# Beyond Alhazen's Problem: Analytical Projection Model for Non-Central Catadioptric Cameras with Quadric Mirrors


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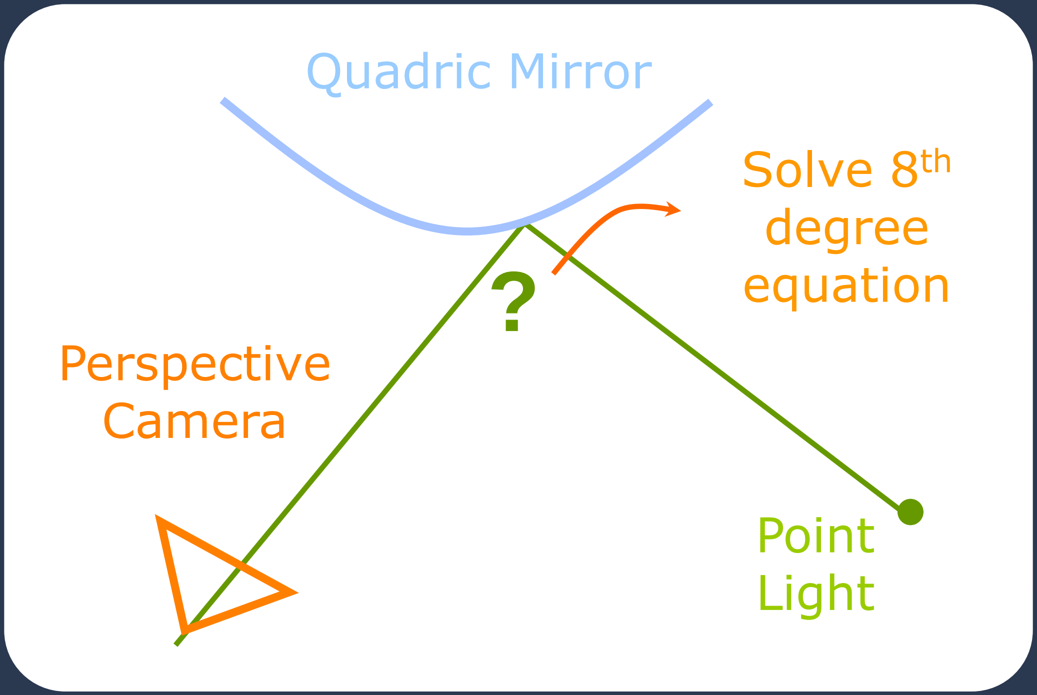


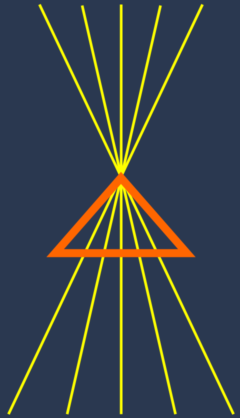

What is the point on the mirror where reflection happens?

Analytical Projection Model for Non-central Catadioptric Cameras

# Perspective Cameras (Central)



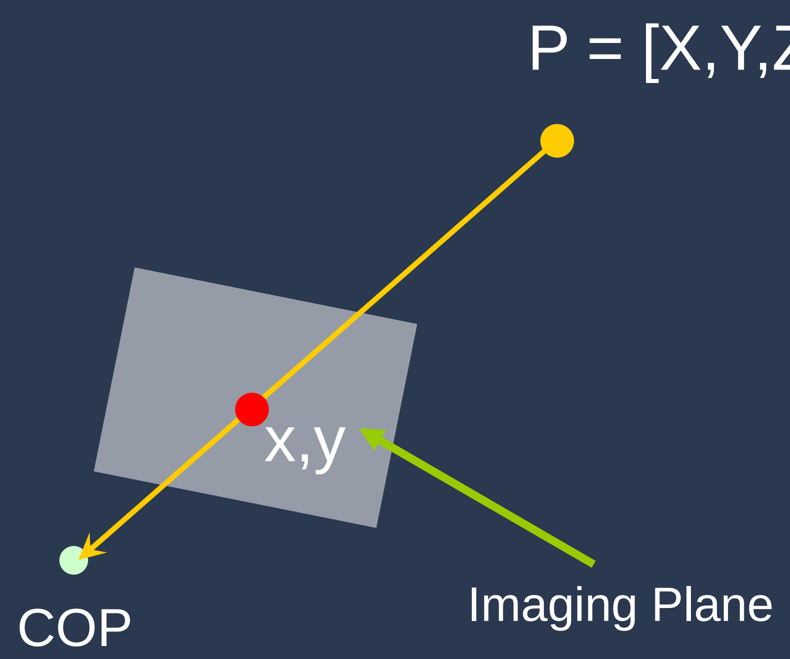
Single Viewpoint
(Central)



Perspective Camera

*Single Viewpoint*

# Projection of a 3D Point? Easy for Perspective Camera



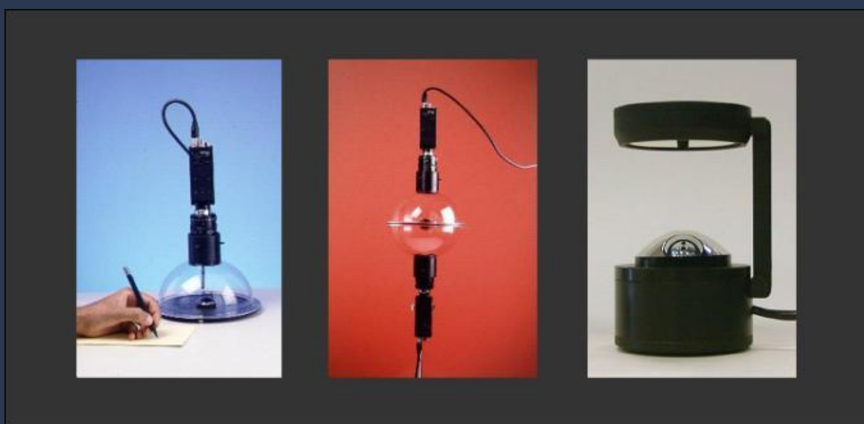

$x = f*X/Z$

$y = f*Y/Z$

Perspective Projection Equations

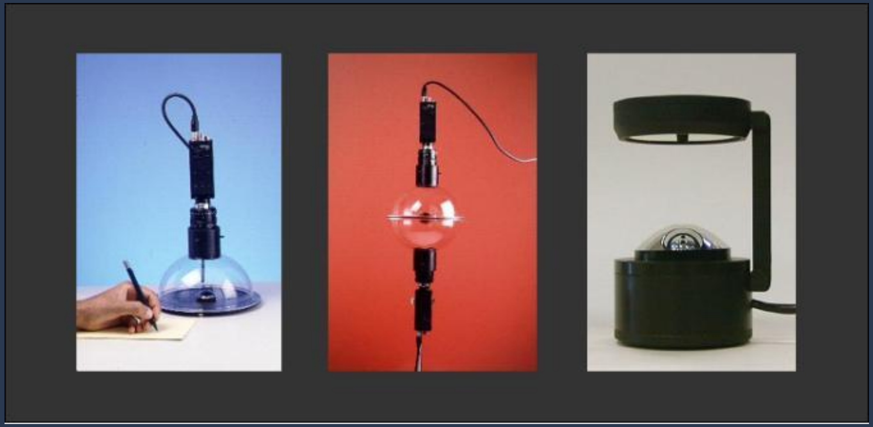
# Catadioptric Cameras

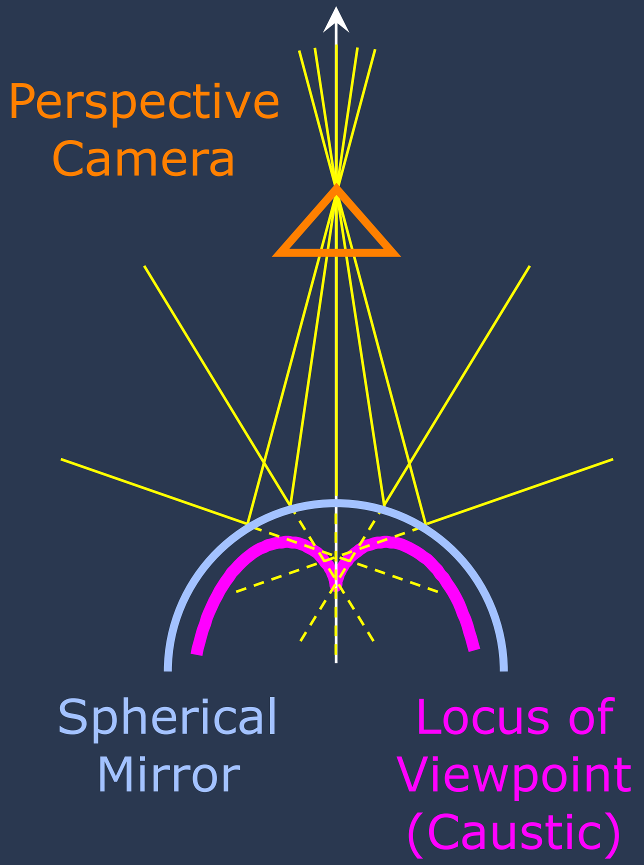
- Mirror + Perspective Camera
- Wide Field of View



