Mediation in Extensive-Form Games

Brian Hu Zhang

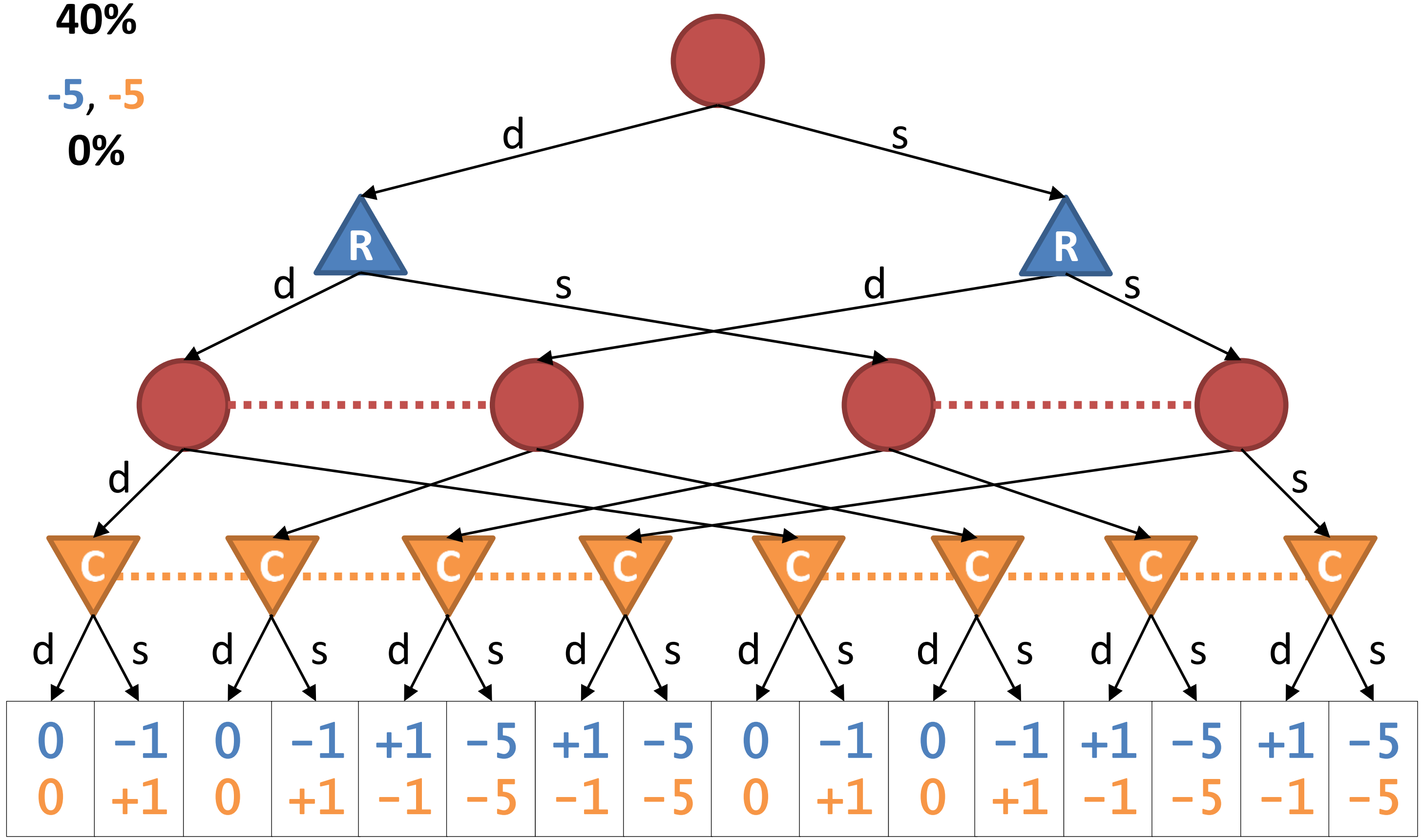
Primarily based on:

Zhang and Sandholm (NeurIPS 2022, to appear), "Polynomial-Time Optimal Equilibria with a Mediator in Extensive-Form Games"
https://arxiv.org/abs/2206.15395

Warm-up: Correlated Equilibria in Normal-Form Games (via Extensive Form!)

Chicken

	Dodge	Straight
Dodge	0, 0 **20%**	-1, +1 **40%**
Straight	+1, -1 **40%**	-5, -5 **0%**



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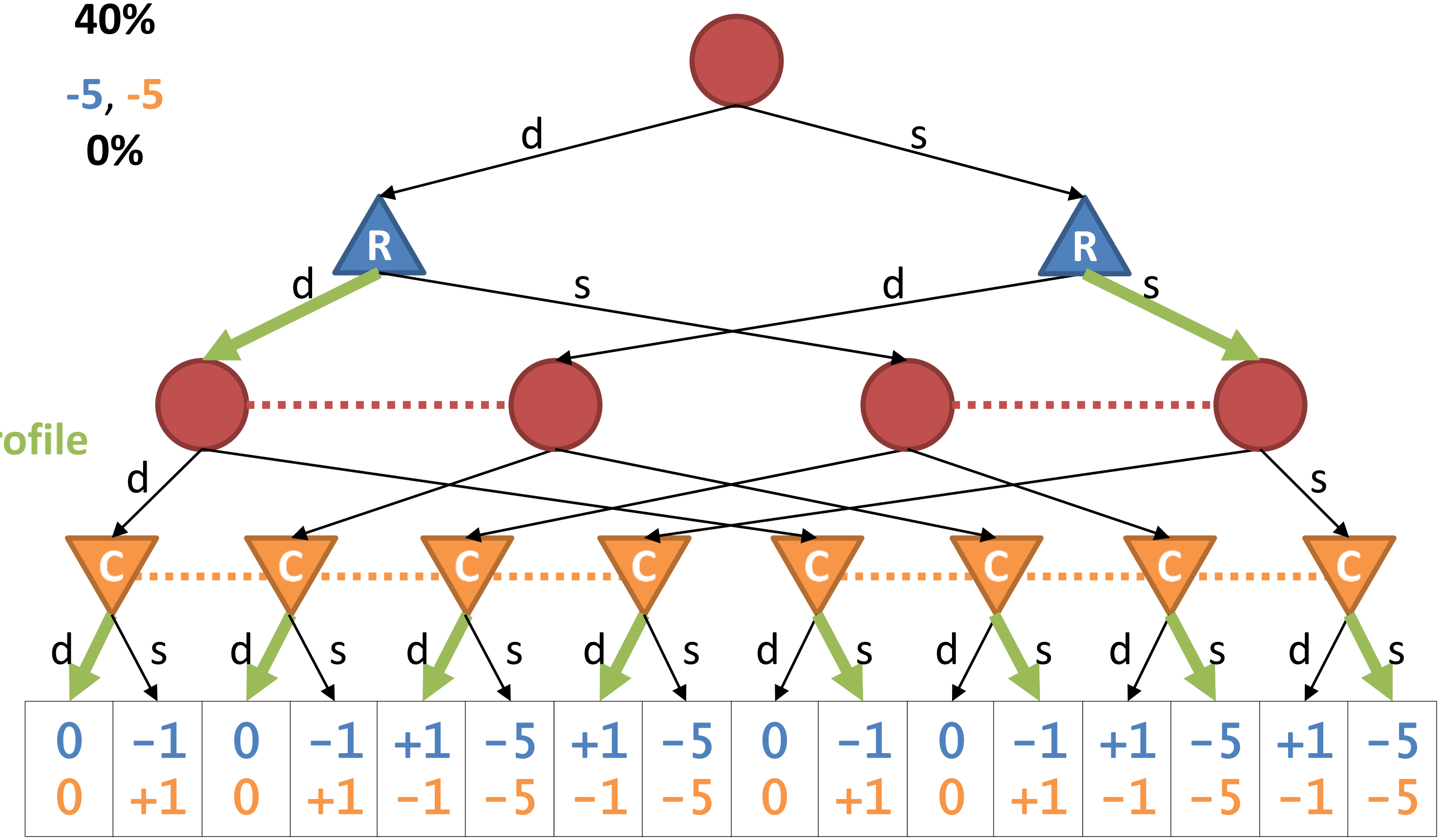
To find a correlated equilibrium: find a **strategy for the mediator** such that, in the game among the players that results from holding the mediator's strategy fixed, **the obedient strategy profile is a Nash equilibrium.**

obedient strategy profile



Warm-up: Correlated Equilibria in Normal-Form Games (via Extensive Form!)

