Lexical Analysis
&
Parsing (1)

15-411/15-611 Compiler Design

Seth Copen Goldstein

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Today

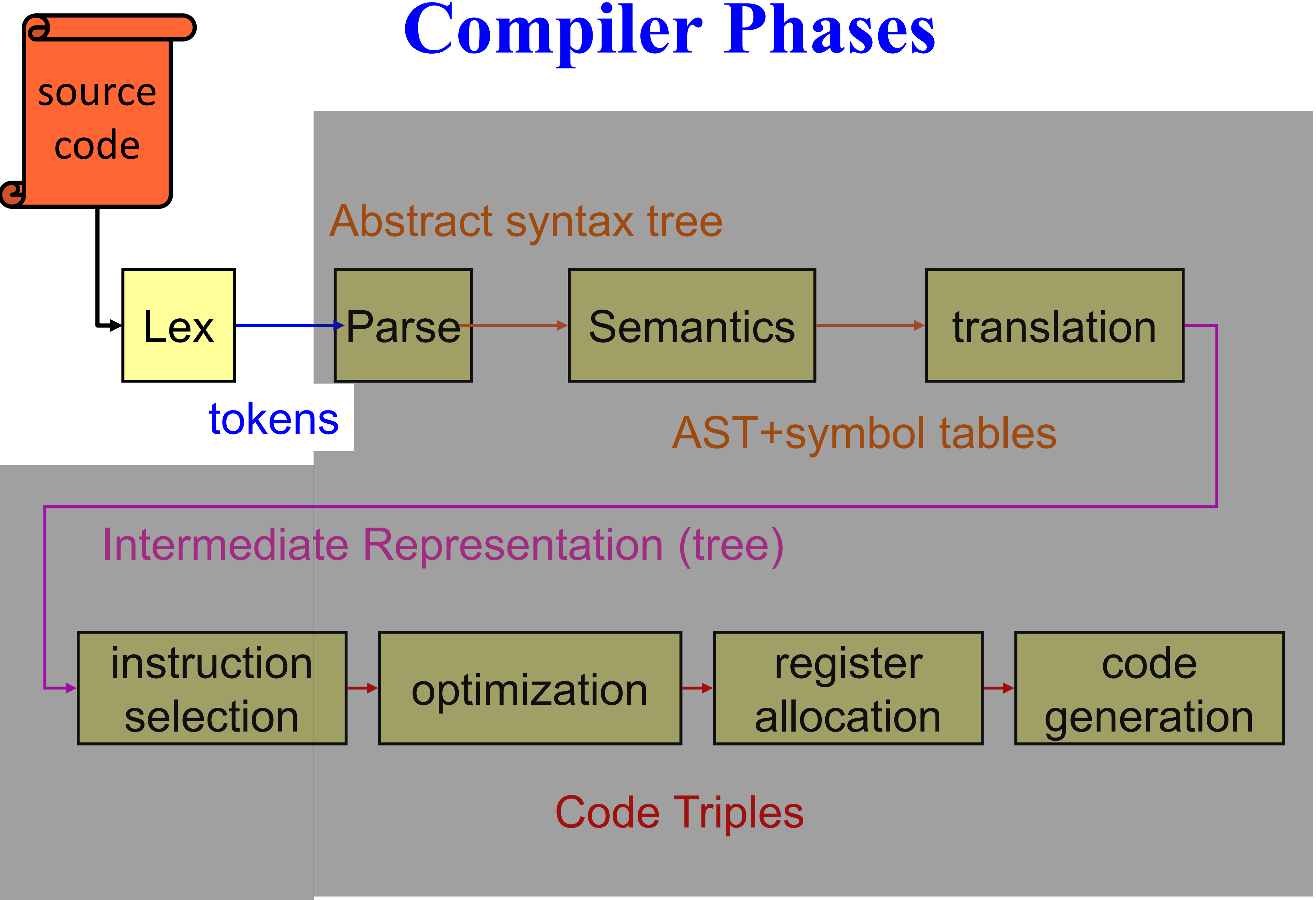
- Lexing
- Parsing

Today – part 1

Lexing

- Flex & other scanner generators
- Regular Expressions
- Finite Automata
- RE $\rightarrow$ NFA
- NFA $\rightarrow$ DFA
- DFA $\rightarrow$ Minimized DFA
- Limits of Regular Languages

Compiler Phases



source code

Lex

tokens

Abstract syntax tree

Parse

Semantics

translation

AST+symbol tables

Intermediate Representation (tree)

instruction selection

optimization

register allocation

code generation

Code Triples

The Lexer

- Turn stream of characters into a stream of tokens

```
// create a user friendly descriptor for this arg.
// if key is absent, then use it.  Otherwise use longkey

char*
ArgDesc::helpkey(WhichKey keytype, bool includebraks)
{
    static char buffer[128];  /* format buffer */
    char* p = buffer;
    …
```

```
CHAR STAR ID DOUBLE_COLON ID LPARIN ID ID COMMA BOOL ID
RPARIN LBRACE STATIC CHAR ID LBRAK INTCONST RBRAK SEMI
CHAR STAR ID EQ ID SEMI …
```

The Lexer

- Turn stream of characters into a stream of tokens
  - Strips out “unnecessary characters”
    - comments
    - whitespace
  - Classify tokens by type
    - keywords
    - numbers
    - punctuation
    - identifiers
  - Track location
  - Associate with syntactic information

The Lexer

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The Lexer

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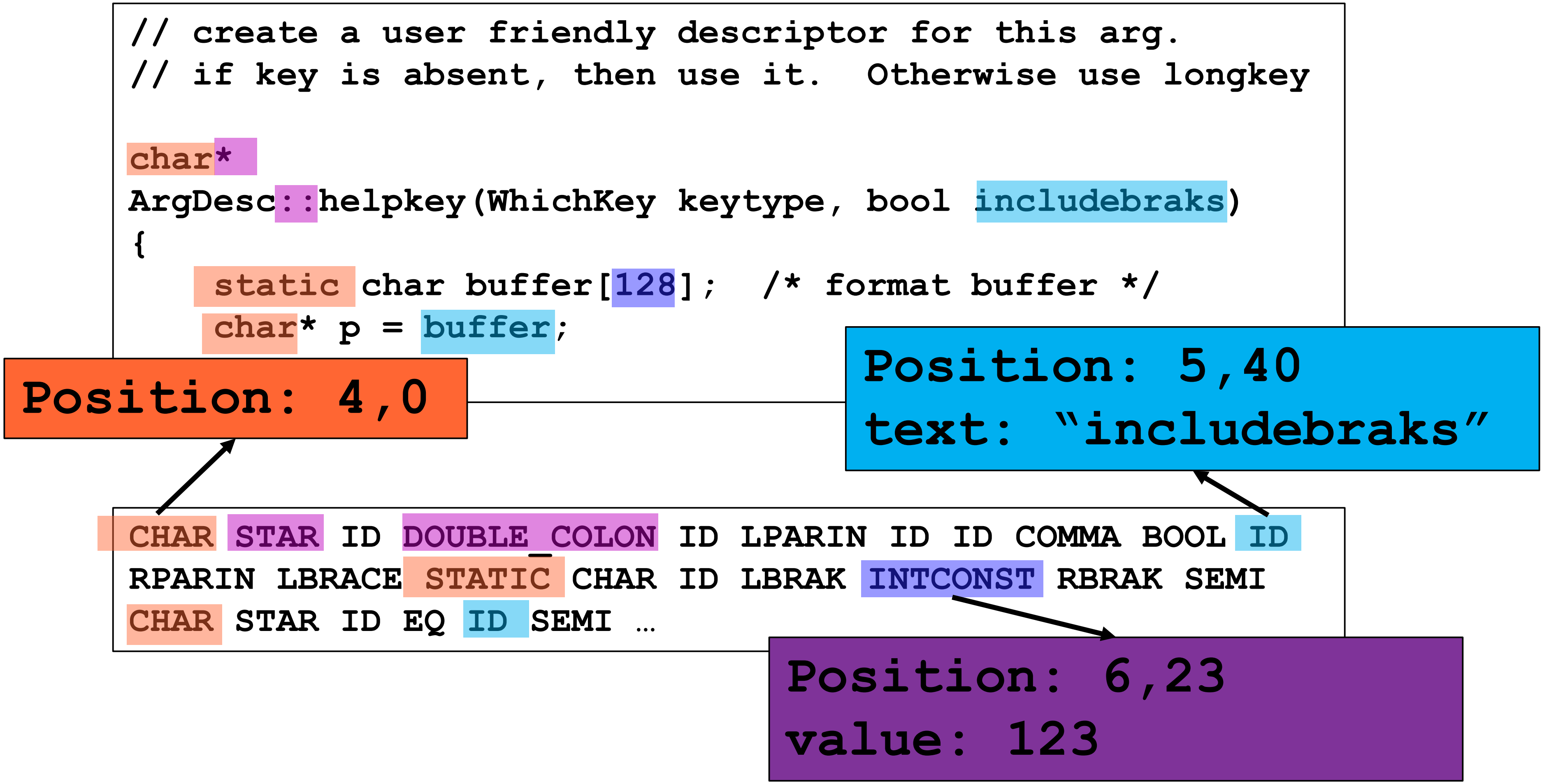

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```

Position: 4,0

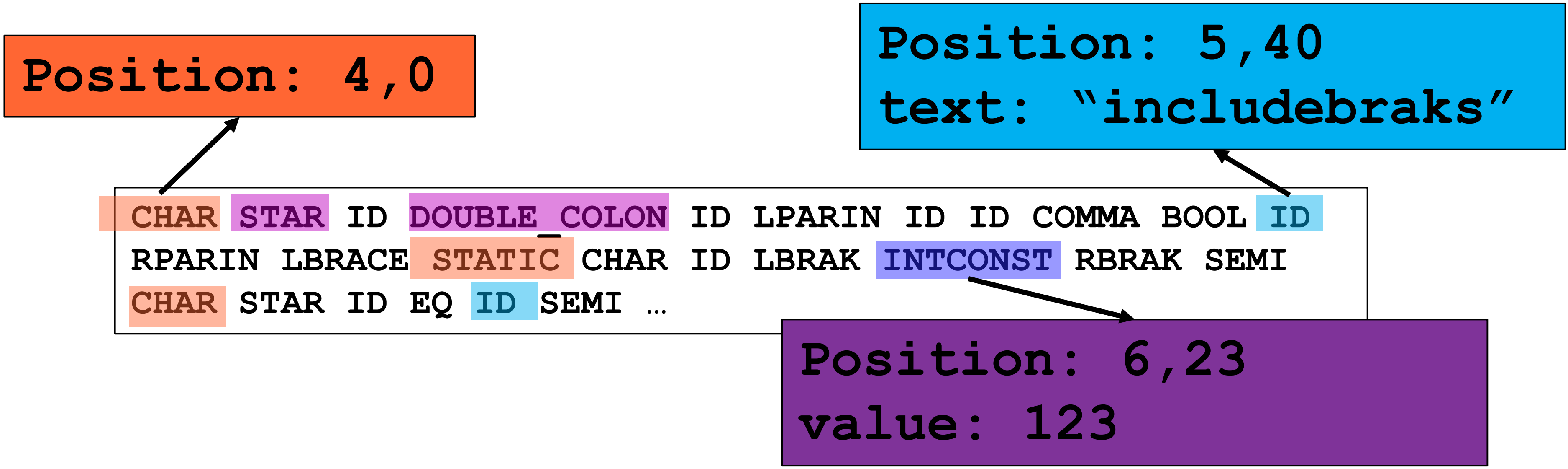
Position: 5,40
text: “includebraks”

```
CHAR STAR ID DOUBLE_COLON ID LPARIN ID ID COMMA BOOL ID
RPARIN LBRACE STATIC CHAR ID LBRAK INTCONST RBRAK SEMI
CHAR STAR ID EQ ID SEMI …
```

Position: 6,23
value: 123

The Lexer

- Turn stream of characters into a stream of tokens
  - More concise
  - Easier to parse



Position: 4,0

Position: 5,40
text: “includebraks”

`CHAR STAR ID DOUBLE_COLON ID LPARIN ID ID COMMA BOOL ID`
`RPARIN LBRACE STATIC CHAR ID LBRAK INTCONST RBRAK SEMI`
`CHAR STAR ID EQ ID SEMI …`

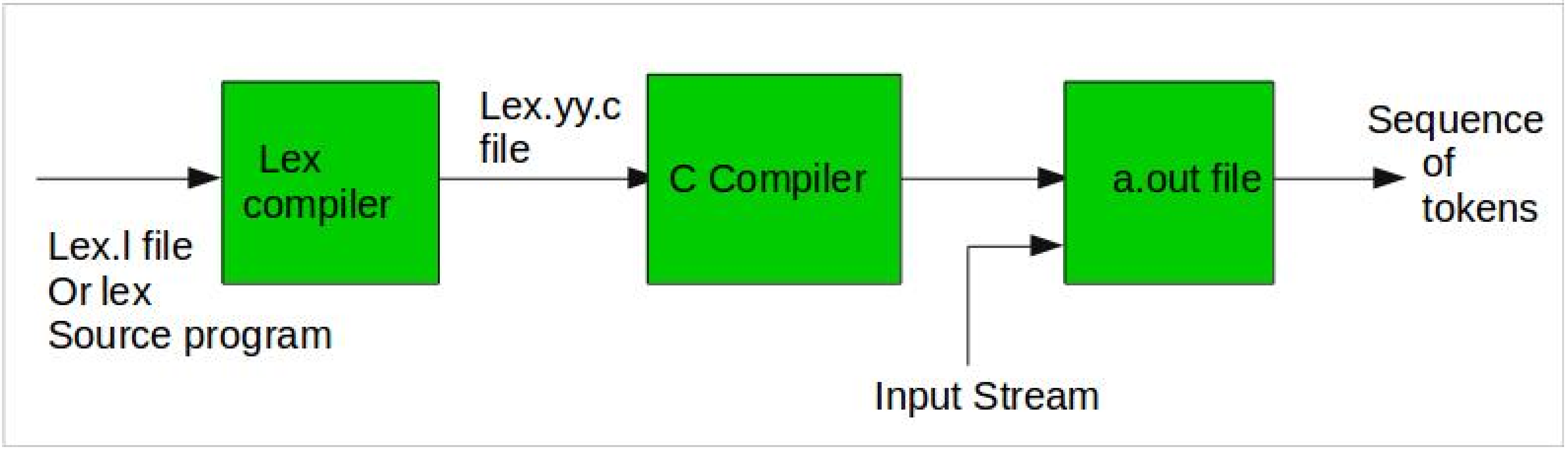
Position: 6,23
value: 123

Lexical Analyzers

- Input: stream of characters
- Output: stream of tokens (with information)
- How to build?
  - By hand is tedious
  - Use Lexical Analyzer Generator, e.g., flex
- Define tokens with regular expressions
- Flex turns REs into Deterministic Finite Automata (DFA) which recognizes and returns tokens.