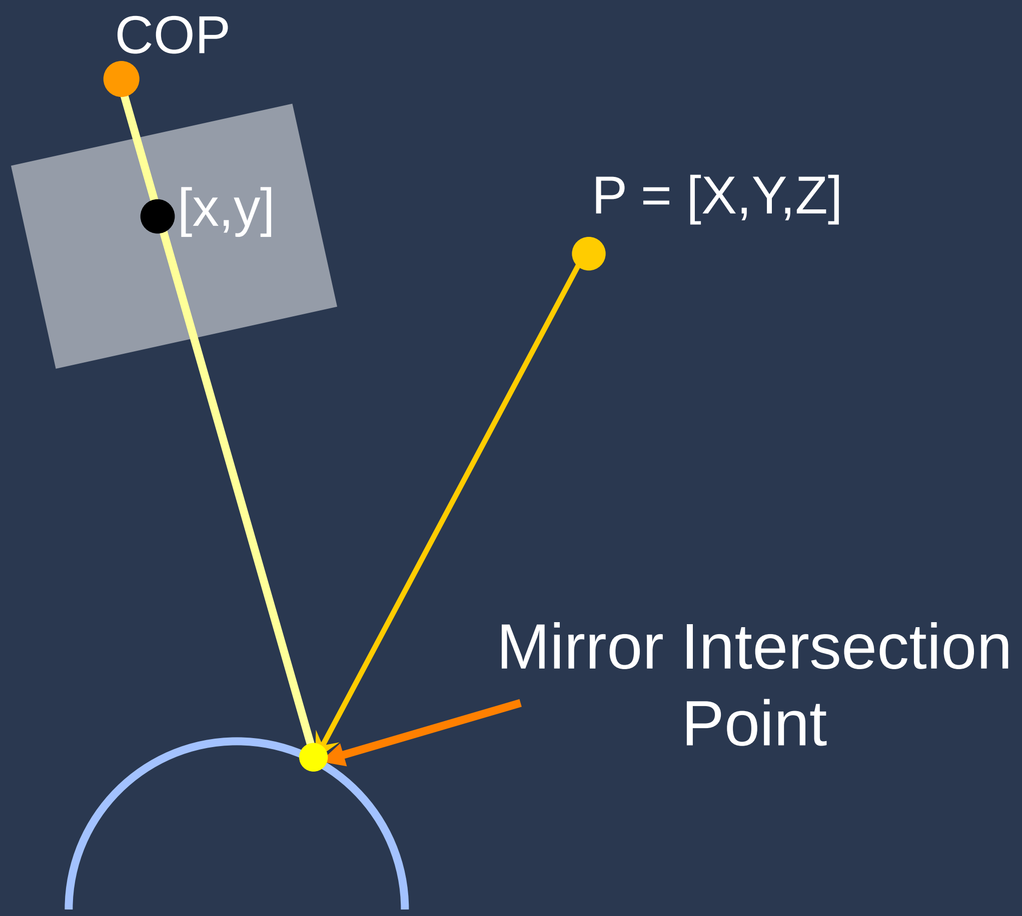
# Catadioptric Cameras

- Mirror + Perspective Camera
- Wide Field of View
- Non-central



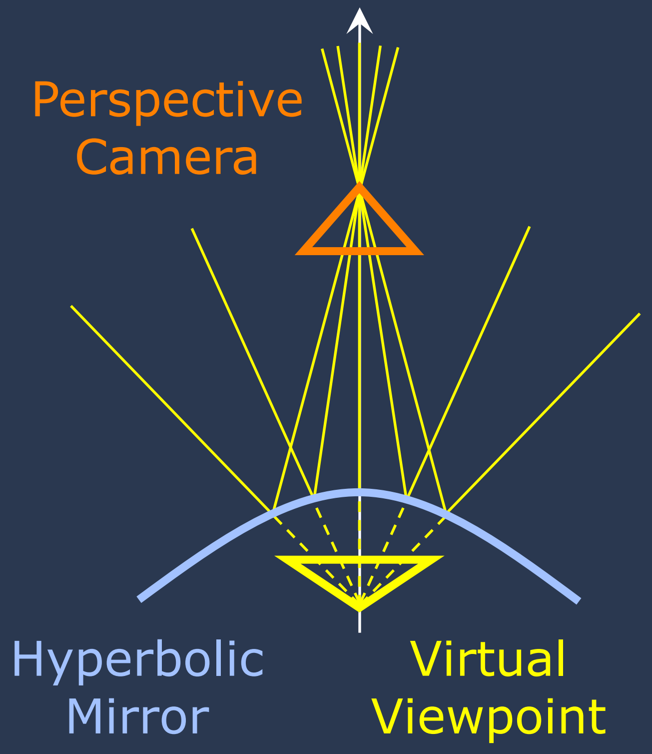


# Projection of a 3D Point?



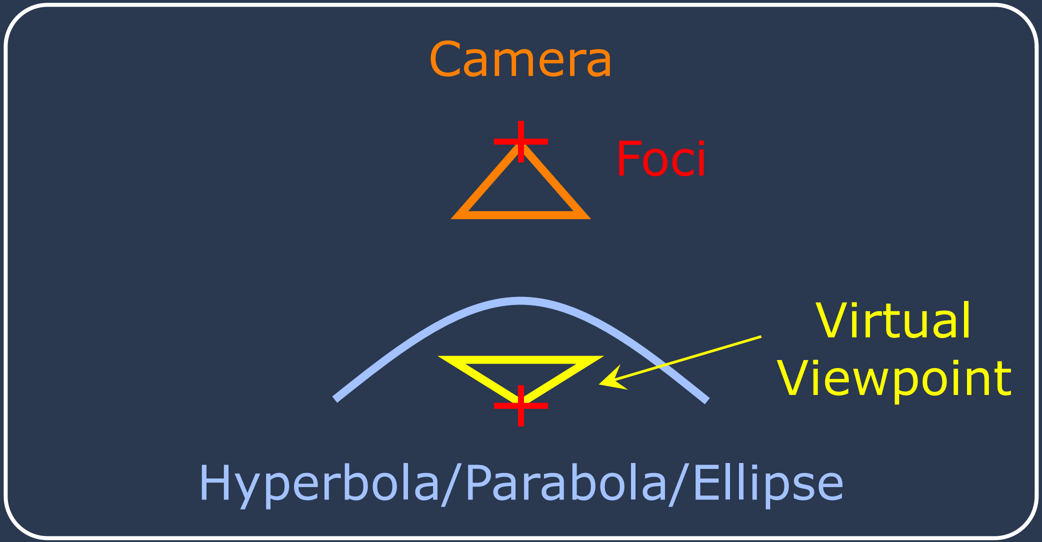
# Single-Viewpoint Catadioptric Cameras

- Have an effective single center of projection
- Avoid the problem
- Easy to Model