To find a correlated equilibrium: find a **strategy for the mediator** such that, in the game among the players that results from holding the mediator's strategy fixed, **the obedient strategy profile is a Nash equilibrium.**

find mediator strategy $\boldsymbol{\mu}$
such that for all players i
obeying is a best response to $\boldsymbol{\mu}$
if all other players are also obedient

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find mediator strategy $\boldsymbol{\mu}$

such that for all players i

$$\max_{\boldsymbol{x}_i \in X_i} \underbrace{u_i(\boldsymbol{\mu}, \boldsymbol{x}_i, \boldsymbol{x}^*_{-i})} \le \underbrace{u_i(\boldsymbol{\mu}, \boldsymbol{x}^*_i, \boldsymbol{x}^*_{-i})}$$

Bilinear function of $\boldsymbol{\mu}$ and $\boldsymbol{x}_i$ — Constants! — Linear function of $\boldsymbol{\mu}$

Notation:

$X_i = \{\boldsymbol{x}_i \ge \boldsymbol{0} : \boldsymbol{F}_i \boldsymbol{x}_i = \boldsymbol{f}_i\}$: strategy space of player i

$\boldsymbol{x}^*_i \in X_i$: the obedient strategy of player i

u_i: the payoff of player i in the given strategy profile

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find mediator strategy $\boldsymbol{\mu}$

such that for all players i

$$\max_{\boldsymbol{x}_i \in X_i} \quad \boldsymbol{\mu}^\top \boldsymbol{A}_i \boldsymbol{x}_i \quad \leq \boldsymbol{b}_i^\top \boldsymbol{\mu}$$

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$\boldsymbol{\mu}^\top \boldsymbol{A}_i \boldsymbol{x}_i = u_i(\boldsymbol{\mu}, \boldsymbol{x}_i, \boldsymbol{x}_{-i}^*)$ $\quad$ $\boldsymbol{b}_i^\top \boldsymbol{\mu} = u_i(\boldsymbol{\mu}, \boldsymbol{x}_i^*, \boldsymbol{x}_{-i}^*)$

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To find a correlated equilibrium: find a **strategy for the mediator** such that, in the game among the players that results from holding the mediator's strategy fixed, **the obedient strategy profile is a Nash equilibrium.**

Apply LP duality…

find mediator strategy $\boldsymbol{\mu}$

such that for all players i

$$\max_{\substack{\boldsymbol{x}_i \geq \boldsymbol{0} \\ \boldsymbol{F}_i \boldsymbol{x} = \boldsymbol{f}_i}} \boldsymbol{\mu}^\top \boldsymbol{A}_i \boldsymbol{x}_i \leq \boldsymbol{b}_i^\top \boldsymbol{\mu}$$

find mediator strategy $\boldsymbol{\mu}$

and dual multipliers $\boldsymbol{v}_i$

such that for all players i

$$\boldsymbol{F}_i^\top \boldsymbol{v}_i \geq \boldsymbol{A}_i^\top \boldsymbol{\mu}, \qquad \boldsymbol{f}_i^\top \boldsymbol{v}_i \leq \boldsymbol{b}_i^\top \boldsymbol{\mu}$$

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To find a correlated equilibrium: find a **strategy for the mediator** such that, in the game among the players that results from holding the mediator's strategy fixed, **the obedient strategy profile is a Nash equilibrium.**

This is a linear program! Solve it with any LP solver

$$\max_{\substack{\text{mediator strategy } \boldsymbol{\mu} \\ \text{dual multipliers } \boldsymbol{v}_i}} \quad \boldsymbol{c}^\top \boldsymbol{\mu}$$

such that for all players i

$$\boldsymbol{F}_i^\top \boldsymbol{v}_i \geq \boldsymbol{A}_i^\top \boldsymbol{\mu}, \qquad \boldsymbol{f}_i^\top \boldsymbol{v}_i \leq \boldsymbol{b}_i^\top \boldsymbol{\mu}$$

arbitrary objective, for example, social welfare

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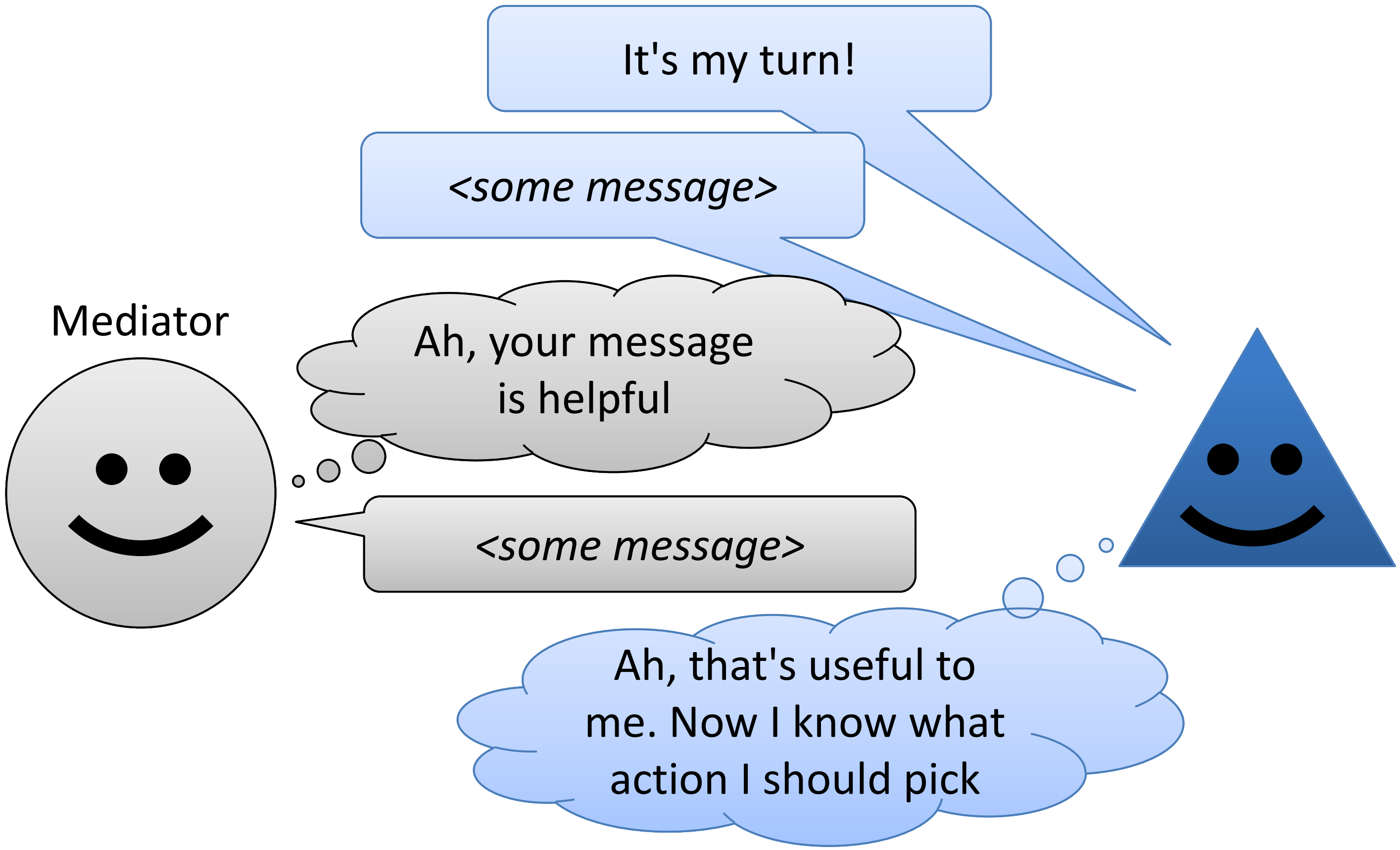
Can we generalize this idea to other useful problems?

Idea: In an extensive-form game, **add a mediator** with the power to both **send and receive** messages from players.

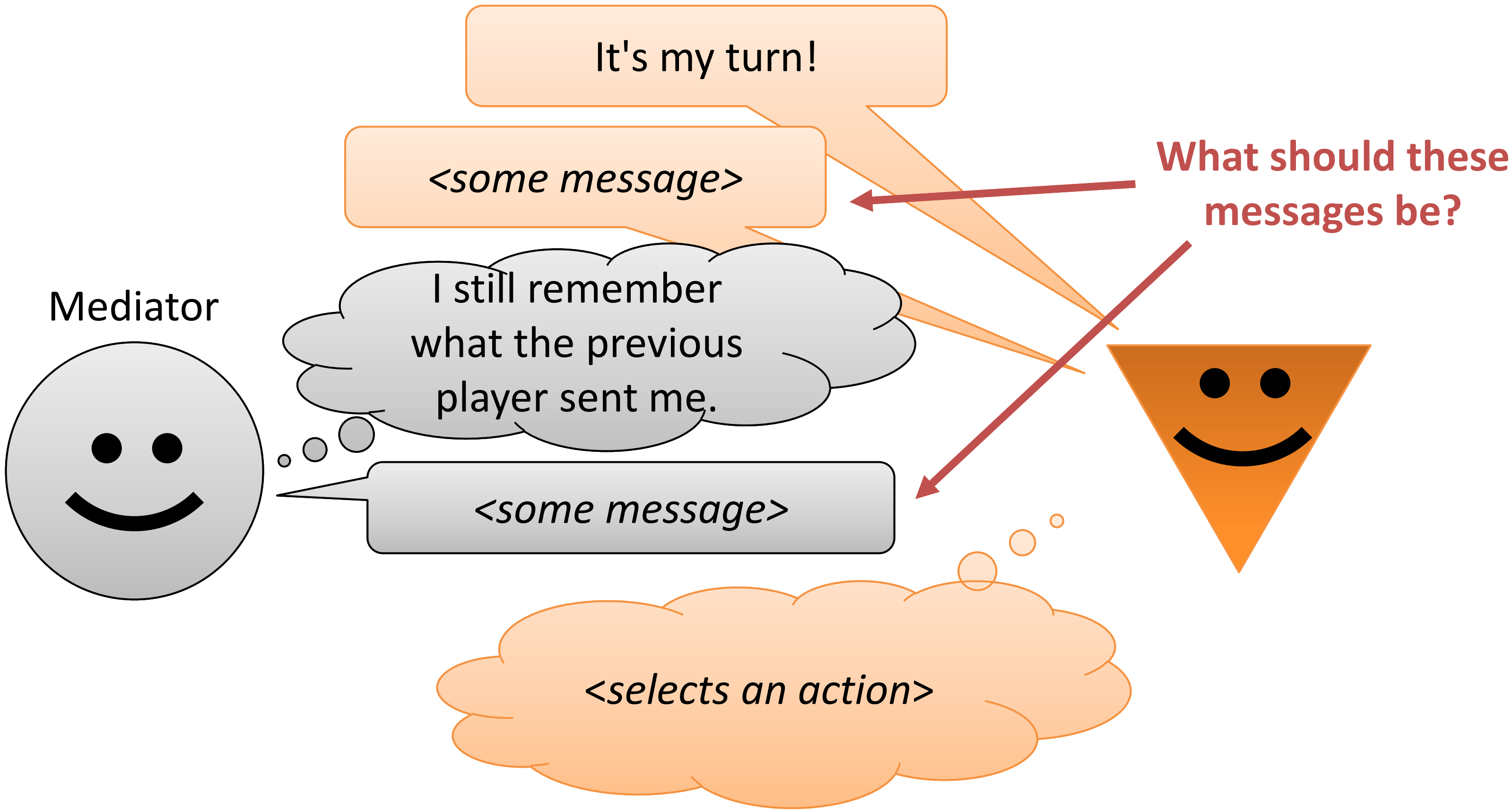
By varying exactly how the messaging system works, **we will recover algorithms for all sorts of different problems that are seemingly unrelated**