FLEX

- Define tokens
- Generate scanner code
- Main interface: **`yylex()`** which reads from **yyin** and returns tokens til EOF



2. Flex Program Format

- A flex program has three sections:

  Definitions
  %%
  RE rules & actions
  %%
  User code

wc As a Flex Program

```
%{
  int charCount=0, wordCount=0, lineCount=0;
%}
word    [^ \t\n]+
%%
{word} {wordCount++; charCount += yyleng; }
[\n]    {charCount++; lineCount++;}
.       {charCount++;}
%%
int main(void) {
   yylex();
   printf(“Chars %d, Words: %d, Lines: %d\n”,
      charCount,  wordCount,  lineCount);
   return 0;
}
```

A Flex Program

```
%{
  int charCount=0, wordCount=0, lineCount=0;
%}
word    [^ \t\n]+
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{word} {wordCount++; charCount += yyleng; }
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   return 0;
}
```

1) Definitions

2) Rules & Actions

3) User Code

skip

Section 1: RE Definitions

- Format:

```
name      RE
```

- Examples:

```
digit       [0-9]
letter      [A-Za-z]
id          {letter} ({letter}|{digit})*

word        [^ \t\n]+
```

Regular Expressions in Flex

`x`	match the char **x**
`\.`	match the char **.**
`"string"`	match contents of string of chars
`.`	match any char except \n
`^`	match beginning of a line
`$`	match the end of a line
`[xyz]`	match one char **x**, **y**, or **z**
`[^xyz]`	match any char except **x**, **y**, and **z**
`[a-z]`	match one of **a** to **z**

Regular Expressions in Flex (cont)

`r*`	closure (match 0 or more r's)
`r+`	positive closure (match 1 or more r's)
`r?`	optional (match 0 or 1 r)
`r1 r2`	match *r*1 then *r*2 (concatenation)
`r1 | r2`	match *r*1 or *r*2 (union)
`( r )`	grouping
`r1 \ r2`	match *r*1 when followed by *r*2
`{ `*`name`*` }`	match the RE defined by name

Some number REs

`[0-9]`	A single digit.
`[0-9]+`	An integer.
`[0-9]+ (\.[0-9]+)?`	An integer or fp number.
`[+-]? [0-9]+ (\.[0-9]+)? ([eE][+-]?[0-9]+)?`	Integer, fp, or scientific notation.

Section 2: RE/Action Rule

- A rule has the form:

```
name        { action }
re          { action }
```

  - the name must be defined in section 1
  - the action is any C code

- If the named RE matches* an input character sequence, then the C code is executed.

* Some caveats here

Rule Matching

- Longest match rule.

```
"int"      { return INT; }
"integer"  { return INTEGER; }
```

- If rules can match same length input, first rule takes priority.

```
"int"      { return INT; }
[a-z]+     { return ID; }
[0-9]+     { return NUM; }
```

Section 3: C Functions

- Added to end of the lexical analyzer

Removing Whitespace

```
whitespace        [ \t\n]
%%

{whitespace}      ;

.                 { ECHO; }

%%


int main(void)
{
   yylex();
   return 0;
}
```

name

empty action

RE

ECHO macro

Printing Line Numbers

```
%{
   int lineno = 1;
%}
%%
^(.*)\n   { printf("%4d\t%s", lineno, yytext);
            lineno++;}
%%
int main(int argc, char *argv[])
{
  // proper arg processing & error handling, …
  yyin = fopen(argv[1], "r");
  yylex();
  return 0;
}
```

the matched text

Today – part 1

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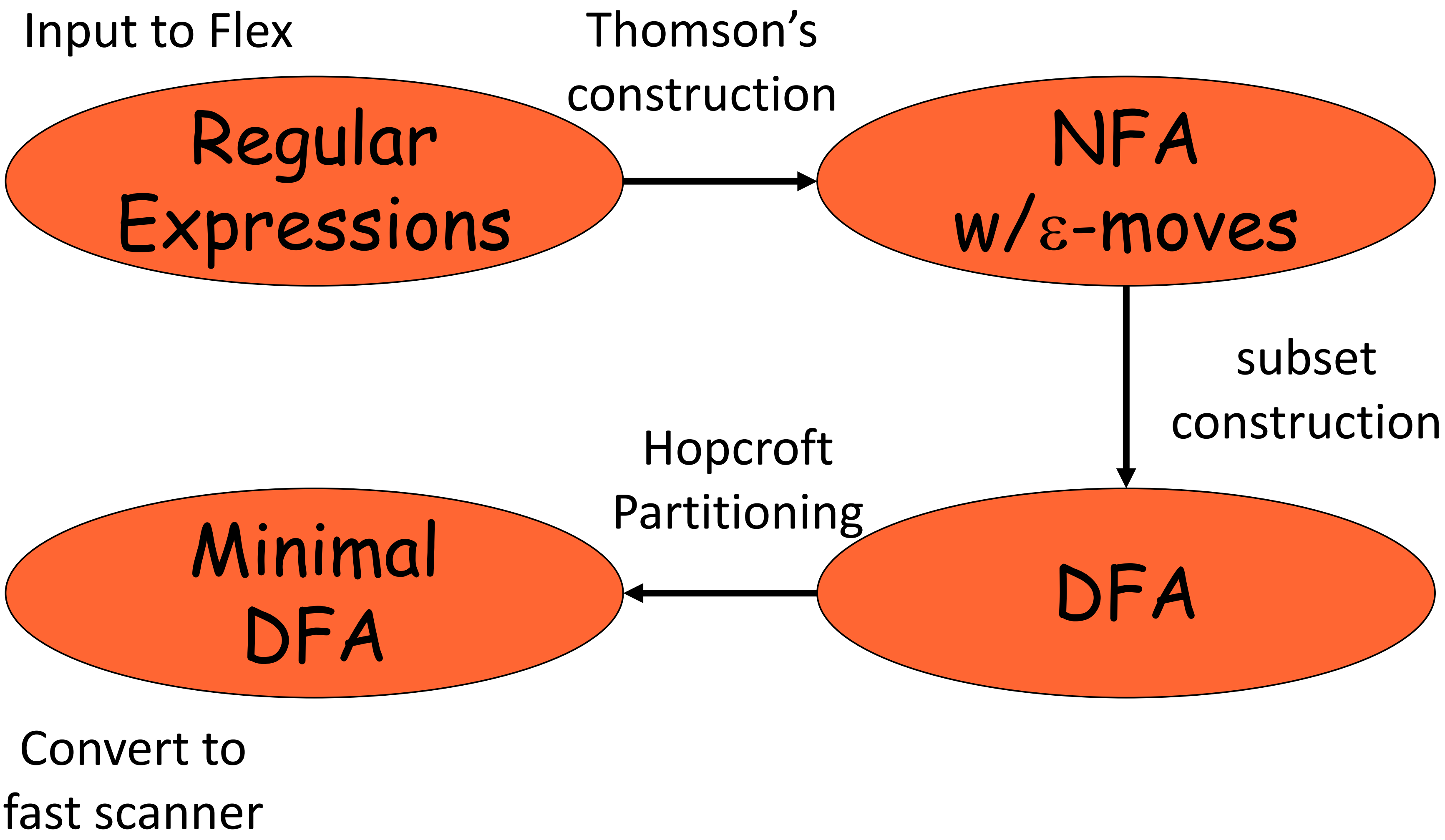
- Flex & other scanner generators
- **Regular Expressions**
- Finite Automata
- RE → NFA
- NFA → DFA
- DFA → Minimized DFA
- Limits of Regular Languages

Under The Covers

- How to go from REs to a working scanner?



Regular Languages

- Finite Alphabet, Σ, of symbols.
- word (or string), a finite sequence of symbols from Σ.
- Language over Σ is a set of words from Σ.
- Regular Expressions describe Regular Languages.
  - easy to write down, but hard to use directly
- The languages accepted by Finite Automata are also Regular.

Regular Expressions defined

- Base Cases:
  - A single character a
  - The empty string ε
- Recursive Rules:
  If R_1 and R_2 are regular expressions
  - Concatenation R_1R_2
  - Union $R_1 | R_2$
  - Closure $R_1{}^*$
  - Grouping (R_1)
- REs describe Regular Languages.

RE Examples

- even a’s

- odd b’s

- even a’s or odd b’s
- even a’s followed by odd b’s

RE Examples

- even a’s

  b* (a b* a b*)*

- odd b’s

  a* b a* (b a* b a*)*

- even a’s or odd b’s
- even a’s followed by odd b’s

RE Examples

- even a’s

$R^A = b^* \, (a \, b^* \, a \, b^* \,)^*$

- odd b’s

$R^B = a^* \, b \, a^* \, (b \, a^* \, b \, a^*)^*$

- even a’s or odd b’s

$R^A \mid R^B$

- even a’s followed by odd b’s

$R^A \; R^B$

Today – part 1

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- **Finite Automata**
- **RE → NFA**
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Finite Automata

- finite set of states
- set of edges from states to states labeled by letter from Σ
- initial state
- set of accepting states
- How it works:
  - Start in initial state, on each character transition goto state using edge labeled for that character.
  - If at end of word we are in accepting state, the word is in language
  - Language accepted are strings that cause FA to end in an accepting state

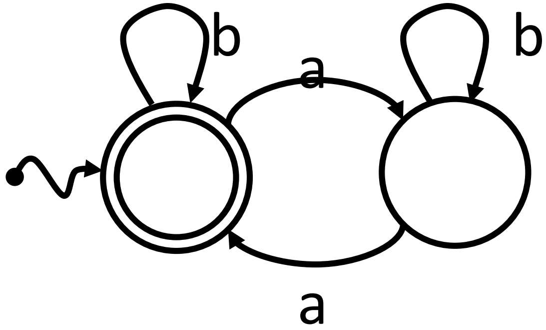
Example REs → FA

- even a’s $b^* (a b^* a b^*)^*$

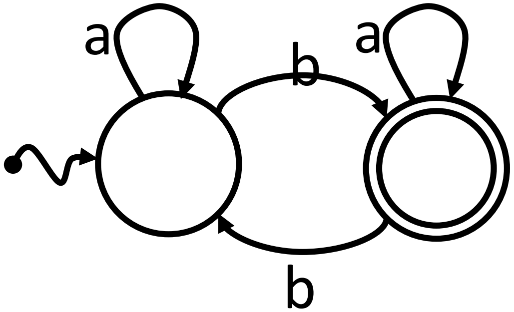
- odd b’s $a^* b a^* (b a^* b a^*)^*$

Example REs → FA

- even a’s b* (a b* a b*)*



- odd b’s a* b a* (b a* b a*)*

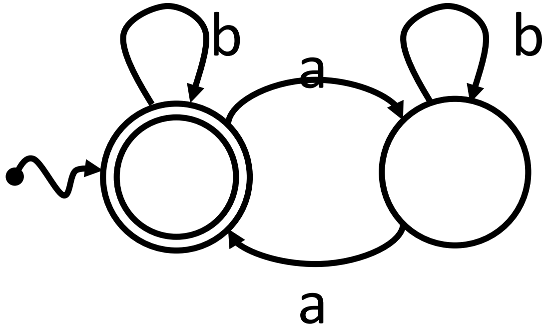


Deterministic Finite Automata
DFA

Ad Hoc

Example REs → FA

- even a's b* (a b* a b*)*



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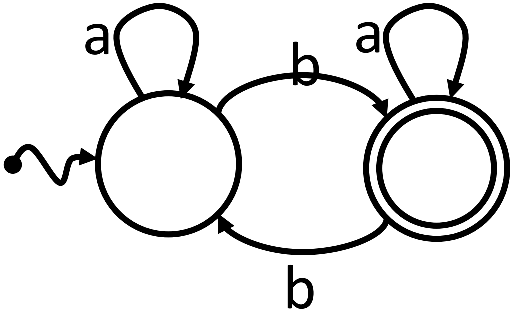


- even a's or odd b's

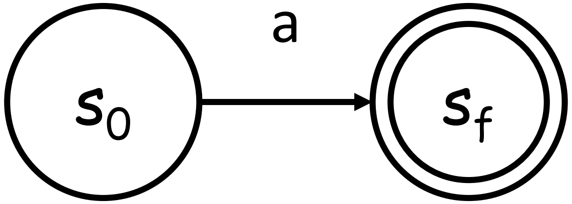
$R^A \mid R^B$

- even a's followed by odd b's

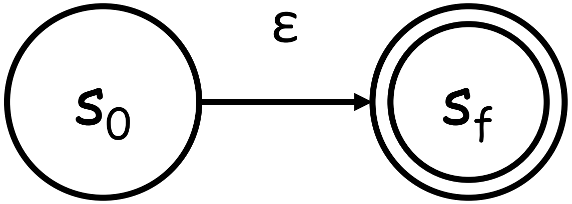
$R^A \; R^B$

Converting RE to NFA: Base Case

- for $a \in \Sigma$ the NFA $M_a = \{\Sigma, \{s_0, s_f\}, \delta, s_0, \{s_f\}\}$

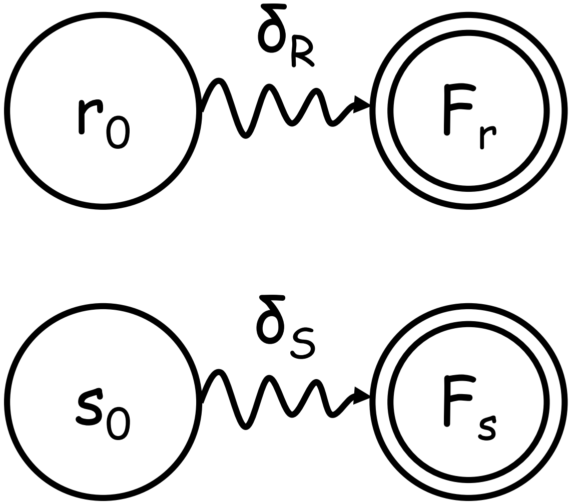


- for ε the NFA $M_\varepsilon = \{\Sigma, \{s_0, s_f\}, \delta, s_0, \{s_f\}\}$



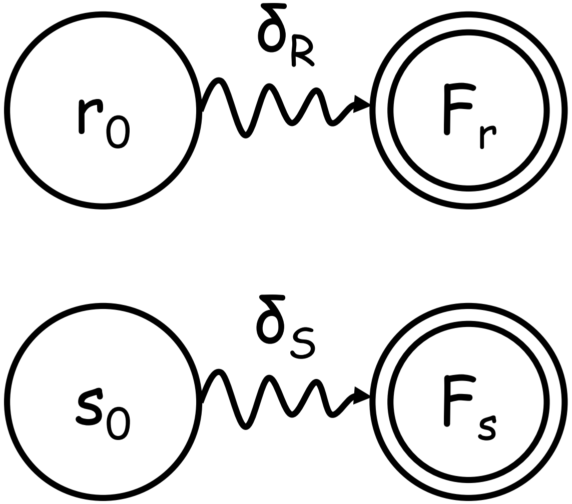
Recursive Case

- for RE R with $M_R = \{\Sigma, s_R, \delta_R, r_0, F_r\}$ and RE S with $M_s = \{\Sigma, s_S, \delta_S, s_0, F_s\}$



R|S

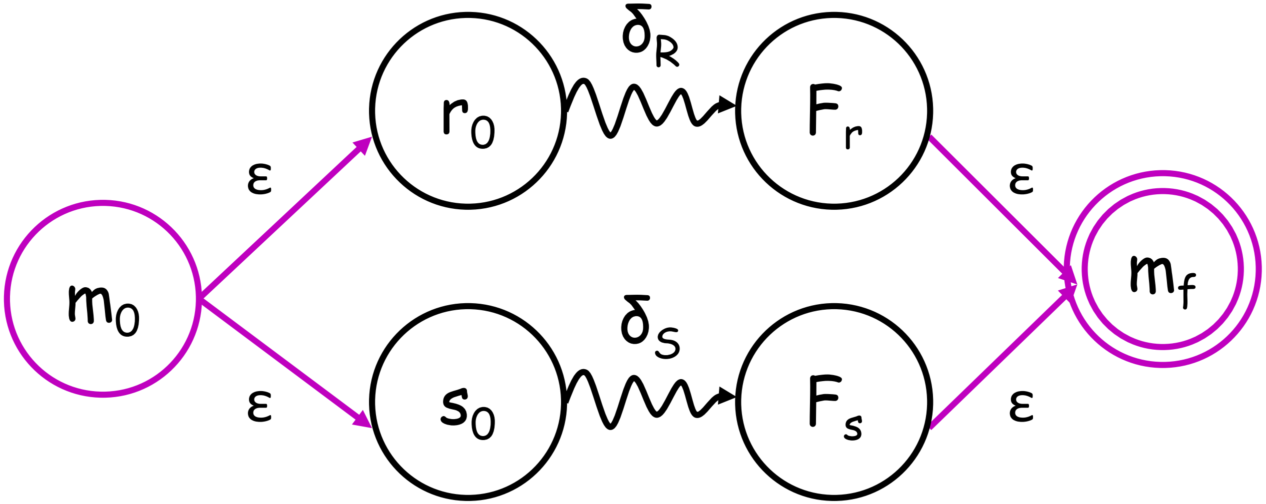
- for RE R with $M_R = \{\Sigma, s_R, \delta_R, r_0, F_r\}$ and
  RE S with $M_s = \{\Sigma, s_S, \delta_S, s_0, F_s\}$



