

Cache Memories

18-213/18-613: Introduction to Computer Systems 10th Lecture, September 26th, 2024

Reminder: AIV Policy

http://www.cs.cmu.edu/~18213/academicintegrity.html

No unauthorized use of information

- Borrowing code: by copying, retyping, looking at a file
- Describing: verbal description of code from one person to another
- Searching the Web for solutions
- Copying code from a previous course or online solution
- Reusing your code from a previous semester (here or elsewhere)

No unauthorized supplying of information

- Providing copy: Giving a copy of a file to someone
- Providing access:
 - Putting material in unprotected directory
 - Putting material in unprotected code repository (e.g., Github)

No collaborations beyond high-level, strategic advice

Anything more than block diagram or a few words

Start early. Make frequent github commits. Plan for stumbling blocks. Use available help. Don't panic: Far better to turn in 25% correct solution than get an AIV.

Today

Cache memory organization and operation	CSAPP 6.4-6.5
Performance impact of caches	
The memory mountain	CSAPP 6.6.1
 Rearranging loops to improve spatial locality 	CSAPP 6.6.2
 Using blocking to improve temporal locality 	CSAPP 6.6.3

Recall: General Cache Concepts



Data in block b is needed

Block b is in cache:

Hit!

General Cache Concepts: Hit



Bryant and O'Hallaron, Computer Systems: A Programmer's Perspective, Third Edition

General Cache Concepts: Miss



Data in block b is needed

Block b is not in cache: *Miss!*

Block b is fetched from memory

Block b is stored in cache

- Placement policy: determines where b goes
- Replacement policy: determines which block gets evicted (victim)

Working Set, Locality, and Caches

Working Set: The set of data a program is currently "working on"

- Definition of "currently" depends on context, e.g., in this loop
- Includes accesses to data and instructions
- Principle of Locality: Programs tend to use data and instructions with addresses near or equal to those they have used recently
 - Nearby addresses: Spatial Locality
 - Equal addresses: Temporal locality

Caches take advantage of temporal locality by storing recently used data, and spatial locality by copying data in block-sized transfer units

- Locality reduces working set sizes
- Caches are most effective when the working set fits in the cache

Recall: 3 Types of Cache Misses

Cold (compulsory) miss

 Cold misses occur because the cache starts empty and this is the first reference to the block.

Capacity miss

 Occurs when the set of active cache blocks (working set) is larger than the cache.

Conflict miss

- Occurs when the cache is large enough, but too many data objects all map (by the placement policy) to the same limited set of blocks
 - E.g., if the placement policy maps both 0 and 8 to the same block, then referencing 0, 8, 0, 8, 0, 8, ... would miss every time.

CPU Cache Memories

Cache memories are small, fast SRAM-based memories managed automatically in hardware

- Hold frequently accessed blocks of main memory
- CPU looks first for data in cache
- Typical system structure:



What it Really Looks Like







Core i7-3960X



What it Really Looks Like (Cont.)



https://twitter.com/aschilling/status/1453391035577495553

Die shot interpretation by Locuza, October 2021

Intel Alder Lake (2021)

8 P-cores + 8 E-cores

L1 caches per P-core: 32KB Instruction & 48KB Data L1 caches per E-core: 64KB Instruction & 32KB Data L2 caches: 1.25MB per P-core, 2MB per four E-cores L3 cache: 30MB shared among all cores

General Cache Organization (S, E, B)



Bryant and O'Hallaron, Computer Systems: A Programmer's Perspective, Third Edition

• Locate set

• Check if any line in set

Cache Read



Bryant and O'Hallaron, Computer Systems: A Programmer's Perspective, Third Edition

Example: Direct Mapped Cache (E = 1)

Direct mapped: One line per set Assume: cache block size B=8 bytes



Example: Direct Mapped Cache (E = 1)

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Example: Direct Mapped Cache (E = 1)

Direct mapped: One line per set Assume: cache block size B=8 bytes



If tag doesn't match (= miss): old line is evicted and replaced

Direct-Mapped Cache Simulation

t=1	s=2	b=1
X	XX	X

4-bit addresses (address space size M=16 bytes) S=4 sets, E=1 Blocks/set, B=2 bytes/block