Communication Equilibria



Forges (Econometrica 1986), "An approach to communication equilibria"

Communication Equilibria



A **communication equilibrium** is a **mediator strategy μ**, and a **strategy profile $(x_1, \dots, x_n)$ for the players** such that, with the mediator's strategy held fixed, $(x_1, \dots, x_n)$ is a Nash equilibrium for the players.

Forges (Econometrica 1986), "An approach to communication equilibria"

The Revelation Principle

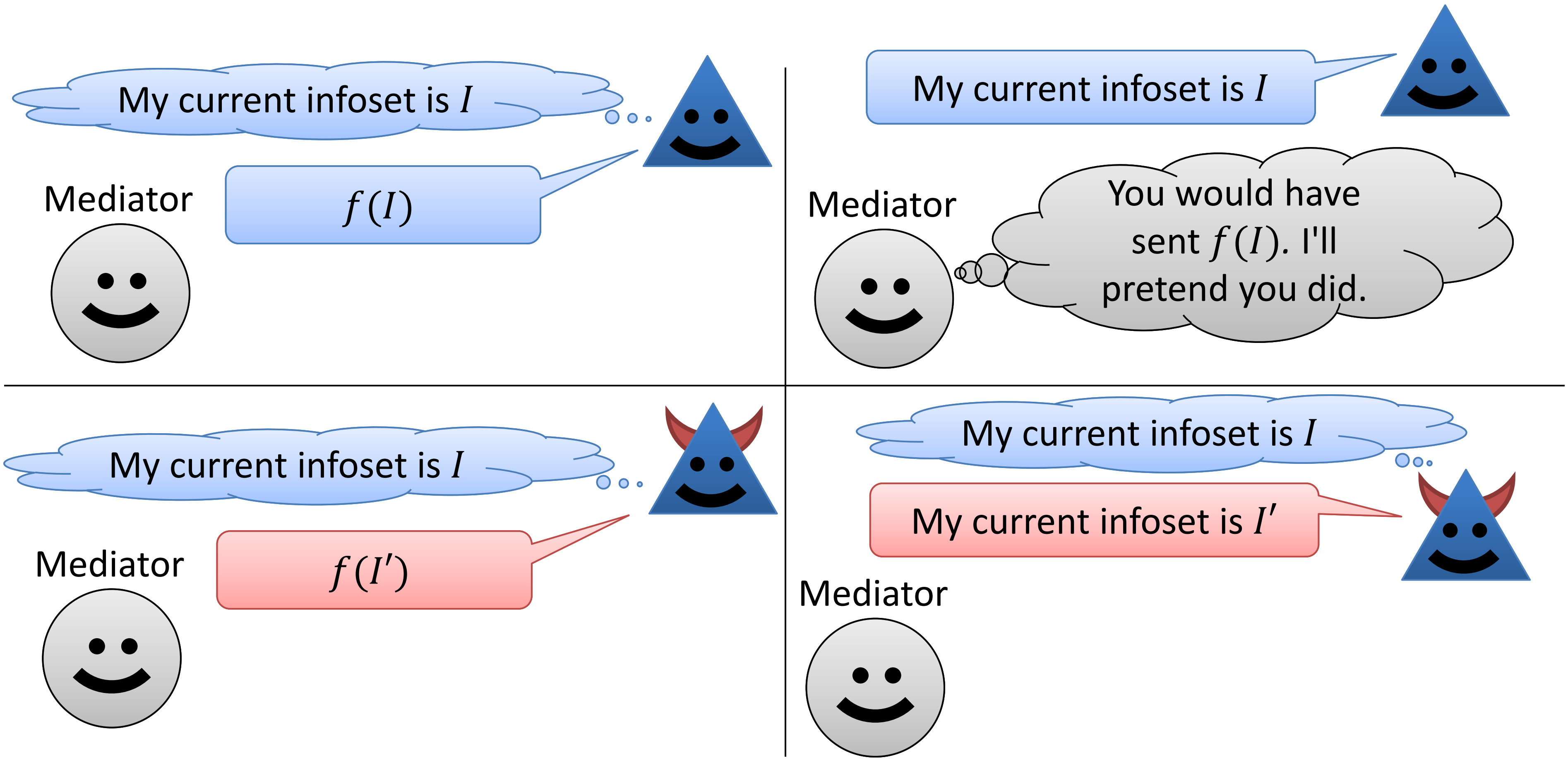
Theorem [Revelation Principle]: For any "reasonably nice" notion of equilibrium with a mediator, the following assumptions can be made without loss of generality.

1. In equilibrium, players always send their true information to the mediator.
2. The mediator's messages to the player are action recommendations.
3. In equilibrium, players always obey action recommendations.

without changing the space of possible equilibrium outcomes--in particular, without changing the optimal equilibrium under any objective

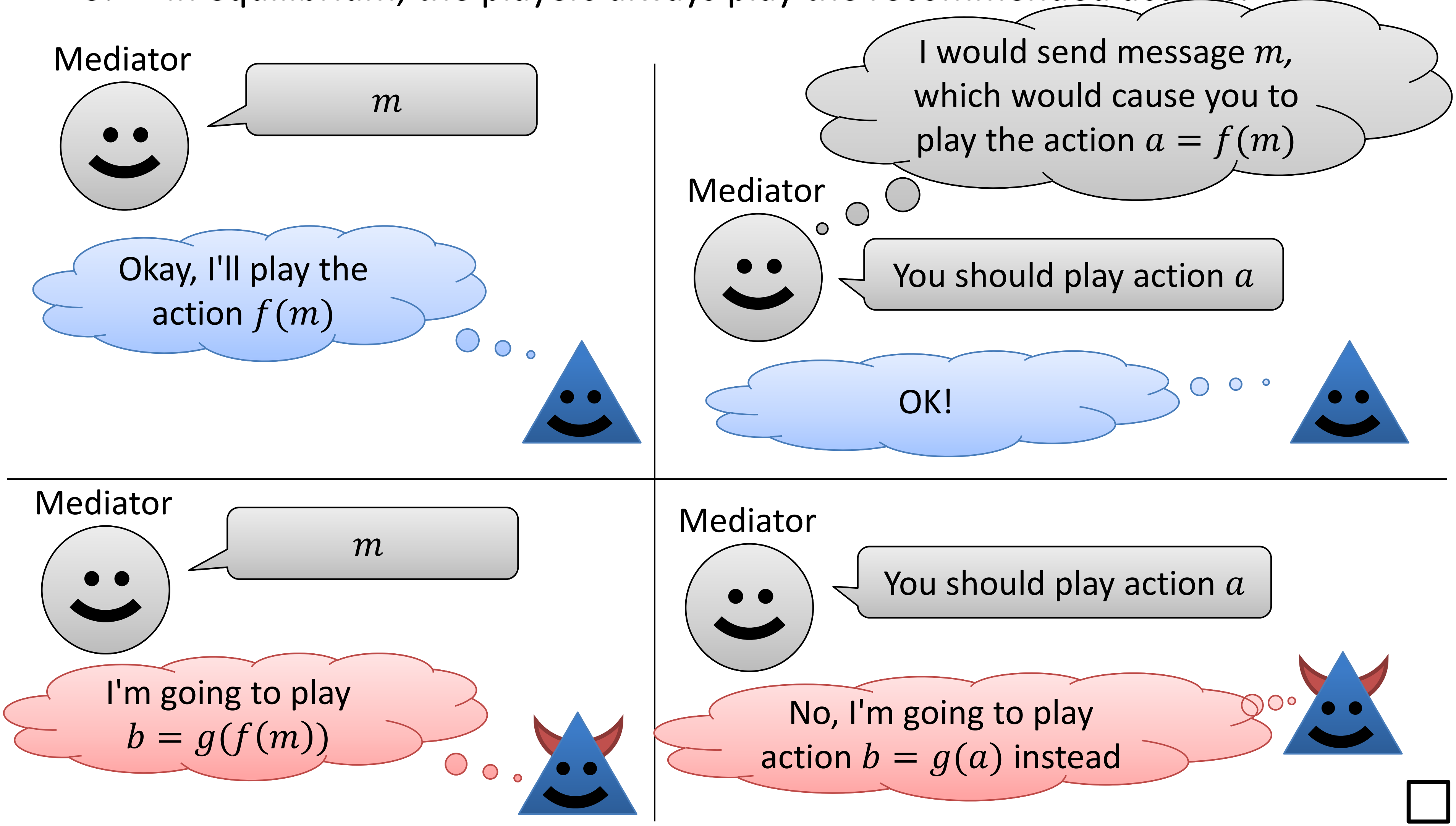
The Revelation Principle: Proof Sketch

1. In equilibrium, players always send their true information to the mediator.



The Revelation Principle: Proof Sketch

2. The mediator's messages to the player are action recommendations.
3. In equilibrium, the players always play the recommended actions.