- $M_{R|S} = \{\Sigma, s_R \cup s_s \cup \{m_0 , m_f\}, \delta_{R|S}, m_0, m_f\}$

R|S

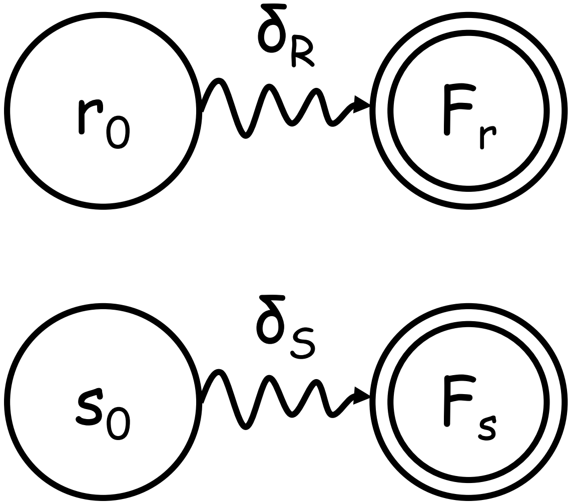
- for RE R with $M_R = \{\Sigma, s_R, \delta_R, r_0, F_r\}$ and RE S with $M_s = \{\Sigma, s_S, \delta_S, s_0, F_s\}$



- $M_{R|S} = \{\Sigma, s_R \cup s_S \cup \{m_0, m_f\}, \delta_{R|S}, m_0, m_f\}$

RS

- for RE R with $M_R = \{\Sigma, s_R, \delta_R, r_0, F_r\}$ and RE S with $M_s = \{\Sigma, s_S, \delta_S, s_0, F_s\}$



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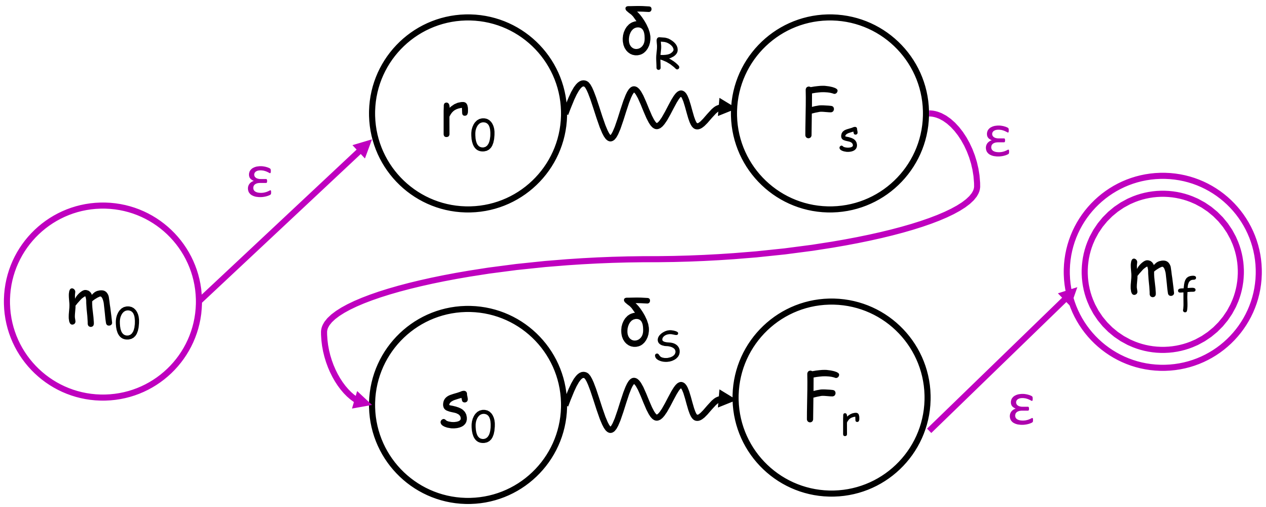
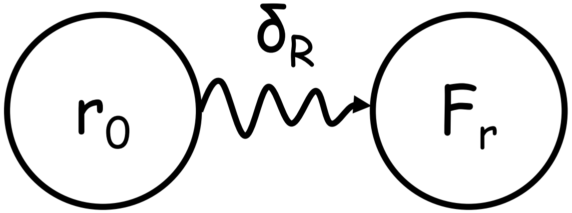


- $M_{RS} = \{\Sigma, s_R \cup s_S \cup \{m_0 , m_f\}, \delta_{RS}, m_0, m_f\}$

R*

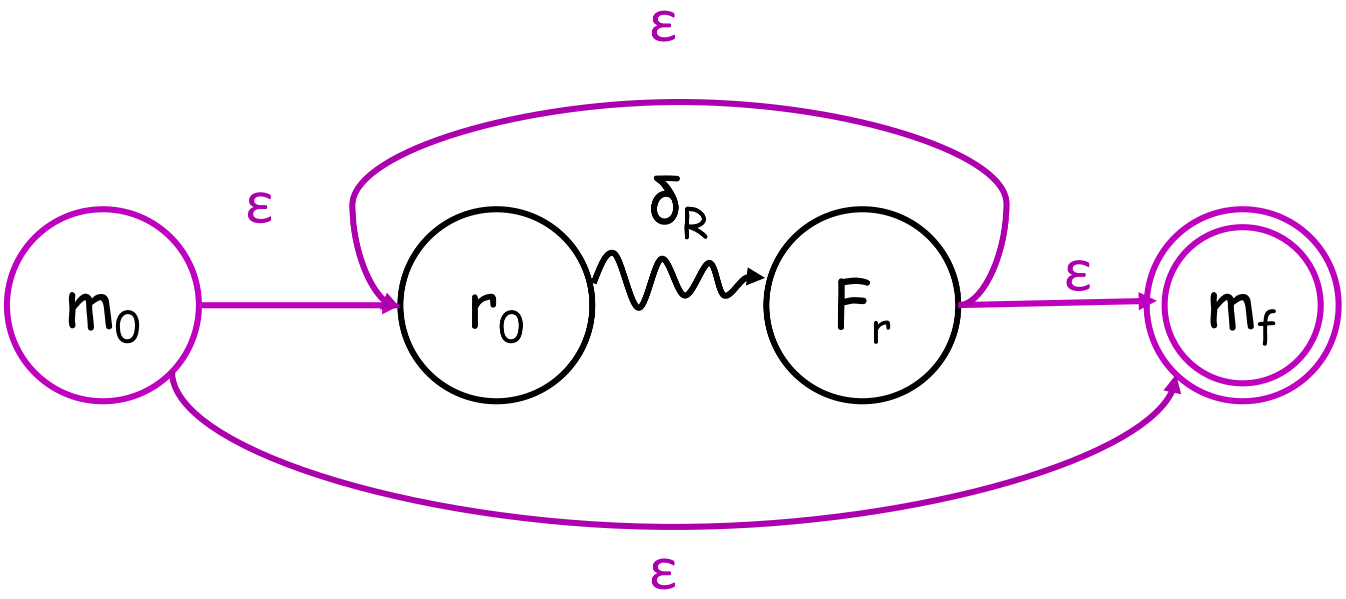
- for RE R with $M_R = \{\Sigma, s_R, \delta_R, r_0, F_r\}$



- $M_{R*} = \{\Sigma, s_R \cup \{m_0 , m_f\}, \delta_{R*}, m_0, m_f\}$

R*

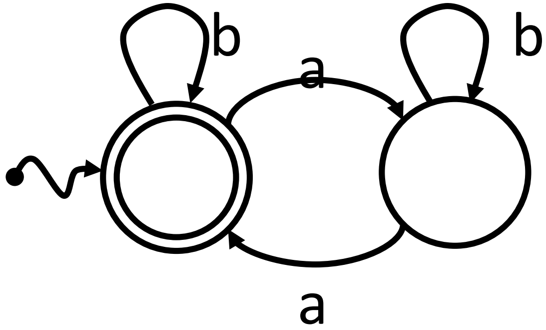
- for RE R with $M_R = \{\Sigma, s_R, \delta_R, r_0, F_r\}$



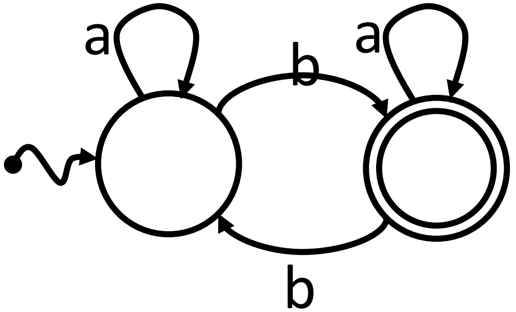
- $M_{R*} = \{\Sigma, s_R \cup \{m_0 , m_f\}, \delta_{R*}, m_0, m_f\}$

Example REs → FA

- even a’s b* (a b* a b*)*



- odd b’s a* b a* (b a* b a*)*

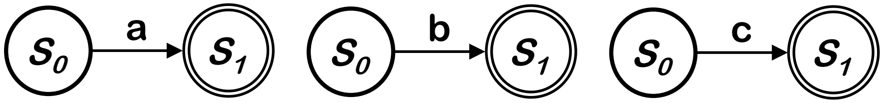


- even a’s or odd b’s
  $R^A \mid R^B$
- even a’s followed by odd b’s
  $R^A \; R^B$

Example of Thompson’s Construction

Let’s try a (b | c)*

1. a, b, & c



2. b | c

3. (b | c)*

Example of Thompson's Construction

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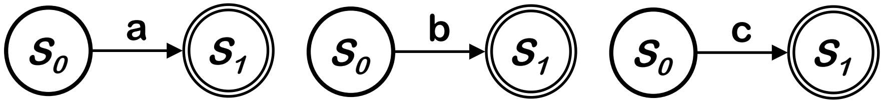
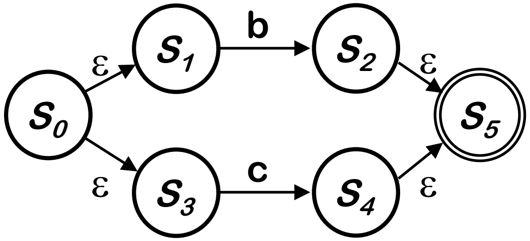
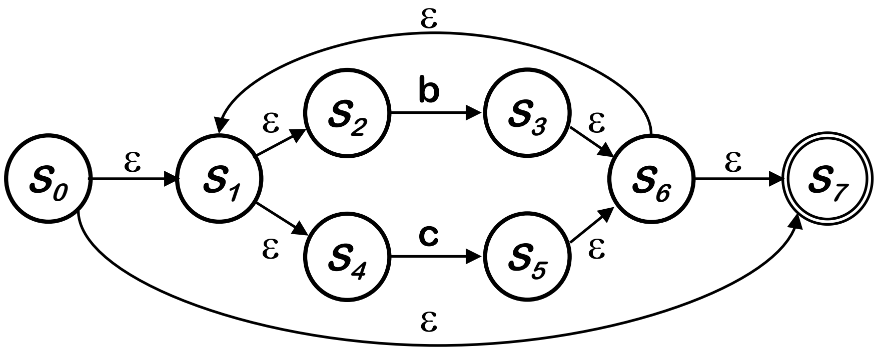


2. b | c

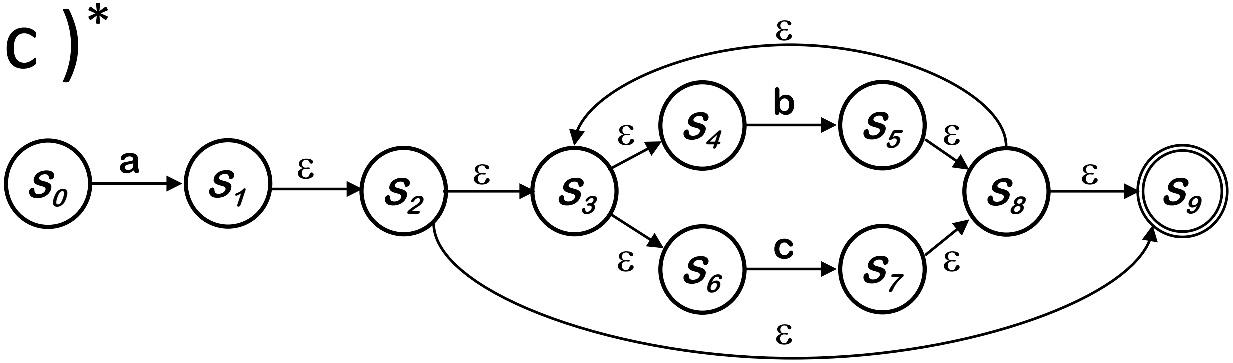


3. (b | c)*



Example of Thompson’s Construction

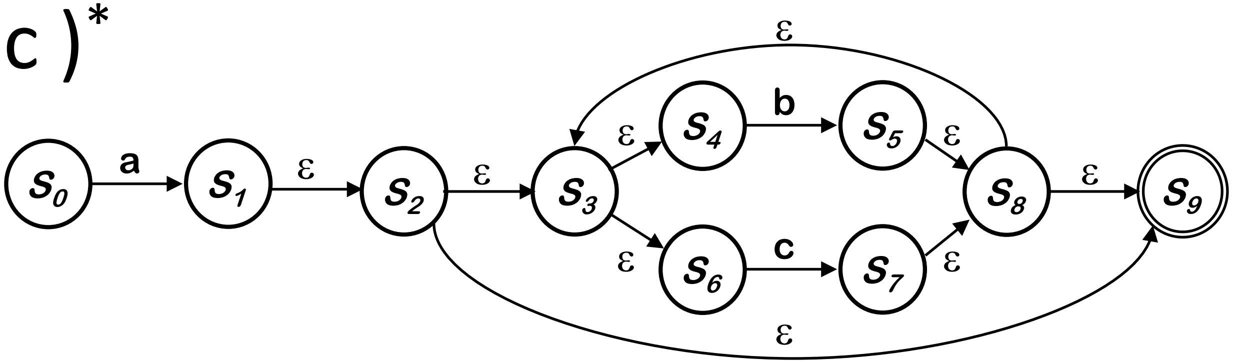
4. a (b | c)*



We could do a bit better. ☺

Example of Thompson’s Construction

4. a (b | c)*



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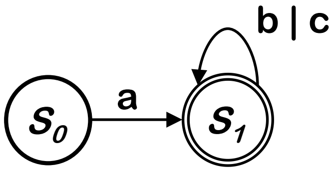