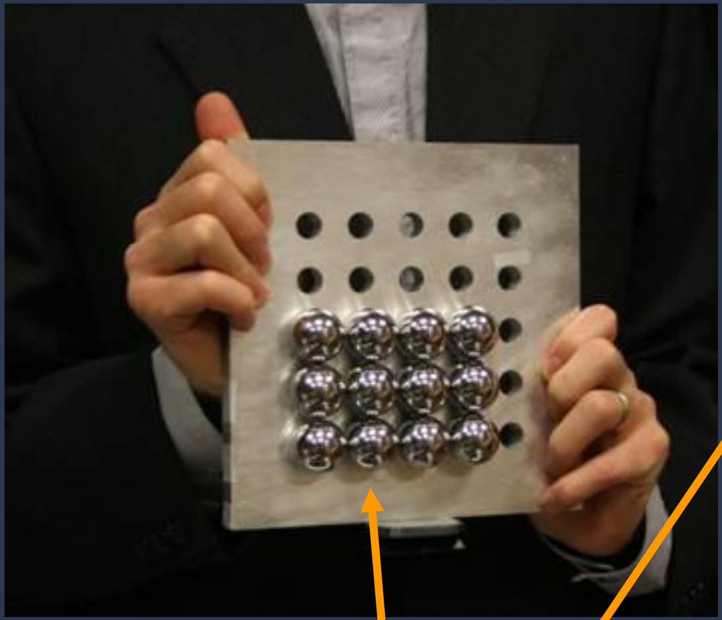
# Single-Viewpoint Catadioptric Cameras

[Baker & Nayar 99]




- Only a few single-viewpoint configurations
- Other configurations lead to non-single viewpoint
  - Spherical mirror
  - Camera placed freely (off-axis, not on foci)
  - Multiple mirrors

# Example: Wide Angle Light Field Camera Using Spherical Mirrors



Array of Spherical Mirrors

Taguchi et al. SIGGRAPH Asia 2010

Captured Photo

Light Field Views (100°x100°)

Depth Map

# Wide Angle Digital Refocusing

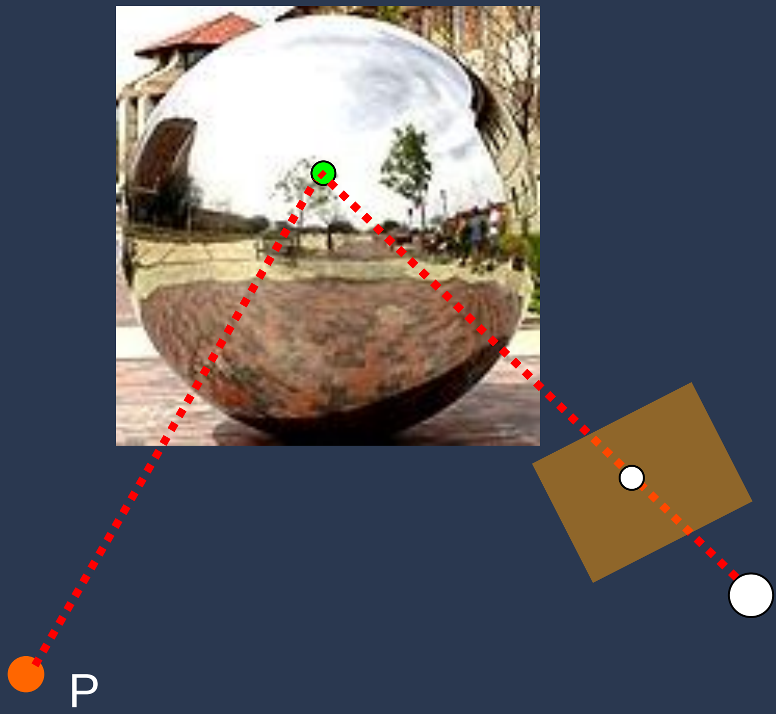
# Multiple Non-Central Cameras!!

# Goal

- Exact analytical modeling of non-central cameras
  - Rotationally Symmetric Quadric Mirrors
  - Camera can be placed anywhere (off-axis)
- Avoid approximations in modeling
  - Central Approximation
  - General linear cameras (GLC) approximation
    - Yu and McMillan, ECCV 2004
- Fast and easy processing
  - Similar to perspective cameras
  - Can apply bundle-adjustment pipeline to catadioptric image

150 A.D., Ptolemy

# Alhazen's Problem

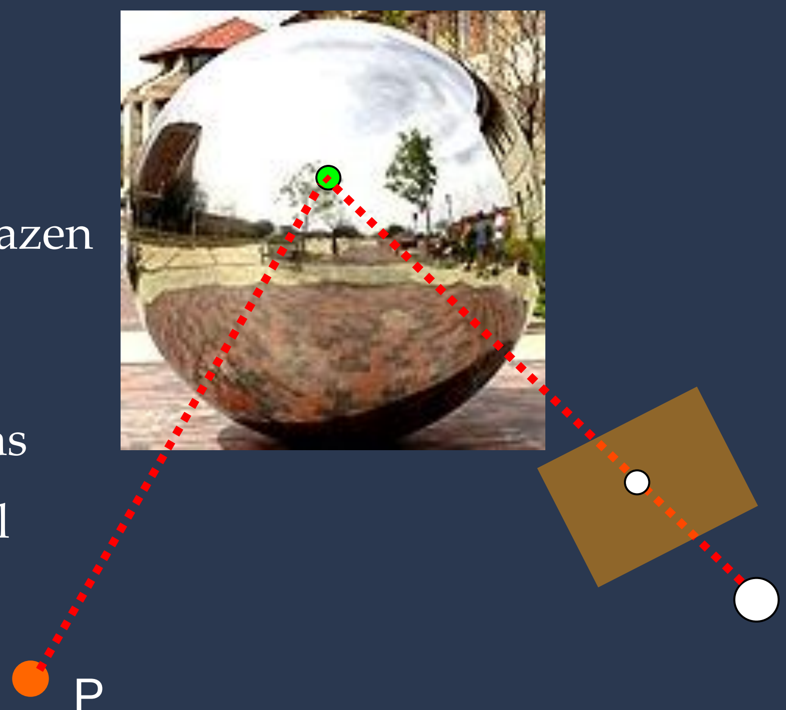


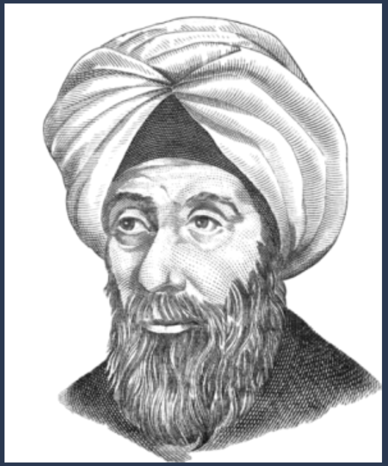


"Given a light source and a spherical mirror, find the point on the mirror where the light will be reflected to the eye of an observer." - *Ptolemy*

# Alhazen's Problem

- 150 A.D., Ptolemy
- 1000 A.D., Alhazen
- 1669, Barrow
- 1673, Huyghens
- 1720, L'Hopital
- 1777, Kaestner
- 1817, Hutton
- 1817, Leybourn
- 1881, Baker



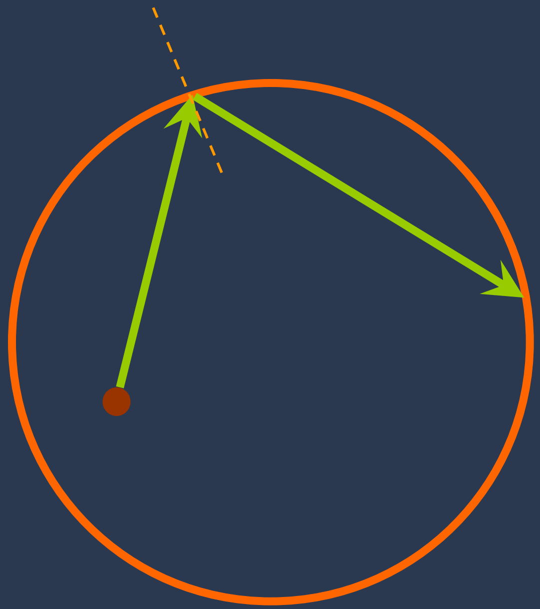



Ibn al-Haytham
(Alhazen)