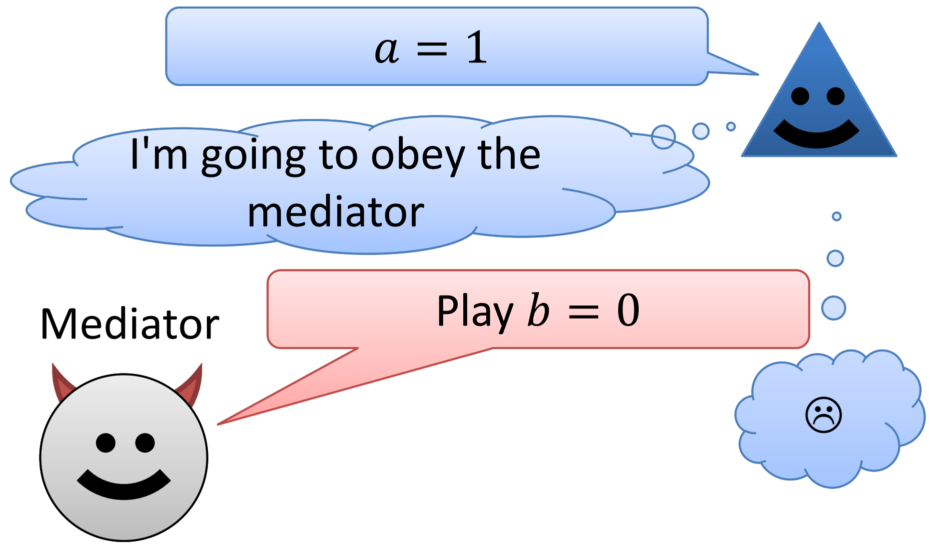
□

The Revelation Principle: Commitment

The revelation principle fundamentally relies on the ability of the mediator to **commit** to a strategy before the players decide what to do.

Example game:

- Nature picks $a \in \{0,1\}$ uniformly at random and tells the player
- The player and mediator undergo one round of communication
- Player picks $b \in \{0,1\}$
- Player gets utility 1 if and only if $a = b$ (and 0 otherwise)
- Mediator gets utility 1 if and only if $a \neq b$ (and 0 otherwise)



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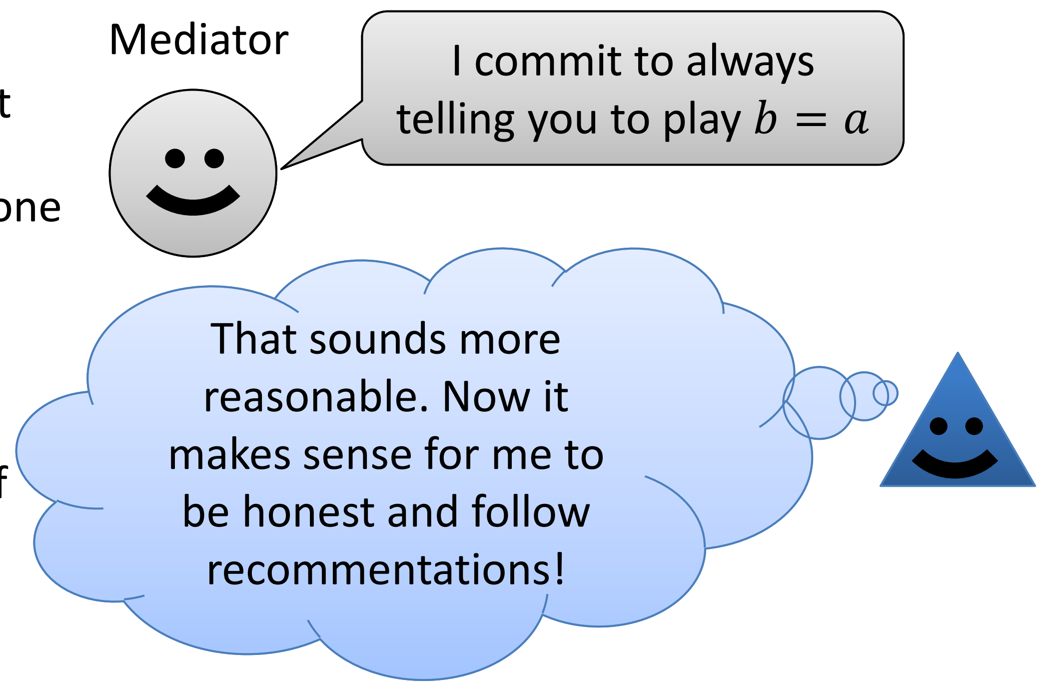

Polynomial-time Communication Equilibria

Theorem [**Zhang** and Sandholm 2022]: Given an extensive-form game Γ with N nodes, an optimal communication equilibrium can be computed in time polynomial in N.

Proof Sketch

1. Consider the augmented extensive-form game $\hat{\Gamma}$ in which the mediator is an explicit extra player, like we did for normal-form correlated equilibria.
2. Prove a slightly stronger version of the revelation principle that ensures that $\hat{\Gamma}$ can be simplified to have size $O(N^2)$
3. Use the LP we already wrote down to compute an optimal strategy for the mediator in $\hat{\Gamma}$ such that the player strategy profile in which every player reports information honestly and plays recommended actions is an equilibrium. □

Zhang and Sandholm (NeurIPS 2022), "Polynomial-Time Optimal Equilibria with a Mediator in Extensive-Form Games"

Lots of related useful problems!

- ✓ Optimal correlated equilibria in normal-form games
- Optimal mediated equilibrium
- Optimal Bayesian persuasion (information design) in extensive-form games
- Optimal automated mechanism design
- Optimal extensive-form correlated equilibria in extensive-form games

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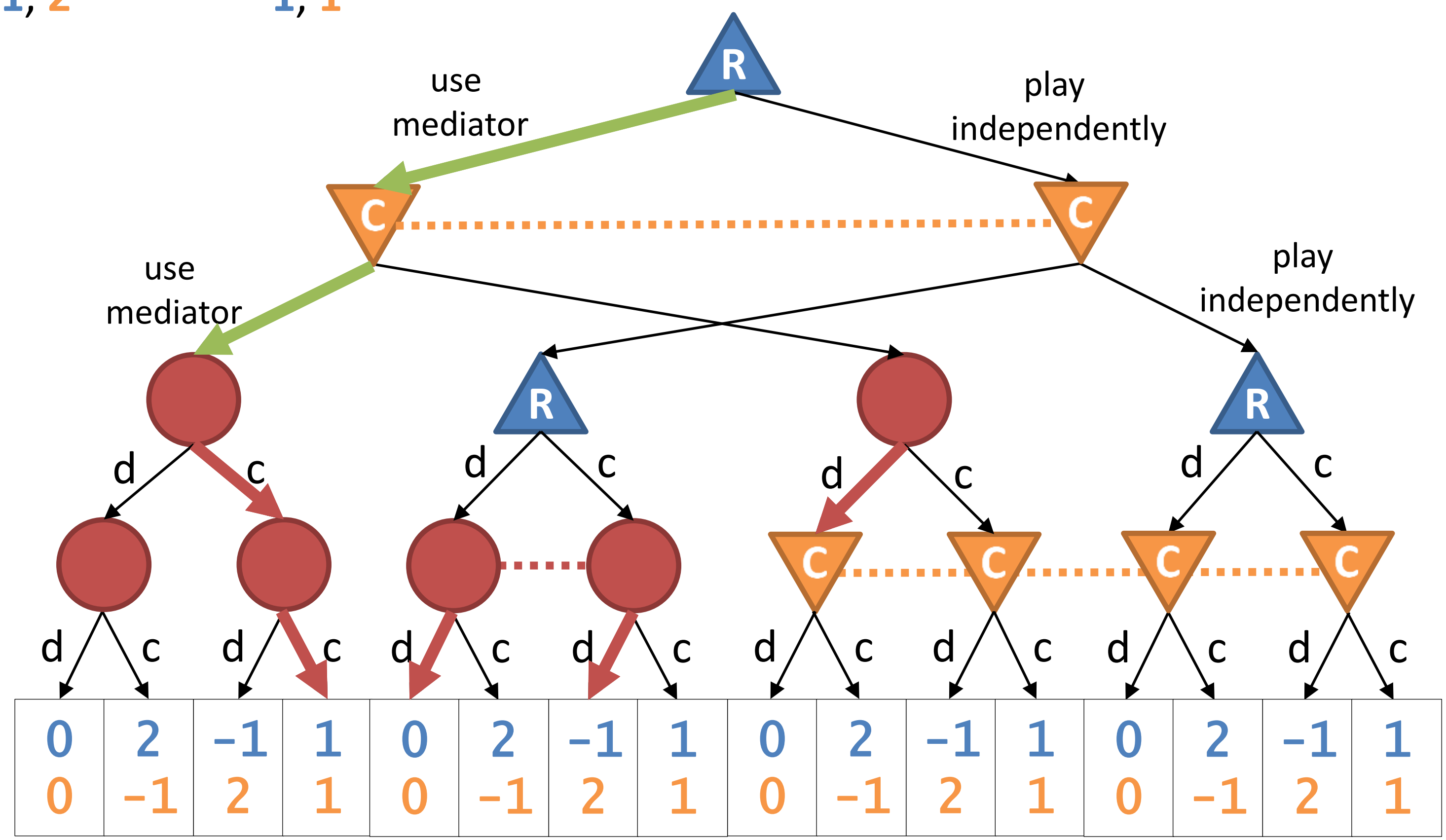
Mediated Equilibrium