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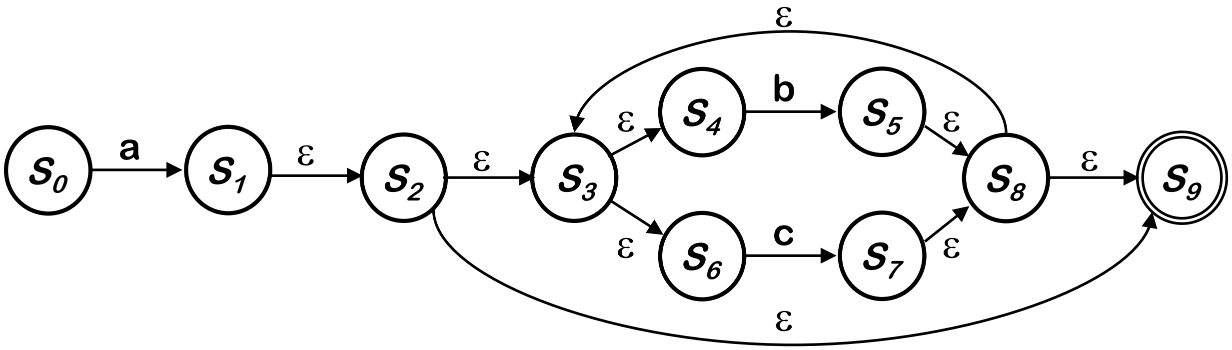
- RE → NFA
- **NFA → DFA**
- DFA → Minimized DFA
- Limits of Regular Languages

RE → NFA → DFA

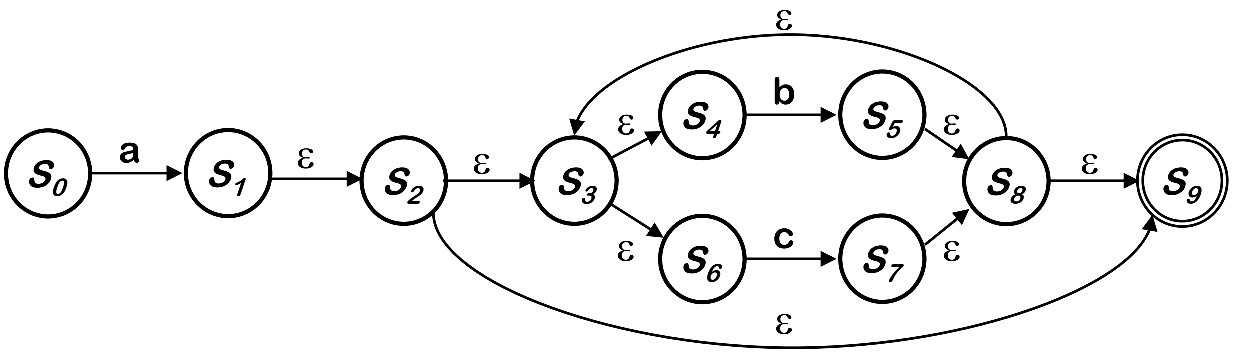
- Can’t directly execute Non-deterministic FA
- Need to convert NFA to DFA
- Essentially, we will build a DFA that **simulates** the NFA



- Key idea: Keep track of all possible NFA states we could be in at each step:
  the **set of all possible NFA states** becomes the DFA state

Subset construction

- start in state { s_0 }.
- For each edge create a set of all states that can be reached. Continue until done.
- All sets that contain an NFA accepting state are accepting.



Lets first deal with ε edges

- ε-closure: all states that can be reached only along ε-edges:
- Computing ε-closure(s) for s∈S:
  - initialize all ε-closure(s) = { s }
  - while some ε-closure(s) changed

```
        foreach s∈S:
            foreach q ∈ ε-closure(s) :
                ε-closure(s) = ε-closure(s) ∪ δ(q, ε)
```

- Terminates?

Subset Construction

- NFA: $\{\Sigma, Q, \delta, q_0, F\} \rightarrow$ DFA: $\{\Sigma, S, \Delta, s_0, F'\}$

```
$s_0 \leftarrow \varepsilon$-closure($q_0$)
while $\exists$ unmarked s $\in$ S:
    mark s
    foreach a $\in \Sigma$
        t $\leftarrow \varepsilon$-closure(Move(s, a))
        if t $\notin$ S:
            add t to S
        $\Delta$ (s,a) $\leftarrow$ t
```

Subset Construction

- NFA: $\{\Sigma, Q, \delta, q_0, F\} \rightarrow$ DFA: $\{\Sigma, S, \Delta, s_0, F'\}$

