maximization and the dominance principle.

- True
- **False**

Playing Matching Pennies against Newcomb's Demon (cf. "Death in Damascus")

	Heads	Tails
Heads	+1,-1	-1,+1
Tails	-1,+1	+1,-1

Ratificationism

You can choose $\pi \in \Delta(A)$.
Then $a \sim \pi$ is sampled.
Your reward/utility is determined by $R(a,\pi)$.

Examples:

- Newcomb's problem
  $R(\text{one-box}, p_{\text{one-box}}) = p_{\text{one-box}} * \$1m$
  $R(\text{two-box}, p_{\text{one-box}}) = p_{\text{one-box}} * \$1m + \$1k$
- Matching Pennies against Newcomb's demon
  $R(\text{Heads}, p_H) = p_H * (-1) + (1-p_H) * (+1)$
  $R(\text{Tails}, p_H) = p_H * (+1) + (1-p_H) * (-1)$

Ratificationism

Definition: A policy π is ratifiable if $\pi(a)>0 \Rightarrow a \in \operatorname{argmax}_{a'} R(a', \pi)$.

Examples:

- Newcomb's problem → Two-boxing
- Matching Pennies against Newcomb's Demon → ½ – ½

Usually assume:

- Effect of *a* is causal.
- Effect of π is non-causal.

Existence of ratifiable policies

Theorem (Bell, Linsefors, Oesterheld, Skals 2021): Let A be finite and R: A x $\Delta(A) \to \mathbb{R}$ be continuous. Then there is a ratifiable policy for R.

Kakutani's fixed point theorem: Let S be a non-empty, compact and convex subset of $\mathbb{R}^n$. Let f: $S \to 2^S$ be a set-valued S s.t.

- The graph of f (i.e., $\{(x,y) \mid x \in S, y \in f(x)\}$) is closed.
- For all $x \in S$, f(x) is closed and convex.

Then f has a fixed point, i.e., there exists $x \in S$ s.t. $x \in f(x)$.

Proof of Theorem: Given R, consider
f: $\Delta(A) \to 2^{\Delta(A)}$
: $\pi \mapsto \{\pi' \in \Delta(A) \mid \pi'(a)>0 \Rightarrow a \in \text{argmax}_{a'} R(a',\pi)\} = \Delta(\text{argmax}_{a'} R(a',\pi))$
Notice that π is ratifiable iff it is a fixed point.
Apply Kakutani's theorem and we're done!

Existence of Nash equilibria follows from Kakutani!

Nash (1950):

The correspondence of each n-tuple with its set of countering n-tuples gives a one-to-many mapping of the product space into itself. From the definition of countering we see that the set of countering points of a point is convex. By using the continuity of the pay-off functions we see that the graph of the mapping is closed. The closedness is equivalent to saying: if $P_1, P_2, \ldots$ and $Q_1, Q_2, \ldots, Q_n, \ldots$ are sequences of points in the product space where $Q_n \to Q$, $P_n \to P$ and Q_n counters P_n then Q counters P.

Since the graph is closed and since the image of each point under the mapping is convex, we infer from Kakutani's theorem[1] that the mapping has a fixed point (i.e., point contained in its image). Hence there is an equilibrium point.

Updatelessness

H

T

"Pay me $1,000!"

You get $5,000 if it is predicted that you would have paid, had the coin come up Heads.

Normal perspective: Obviously, don't pay!

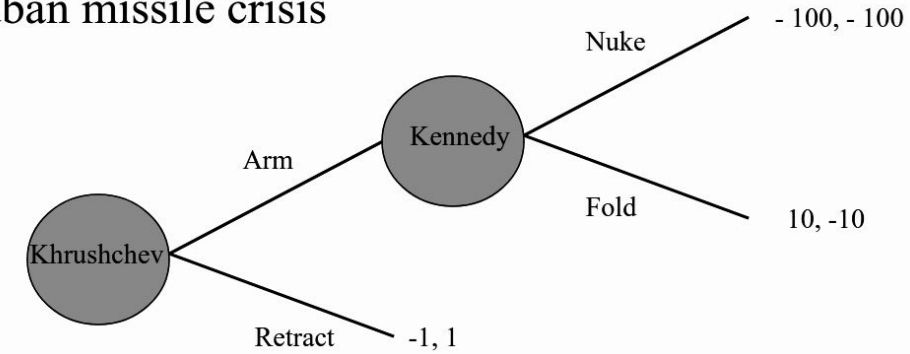
Updatelessness: Choose what's best from the prior / *ex ante* perspective – pay!