Prisoner's Dilemma

	Defect	Cooperate
Defect	0, 0	2, -1
Cooperate	-1, 2	1, 1

find mediator strategy $\mu \in X_{\text{mediator}}$
such that for all players i
x_i^* is a best response to (μ, x_{-i}^*)



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Information Design

- n players playing an extensive-form imperfect-information game
- **The mediator has an informational advantage over the players.** In particular, the mediator always knows *at least* the infoset of the acting player, and has perfect recall.
  - Therefore, the players don't need to send messages to the mediator because that would be pointless (the mediator knows the acting player's information already)
- **Question:** How should the mediator send signals so as to *persuade* the players to act in some desirable way?
  - **Revelation principle:** WLOG, signals are *action recommendations*

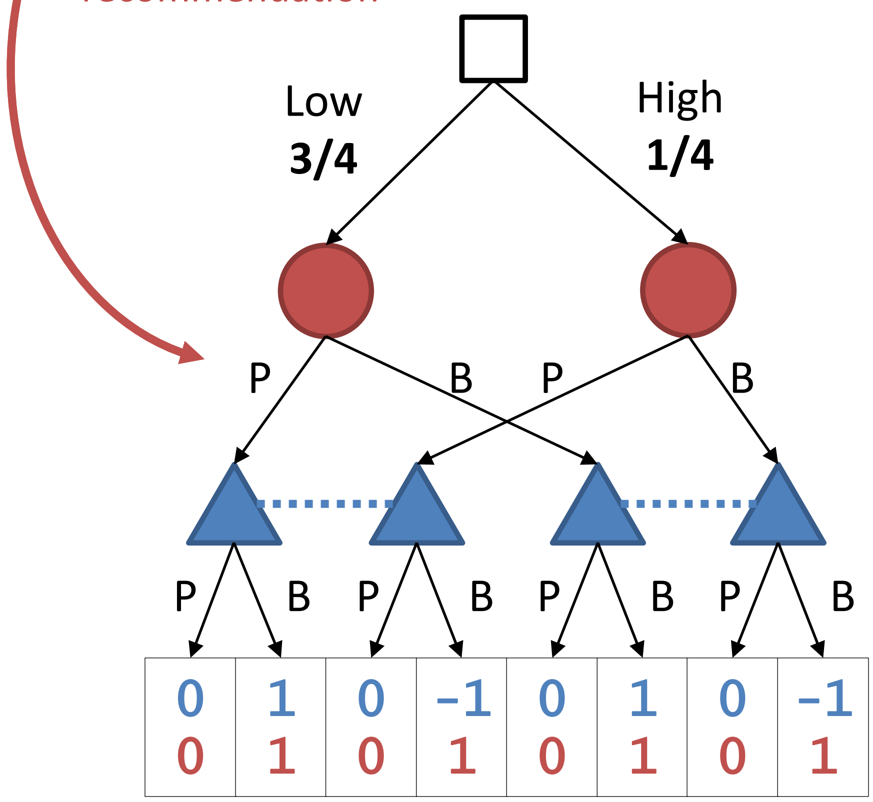
Kamenica and Gentzkow (American Economic Review 2011), "Bayesian Persuasion"

Information Design: An Example

- A **seller** ("mediator") wishes to persuade a **buyer** ("player") to purchase an item
- The item's quality is either **low** (p=3/4) or **high** (p=1/4). The **seller** knows the quality of the item, but the **buyer** does not.
- The **seller** scores **1** if the buyer buys the item. The seller can commit to a messaging scheme.
- The **buyer** can pass (P) or buy (B). The buyer wants to buy only high-quality items: she scores **1** if she buys a high-quality item and **-1** if she buys a low-quality item

The seller learns the quality of the item and sends a signal to the buyer

Revelation principle: seller's signal is a recommendation



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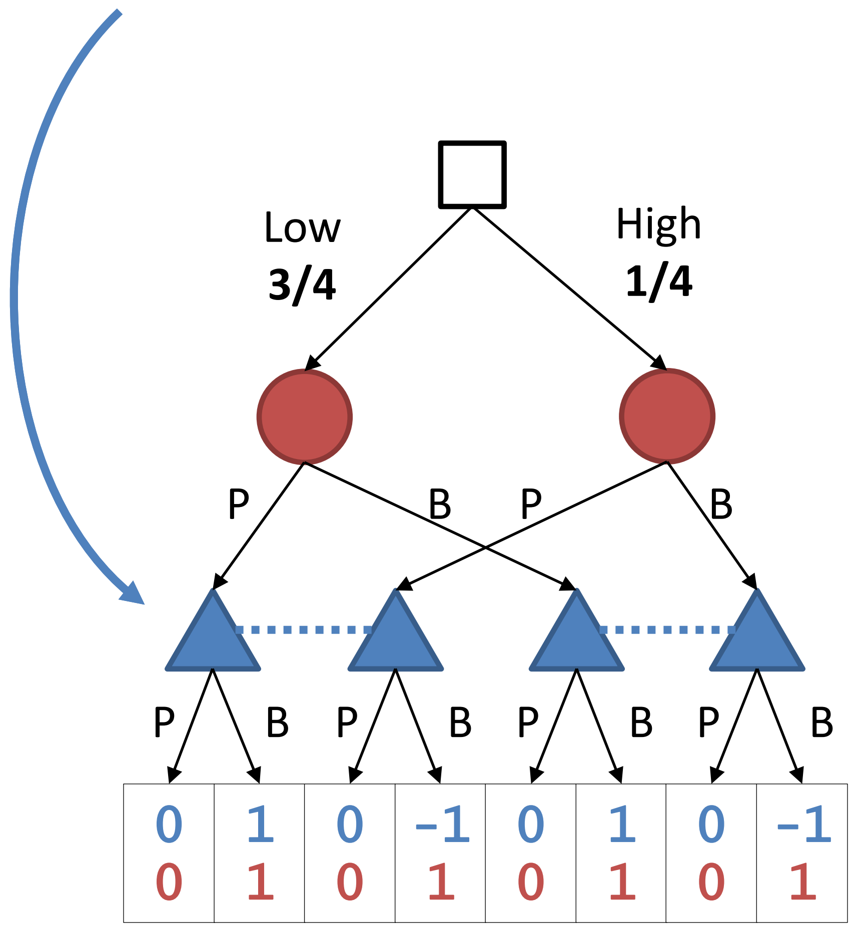
The buyer learns the seller's message, but not the true item quality, and decides how to act



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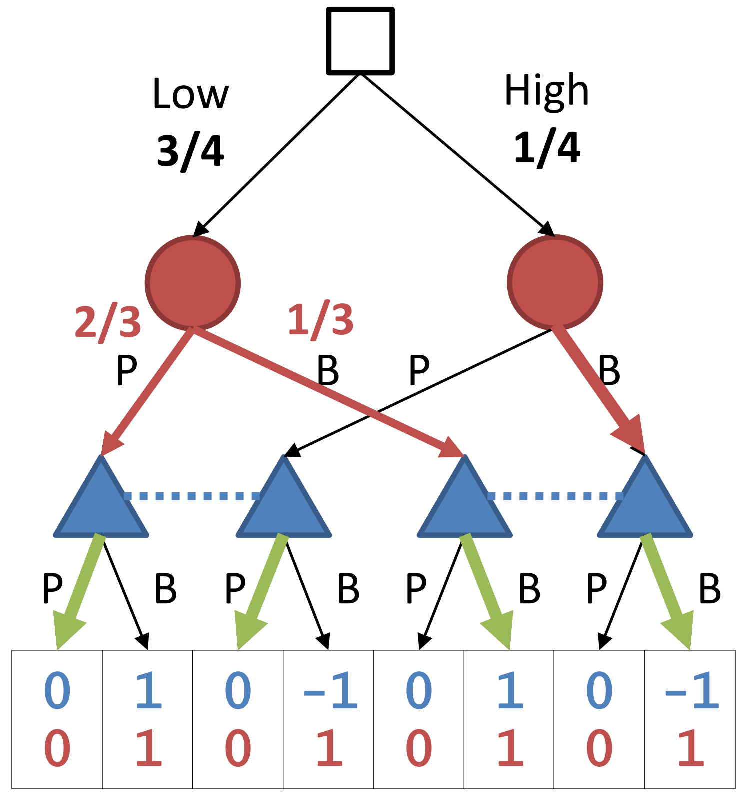