Move(s, a)
the set of states
reachable from s by a

```
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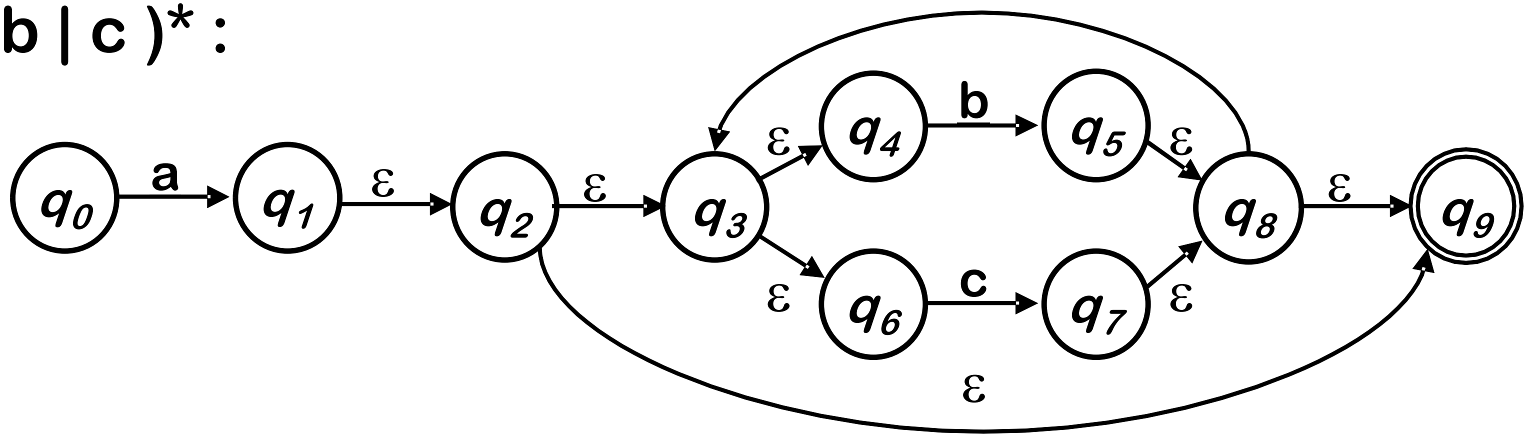
Why does this terminate?

Subset Construction

- NFA: $\{\Sigma, Q, \delta, q_0, F\} \rightarrow$ DFA: $\{\Sigma, S, \Delta, s_0, F'\}$
- Example of a fixed point computation
  - S is finite, at most ?
  - Always add to S, i.e., while loop is monotone
  - no duplicates in S
  - stop when S stops changing
- Other fixed point computations:
  - Constructing LR(1) items
  - Many Dataflow analysis (e.g., liveness)

example of subset construction

a (b | c)* :

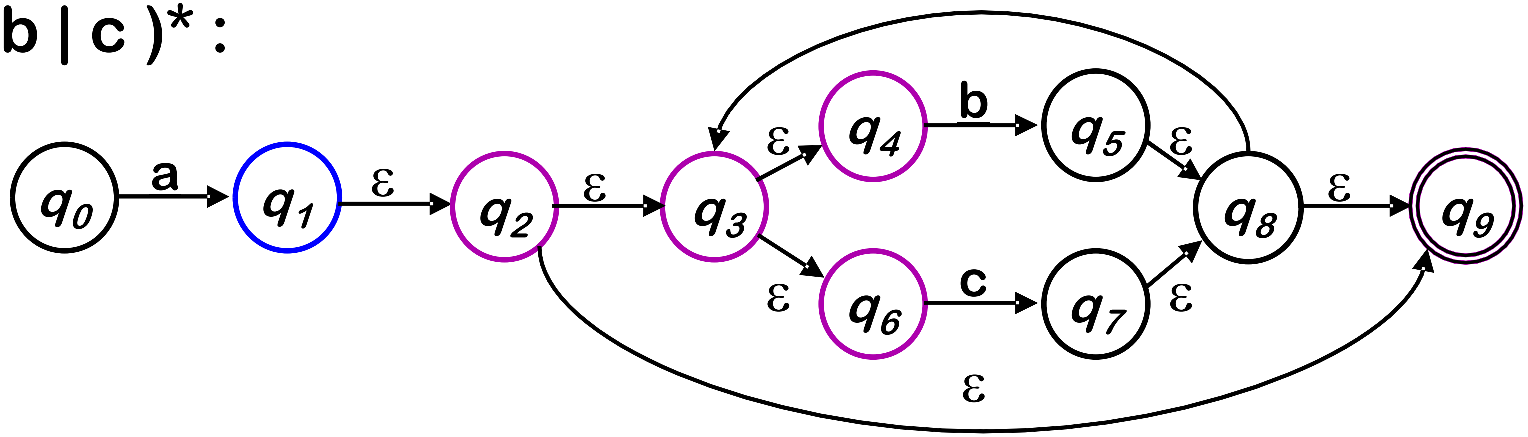


DFA States	NFA States	a	b	c
s_0	0			

Move(s_0,a)?

example of subset construction

a (b | c)* :



DFA States	NFA States	a	b	c
s_0	0	1, 2, 3, 4, 6, 9	-	-
s_1				

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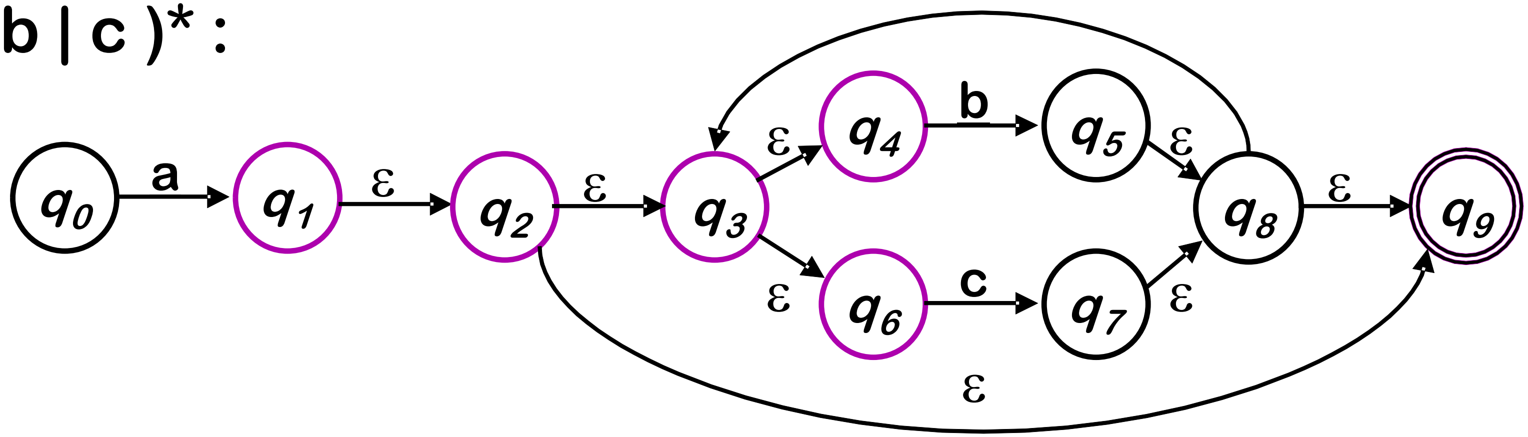
example of subset construction

a (b | c)* :

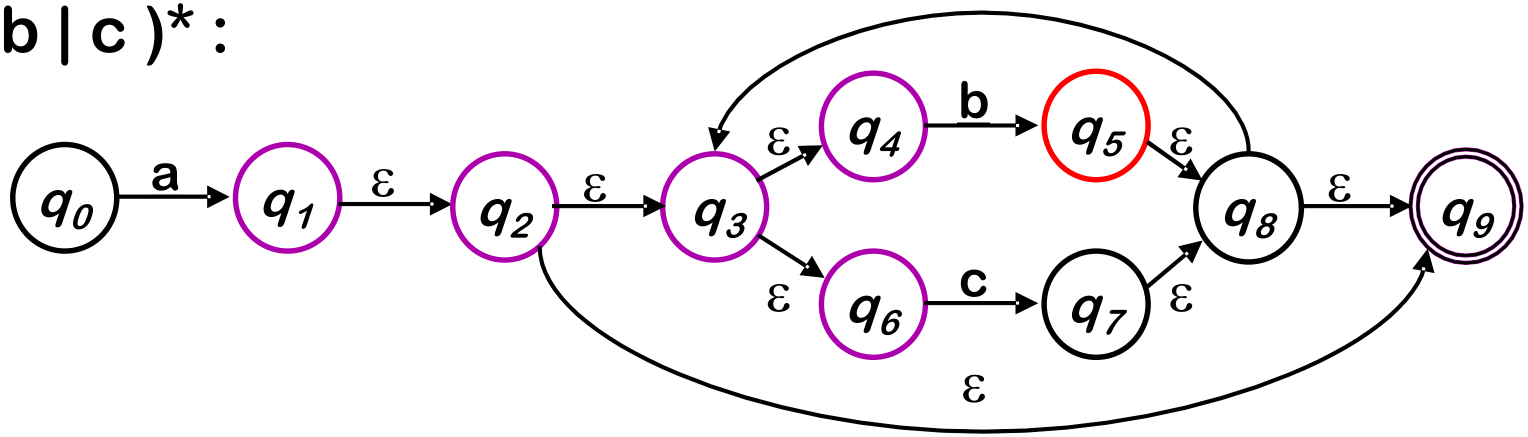


DFA States	NFA States	a	b	c
s_0	0	1, 2, 3, 4, 6, 9	-	-
s_1	1, 2, 3, 4, 6, 9	-		

b?

example of subset construction

a (b | c)* :

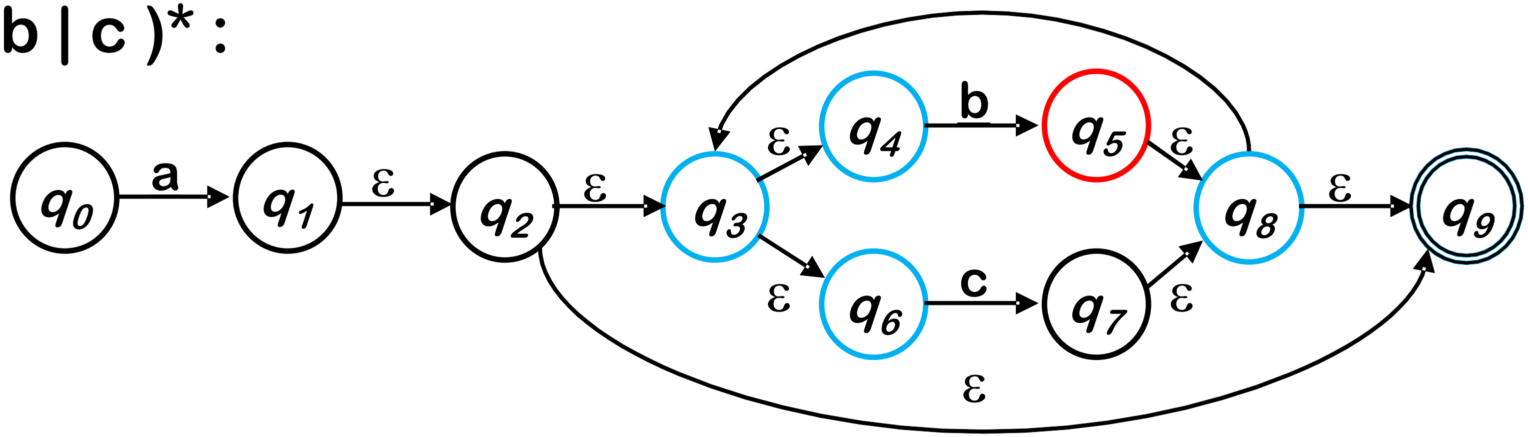


DFA States	NFA States	a	b	c
s_0	0	1, 2, 3, 4, 6, 9	-	-
s_1	1, 2, 3, 4, 6, 9	-	5	

ε-closure?

example of subset construction

a (b | c)* :

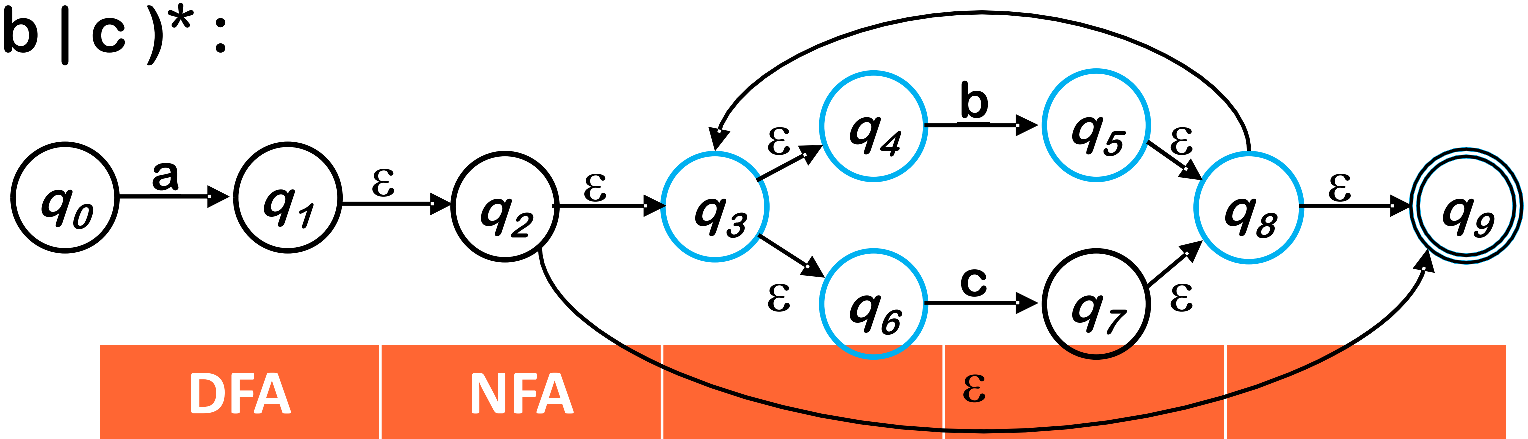


DFA States	NFA States	a	b	c
s_0	0	1, 2, 3, 4, 6, 9	-	-
s_1	1, 2, 3, 4, 6, 9	-	5, 3, 4, 6, 8, 9	

c?

example of subset construction

a (b | c)* :



DFA States	NFA States	a	b	c
s_0	0	1, 2, 3, 4, 6, 9	-	-
s_1	1, 2, 3, 4, 6, 9	-	5, 3, 4, 6, 8, 9	7, 3, 4, 6, 8, 9
s_2	5, 3, 4, 6, 8, 9			
s_3	7, 3, 4, 6, 8, 9			

s_2,a?

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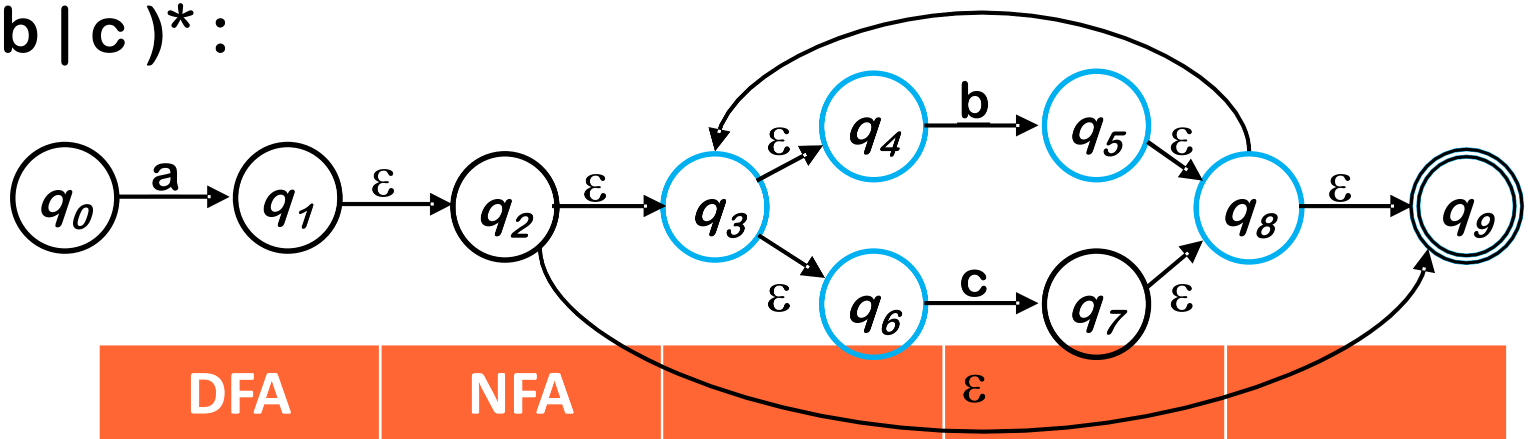
example of subset construction

a (b | c)* :

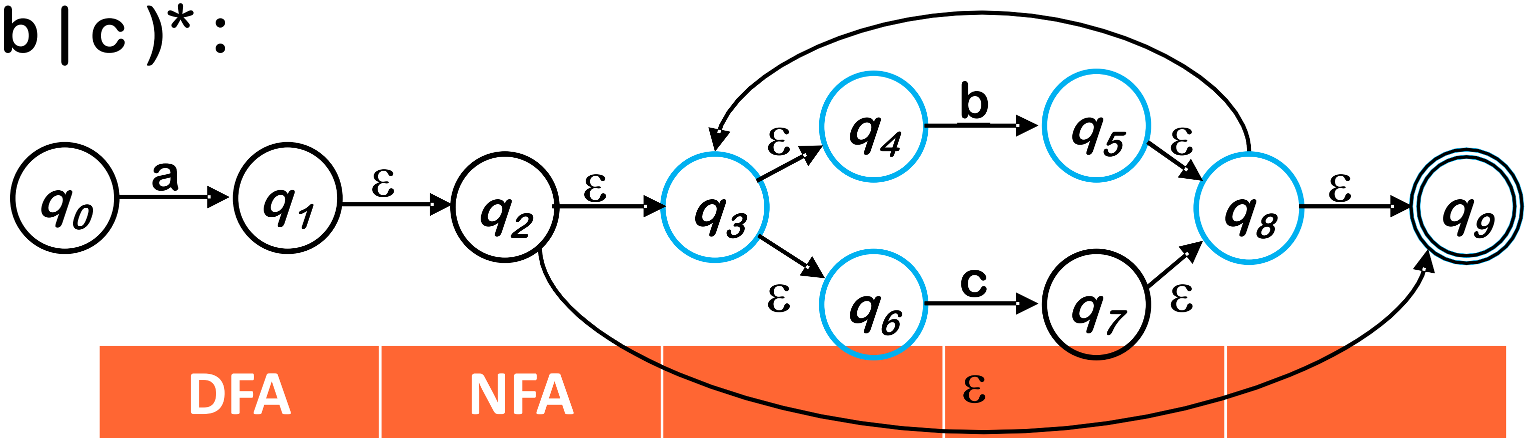


DFA States	NFA States	a	b	c
s_0	0	1, 2, 3, 4, 6, 9	-	-
s_1	1, 2, 3, 4, 6, 9	-	5, 3, 4, 6, 8, 9	7, 3, 4, 6, 8, 9
s_2	5, 3, 4, 6, 8, 9	-		
s_3	7, 3, 4, 6, 8, 9			

s_2,b?

example of subset construction

a (b | c)* :

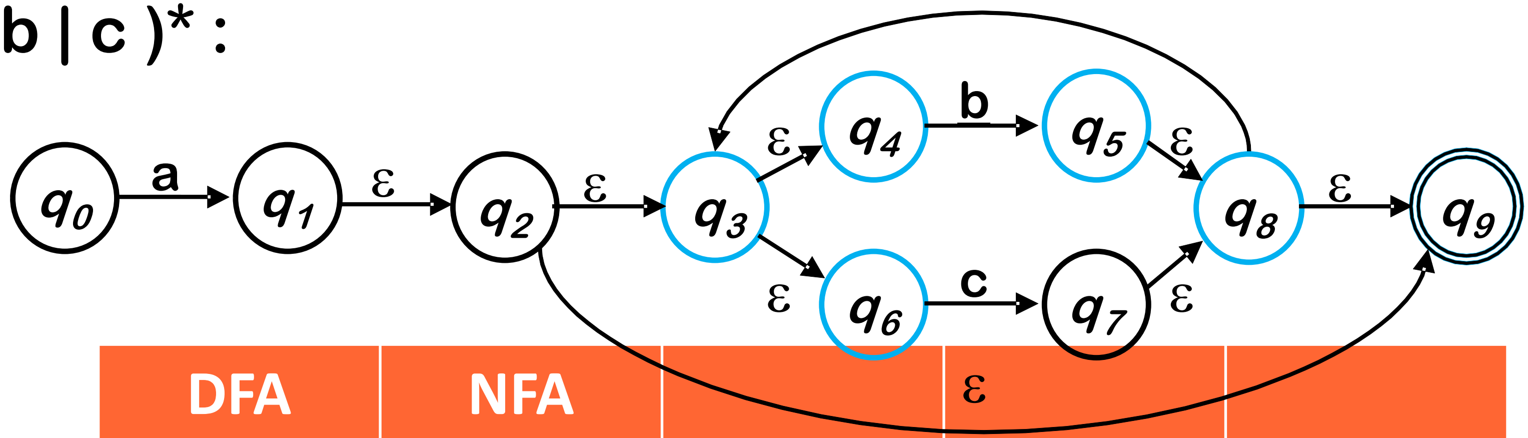


DFA States	NFA States	a	b	c
s_0	0	1, 2, 3, 4, 6, 9	-	-
s_1	1, 2, 3, 4, 6, 9	-	5, 3, 4, 6, 8, 9	7, 3, 4, 6, 8, 9
s_2	5, 3, 4, 6, 8, 9	-	s_2	
s_3	7, 3, 4, 6, 8, 9			

s_2,c?

example of subset construction

a (b | c)* :