Address trace (reads, one byte per read):

0	[<mark>0<u>00</u>0₂],</mark>	miss	(cold)
1	[<mark>0<u>00</u>1₂],</mark>	hit	
7	$[0111_2],$	miss	(cold)
8	[1 <u>00</u> 0 ₂],	miss	(cold)
0	$[0000_2]$	miss	(conflict)

	V	Tag	Block
Set 0	1	0	M[0-1]
Set 1	0		
Set 2	0		
Set 3	1	0	M[6-7]

E-way Set Associative Cache (Here: E = 2)

E = 2: Two lines per set

Assume: cache block size B=8 bytes





Address of short int:

E-way Set Associative Cache (Here: E = 2)

E = 2: Two lines per set

Assume: cache block size B=8 bytes



block offset

Address of short int:

E-way Set Associative Cache (Here: E = 2)

E = 2: Two lines per set

Assume: cache block size B=8 bytes



short int (2 Bytes) is here

No match or not valid (= miss):

- One line in set is selected for eviction and replacement
- Replacement policies: random, least recently used (LRU), ...

2-Way Set Associative Cache Simulation

t=2	s=1	b=1
XX	X	X

4-bit addresses (M=16 bytes) S=2 sets, E=2 blocks/set, B=2 bytes/block

Address trace (reads, one byte per read):

0	[<mark>000</mark> 0 ₂],	miss
1	[<mark>000</mark> 1 ₂],	hit
7	[<mark>01<u>1</u>1₂],</mark>	miss
8	[<mark>100</mark> 0 ₂],	miss
0	[<mark>000</mark> 0 ₂]	hit



What about writes?

Multiple copies of data exist:

L1, L2, L3, Main Memory, Disk

What to do on a write-hit?

- Write-through (write immediately to memory)
- Write-back (defer write to memory until replacement of line)
 - Each cache line needs a dirty bit (set if data differs from memory)

What to do on a write-miss?

- Write-allocate (load into cache, update line in cache)
 - Good if more writes to the location will follow
- No-write-allocate (writes straight to memory, does not load into cache)

Typical

- Write-through + No-write-allocate
- Write-back + Write-allocate



Why Index Using Middle Bits?

Direct mapped: One line per set Assume: cache block size 8 bytes



Illustration of Indexing Approaches

- 64-byte memory
 - 6-bit addresses
- 16 byte, direct-mapped cache
- Block size = 4. (Thus, 4 sets; why?)
- 2 bits tag, 2 bits index, 2 bits offset





Middle Bits Indexing

- Addresses of form TTSSBB
 - **TT** Tag bits
 - SS Set index bits
 - BB Offset bits
- Makes good use of spatial locality





High Bits Indexing

- Addresses of form SSTTBB
 - SS Set index bits
 - **TT** Tag bits
 - BB Offset bits
- Program with high spatial locality would generate lots of conflicts





Intel Core i7 Cache Hierarchy

Processor package



L1 i-cache and d-cache:

32 KB, 8-way, Access: 4 cycles

L2 unified cache: 256 KB, 8-way, Access: 10 cycles

L3 unified cache: 8 MB, 16-way, Access: 40-75 cycles

Block size: 64 bytes for all caches.

Example: Core i7 L1 Data Cache



	t ner	cimal any
X1		0000
1	1	0001
2	2	0010
3	3	0011
4	4	0100
5	5	0101
6	6	0110
7	7	0111
8	8	1000
9	9	1001
A	10	1010
В	11	1011
С	12	1100
D	13	1101
E	14	1110
F	15	1111

Address of word: t bits s bits b bits block set tag **Block offset: Stack Address:** 0x?? index offset Set index: 0x00007f7262a1e010 0x?? Block offset: . bits Tag: 0x?? Set index: . bits Tag: . bits

Example: Core i7 L1 Data Cache



He	t Der	timal Binary
0	0	0000
1	1	0001
2	2	0010
3	3	0011
4	4	0100
5	5	0101
6	6	0110
7	7	0111
8	8	1000
9	9	1001
Α	10	1010
В	11	1011
С	12	1100
D	13	1101
Ε	14	1110
F	15	1111