Precommitment and CDT

That is, the causal decision theorist says: If I anticipate playing Newcomb's problem in the future, I should commit to one-boxing.

Subgame perfect equilibrium [Selten 72] & credible threats

- Proper subgame = subtree (of the game tree) whose root is alone in its information set
- Subgame perfect equilibrium = strategy profile that is in Nash equilibrium in every proper subgame (including the root), whether or not that subgame is reached along the equilibrium path of play
- E.g. Cuban missile crisis

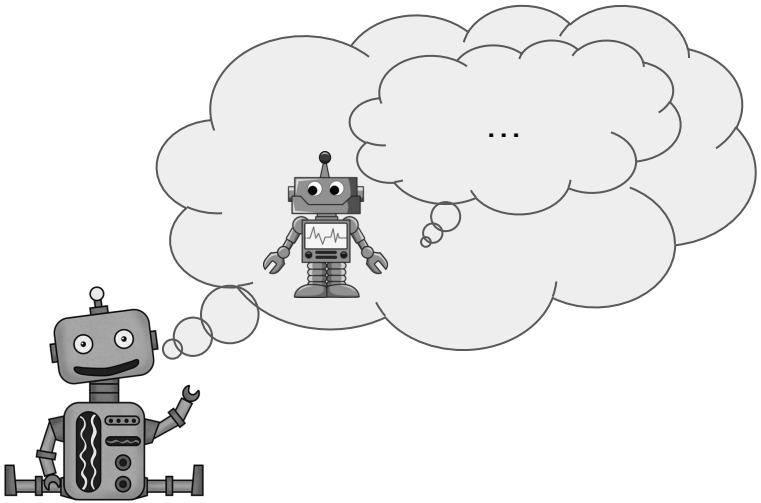


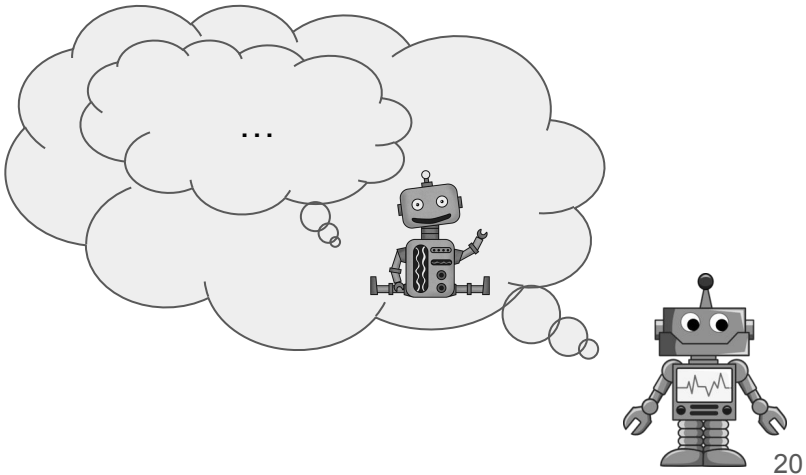
- Pure strategy Nash equilibria: (Arm,Fold), (Retract,Nuke)
- Pure strategy subgame perfect equilibria: (Arm,Fold)
- Conclusion: Kennedy's Nuke threat was not *credible*

Updatelessness: Assume the prior perspective – play it as a normal-form game!

Recall: Two perspectives on program games

1. (taken throughout this lecture) Players play a normal-form game.
    - The normal form game happens to consist in choosing programs that can access each other's code…
    - … but we can analyze it using standard concepts (Nash equilibrium).
2. How should you reason/learn/choose when your source code is (at least partially) known to others? ← Updatelessness: Use 1 as an answer to 2!





A valid critique of updatelessness?