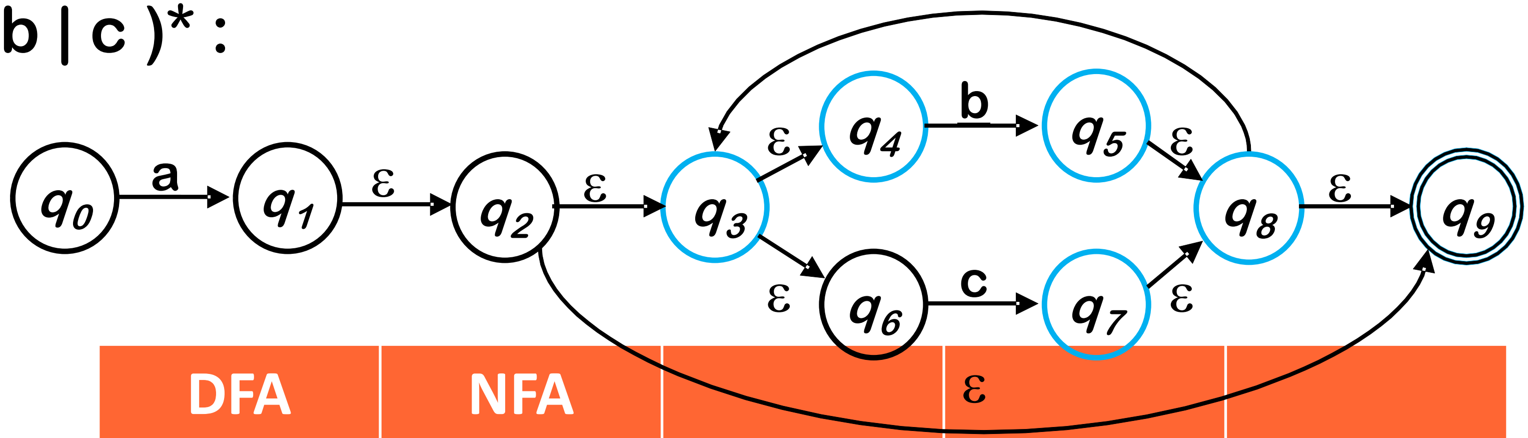


DFA States	NFA States	a	b	c
s_0	0	1, 2, 3, 4, 6, 9	-	-
s_1	1, 2, 3, 4, 6, 9	-	5, 3, 4, 6, 8, 9	7, 3, 4, 6, 8, 9
s_2	5, 3, 4, 6, 8, 9	-	s_2	s_3
s_3	7, 3, 4, 6, 8, 9			

rest?

example of subset construction

a (b | c)* :

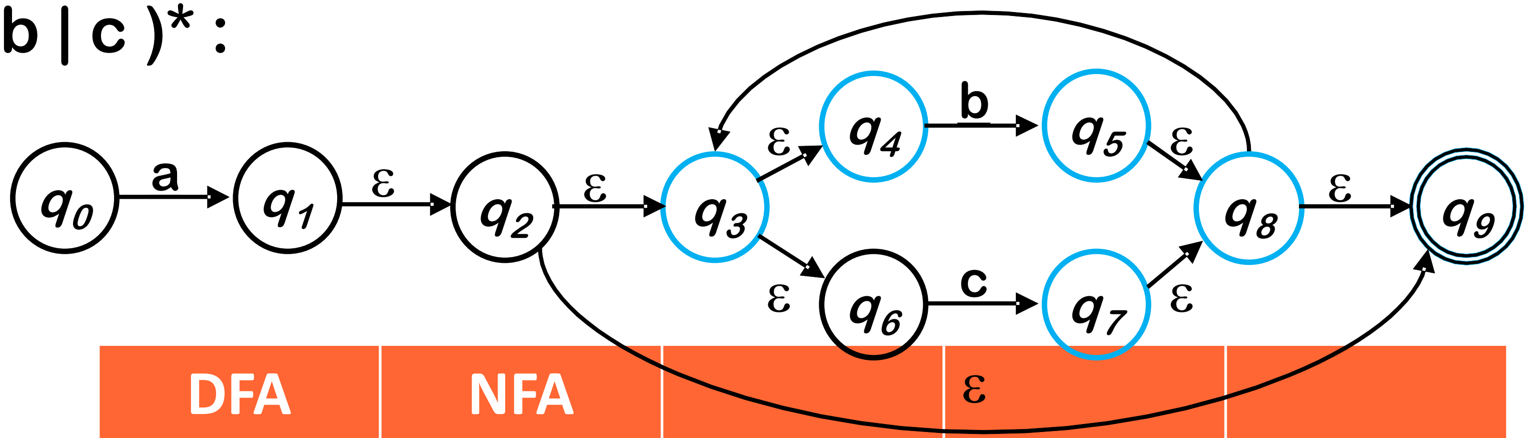


DFA States	NFA States	a	b	c
s_0	0	1, 2, 3, 4, 6, 9	-	-
s_1	1, 2, 3, 4, 6, 9	-	5, 3, 4, 6, 8, 9	7, 3, 4, 6, 8, 9
s_2	5, 3, 4, 6, 8, 9	-	s_2	s_3
s_3	7, 3, 4, 6, 8, 9	-	s_2	s_3

Final?

example of subset construction

a (b | c)* :

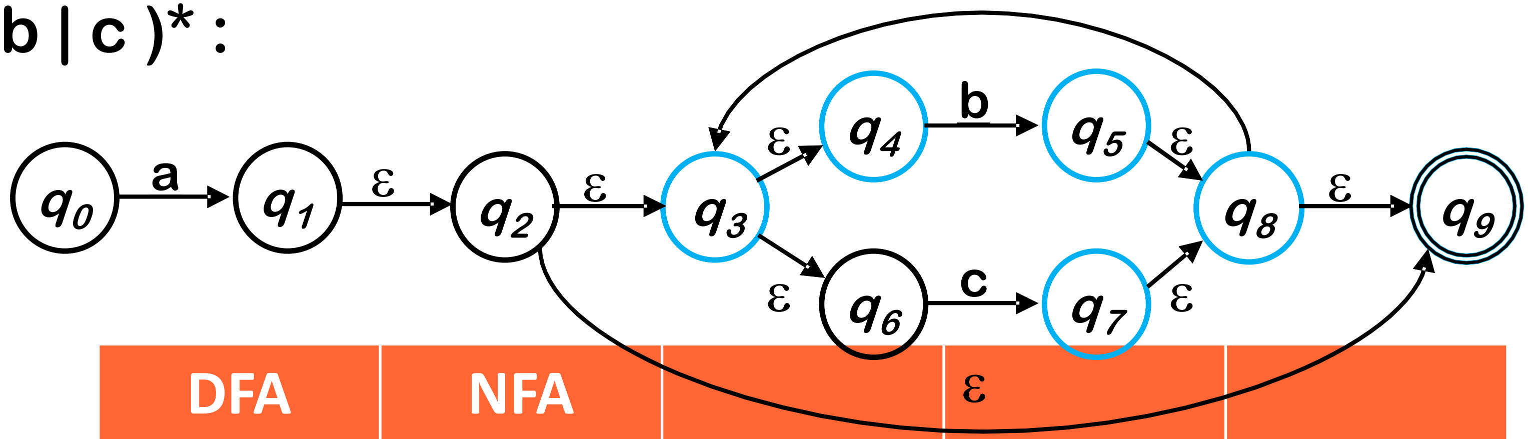


DFA States	NFA States	a	b	c
s_0	0	1, 2, 3, 4, 6, 9	-	-
s_1	1, 2, 3, 4, 6, **9**	-	5, 3, 4, 6, 8, 9	7, 3, 4, 6, 8, 9
s_2	5, 3, 4, 6, 8, **9**	-	s_2	s_3
s_3	7, 3, 4, 6, 8, **9**	-	s_2	s_3

Final?

example of subset construction

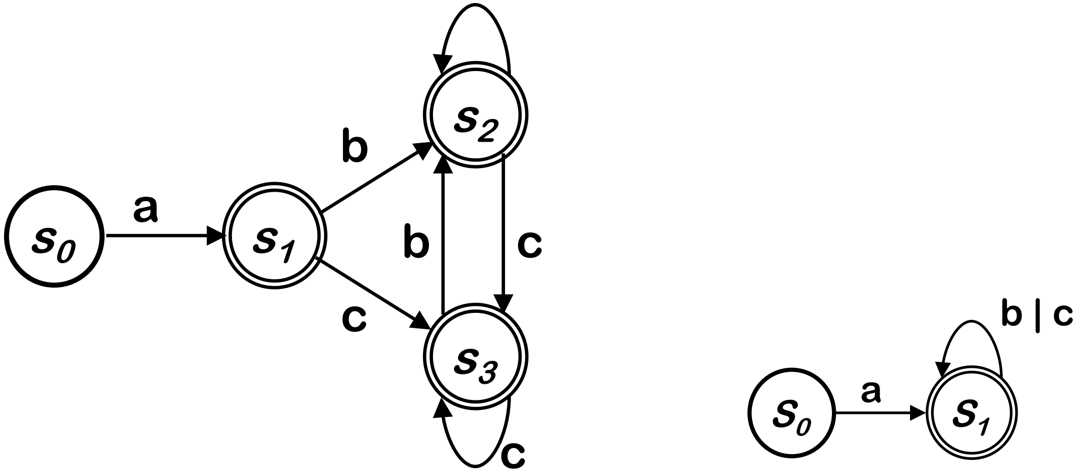
a (b | c)* :



DFA States	NFA States	a	b	c
s_0	0	1, 2, 3, 4, 6, 9	-	-
s_1	1, 2, 3, 4, 6, **9**	-	5, 3, 4, 6, 8, 9	7, 3, 4, 6, 8, 9
s_2	5, 3, 4, 6, 8, **9**	-	s_2	s_3
s_3	7, 3, 4, 6, 8, **9**	-	s_2	s_3

example of subset construction

a (b | c)* :



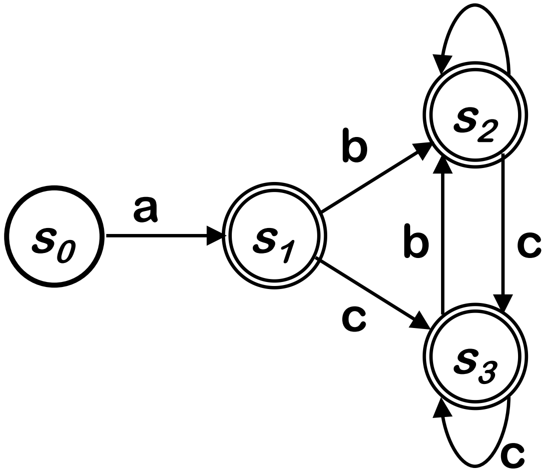
DFA States	NFA States	a	b	c
s_0	0	1, 2, 3, 4, 6, 9	-	-
s_1	1, 2, 3, 4, 6, **9**	-	5, 3, 4, 6, 8, 9	7, 3, 4, 6, 8, 9
s_2	5, 3, 4, 6, 8, **9**	-	s_2	s_3
s_3	7, 3, 4, 6, 8, **9**	-	s_2	s_3

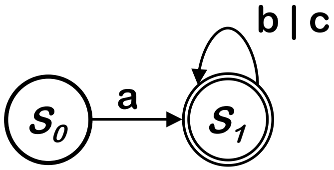
Today – part 1

- Lexing
- Flex & other scanner generators
- Regular Expressions
- Finite Automata
- RE → NFA
- NFA → DFA
- **DFA → Minimized DFA**
- Limits of Regular Languages

DFA Minimization

- Partition states into equivalent sets
- Two states are equivalent iff:
  - paths entering them are the same
  - $\forall$ a $\in \Sigma$, transitions lead to equivalent states
- transition on a to different sets $\Rightarrow$ different states.





DFA Minimization

- Plan:
  - start with maximal sets: { Q } and { Q – F }
  - partition sets for each a $\in\Sigma$ until no change
  - paritions become new states of minimized DFA

- Partitioning a set on "α"
  - Assume q_a, & $q_b \in$ s, and $\delta(q_a,\alpha) = q_x$ & $\delta(q_b, \alpha) = q_y$
  - If q_x & q_y are not in the same set, then s must be split (q_a has transition on α, q_b does not $\Rightarrow$ α splits s)
- One state in the final DFA cannot have two transitions on α

DFA Minimization