Solution for Spherical Mirror

4th degree equation

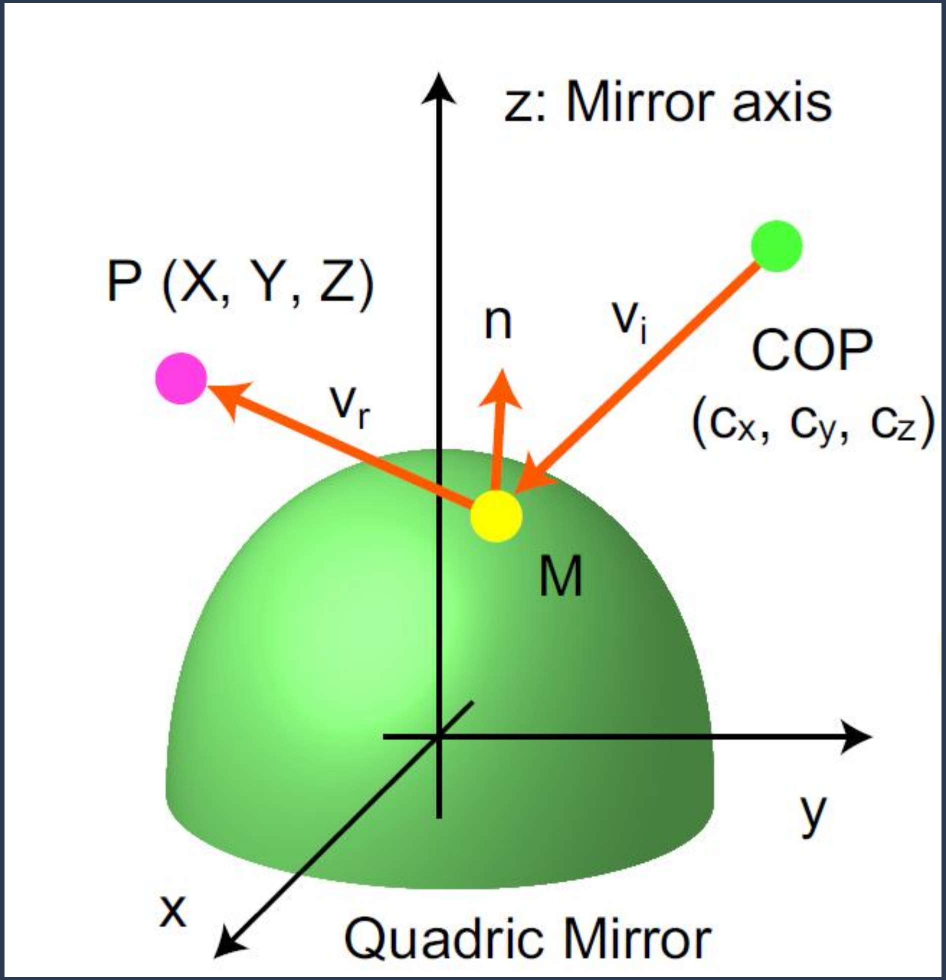
# Also known as Circular Billiard Problem

# Beyond Alhazen's Problem

- General Quadric Mirrors
- Agrawal, Taguchi & Ramalingam, ECCV 2010
  - Camera on Axis
  - $6^{th}$ degree equation
  - Special case of this paper
- Bertrand Vandeportaele, 2006
  - Phd Thesis (In French)
  - Off-axis Camera + Quadric Mirrors
- Search/Optimization
  - Gonçalves & Nogueira, OMNIVIS 2009
  - Micusik & Pajdla, CVPR 2004

# Finding the Mirror Intersection Point



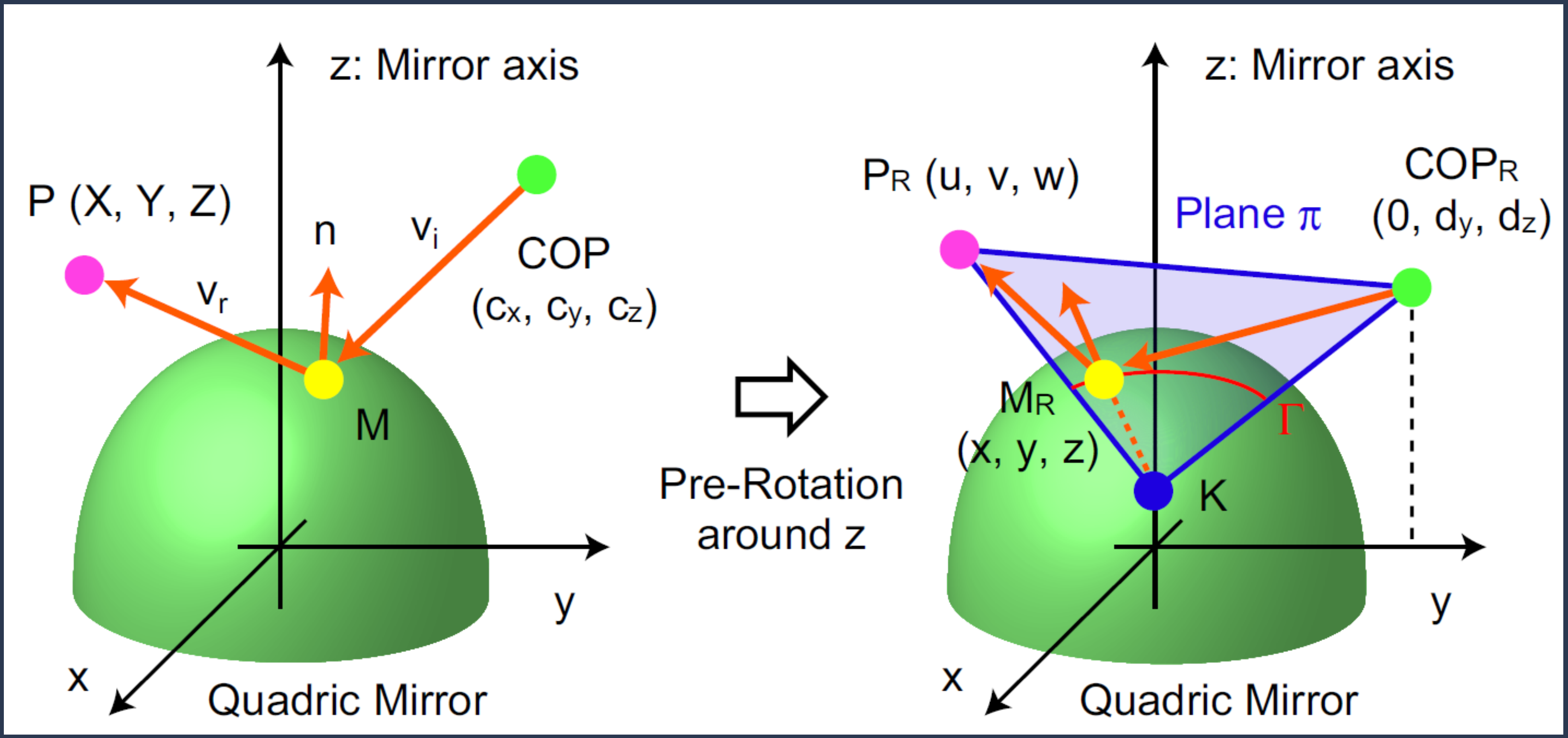

Knowns:

(A, B, C), COP, P

Unknown: M = (x,y,z)