> [S]uppose I'm choosing between an avocado sandwich and a hummus sandwich, and my prior was that I prefer avocado, but I've since tasted them both and gotten evidence that I prefer hummus. The choice that does best in terms of expected utility with respect to my prior for the decision problem under consideration is the avocado sandwich [...]. But, uncontroversially, I should choose the hummus sandwich, because I prefer hummus to avocado.
>
> Will MacAskill on LessWrong

Updatelessness in the Avocado–Hummus game

Observations: {OA,OH}

Prior over observations: P(OA)=0.6, P(OH)=0.4

Actions: {CA,CH}

Utilities: 1 if you choose your favorite and 0 otherwise.

Set of policies and their utilities:

- OA→CH, OH→CH; ex ante expected utility: 0.6*0+0.4*1=0.4
- OA→CA, OH→CA; ex ante expected utility: 0.6*1+0.4*0=0.6
- OA→CA, OH→CH; ex ante expected utility: 0.6*1+0.4*1=1
- OA→CH, OH→CA; ex ante expected utility: 0.6*0+0.4*0=0

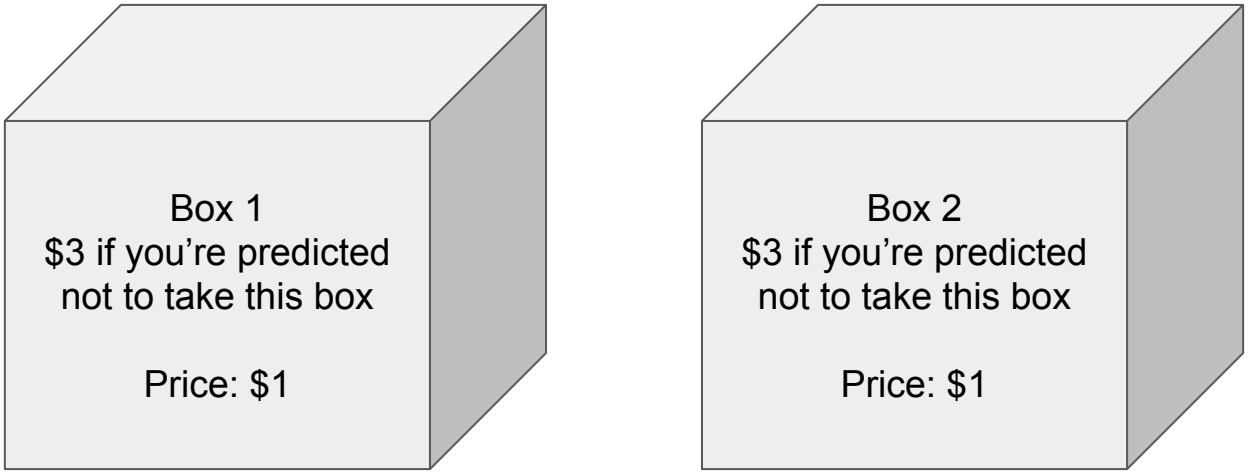
In general: in “normal scenarios” (no predictions, computational restrictions, etc.), the ex-ante optimal policy is the same as the Bayesian updating-based answer

Smoking lesion and the tickle defense

- Imagine that smoking doesn’t cause cancer. Instead, smoking and cancer have a common cause, e.g. a particular gene. Smoking in itself is beneficial! But smoking is evidence that you have cancer...
- It would seem that EDT advises against smoking.
- This is arguably unreasonable.
- CDT, on the other hand, recommends smoking.
- Most common response: The Tickle Defense; see Oesterheld (2018) for an introduction.

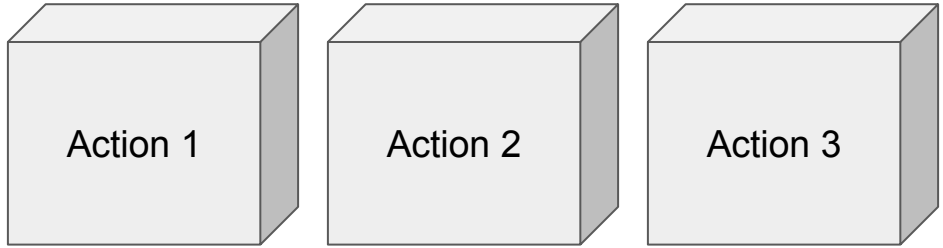
Adversarial Offer

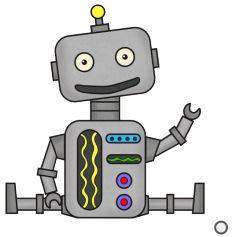
Two boxes are offered. You can buy at most one.