Address of word:



Stack Address: 0x00007f7262a1e <u>010</u>	Block offset: Set index: Tag:	0x?? 0x?? 0x??
	148.	•••••

Decimal

Het

Α

Β

С

D

Ε

F

Binary

Example: Core i7 L1 Data Cache





Bryant and O'Hallaron, Computer Systems: A Programmer's Perspective, Third Edition

Cache Performance Metrics

Miss Rate

- Fraction of memory accesses not found in cache (misses / accesses)
 = 1 hit rate
- Typical numbers (as %):
 - 3-10% for L1
 - can be quite small (e.g., < 1%) for L2, depending on size, etc.

Hit Time

- Time to deliver a cached block to the processor
 - includes time to determine whether line is in cache
- Typical numbers:
 - 4 clock cycle for L1
 - 10 clock cycles for L2

Miss Penalty

- Additional time required because of a miss
 - typically 50-200 cycles for main memory (Trend: increasing!)

How Bad Can a Few Cache Misses Be?

Huge difference between a hit and a miss

Could be 100x, if just L1 and main memory

Would you believe 99% hits is twice as good as 97%?

- Consider this simplified example: cache hit time of 1 cycle miss penalty of 100 cycles
- Average access time:

97% hits: 1 cycle + 0.03 x 100 cycles = **4 cycles**

99% hits: 1 cycle + 0.01 x 100 cycles = 2 cycles

This is why "miss rate" is used instead of "hit rate"

Writing Cache Friendly Code

- Make the common case go fast
 - Focus on the inner loops of the core functions
- Minimize the misses in the inner loops
 - Repeated references to variables are good (temporal locality)
 - Stride-1 reference patterns are good (spatial locality)

Key idea: Our qualitative notion of locality is quantified through our understanding of cache memories

Quiz Time!

Canvas Quiz: Day 10 – Cache Memories

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Today

Cache organization and operation

Performance impact of caches

- The memory mountain
- Rearranging loops to improve spatial locality
- Using blocking to improve temporal locality
The Memory Mountain

Read throughput (read bandwidth)

- Number of bytes read from memory per second (MB/s)
- Memory mountain: Measured read throughput as a function of spatial and temporal locality.
 - Compact way to characterize memory system performance.

Memory Mountain Test Function

```
long data[MAXELEMS]; /* Global array to traverse */
/* test - Iterate over first "elems" elements of
 *
          array "data" with stride of "stride",
          using 4x4 loop unrolling.
 *
 */
int test(int elems, int stride) {
    long i, sx2=stride*2, sx3=stride*3, sx4=stride*4;
    long acc0 = 0, acc1 = 0, acc2 = 0, acc3 = 0;
    long length = elems, limit = length - sx4;
    /* Combine 4 elements at a time */
    for (i = 0; i < limit; i += sx4) {</pre>
        acc0 = acc0 + data[i];
        acc1 = acc1 + data[i+stride];
        acc2 = acc2 + data[i+sx2];
        acc3 = acc3 + data[i+sx3];
    }
    /* Finish any remaining elements */
    for (; i < length; i++) {</pre>
        acc0 = acc0 + data[i];
    return ((acc0 + acc1) + (acc2 + acc3));
                               mountain/mountain.c
```

Call test() with many combinations of elems and stride.

For each elems and stride:

1. Call test() once to warm up the caches.

2. Call test() again and measure the read throughput(MB/s)





Throughput for size = 128K 35000 30000 Miss rate = stride/8 25000 20000 **MB/sec** Miss rate = 1.0 Measured 15000 10000 8 elems per 5000 cache block 0 s2 s1 s3 s10 s11 s12 s4 s5 s6 s7 s8 s9 stride

Closer Look at Stride Effects

Bryant and O'Hallaron, Computer Systems: A Programmer's Perspective, Third Edition

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Matrix Multiplication Example

Description:

- Multiply N x N matrices
- Matrix elements are doubles (8 bytes)
- O(N³) total operations
- N reads per source element
- N values summed per destination
 - but may be able to hold in register