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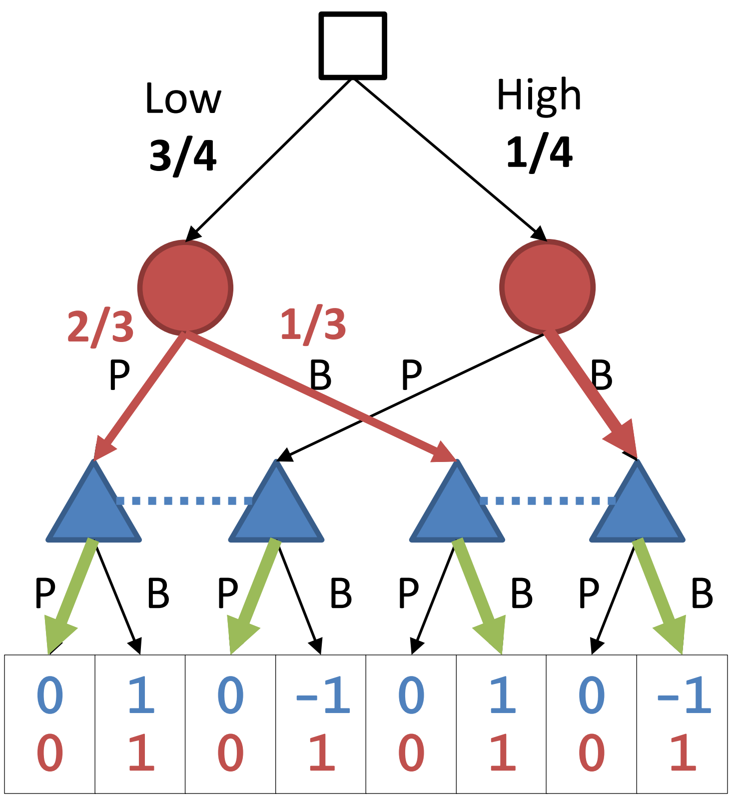
In the optimal signaling scheme, the seller gets the buyer to buy a low-quality item 1/3 of the time!



Kamenica and Gentzkow (American Economic Review 2011), "Bayesian Persuasion"

Lots of related useful problems!

- ✓ Optimal correlated equilibria in normal-form games
- ✓ Optimal mediated equilibrium
- ✓ Optimal Bayesian persuasion (information design) in extensive-form games
- **Optimal automated mechanism design**
- Optimal extensive-form correlated equilibria in extensive-form games

Mechanism Design

- n players, with private information
- **The mediator takes all actions in the game** but doesn't have any information
  - Therefore, the mediator relies on the players to provide information
- The players' and mediator's payoffs depend on the players' private information and the mediator's actions.
- **Question:** How should the mediator play?
  - **Revelation principle:** WLOG, the mediator should play in such a way that all players are incentivized to report their information honestly

Mechanism Design: An Example

Role reversal!

A few minutes ago…

A seller ("mediator") wishes to persuade a buyer ("player") to purchase an item

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Mechanism Design: An Example

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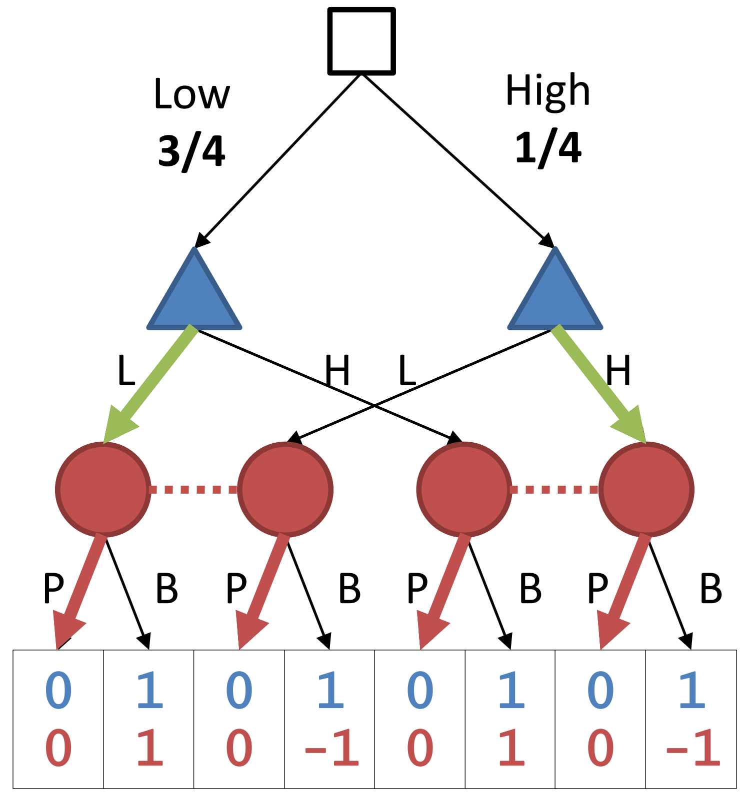
We've recovered an algorithm equivalent to the (randomized) mechanism design algorithm of Conitzer and Sandholm (2002)!

Revelation principle: seller's signal is **the true quality**

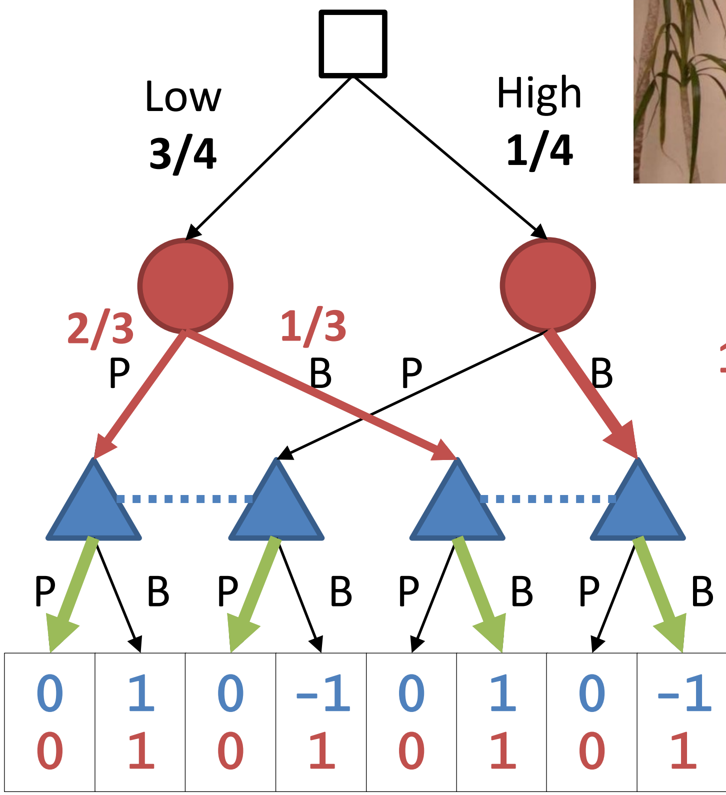
find mediator strategy $\mu \in X_{\text{mediator}}$ such that for all players i x_i^* is a best response to (μ, x_{-i}^*)



Conitzer and Sandholm (UAI 2002), "Complexity of Mechanism Design"

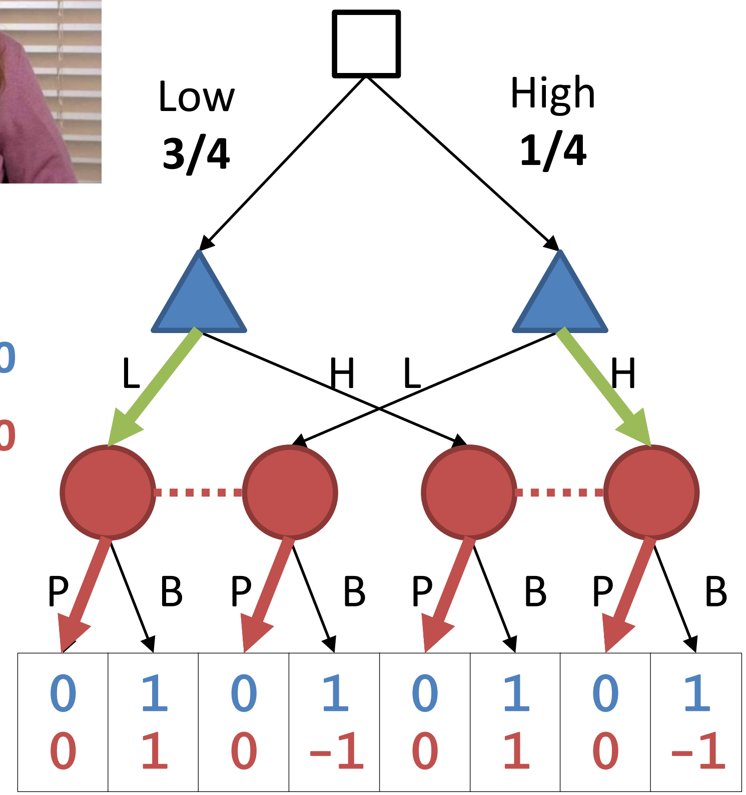
What happened?