```
$P \leftarrow \{ F, \{Q-F\}\}$
while ( P is still changing)
    $T \leftarrow \{ \}$
    for each set $S \in P$
        for each $\alpha \in \Sigma$
            partition S by $\alpha$ into $S_1, S_2, \ldots, S_k$
            $T \leftarrow T \cup S_1 \cup S_2 \cup \ldots \cup S_k$
    if $T \neq P$ then
        $P \leftarrow T$
```

DFA Minimization

```
P $\leftarrow$ { F, {Q-F}}
while ( P is still changing)
    T $\leftarrow$ { }
    for each set S $\in$ P
        for each $\alpha \in \Sigma$
            partition S by $\alpha$ into $S_1, S_2, \ldots, S_k$
            T $\leftarrow$ T $\cup S_1 \cup S_2 \cup \ldots \cup S_k$
    if T $\neq$ P then
        P $\leftarrow$ T
```

Another Fixed Point Alg

Terminates:

- maximum of $2^{|Q|}$ sets
- Always adding to P
- Never combining sets in P

Initial partition ensures that final states remain final.

Hopcroft’s worklist algorithm is efficient.

Today – part 1

- Lexing
- Flex & other scanner generators
- Regular Expressions
- Finite Automata
- RE → NFA
- NFA → DFA
- DFA → Minimized DFA
- **Limits of Regular Languages**

Regular Languages

- Regular Expressions are great
  - concise notation
  - automatic scanner generation
  - lots of useful languages
- But, …
  - Not all languages are regular
    - Context Free Languages
    - Context Sensitive Languages
  - Even simple things like balanced parenthesis, e.g., L = { A^kB^k } (or nested comments!)
  - RL can't count

Not all Scanning is easy

- Language design should start with lexemes
  - My favorite example from PL/I

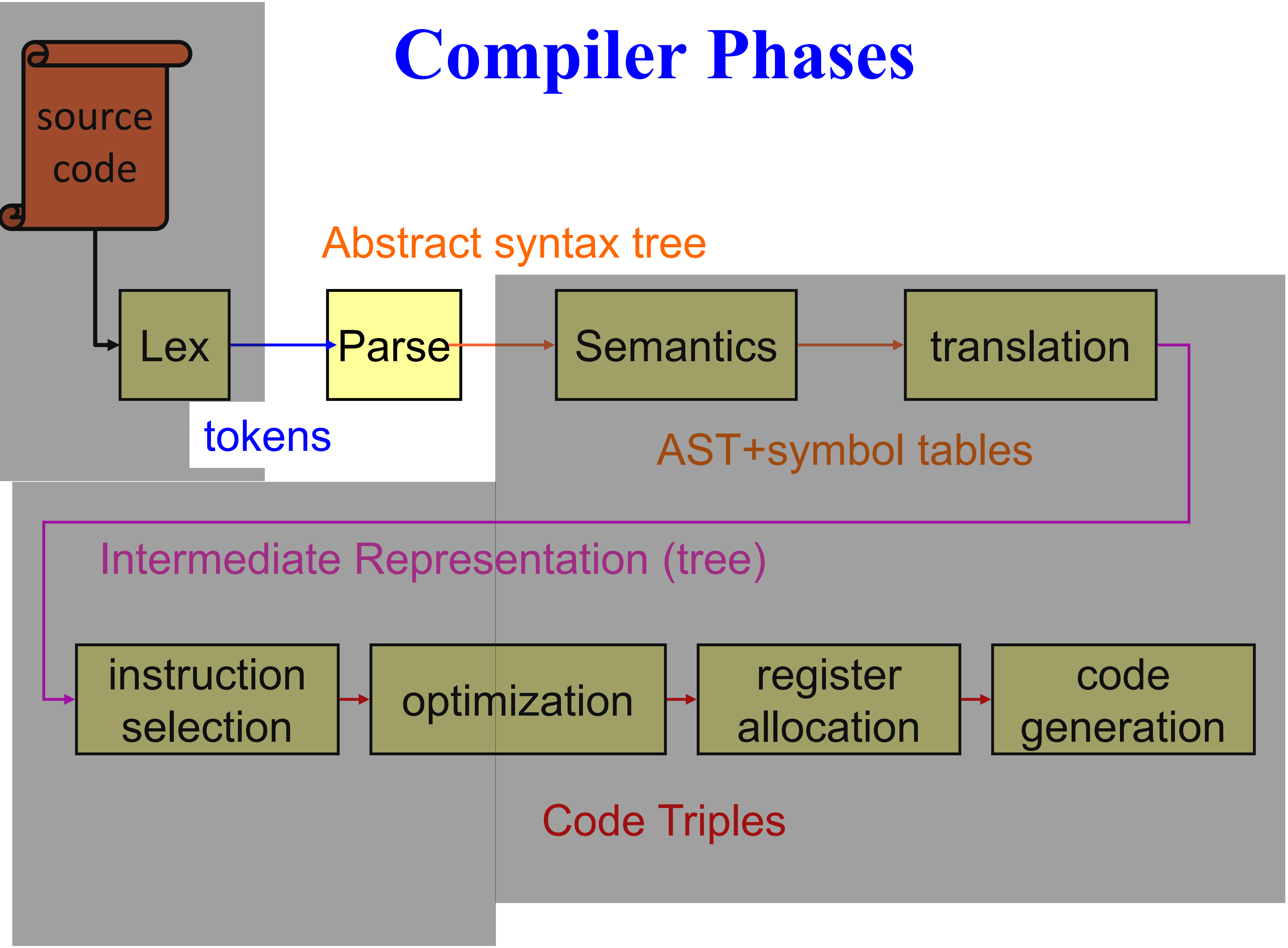
    `if then then then = else; else else = then`
- blanks not important in Fortran
- nested comments in C
- limited identifier lengths in Fortran

Today – part 2

Parsing

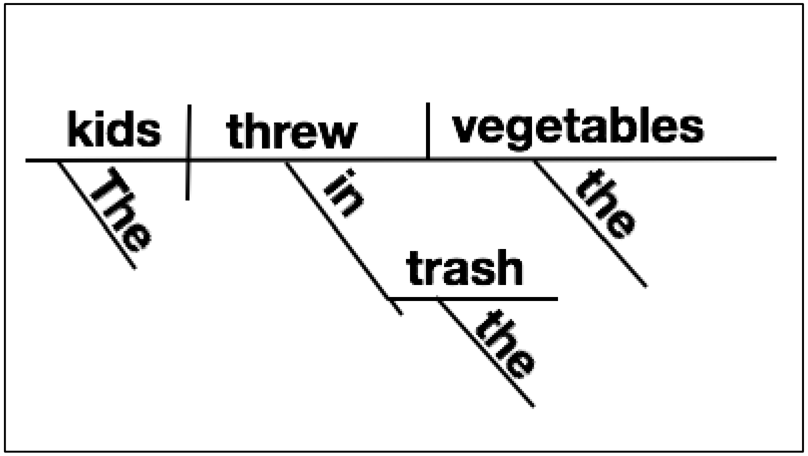
- Languages and Grammars
- Context Free Grammars
- Derivations & Parse Trees
- Ambiguity
- Top-down parsers
- FIRST, FOLLOW, and NULLABLE
- Bottom-up parsers

Compiler Phases



Languages

- Compiler translates from sequence of characters to an executable.
- A series of language transformations
- lexing: characters → tokens
- parsing: tokens → “sentences”



Languages

- Compiler translates from sequence of characters to an executable.
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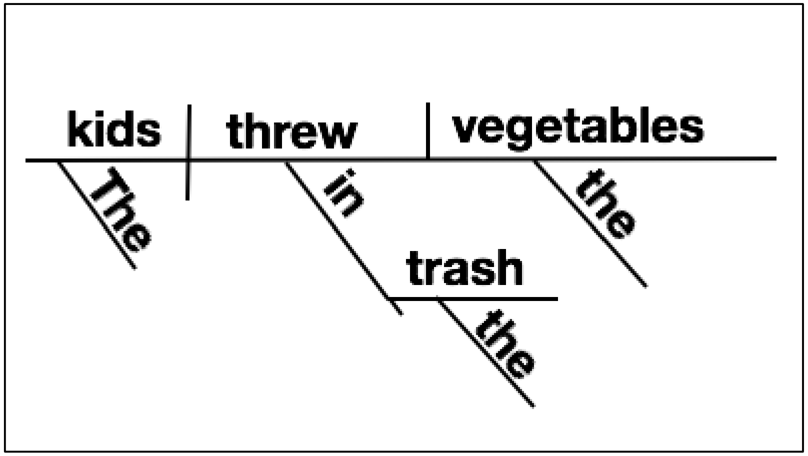

Grammars and Languages

- A grammar, G, recognizes a language, L(G)
  - Σ set of terminal symbols
  - A set of non-terminals
  - S the start symbol, a non-terminal
  - P a set of productions
- Usually,
  - $\alpha, \beta, \gamma, \ldots$ strings of terminals and/or non-terminals
  - A, B, C, … are non-terminals
  - a, b, c, … are terminals
- General form of a production is: $\alpha \to \beta$

Derivation

- A sequence of applying productions starting with S and ending with *w*

$$S \rightarrow \gamma_1 \rightarrow \gamma_2 \dots \rightarrow \gamma_{n-1} \rightarrow w$$

$$S \rightarrow^* w$$

- *L(G) are all the w that can be derived from S*

→ ε

Regular Grammar (NFA)

- Regular expressions and NFAs can be described by a regular grammar
- E.G., a*bc*

S → aS
S → bA
A → ε
A → cA

- An example derivation of aabc:

S → aS

→ ε

Regular Grammar (NFA)

- Regular expressions and NFAs can be described by a regular grammar
- E.G., a*bc*

  S → aS
  S → bA
  A → ε
  A → cA

- An example derivation of aabc:

  S → aS→ aaS

→ ε

Regular Grammar (NFA)

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S → bA
A → ε
A → cA

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S → aS→ aaS→ aabA

→ ε

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A → cA

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→ ε

Regular Grammar (NFA)

- Regular expressions and NFAs can be described by a regular grammar
- E.G., a*bc*

S → aS
S → bA
A → ε
A → cA

- An example derivation of aabc:

S → aS→ aaS→ aabA→ aabcA → aabc

→ ε

Regular Grammar (NFA)

- Regular expressions and NFAs can be described by a regular grammar
- E.G., a*bc*

S → aS
S → bA
A → ε
A → cA

- Above is a right-regular grammar
- All rules are of form:

A → a
A → aB
A → ε

→ ε

Regular Grammar (NFA)

- Regular expressions and NFAs can be described by a regular grammar
- right regular grammar:
  $A \rightarrow a$
  $A \rightarrow aB$
  $A \rightarrow \varepsilon$
- left regular grammar:
  $A \rightarrow a$
  $A \rightarrow Ba$
  $A \rightarrow \varepsilon$
- Regular grammars are either right-regular or left-regular.

Expressiveness

- Restrictions on production rules limit expressiveness of grammars.
- No restrictions allow a grammar to recognize all recursively enumerable languages
- A bit too expressive for our uses ☺
- Regular grammars cannot recognize $a^n b^n$
- We need something more expressive

Chomsky Hierarchy

Class	Language	Automaton	Form	"word" problem	Example
0	Recursively Enumerable	Turing Machine	any	undecidable	Post's Corresp. problem
1	Context Sensitive	Linear-Bounded TM	$\alpha A\beta\rightarrow\alpha\gamma\beta$	PSPACE-complete	$a^nb^nc^n$
2	Context Free	Pushdown Automata	$A\rightarrow\alpha$	cubic	a^nb^n
3	Regular	NFA	$A\rightarrow a$ $A\rightarrow aB$	linear	a^*b^*

Today – part 2

- Languages and Grammars
- Context Free Grammars
- Derivations & Parse Trees
- Ambiguity
- Top-down parsers
- FIRST, FOLLOW, and NULLABLE
- Bottom-up parsers

Context-Free Grammar

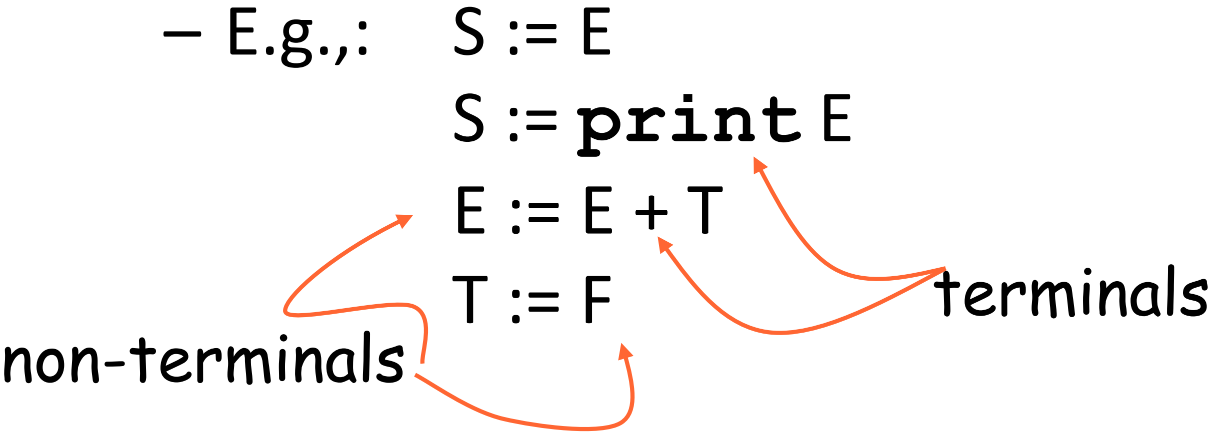
- A context-free grammar, G, is described by:
  - Σ, a set of terminals (which are just the set of possible tokens from the lexer)
    e.g., **`if`**, **`then`**, **`while`**, **`id`**, **`int`**, **`string`**, …
  - A, a set of non-terminals.
    Non-terminals are syntactic variables which define sets of strings in the language
    e.g., stmt, expr, term, factor, vardecl, …
  - S
  - P

Context-Free Grammar

- A context-free grammar, G, is described by:
  - Σ, a set of terminals …
  - A, a set of non-terminals.
  - S, $S \in A$, the start symbol
    The set of strings derived from S are the valid string in the language.
  - P, set of productions that specify how terminals and non-terminals combine to form strings in the language
    a production, p, has the form: $A \rightarrow \alpha$

Context-Free Grammar

- A context-free grammar, G, is described by:
  - Σ, a set of terminals …
  - A, a set of non-terminals.
  - S, S ∈ A, the start symbol
  - P, set of productions …
    a production, p, has the form: : A→ α
  - E.g.,:



S := E
S := **`print`** E
E := E + T
T := F

What makes a grammar CF?

- Only one NT on left-hand side → context-free
- What makes a grammar context-sensitive?
- $\alpha A\beta \rightarrow \alpha\gamma\beta$ where
  - α or β may be empty,
  - but γ is not-empty
- Are context-sensitive grammars useful for compiler writers?

Simple Grammar of Expressions

S := Exp

Exp := Exp + Exp

Exp := Exp - Exp

Exp := Exp * Exp

Exp := Exp / Exp

Exp := **`id`**

Exp := **`int`**

Describes a language of expressions. e.g.: 2+3*x

Derivations

- A sequence of steps in which a non-terminal is replaced by its right-hand side.

There are possibly many derivations determined by the NT chosen to expand.

1 S := Exp

2 Ex

3 Ex

4 Exp:= Exp * Exp

5 Exp:= Exp / Exp

6 Exp:= **id**

7 Exp:= **int**

S

xp

by 6 ⇒ Exp * $\texttt{id}_x$

by 2 ⇒ Exp + Exp * $\texttt{id}_x$

by 7 ⇒ $\texttt{int}_2$ + Exp * $\texttt{id}_x$

by 7 ⇒ $\texttt{int}_2$ + $\texttt{int}_3$ * $\texttt{id}_x$

Leftmost Derivations

- Leftmost derivation: leftmost NT always chosen

1 S := Exp

2 Exp:= Exp + Exp

3 Exp:= Exp - Exp

4 Exp:= Exp * Exp

5 Exp:= Exp / Exp

6 Exp:= `id`

7 Exp:= `int`

S

by 1 $\Rightarrow$ Exp

by 4 $\Rightarrow$ Exp * Exp

by 2 $\Rightarrow$ Exp + Exp * Exp

by 7 $\Rightarrow$ $\texttt{int}_2$ + Exp * Exp

by 7 $\Rightarrow$ $\texttt{int}_2$ + $\texttt{int}_3$ * Exp

by 6 $\Rightarrow$ $\texttt{int}_2$ + $\texttt{int}_3$ * $\texttt{id}_x$

Rightmost Derivations

- Rightmost derivation: rightmost NT always chosen

1 S := Exp
2 Exp:= Exp + Exp
3 Exp:= Exp - Exp
4 Exp:= Exp * Exp
5 Exp:= Exp / Exp
6 Exp:= **id**
7 Exp:= **int**

S
by 1 ⇒ Exp
by 4 ⇒ Exp * Exp
by 6 ⇒ Exp * $\mathbf{id}_x$
by 2 ⇒ Exp + Exp * $\mathbf{id}_x$
by 7 ⇒ Exp + $\mathbf{int}_3$ * $\mathbf{id}_x$
by 7 ⇒ $\mathbf{int}_2$ + $\mathbf{int}_3$ * $\mathbf{id}_x$

Parse Trees

- symbols in rhs are children of NT being rewritten

S

by 1 ⇒ Exp

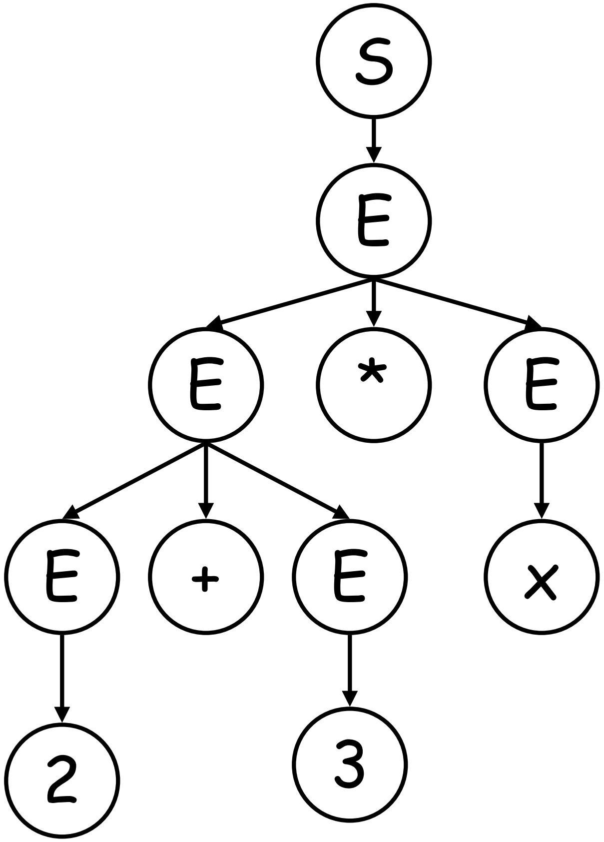
by 4 ⇒ Exp * Exp

by 2 ⇒ Exp + Exp * Exp

by 7 ⇒ $\texttt{int}_2$ + Exp * Exp

by 7 ⇒ $\texttt{int}_2$ + $\texttt{int}_3$ * Exp

by 6 ⇒ $\texttt{int}_2$ + $\texttt{int}_3$ * $\texttt{id}_x$



Parse Trees

- parse tree for rightmost derivation

S

by 1 ⇒ Exp

by 4 ⇒ Exp *

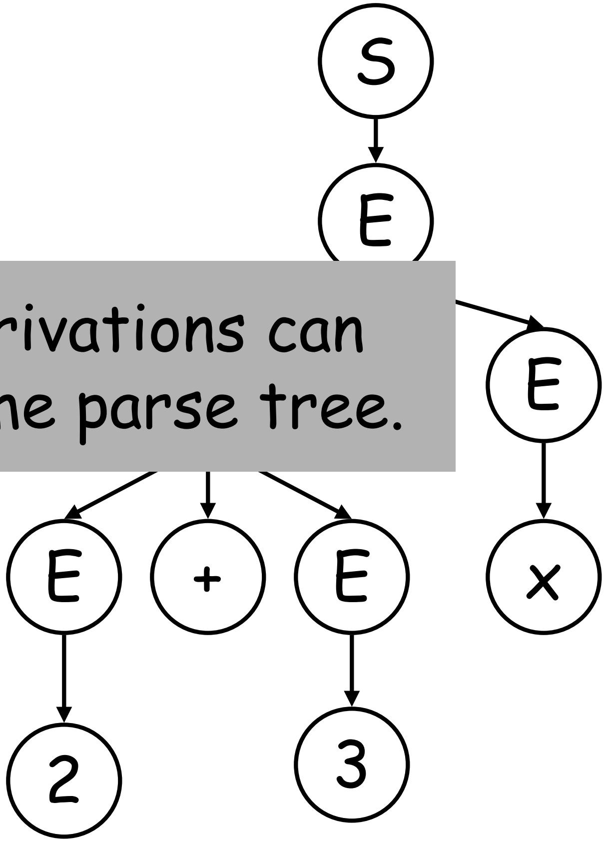
by 6 ⇒ Exp *

by 2 ⇒ Exp + Exp * $\mathtt{id}_x$

by 7 ⇒ Exp + $\mathtt{int}_3$ * $\mathtt{id}_x$

by 7 ⇒ $\mathtt{int}_2$ + $\mathtt{int}_3$ * $\mathtt{id}_x$

Different derivations can lead to the same parse tree.

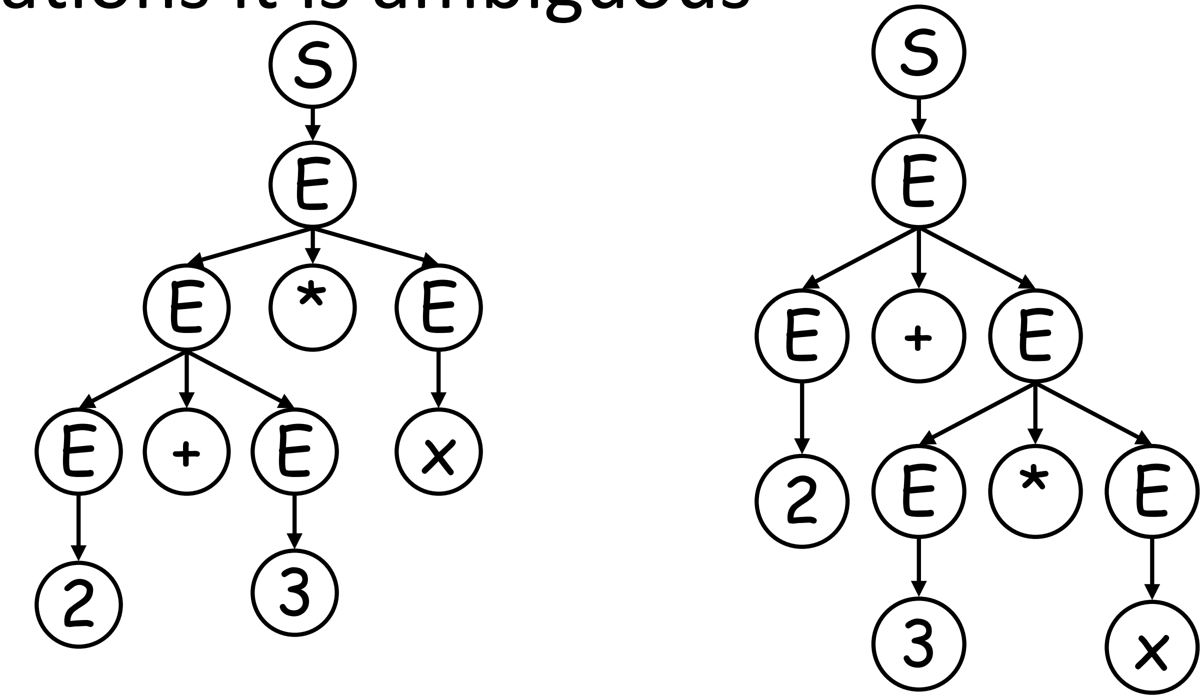


What about different parse trees for same sentence?

Ambiguous Grammars

What does ambiguity point out?

- A gra a sentence with >1 parse trees. or,
- If grammer has >1 leftmost (rightmost) derivations it is ambiguous



Converting Expression Grammar

- Adding precedence with more non-terminals
- One for each level of precedence:
  - (+, -) exp
  - (*, /) term
  - (**`id`**, **`int`**) factor
  - Make sure parse derives sentences that respect the precedence
  - Make sure that extra levels of precedence can be bypassed, i.e., “x” is still legal

A Better Exp Grammar

1 S := Exp
2 Exp := Exp + Term
3 Exp := Exp - Term
4 Exp := Term
5 Term := Term * Factor
6 Term := Term / Factor
7 Term := Factor
8 Factor := **id**
9 Factor := **int**

S
by 1 ⇒ Exp
by 2 ⇒ Exp + Term
by 4 ⇒ Term + Term
by 7 ⇒ Factor + Term
by 9 ⇒ $\mathbf{int}_2$ + Term
by 5 ⇒ $\mathbf{int}_2$ + Term * Factor
by 7 ⇒ $\mathbf{int}_2$ + Factor * Factor
by 9 ⇒ $\mathbf{int}_2$ + $\mathbf{int}_3$ * Factor
by 8 ⇒ $\mathbf{int}_2$ + $\mathbf{int}_3$ * $\mathbf{id}_x$

What is the parse tree?

Another Ambiguous Grammer

S := **`if`** E **`then`** S

| **`if`** E **`then`** S **`else`** S

| **`other`**

- What is the parse tree for:

  `if` E **`then`** **`if`** E **`then`** S **`else`** S?

- What is the language designers intention?
- Is there a context-free solution?

Dangling Else Grammar

S := matchedS
| unmatchedS
unmatchedS := **if** E **then** S
| if E then matchedS else unmatchedS
matchedS := **if** E **then** matchedS **else** matchedS
| **other**

- Is this clearer?
- What is parse tree for: **if** E **then** **if** E **then** S **else** S?

Parser generators provide a better way

Parsing a CFG

- Top-Down
  - start at root of parse-tree
  - pick a production and expand to match input
  - may require backtracking
  - if no backtracking required, predictive
- Bottom-up
  - start at leaves of tree
  - recognize valid prefixes of productions
  - consume input and change state to match
  - use stack to track state

Top-down Parsers

- Starts at root of parse tree and recursively expands children that match the input
- In general case, may require backtracking
- Such a parser uses recursive descent.
- When a grammar does not require backtracking a predictive parser can be built.

A Predictive Parser

```
S  :=  B S F
    |
B  :=  b
F  :=  f
```

Idea is for parser to do something besides recognize legal sentences.