At least one of the boxes contains money.
=> The average box contains at least $1.50 in expectation
=> The average CDT-expected value of the boxes is at least $1.50
=> The CDT-expected value of at least one of the two boxes is >=$1.50
=> CDT buys a box
=> Predictors can use CDT agents as money pumps

Naive learning





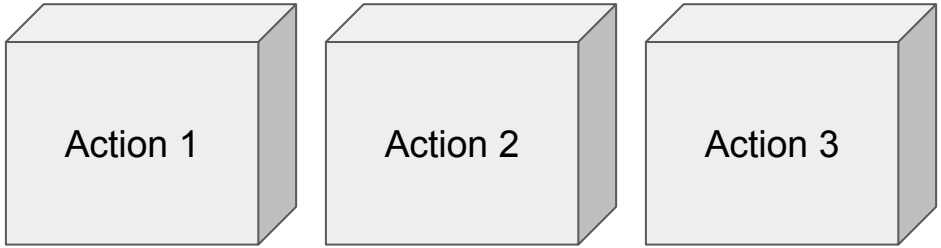
Take Action 1. Reward 1.
Take Action 2. Reward 2.
Take Action 3. Reward 3.
Take Action 1. Reward 2.
Take Action 2. Reward 1.
Take Action 3. Reward 2.
…

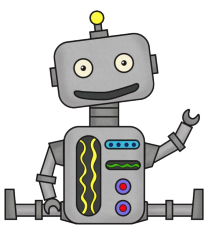
I guess Action 3 is best.

One-boxing in Newcomb's problem will look better (assuming **predictability** of exploration).

Cf. Oesterheld, Demski, Conitzer (2022) for a less naive version of this.

Randomized trials



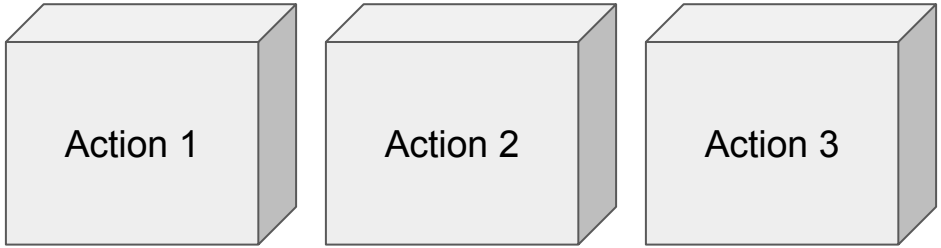


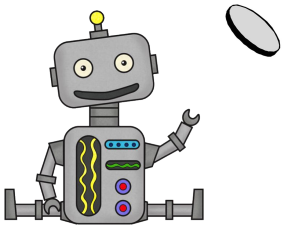
Take Action 2. Reward 3.
Take Action 3. Reward 2.
Take Action 2. Reward 3.
Take Action 1. Reward 3.
Take Action 2. Reward 2.
Take Action 1. Reward 2.
…

I guess Action 2 is best.

Two-boxing in Newcomb's problem will look better (assuming **unpredictability** of exploration).

Randomized trials – Q-learning





Trial 1: ⅓ – ⅓ – ⅓. Average rewards: 3, 3, 4
Trial 2: ¼ – ¼ – ½. Average rewards: 2, 2, 1
…

When you trial with different probabilities it gets complicated…

Bell, Linsefors, Oesterheld, Skalse (2021): Q learning can only converge to ratificationist policies!

Regret learning and ratificationism

$\text{Regret}(\pi) := \max_{a^*} R(a^*,\pi) - \mathbf{E}_{a\sim\pi}[R(a,\pi)]$

Then, $\text{Regret}(\pi) = 0$ if and only if π is ratifiable.

References

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