16831 Statistical Techniques, Fall 2014: Problem Set 1

Due: Thursday, September 11, beginning of the class

Instructions

There are 2 questions on this assignment (3 pages). These are short, simple problems. Included is the maximum answer length.

1 Markov Assumption

This is not meant to be a tricky question, just one to get you thinking about an important assumption that is often made. One paragraph each at most.

1.1

Give one robotic example where the Markov assumption is used (correctly or not). Is the assumption valid or not? Explain.

1.2

Give one real-world example where the Markov assumption is used (correctly or not). Is the assumption valid or not? Explain.

2 Bayes Filter Derivation

Recall the derivation for the Bayes Filter in the slides:

$$
\begin{aligned}
Bel\left(x_t\right) &= P\left(x_t|u_{1:t}, z_{1:t}\right) && (1)\\
&\ldots && (2)\\
&\propto P\left(z_t|x_t\right)\int P\left(x_t|u_t, x_{t-1}\right)P\left(x_{t-1}|u_{1:t}, z_{1:t-1}\right)dx_{t-1} && (3)\\
&\propto P\left(z_t|x_t\right)\int P\left(x_t|u_t, x_{t-1}\right)P\left(x_{t-1}|u_{1:t-1}, z_{1:t-1}\right)dx_{t-1} && (4)\\
&\propto P\left(z_t|x_t\right)\int P\left(x_t|u_t, x_{t-1}\right)Bel\left(x_{t-1}\right)dx_{t-1} && (5)
\end{aligned}
$$

2.1

In the slides, the Markov assumption is invoked between lines (3) and (4) to drop u_t. Why is this incorrect? (Why does the Markov assumption not enable you to drop u_t)

2.2

Provide a counter-example where knowing u_t gives you information about the state x_{t-1}.

2.3

What does the book assume about the controls u in order to drop u_t from the derivation of the Bayes Filter? Is this assumption reasonable? Why or why not?

2.4

There are weaker assumptions you can make about the controls to still drop u_t from the Bayes Filter derivation. Think conditional independence, and derive how u_t is dropped between lines (3) and (4).