```
S() {
        if match('b') -> B(); S(); F(); action();
        else return;
}
B() {   mustMatch('b');  action(); return;}
F() {   mustMatch('f'); action();  return;}
```

Top-Down parsing

- Start with root of tree, i.e., S
- Repeat until entire input matched:
  - pick a non-terminal, A, and pick a production $A \rightarrow \gamma$ that can match input, and expand tree
  - if no such rule applies, backtrack
- Key is obviously selecting the right production

Top-down for Exp Grammar

1 S := E
2 E := E + T
3 E := E - T
4 E := T
5 T := T * F
6 T := T / F
7 T := F
8 F := **id**
9 F := **int**

S | $\mathbf{int}_2$ - $\mathbf{int}_3$ * $\mathbf{id}_x$

by 1 ⇒ E | $\mathbf{int}_2$ - $\mathbf{int}_3$ * $\mathbf{id}_x$

Top-down for Exp Grammar

1 S := E
2 E := E + T
3 E := E - T
4 E := T
5 T := T * F
6 T := T / F
7 T := F
8 F := id
9 F := int

	S	| $\mathtt{int}_2 - \mathtt{int}_3 * \mathtt{id}_x$
by 1 ⇒	E	| $\mathtt{int}_2 - \mathtt{int}_3 * \mathtt{id}_x$
by 2 ⇒	E + T	| $\mathtt{int}_2 - \mathtt{int}_3 * \mathtt{id}_x$
by 4 ⇒	T + T	| $\mathtt{int}_2 - \mathtt{int}_3 * \mathtt{id}_x$
by 7 ⇒	F + T	| $\mathtt{int}_2 - \mathtt{int}_3 * \mathtt{id}_x$
by 9 ⇒	$\mathtt{int}_2$ + T	$\mathtt{int}_2$ | $- \mathtt{int}_3 * \mathtt{id}_x$

Must backtrack here!

Top-down for Exp Grammar

1 S := E
2 E := E + T
3 E := E - T
4 E := T
5 T := T * F
6 T := T / F
7 T := F
8 F := **id**
9 F := **int**

	S	▌**int**$_2$ - **int**$_3$ * **id**$_x$
by 1 ⇒	E	▌**int**$_2$ - **int**$_3$ * **id**$_x$
by 2 ⇒	E + T	▌**int**$_2$ - **int**$_3$ * **id**$_x$
by 4 ⇒	T + T	▌**int**$_2$ - **int**$_3$ * **id**$_x$
by 7 ⇒	F + T	▌**int**$_2$ - **int**$_3$ * **id**$_x$
by 9 ⇒	**int**$_2$ + T	**int**$_2$▌- **int**$_3$ * **id**$_x$
by 3 ⇒	E - T	▌**int**$_2$ - **int**$_3$ * **id**$_x$
by 4 ⇒	T - T	▌**int**$_2$ - **int**$_3$ * **id**$_x$
by 7 ⇒	F - T	▌**int**$_2$ - **int**$_3$ * **id**$_x$
by 9 ⇒	**int**$_2$ - T	**int**$_2$▌- **int**$_3$ * **id**$_x$
by 5 ⇒	**int**$_2$ - T * F	**int**$_2$ - ▌**int**$_3$ * **id**$_x$

Top-down for Exp Grammar

1 S := E
2 E := E + T
3 E := E - T
4 E := T
5 T := T * F
6 T := T / F
7 T := F
8 F := id
9 F := int

	S	▌int$_2$ - int$_3$ * id$_x$
by 1 ⇒	E	▌int$_2$ - int$_3$ * id$_x$
by 2 ⇒	E + T	▌int$_2$ - int$_3$ * id$_x$
by 4 ⇒	T + T	▌int$_2$ - int$_3$ * id$_x$
by 7 ⇒	F + T	▌int$_2$ - int$_3$ * id$_x$
by 9 ⇒	int$_2$ + T	int$_2$▌- int$_3$ * id$_x$
by 3 ⇒	E - T	▌int$_2$ - int$_3$ * id$_x$
by 4 ⇒	T - T	▌int$_2$ - int$_3$ * id$_x$
by 7 ⇒	F - T	▌int$_2$ - int$_3$ * id$_x$
by 9 ⇒	int$_2$ - T	int$_2$▌- int$_3$ * id$_x$
		nt$_2$ -▌int$_3$ * id$_x$

What kind of derivation is this parsing?

Top-down for Exp Grammar

1 S := E
2 E := E + T
3 E := E - T
4 E := T
5 T := T * F
6 T := T / F
7 T := F
8 F := id
9 F := int

	S	| $int_2 - int_3 * id_x$
by 1 ⇒	E	| $int_2 - int_3 * id_x$
by 2 ⇒	E + T	| $int_2 - int_3 * id_x$
by 2 ⇒	E + E + T	| $int_2 - int_3 * id_x$
by 2 ⇒	E + E + E + T	| $int_2 - int_3 * id_x$

Will not terminate! Why?

grammar is left-recursive

What should we do about it?

Eliminate left-recursion

Does this work?

```
1    S := E
2    E := E + T
3    E := E - T
4    E := T
5    T := T * F
6    T := T / F
7    T := F
8    F := id
9    F := int
```

```
1    S := E
2    E := T + E
3    E := T - E
4    E := T
5    T := F * T
6    T := F / T
7    T := F
8    F := id
9    F := int
```

It is right recursive, but also right associative!

Eliminating Left-Recursion

- Given 2 productions:

  $A := A\ \alpha \mid \beta$

  Where neither α nor β start with A

  (e.g., For example, E := E + T | T)

  α β

- Make it right-recursive:

  $$\boxed{\begin{array}{l} A := \beta\ R \\ R := \alpha\ R \\ \quad \mid \end{array}}$$

  R is right recursive

- Extends to general case.

Rewriting Exp Grammar

1 S := E
2 E := E + T
3 E := E - T
4 E := T
5 T := T * F
6 T := T / F
7 T := F
8 F := `id`
9 F := `int`

1 S := E
2' E' := + T E'
3' E' := - T E'
4' E' :=
5' T' := * F T'
6' T' := / F T'
7' T' :=
8 F := `id`
9 F := `int`

2 E := T E'

5 T := F T'

Is this legible?

Try again

1 S := E
2 E := T E′
2′ E′ := + T E′
3′ E′ := - T E′
4′ E′ :=
5 T := F T′
5′ T′ := * F T′
6′ T′ := / F T′
7′ T′ :=
8 F := id
9 F := int

	S	● $\mathtt{int}_2 - \mathtt{int}_3 * \mathtt{id}_x$
by 1 ⇒	E	● $\mathtt{int}_2 - \mathtt{int}_3 * \mathtt{id}_x$
by 2 ⇒	T E′	● $\mathtt{int}_2 - \mathtt{int}_3 * \mathtt{id}_x$
by 5 ⇒	F T′ E′	● $\mathtt{int}_2 - \mathtt{int}_3 * \mathtt{id}_x$
by 9 ⇒	2 T′ E′	$\mathtt{int}_2$ ● $- \mathtt{int}_3 * \mathtt{id}_x$
by 7′ ⇒	2 E′	$\mathtt{int}_2$ ● $- \mathtt{int}_3 * \mathtt{id}_x$
by 3′ ⇒	2 - T E′	$\mathtt{int}_2 -$ ● $\mathtt{int}_3 * \mathtt{id}_x$
by 5 ⇒	2 - F T′ E′	$\mathtt{int}_2 -$ ● $\mathtt{int}_3 * \mathtt{id}_x$
by 9 ⇒	2 - 3 T′ E′	$\mathtt{int}_2 - \mathtt{int}_3$ ● $* \mathtt{id}_x$
by 5′ ⇒	2 - 3 * F T′ E′	$\mathtt{int}_2 - \mathtt{int}_3 *$ ● $\mathtt{id}_x$
by 8 ⇒	2 - 3 * x T′ E′	$\mathtt{int}_3 * \mathtt{id}_x$ ●
		$\mathtt{int}_3 * \mathtt{id}_x$ ●
		$\mathtt{int}_3 * \mathtt{id}_x$ ●

Unlike previous time we tried this, it appears that only one production applies at a time. I.e., no backtracking needed. Why?

Lookahead

- How to pick right production?
- Lookahead in input stream for guidance
- General case: arbitrary lookahead required
- Luckily, many context-free grammars can be parsed with limited lookahead
- If we have $A \to \alpha \mid \beta$, then we want to correctly choose either $A \to \alpha$ or $A \to \beta$
- define FIRST(α) as the set of tokens that can be first symbol of α, i.e.,

  $a \in \text{FIRST}(\alpha)$ iff $\alpha \to^* a\gamma$ for some γ

Lookahead

- How to pick right production?
- If we have $A \rightarrow \alpha \mid \beta$, then we want to correctly choose either $A \rightarrow \alpha$ or $A \rightarrow \beta$
- define FIRST(α) as the set of tokens that can be first symbol of α, i.e.,

  $a \in \text{FIRST}(\alpha)$ iff $\alpha \rightarrow^* a\gamma$ for some γ
- If $A \rightarrow \alpha \mid \beta$ we want:

  $\text{FIRST}(\alpha) \cap \text{FIRST}(\beta) = \emptyset$
- If that is always true, we can build a predictive parser.

Computing FIRST(α)

- Given X := A B C, FIRST(X) = FIRST(A B C)
- Can we ignore B or C?
- Consider:

```
A := a
   |
B := b
   | A
C := c
```

Computing FIRST(α)

- Given X := A B C, FIRST(X) = FIRST(A B C)
- Can we ignore B or C?
- Consider:

```
A := a
   |
B := b
   | A
C := c
```

- FIRST(X) must also include FIRST(C)
- IOW:
  - Must keep track of NTs that are nullable
  - For nullable NTs, determine FOLLOWS(NT)

nullable(A)

- nullable(A) is
  - true if A can derive the empty string
  - false otherwise
- For example:

```
B := X Y b
X := x
   |  Y Y
Y :=
```

In this case, nullable(X) = nullable(Y) = true

nullable(B) = false

FOLLOW(A)

- FOLLOW(A) is the set of terminals that can immediately follow A in a sentential form.
- I.e.,
  $a \in$ FOLLOW(A) iff $S \Rightarrow^* \alpha A a \beta$ for some α and β

Building a Predictive Parser

- We want to know for each non-terminal which production to choose based on the next input character.
- Build a table with rows labeled by non-terminals, A, and columns labeled by terminals, a. We will put the production, A := α , in (A, a) iff
  - FIRST(α) contains a or
  - nullable(α) and FOLLOW(A) contains a

The table for the robot

S := B S F

|

B := b

F := f

	FIRST	FOLLOW	nullable
S	b	$	yes
B	b	b,f	no
F	f	f,$	no

	b	f	$
S			
B			
F			

The table for the robot

S := B S F

|

B := b

F

FIRST(BSF) = b

	FIRST	FOLLOW	nullable
S	b	$	yes
B	b	b,f	no
F	f	f,$	no

nullable(ε)=true
and
FOLLOW(S) = $

	b	f	$
S	S:=BSF		S:=
B	B:=b		
F		F:=f	

Table f

1 S := E
2 E := T E′
2′ E′ := + T E′
3′ E′ := - T E′
4′ E′ :=
5 T := F T′
5′ T′ := * F T′
6′ T′ := / F T′
7′ T′ :=
8 F := `id`
9 F := `int`

	FIRST	FOLLOW	nullable
S	id, int	$	
E	id, int	$	
E′	+, -	$	yes
T	id, int	+,-,$	
T′	/, *	+,-,$	yes
F	id, int	/, *,$	

	+	-	*	/	id	int	$
S							
E							
E′							
T							
T′							
F							

Table f

1 S := E
2 E := T E′
2′ E′ := + T E′
3′ E′ := - T E′
4′ E′ :=
5 T := F T′
5′ T′ := * F T′
6′ T′ := / F T′
7′ T′ :=
8 F := `id`
9 F := `int`

	FIRST	FOLLOW	nullable
S	id, int	$	
E	id, int	$	
E′	+, -	$	yes
T	id, int	+,-,$	
T′	/, *	+,-,$	yes
F	id, int	/, *,$	

	+	-	*	/	id	int	$
S					:=E	:=E	
E					:=TE′	:=TE′	
E′	:=+TE′	:=-TE′					:=
T					:=FT′	:=FT′	
T′	:=	:=	:=*FT′	:=/FT′			:=
F					:=id	:=int	

Using the Table

- Each row in the table becomes a function
- For each input token with an entry: Create a series of invocations that implement the production, where
  - a non-terminal is eaten
  - a terminal becomes a recursive call
- For the blank cells implement errors

Example function

	+	-	*	/	id	int	$
S					:=E	:=E	
E					:=TE'	:=TE'	
E'	:=+TE'	:=-TE'			:=TE'	:=TE'	:=
T							
T'	:=	:=	:=*FT				
F					:=id	:=int	

How to handle errors?

```
Eprime() {
    switch (token) {
    case PLUS:     eat(PLUS); T(); Eprime(); break;
    case MINUS:    eat(MINUS); T(); Eprime(); break;
    case ID:       T(); Eprime();
    case INT:      T(); Eprime();
    default:       error();
}
```

Left-Factoring

- Predictive parsers need to make a choice based on the next terminal.
- Consider:

  S := `if E then S else S`
  | `if E then S`

- When looking at `if`, can't decide
- so left-factor the grammar

S := `if E then S X`
X := `else S`
|

Top-Down Parsing

- Can be constructed by hand
- LL(k) grammars can be parsed
  - Left-to-right
  - Leftmost-derivation
  - with k symbols lookahead
- Often requires
  - left-factoring
  - Elimination of left-recursion