Incentive design example



1/2	Seller EV	0
0	Buyer EV	0

Mechanism design example



Answer: It matters who has the commitment power—the mediator enjoys a Stackelberg commitment advantage!

Lots of related useful problems!

- ✓ Optimal correlated equilibria in normal-form games
- ✓ Optimal mediated equilibrium
- ✓ Optimal Bayesian persuasion (information design) in extensive-form games
- ✓ Optimal automated mechanism design
- **Optimal extensive-form correlated equilibria in extensive-form games**

Extensive-Form Correlated Equilibria Imperfect-Recall Mediators

Information design:

- n players playing an extensive-form imperfect-information game
- **The mediator has an informational advantage over the players.** In particular, the mediator always knows at least the infoset of the acting player (and perhaps more), and has perfect recall.
  Therefore, the players don't need to send messages to the mediator because that would be pointless (the mediator knows the acting player's information already)
- **Question:** How should the mediator give recommendations?
- **Answer:** As usual, build an augmented game and solve for an optimal incentive-compatible mediator strategy

Extensive-Form Correlated Equilibria
Imperfect-Recall Mediators

~~Information design~~ Extensive-Form Correlation:

- n players playing an extensive-form imperfect-information game
- ~~**The mediator has an informational advantage over the players.**~~ In particular, the mediator always knows ~~at least~~ the infoset of the acting player (and ~~perhaps~~ **no** more), and has **im**perfect recall.

  Therefore, the players don't need to send messages to the mediator because that would be pointless (the mediator knows the acting player's information already)
- **Question:** How should the mediator give recommendations?
- **Answer:** As usual, build an augmented game and solve for an optimal incentive-compatible mediator strategy
- In other words, **the mediator cannot leak information between players—** its recommendations can only depend on what the player already knows (and some shared randomness)

Problem: How do we optimize over the decision space of the mediator when the mediator has imperfect recall?

Finding Optimal EFCEs

- In general, computing optimal strategies for a player with imperfect recall is NP-hard, and therefore so is computing an optimal EFCE, for the same reason [Chu and Halpern 2001]
- For EFCE, there is a **fixed-parameter tractable** representation of the decision space, with size $O^*\left((b+d)^k\right)$, where
  - b = branching factor
  - d = depth
  - k = "information complexity" of game ≈ "amount of private information that is not public"
  - O^* hides factors polynomial in the size of the game

  [**Zhang**, Farina, Celli, and Sandholm 2022]

  Therefore, optimal equilibria can be found by solving an LP of that size
- In **two-player games with public chance actions**, there is a **polynomially-sized** representation of the decision space

  [Farina and Sandholm 2020; **Zhang**, Farina, Celli, and Sandholm 2022]
- In any case, finding *one* EFCE is easy, by either an **ellipsoid-based method** [Huang and von Stengel 2008] or **regret minimization** [Farina, Celli, Marchesi, and Gatti 2020]

Zhang, Farina, Celli, and Sandholm (EC 2022), "Optimal Correlated Equilibria in General-Sum Extensive-Form Games: Fixed-Parameter Algorithms, Hardness, and Two-Sided Column-Generation"

Farina and Sandholm (NeurIPS 2020), " Polynomial-Time Computation of Optimal Correlated Equilibria in Two-Player Extensive-Form Games with Public Chance Moves and Beyond"

Farina, Celli, Marchesi, and Gatti (NeurIPS 2020), "Simple Uncoupled No-Regret Learning Dynamics for Extensive-Form Correlated Equilibrium"

Huang and von Stengel (WINE 2008), "Computing an extensive-form correlated equilibrium in polynomial time"

Chu and Halpern (Int J Game Theory 2001), "On the NP-completeness of Finding an Optimal Strategy in Games with Common Payoffs"

Lots of related useful problems!

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- ✓ Optimal mediated equilibrium
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- ✓ Optimal automated mechanism design
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Takeaway 1: Several seemingly disparate problems can in fact be grouped under the same umbrella and solved using very similar techniques

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Takeaway 2: The hardness of optimal equilibrium computation, at least among these equilibrium concepts, is driven by **the imperfect recall of the mediator**

More Scenarios

Certifiable messages [Forges and Koessler 2005]

- The messages that a player can send are a function of the player's current information. Thus, some messages are *certifiable*. For example, if a player can only send a certain message m from one information set I, then sending m certifies that the player is at I.
- Under the *nested range condition* [Green and Laffont 1977], the revelation principle holds, and therefore our algorithm runs in polytime [**Zhang** and Sandholm 2022]
  - The *nested range condition* asserts that, if a player at infoset I can send a message pretending to be at information set I', then the player at I must also be able to send every message that she would be able to send at I'.
  - It is the condition that we need to ensure that the revelation principle holds!
  - LP size for certification is **basically linear** (unlike LP size for communication, which is quadratic) because players have far fewer deviations!
- Special case: **full**-certification equilibria are equilibria in which, at any information set I, the player's only options are to send I or send a special "empty message" $\perp$. That is, all information sent by the player is certifiable.

Zhang and Sandholm (arXiv preprint 2022), "Polynomial-Time Optimal Equilibria with a Mediator in Extensive-Form Games"
Forges and Koessler (J Math Econ 2005), "Communication equilibria with partially verifiable types"
Green and Laffont (Econometrica 1977), "Characterization of satisfactory mechanisms for the revelation of preferences for public goods."

More Scenarios

Coarse deviations

- Extensive-form coarseness: Players must decide whether to obey recommendations *before* seeing them.
- Normal-form coarseness: Players must decide, at the beginning of the game, whether to play the obedient strategy or to play a different strategy; in the latter case, the player does not communicate with the mediator at all.

  In the mechanism design and information design contexts, this is sometimes called "*ex-ante* incentive compatibility", as opposed to "*ex-interim* incentive compatibility"
- Coarseness can be applied to various equilibrium concepts, in particular to communication and certification [**Zhang** and Sandholm 2022], and to correlation [Farina, Bianchi, and Sandholm 2020]

Zhang and Sandholm (arXiv preprint 2022), "Polynomial-Time Optimal Equilibria with a Mediator in Extensive-Form Games"
Farina, Bianchi, and Sandholm (AAAI 2020), "Coarse correlation in extensive-form games"

More Scenarios

Normal-form correlated equilibria??

- In an extensive-form game Γ, a **normal-form correlated equilibrium** is a correlated equilibrium of Γ expressed in normal form. That is, it is an equilibrium where the players are told at the start of the game their *whole strategy* and then can decide whether or not to play it
- This does not fit into our framework of "one round of communication per action taken by the player"
- The complexity of finding *one* normal-form correlated equilibrium in an extensive-form game is an open problem

Optimal (full-)certification equilibria are very fast to compute since the LP has basically linear size

Experiments

game	$\lvert Z \rvert$		NFCCE	[ZFCS'22] EFCCE	EFCE	NF Coarse Cert no pay	NF Coarse Cert pay	Coarse Cert no pay	Coarse Cert pay	*This paper* Cert no pay	Cert pay	Comm no pay	Comm pay
B222	1072	value	0.000	-0.525	-0.525	0.000	0.000	-0.525	-0.333	-0.525	-0.453	-0.750	-0.520
		time	0.02s	0.05s	0.17s	0.01s	0.01s	0.02s	0.07s	0.06s	0.17s	3.80s	4.05s
B322	19116	value	0.000	-0.317	-0.317	-0.000	0.000	-0.317	-0.200	-0.317	-0.226	oom	oom
		time	0.21s	1.38s	5.83s	0.02s	0.15s	0.05s	0.72s	0.28s	4.05s	oom	oom
B323	191916	value	0.000	-0.375	-0.375	0.000	0.000	-0.375	-0.250	-0.375	oom	oom	oom
		time	2.82s	32.84s	1m 55s	0.32s	2.42s	2.77s	16.50s	22.59s	oom	oom	oom
S122	396	value	13.636	9.565	9.078	50.000	50.000	10.000	42.000	10.000	42.000	0.820	42.000
		time	0.01s	0.02s	0.04s	0.01s	0.01s	0.02s	0.11s	0.08s	0.20s	0.85s	1.74s
S123	2376	value	13.636	10.000	10.000	50.000	50.000	10.000	42.000	10.000	42.000	0.820	42.000
		time	0.04s	0.23s	0.65s	0.03s	0.07s	0.12s	0.25s	0.46s	1.04s	1m 13s	1m 49s
S133	5632	value	18.182	15.000	15.000	50.000	50.000	15.000	43.000	15.000	43.000	0.820	oom
		time	0.04s	1.51s	2.46s	0.06s	0.12s	0.31s	0.65s	1.78s	2.89s	17m 7s	oom
RS12	400	value	6.010	6.010	6.010	6.173	6.173	6.173	6.173	6.173	6.173	6.173	6.173
		time	0.02s	0.01s	0.01s	0.00s	0.01s	0.00s	0.01s	0.01s	0.04s	0.91s	1.77s
RS13	4356	value	9.398	9.385	9.367	9.622	9.622	9.622	9.622	9.622	9.622	9.592	9.592
		time	2.82s	1m 28s	12m 31s	0.03s	0.11s	0.07s	0.19s	0.16s	0.55s	3m 9s	6m 42s
RS14	229888	value	oom	oom	oom	10.500	10.500	10.500	10.500	10.500	10.500	oom	oom
		time	oom	oom	oom	0.20s	1.41s	0.66s	3.97s	2.34s	12.04s	oom	oom
RS22	484	value	7.188	7.176	7.176	7.594	7.594	7.594	7.594	7.594	7.594	7.594	7.594
		time	0.20s	0.20s	0.16s	0.00s	0.01s	0.01s	0.02s	0.01s	0.04s	0.90s	1.80s
RS23	4096	value	10.961	10.820	10.791	11.516	11.516	11.513	11.513	11.485	11.485	11.464	11.464
		time	3.12s	56m 31s	6m 35s	0.03s	0.10s	0.06s	0.18s	0.19s	0.63s	7m 43s	12m 1s