/* ijk */
for (i=0; i<n; i++) {
 for (j=0; j<n; j++) {
 sum = 0.0; {
 for (k=0; k<n; k++)
 sum += a[i][k] * b[k][j];
 c[i][j] = sum;
 }
 matmult/mm.c</pre>

Miss Rate Analysis for Matrix Multiply

Assume:

- Block size = 32B (big enough for four doubles)
- Matrix dimension (N) is very large
 - Approximate 1/N as 0.0
- Cache is not even big enough to hold multiple rows

Analysis Method:

Look at access pattern of inner loop



Layout of C Arrays in Memory (review)

- C arrays allocated in row-major order
 - each row in contiguous memory locations
- Stepping through columns in one row:

for
$$(i = 0; i < N; i++)$$

sum += a[0][i];

- accesses successive elements
- if block size (B) > sizeof(a_{ii}) bytes, exploit spatial locality
 - miss rate = sizeof(a_{ii}) / B
- Stepping through rows in one column:
 - for (i = 0; i < n; i++)
 sum += a[i][0];</pre>
 - accesses distant elements
 - no spatial locality!
 - miss rate = 1 (i.e. 100%)

Matrix Multiplication (ijk)



Block size = 32B (four doubles)

(į,j)

Fixed

Matrix Multiplication (jik)



<u>B</u>

1.0

Same analysis as ijk

0.0

Row-wise Columnwise Misses per inner loop iteration:

Inner loop:

Α

(i,*)



*,j)

В

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Α

0.25

Matrix Multiplication (kij)



Miss rate for inner loop iterations:ABC0.00.250.25

Block size = 32B (four doubles)

Matrix Multiplication (jki)



1.0

$\frac{\text{Miss rate for inner loop iterations:}}{\underline{A}} \qquad \underline{B} \qquad \underline{C}$

0.0

Block size = 32B (four doubles)

1.0

Summary of Matrix Multiplication

```
for (i=0; i<n; i++) {
  for (j=0; j<n; j++) {
    sum = 0.0;
    for (k=0; k<n; k++)
        sum += a[i][k] * b[k][j];
    c[i][j] = sum;
  }
}</pre>
```

```
for (k=0; k<n; k++) {
  for (i=0; i<n; i++) {
    r = a[i][k];
    for (j=0; j<n; j++)
      c[i][j] += r * b[k][j];
  }</pre>
```

```
for (j=0; j<n; j++) {
  for (k=0; k<n; k++) {
    r = b[k][j];
    for (i=0; i<n; i++)
        c[i][j] += a[i][k] * r;
  }
}</pre>
```

ijk(&jik):

- 2 loads, 0 stores
- avg misses/iter = 1.25

```
kij(&ikj):
```

- 2 loads, 1 store
- avg misses/iter = 0.5

jki (& kji):

- 2 loads, 1 store
- avg misses/iter = 2.0

Core i7 Matrix Multiply Performance

Cycles per inner loop iteration



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 - Using blocking to improve temporal locality

Example: Matrix Multiplication





Cache Miss Analysis

Assume:

- Matrix elements are doubles
- Cache line = 8 doubles
- Cache size C << n (much smaller than n)</p>



Cache Miss Analysis

Assume:

- Matrix elements are doubles
- Cache line = 8 doubles
- Cache size C << n (much smaller than n)</p>

• Second iteration: • Again: n/8 + n = 9n/8 misses $= \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 2 \\ 1 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 2 \\ 1 & 1$

• $9n/8 n^2 = (9/8) n^3$

Blocked Matrix Multiplication

```
c = (double *) calloc(sizeof(double), n*n);
/* Multiply n x n matrices a and b */
void mmm(double *a, double *b, double *c, int n) {
    int i, j, k;
    for (i = 0; i < n; i+=L)
       for (j = 0; j < n; j+=L)
             for (k = 0; k < n; k+=L)
                /* L x L mini matrix multiplications */
                  for (i1 = i; i1 < i+L; i1++)
                      for (j1 = j; j1 < j+L; j1++)
                          for (k1 = k; k1 < k+L; k1++)
                              c[i1*n+j1] += a[i1*n + k1]*b[k1*n + j1];
                                                         matmult/bmm.c
```