Mirror Equation: $x^2 + y^2 + Az^2 + Bz - C = 0.$

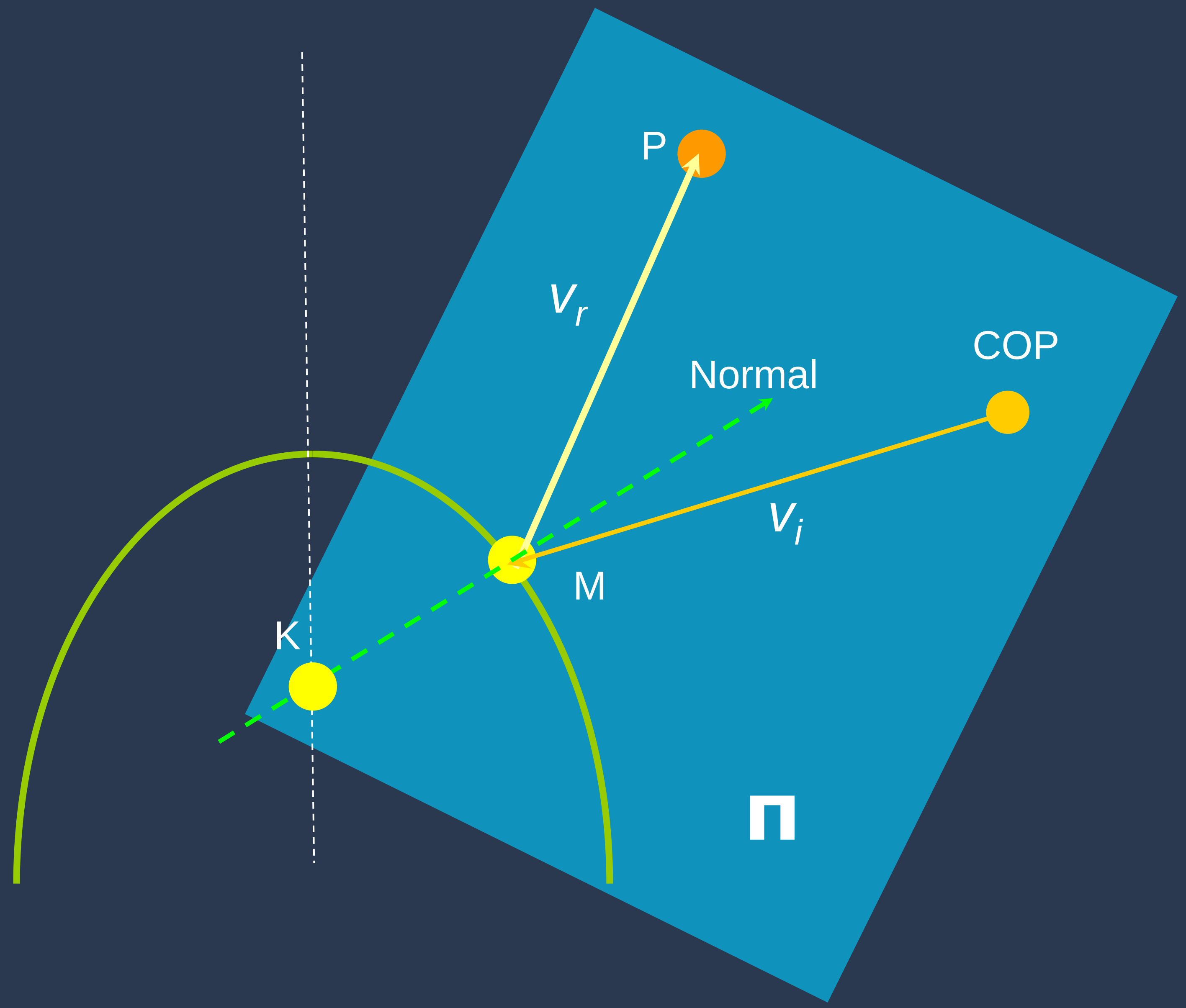
# Key Idea 1: Pre-Rotation




Make the x-coordinate of COP to be zero

COP: $(c_x, c_y, c_z) \rightarrow (0, d_y, d_z)$

# Key Idea 2: Use of Reflection Plane π



# Equation of Reflection Plane π

- Using P, COP and K

$$c_1(z)x + c_2(z)y + c_3(z) = 0$$

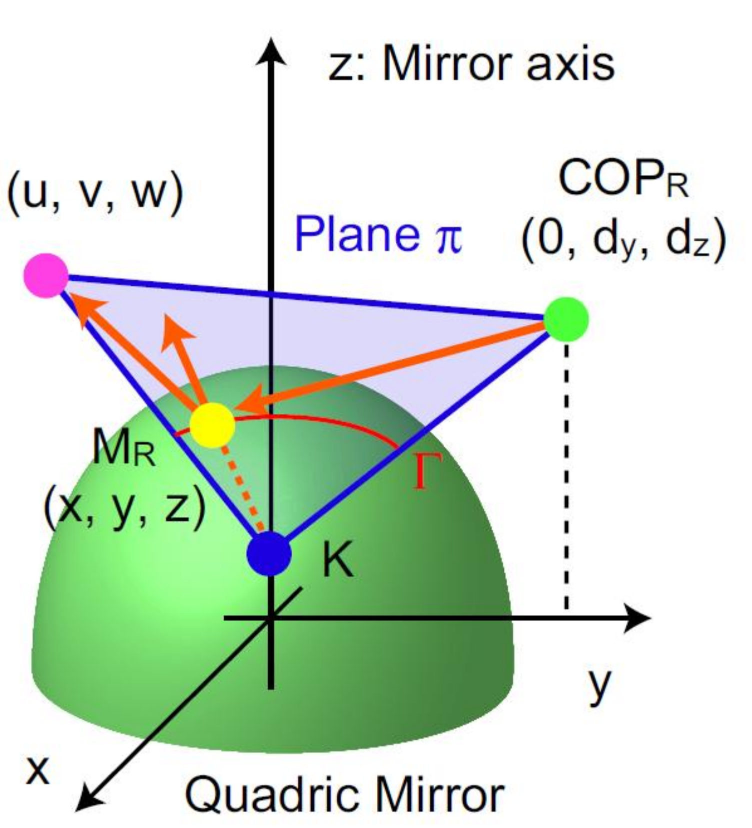
$$x = \frac{-c_2(z)y - c_3(z)}{c_1(z)}$$

Substitute in Mirror Equation

$$\mathbf{IE}_1 : k_{41}(z)y^2 + k_{42}(z)y + k_{43}(z) = 0$$

$$\mathbf{IE}_1 : k_{41}(z)y^2 + k_{42}(z)y + k_{43}(z) = 0$$

- Intersection of Reflection plane π with the Mirror
- Curve Γ on the mirror where reflection can happen



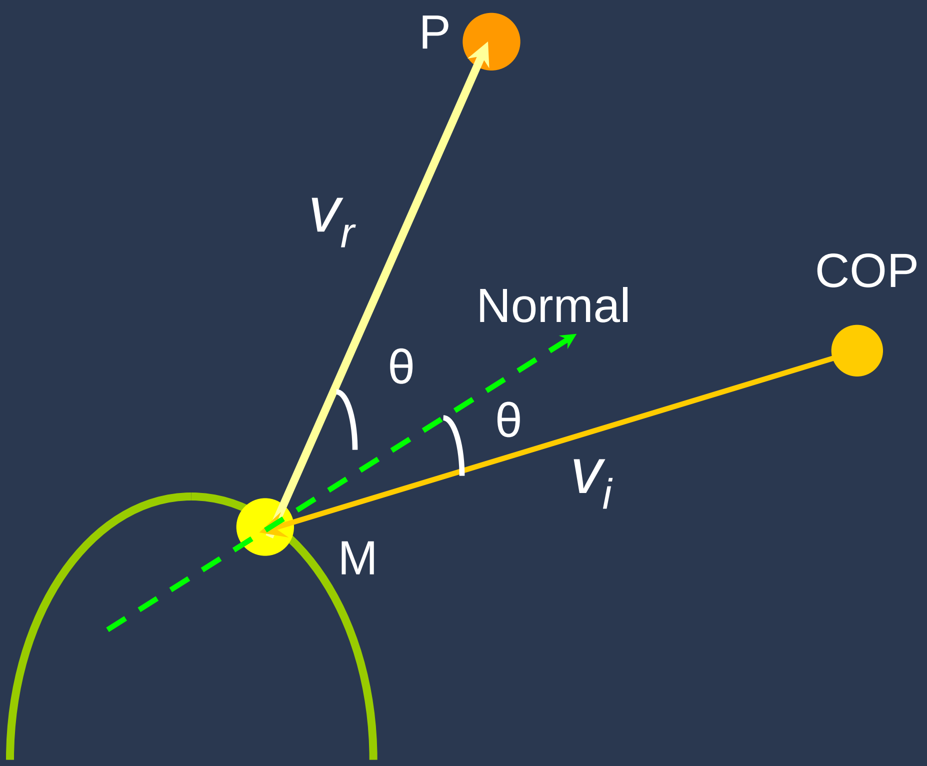
# Law of Reflection

$$\mathbf{v}_r = \mathbf{v}_i - 2\mathbf{n}(\mathbf{v}_i^T \mathbf{n})/(\mathbf{n}^T \mathbf{n})$$

