

16831 Statistical Techniques, Fall 2014

Problem Set 2

Due: Thursday, September 25, beginning of class

Instructions

There are 4 questions on this assignment (3 pages). Included is the maximum answer length.

1 Importance Sampling

Let P be a distribution, and f be a real valued function. In class, we discussed computing $\mathbb{E}_P[f(X)]$ using importance sampling, where one draws M samples $x^{[1]}, \dots, x^{[M]}$ from a different distribution Q (the importance distribution) and *estimates* the expectation of f using:

$$\hat{\mu} = \frac{1}{M} \sum_{m=1}^M w^{[m]} f(x^{[m]})$$

where $w^{[m]} = P(x^{[m]})/Q(x^{[m]})$.

For what choice of importance distribution Q is the variance of $\hat{\mu}$ minimized? Assume that $Q(x)$ is nonzero whenever $P(x)$ is nonzero.

2 Occupancy Maps

We discussed in class viewing occupancy map as a collection of 1D binary Bayes filters.

2.1

a) What is the motion model in this filter?

b) The simplest model of dynamics for this system allows the underlying state x_i of each map cell to transition with Markov dynamics between filled and empty. Consider what modifications that are necessary to the occupancy grid algorithm if the probability of transitioning between filled and empty is symmetric and happens with probability α . Point out the changes. Is this a reasonable model of dynamic obstacles?

2.2

As discussed in class, there is a step in the derivation of the occupancy grid algorithm that is false for any reasonable model of sensors on robots. Go through the derivation of the binary bayes filter in the book and identify this step. (Hint: we discussed that there is something fundamentally wrong with pretending mapping is running a lot of 1-dimensional filters.) Why does this assumption not make sense in the derivation? What are some consequences of making this assumption?