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n/L blocks

X

X

Block size L x L

Cache Miss Analysis

Assume:

- Cache line = 8 doubles. Blocking size $L \ge 8$
- Cache size C << n (much smaller than n)</p>
- Three blocks fit into cache: 3L² < C</p>

First (block) iteration: Misses per block: L²/8 Blocks per Iteration: 2n/L (omitting matrix c) Misses per Iteration: 2n/L x L²/8 = nL/4 Afterwards in cache (schematic)

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Cache Miss Analysis

Assume:

- Cache line = 8 doubles. Blocking size $L \ge 8$
- Cache size C << n (much smaller than n)</p>
- Three blocks fit into cache: 3L² < C</p>



Total misses:

• nL/4 misses per iteration x $(n/L)^2$ iterations = $n^3/(4L)$ misses

Blocking Summary

- No blocking: (9/8) n³ misses
- Blocking: (1/(4L)) n³ misses

Use largest block size L, such that L satisfies 3L² < C</p>

Fit three blocks in cache! Two input, one output.

Reason for dramatic difference:

- Matrix multiplication has inherent temporal locality:
 - Input data: $3n^2$, computation $2n^3$
 - Every array elements used O(n) times!
- But program has to be written properly

Cache Summary

Cache memories can have significant performance impact

You can write your programs to exploit this!

- Focus on the inner loops, where bulk of computations and memory accesses occur.
- Try to maximize spatial locality by reading data objects sequentially with stride 1.
- Try to maximize temporal locality by using a data object as often as possible once it's read from memory.

Supplemental slides





Cache Capacity Effects from Memory Mountain



Core i7 Haswell 3.1 GHz 32 KB L1 d-cache 256 KB L2 cache 8 MB L3 cache 64 B block size

Slice through memory mountain with stride=8

Modeling Block Size Effects from Memory Mountain

Core i7 Haswell 2.26 GHz 32 KB L1 d-cache 256 KB L2 cache 8 MB L3 cache 64 B block size



Throughput for size = 128K

Carnegie Mellon



Matrix Multiplication (ikj)



$\begin{array}{c|c} \underline{Misses \ per \ inner \ loop \ iteration:} \\ \underline{A} & \underline{B} & \underline{C} \\ 0.0 & 0.25 & 0.25 \end{array}$

Block size = 32B (four doubles)

Matrix Multiplication (kji)



Block size = 32B (four doubles)

Recap: Stack and instruction pointers

- The stack pointer (%rsp) points to the top of the stack
- The instruction pointer (%rip) points to the next instruction to be executed
- They are independent
 - But linked by call and ret instructions



push %rax =



push %rax =

- sub %rsp, 8
- mov %rax, (%rsp)



- push %rax =
 - sub %rsp, 8
 - mov %rax, (%rsp)
- call func =



- push %rax =
 - sub %rsp, 8
 - mov %rax, (%rsp)
- call func =
 - sub %rsp, 8
 - mov %rip, (%rsp)
 - jmp func



- push %rax =
 - sub %rsp, 8
 - mov %rax, (%rsp)
- call func =
 - sub %rsp, 8
 - mov %rip, (%rsp)
 - jmp func
- ret =


Recap: stack operations

- push %rax =
 - sub %rsp, 8
 - mov %rax, (%rsp)
- call func =
 - sub %rsp, 8
 - mov %rip, (%rsp)
 - jmp func
- ret =
 - mov (%rsp), %rip
 - add %rsp, 8



Recap: stack operations

- push %rax =
 - sub %rsp, 8
 - mov %rax, (%rsp)
- call func =
 - sub %rsp, 8
 - mov %rip, (%rsp)
 - jmp func
- ret =
 - mov (%rsp), %rip
 - add %rsp, 8
- pop %rax =



Recap: stack operations

- push %rax =
 - sub %rsp, 8
 - mov %rax, (%rsp)
- call func =
 - sub %rsp, 8
 - mov %rip, (%rsp)
 - jmp func
- ret =
 - mov (%rsp), %rip
 - add %rsp, 8
- pop %rax =
 - mov (%rsp), %rax
 - add %rsp, 8