$V_r$ should be same as (P-M)

cross(vr, P-M) = 0

$$\mathbf{IE}_2 : k_{31}(z)y^2 + k_{32}(z)y + k_{33}(z) = 0$$



# Final Equation

- Two quadratic equations in y

$$\mathbf{IE}_1 : k_{41}(z)y^2 + k_{42}(z)y + k_{43}(z) = 0$$

$$\mathbf{IE}_2 : k_{31}(z)y^2 + k_{32}(z)y + k_{33}(z) = 0$$

$$\begin{aligned} &k_{41}(z)\left(k_{43}(z)k_{32}^2(z) - k_{42}(z)k_{32}(z)k_{33}(z) + k_{41}(z)k_{33}^2(z)\right) \\ &- k_{31}(z)(-k_{33}(z)k_{42}^2(z) + k_{43}(z)k_{32}(z)k_{42}(z) \\ &+ 2k_{41}(z)k_{43}(z)k_{33}(z)) + k_{43}^2(z)k_{31}^2(z) = 0. \end{aligned}$$

- $8^{th}$ degree equation in z

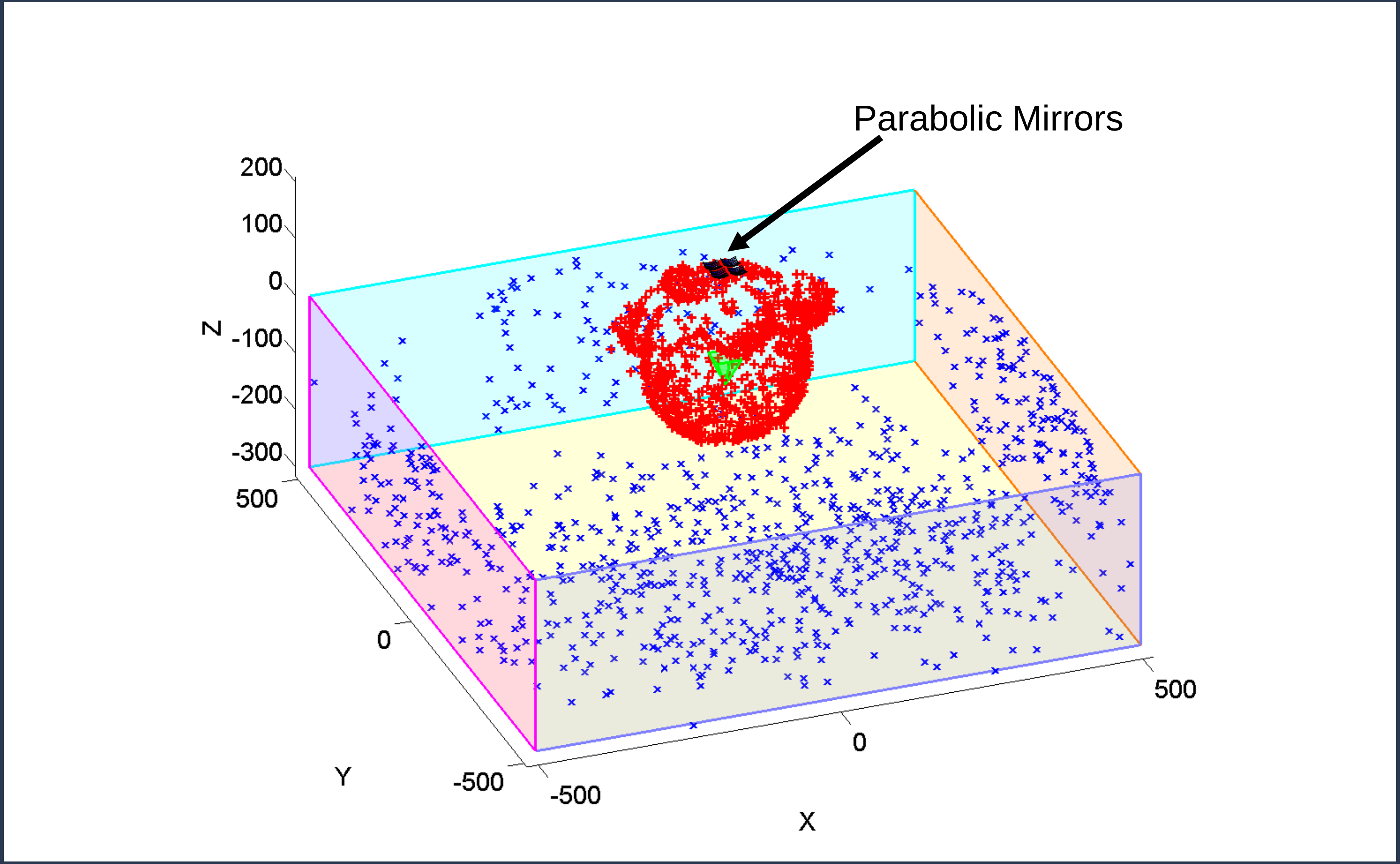
| Mirror Shape | Parameters | Camera Placement | | |
|---|---|---|---|---|
| | | Off-Axis | Axial | |
| | | | NSVP | SVP |
| General | A, B, C | 8 | 6 | - |
| Spherical | $A = 1, B = 0, C > 0$ | | $4^{\ddagger}$ | - |
| Elliptical | $A > 0, B = 0, C > 0$ | 8 | 6 | 2 |
| Hyperbolic | $A < 0, B = 0, C < 0$ | 8 | 6 | 2 |
| Parabolic | $A = 0, C = 0$ | 7 | 5 | $2^{\dagger}$ |
| Conical | $A < 0, B = 0, C = 0$ | 4 | 2 | - |
| Cylindrical | $A = 0, B = 0, C > 0$ | 4 | 2 | - |

# Fast Projection of 3D Points

- Off-axis camera looking at quadric mirror
- Project randomly generated 100,000 points
- Matlab on standard PC
- 40X speed up compared to optimization
- 100X speed up for spherical mirrors

# Sparse 3D Reconstruction

- Can minimize the re-projection error



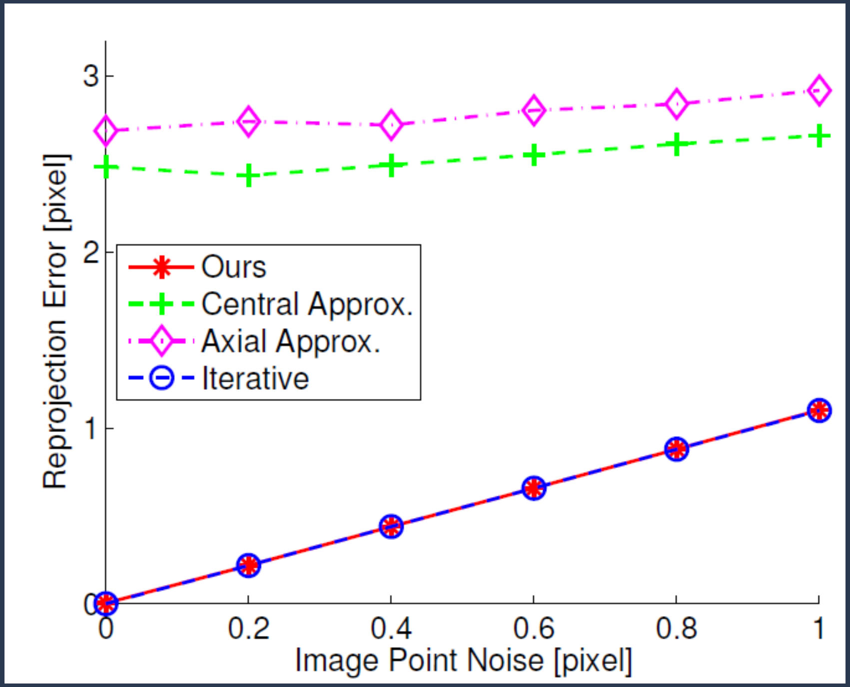
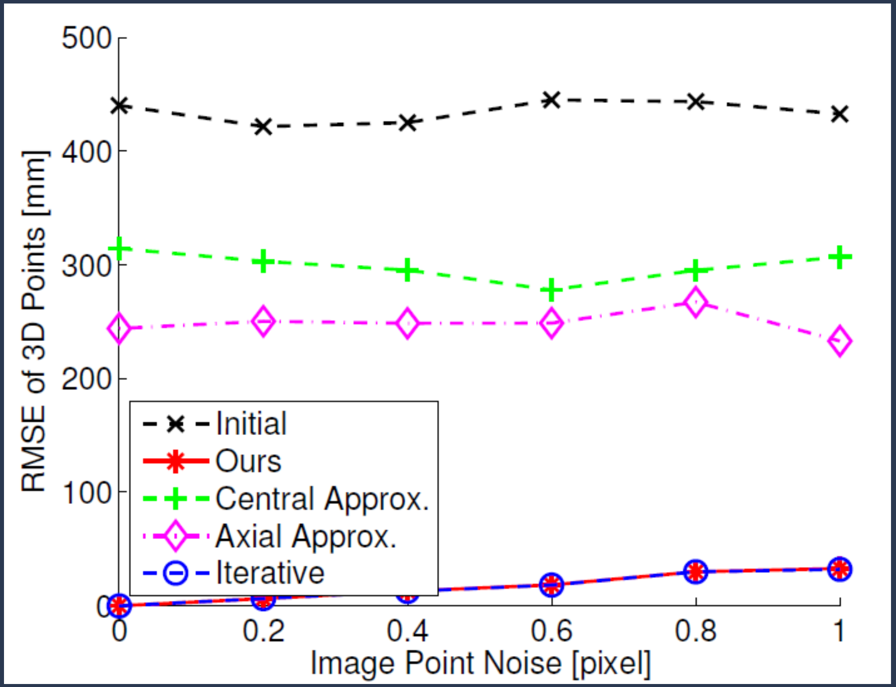
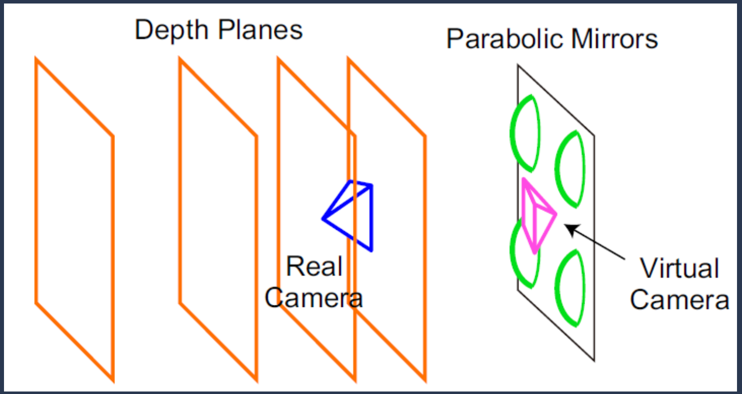