Optimal communication equilibria are much slower to compute since the LP has quadratic size

Experiments

game	|Z|		NFCCE	[ZFCS'22] EFCCE	EFCE	NF Coarse Cert no pay	NF Coarse Cert pay	*This paper* Coarse Cert no pay	Coarse Cert pay	Cert no pay	Cert pay	Comm no pay	Comm pay
B222	1072	value	0.000	-0.525	-0.525	0.000	0.000	-0.525	-0.333	-0.525	-0.453	-0.750	-0.520
		time	0.02s	0.05s	0.17s	0.01s	0.01s	0.02s	0.07s	0.06s	0.17s	3.80s	4.05s
B322	19116	value	0.000	-0.317	-0.317	-0.000	0.000	-0.317	-0.200	-0.317	-0.226	oom	oom
		time	0.21s	1.38s	5.83s	0.02s	0.15s	0.05s	0.72s	0.28s	4.05s	oom	oom
B323	191916	value	0.000	-0.375	-0.375	0.000	0.000	-0.375	-0.250	-0.375	oom	oom	oom
		time	2.82s	32.84s	1m 55s	0.32s	2.42s	2.77s	16.50s	22.59s	oom	oom	oom
S122	396	value	13.636	9.565	9.078	50.000	50.000	10.000	42.000	10.000	42.000	0.820	42.000
		time	0.01s	0.02s	0.04s	0.01s	0.01s	0.02s	0.11s	0.08s	0.20s	0.85s	1.74s
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		time	0.04s	0.23s	0.65s	0.03s	0.07s	0.12s	0.25s	0.46s	1.04s	1m 13s	1m 49s
S133	5632	value	18.182	15.000	15.000	50.000	50.000	15.000	43.000	15.000	43.000	0.820	oom
		time	0.04s	1.51s	2.46s	0.06s	0.12s	0.31s	0.65s	1.78s	2.89s	17m 7s	oom
RS12	400	value	6.010	6.010	6.010	6.173	6.173	6.173	6.173	6.173	6.173	6.173	6.173
		time	0.02s	0.01s	0.01s	0.00s	0.01s	0.00s	0.01s	0.01s	0.04s	0.91s	1.77s
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RS14	229888	value	oom	oom	oom	10.500	10.500	10.500	10.500	10.500	10.500	oom	oom
		time	oom	oom	oom	0.20s	1.41s	0.66s	3.97s	2.34s	12.04s	oom	oom
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		time	0.20s	0.20s	0.16s	0.00s	0.01s	0.01s	0.02s	0.01s	0.04s	0.90s	1.80s
RS23	4096	value	10.961	10.820	10.791	11.516	11.516	11.513	11.513	11.485	11.485	11.464	11.464
		time	3.12s	56m 31s	6m 35s	0.03s	0.10s	0.06s	0.18s	0.19s	0.63s	7m 43s	12m 1s

Which is faster between correlated equilibrium and communication equilibrium is game-dependent

Experiments

game	$\lvert Z \rvert$		NFCCE	[ZFCS'22] EFCCE	EFCE	*This paper* NF Coarse Cert		Coarse Cert		Cert		Comm	
						no pay	pay	no pay	pay	no pay	pay	no pay	pay
B222	1072	value	0.000	-0.525	-0.525	0.000	0.000	-0.525	-0.333	-0.525	-0.453	-0.750	-0.520
		time	0.02s	0.05s	0.17s	0.01s	0.01s	0.02s	0.07s	0.06s	0.17s	3.80s	4.05s
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		time	2.82s	32.84s	1m 55s	0.32s	2.42s	2.77s	16.50s	22.59s	oom	oom	oom
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		time	0.02s	0.01s	0.01s	0.00s	0.01s	0.00s	0.01s	0.01s	0.04s	0.91s	1.77s
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		time	oom	oom	oom	0.20s	1.41s	0.66s	3.97s	2.34s	12.04s	oom	oom
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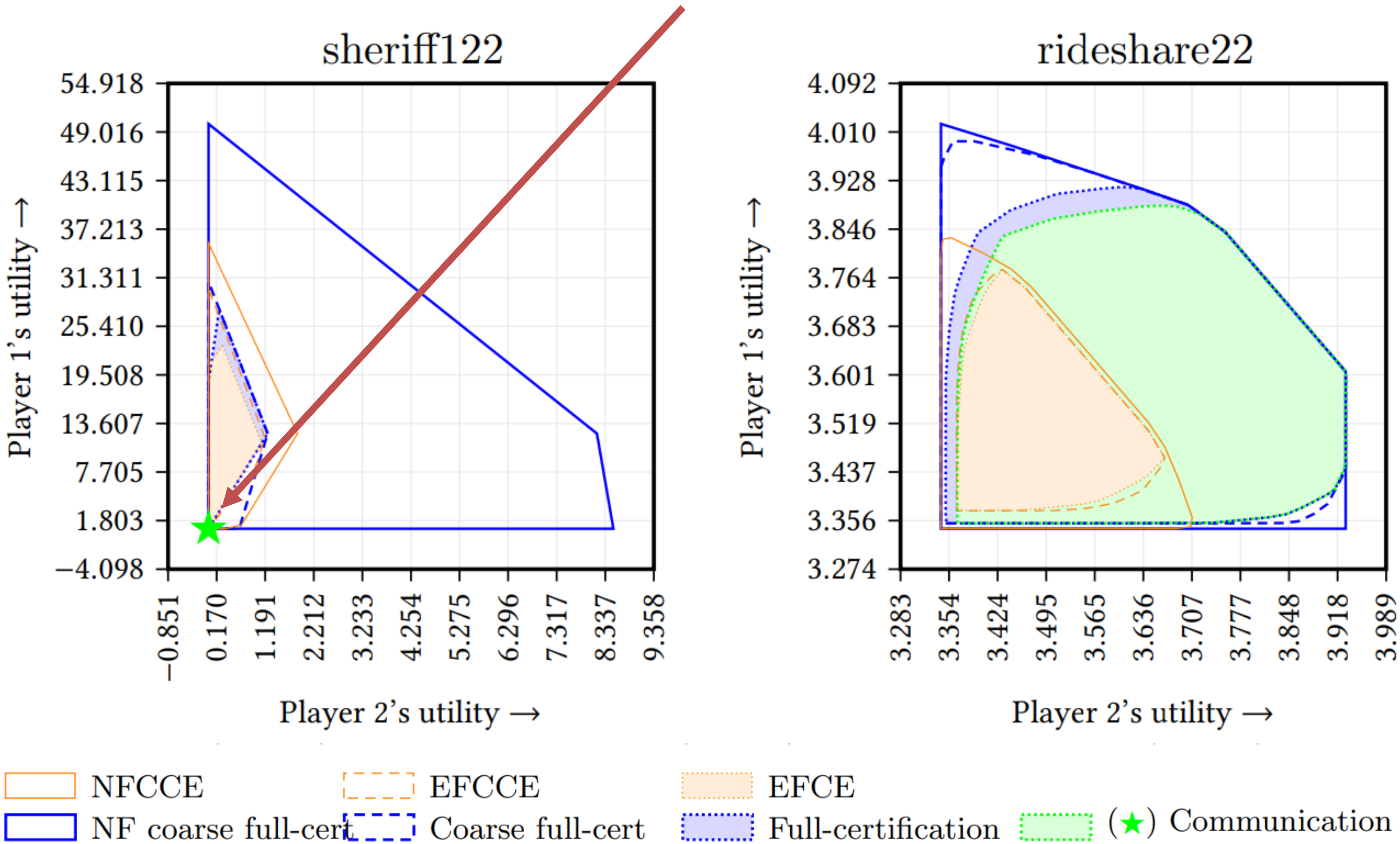
Experiments: Payoff Spaces

In the left game, the set of communication equilibrium payoffs is a single point…
…so that point is also the unique Nash payoff!



Experiments: Payoff Spaces

In the right game, communication equilibria can achieve all EFCE payoffs and more

Therefore, EFCE and communication equilibria are in general incomparable