Image Re-Projection Error



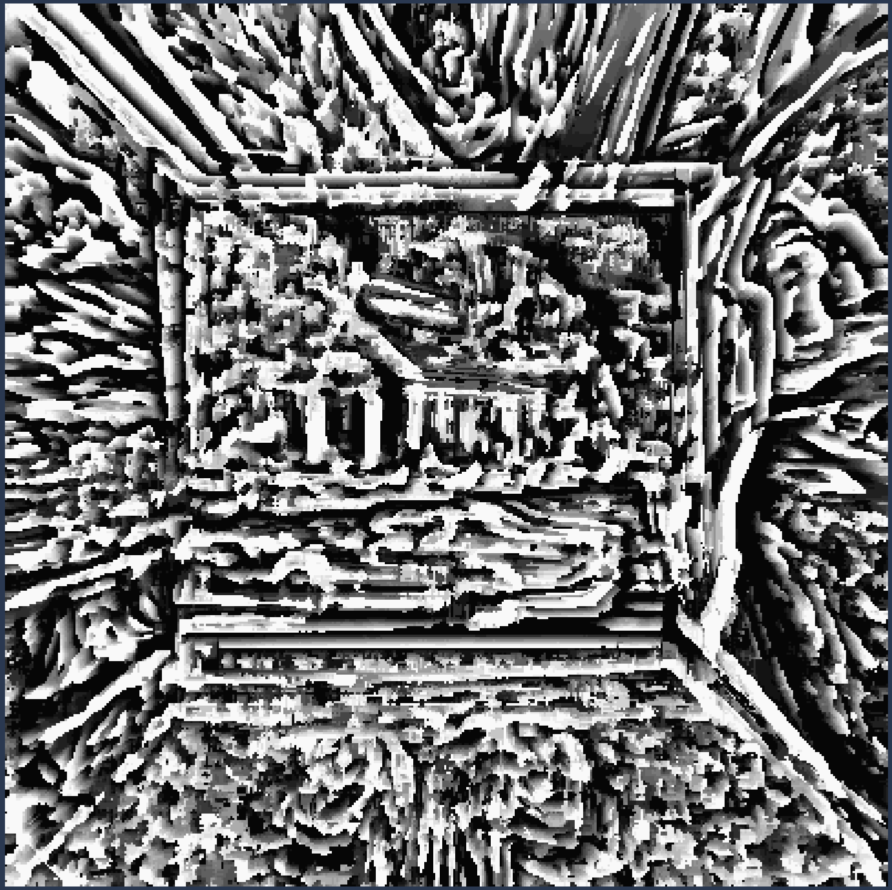

Error of 3D Points

# Dense Volumetric Reconstruction

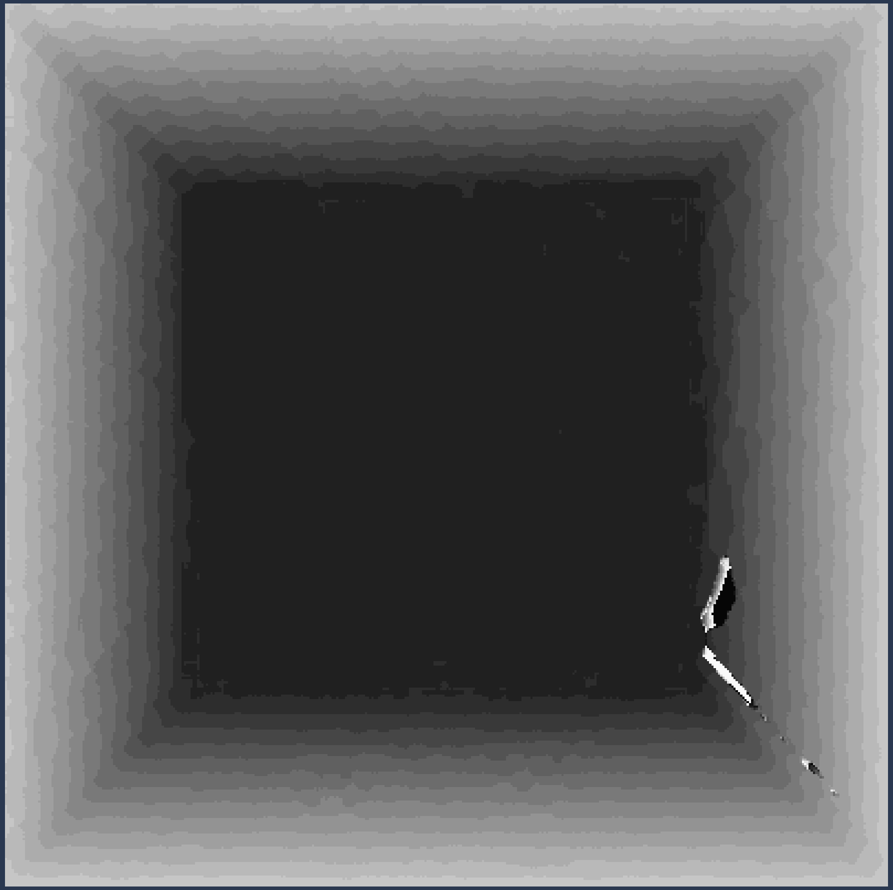



- Plane Sweep, Collins 1996
- Ding, Yu and Strum, ICCV 2009
  - Spherical Mirrors
  - GLC approximation for computing the projection
- We avoid any approximation
- Standard 3D Reconstruction Pipeline
  - Feature detection and matching using SIFT
  - Bundle-adjustment (Refine mirror pose)
  - Dense Reconstruction

# Dense Depth Maps

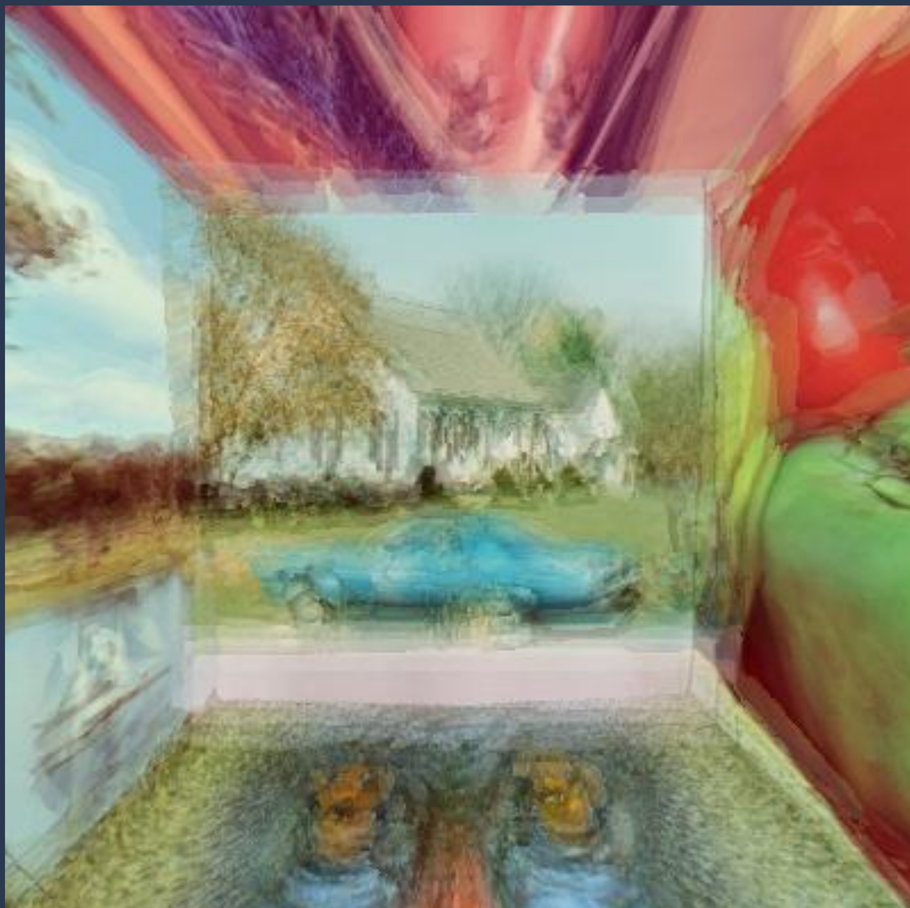


Without Bundle Adjustment



With Bundle Adjustment

# All-in-Focus Image using depth map



Without Bundle Adjustment

With Bundle Adjustment

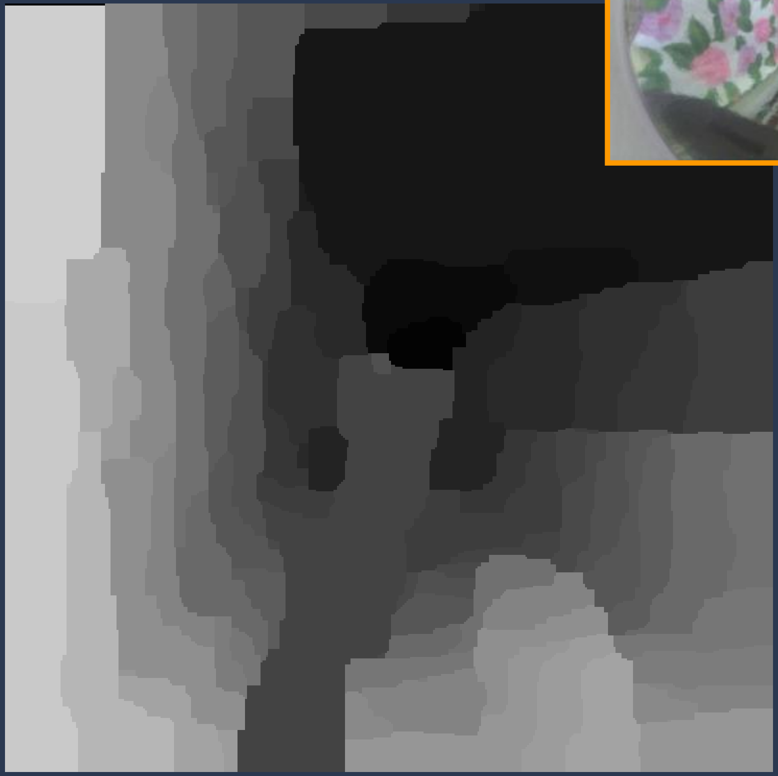
Checking Photo-consistency using the estimated depth map

# Real Result

- Four Parabolic Mirrors
- Single photo using 22 Megapixel camera

# Dense Reconstruction

Without Bundle Adjustment



With Bundle Adjustment

# All-In-Focus Images

Without Bundle Adjustment

With Bundle Adjustment

Checking Photo-consistency using the estimated depth map

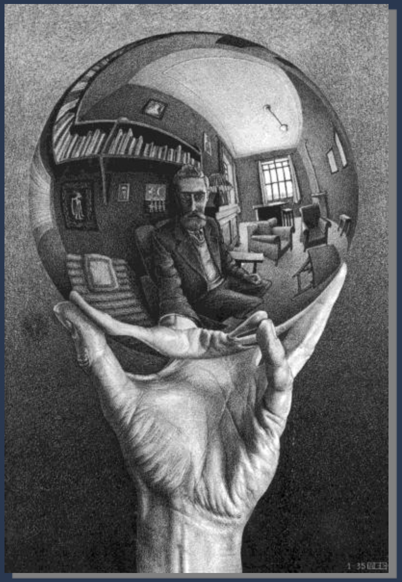
# Summary

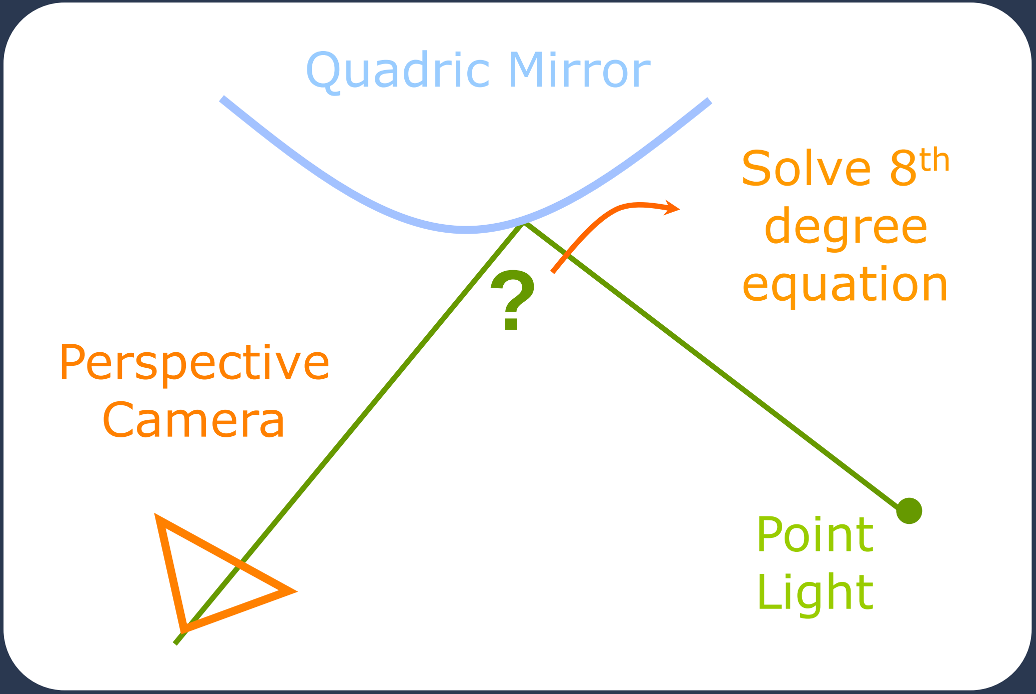
- Analytical Projection Model
  - Quadric Mirrors
    - Hyperbolic, Elliptical, Spherical, Parabolic Mirrors
  - Camera can be placed anywhere (off-axis)
- Avoid central and GLC approximation
  - Can use exact non-central model, 40X speed up
  - Allows to minimize the image re-projection error
- Sparse and Dense 3D reconstruction
  - Same pipeline as used for perspective cameras

# Acknowledgments


- MERL
  - Jay Thornton, Keisuke Kojima, John Barnwell, Joseph Katz
- Mitsubishi Electric, Japan
  - Haruhisa Okuda, Kazuhiko Sumi

# Analytical Projection Model for Non-Central Catadioptric Cameras






google "beyond alhazen"