Boosting

Can we make dumb learners smart?

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Machine Learning 10-315
Nov 9, 2020

Slides Courtesy: Carlos Guestrin, Freund & Schapire

Why boost weak learners?

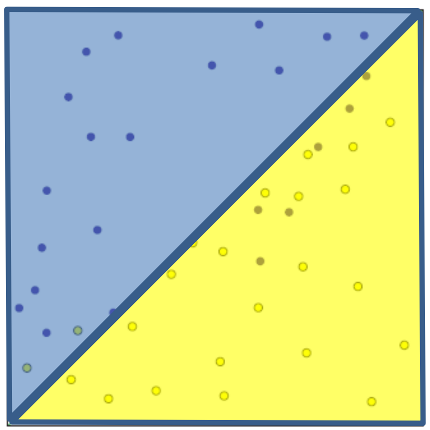
Goal: Automatically categorize type of call requested
(Collect, Calling card, Person-to-person, etc.)

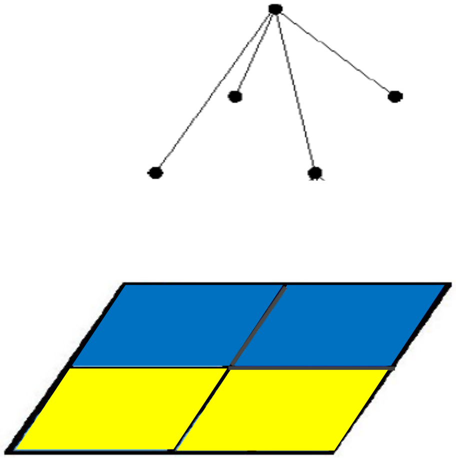
- yes I'd like to place a collect call long distance please (Collect)
- operator I need to make a call but I need to bill it to my office (ThirdNumber)
- yes I'd like to place a call on my master card please (CallingCard)

- **Easy to find "rules of thumb" that are "often" correct.**
  E.g. If 'card' occurs in utterance, then predict 'calling card'
- **Hard to find single highly accurate prediction rule.**

Fighting the bias-variance tradeoff

- **Simple (a.k.a. weak) learners** e.g., naïve Bayes, logistic regression, decision stumps (or shallow decision trees)





Are good ☺ - don't usually overfit

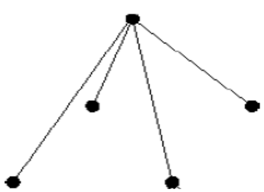
Are bad ☹ - can't solve hard learning problems

- **Can we make weak learners good???**

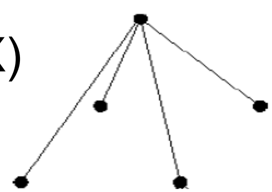
Voting (Ensemble Methods)

- Instead of learning a single (weak) classifier, learn **many weak classifiers** that are **good at different parts of the input space**
- **Output class:** (Weighted) vote of each classifier
  - Classifiers that are most "sure" will vote with more conviction
  - Classifiers will be most "sure" about a particular part of the space
  - On average, do better than single classifier!

$h_1(X)$



$h_2(X)$

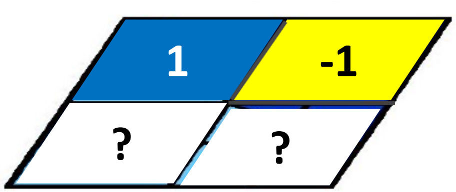


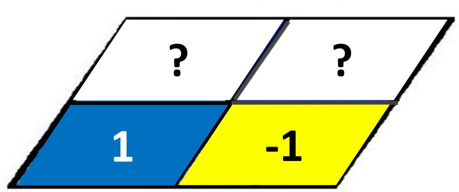
$H: X \rightarrow Y$ (-1,1)

$H(X) = h_1(X)+h_2(X)$

$H(X) = \text{sign}(\sum_t \alpha_t h_t(X))$

weights





Voting (Ensemble Methods)

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- **Output class:** (Weighted) vote of each classifier
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  - On average, do better than single classifier!
- **But how do you ???**
  - force classifiers h_t to learn about different parts of the input space?
  - weigh the votes of different classifiers? α_t

Boosting [Schapire'89]

- **Idea:** given a weak learner, run it multiple times on (reweighted) training data, then let learned classifiers vote

- On each iteration *t*:
  - weight $D_t(i)$ for each training example i, based on how incorrectly it was classified
  - Learn a weak hypothesis – h_t
  - A weight for this hypothesis – α_t

- Final classifier: $\boxed{H(X) = \text{sign}(\sum \alpha_t h_t(X))}$

- **Practically useful**
- **Theoretically interesting**

Learning from weighted data

- **Consider a weighted dataset**
  - D(i) – weight of *i* th training example ($\mathbf{x}^i, y^i$)
  - Interpretations:
    - *i* th training example counts as D(i) examples
    - If I were to "resample" data, I would get more samples of "heavier" data points
- **Now, in all calculations, whenever used, *i* th training example counts as D(i) "examples"**
  - e.g., in MLE redefine *Count(Y=y)* to be weighted count

Unweighted data

$$Count(Y=y) = \sum_{i=1}^{m} \mathbf{1}(Y^i=y)$$

Weights D(i)

$$Count(Y=y) = \sum_{i=1}^{m} D(i)\mathbf{1}(Y^i=y)$$

AdaBoost [Freund & Schapire'95]

Given: $(x_1, y_1), \ldots, (x_m, y_m)$ where $x_i \in X, y_i \in Y = \{-1, +1\}$

Initialize $D_1(i) = 1/m$. **Initially equal weights**

For $t = 1, \ldots, T$:

- Train weak learner using distribution D_t. **Naïve bayes, decision stump**
- Get weak classifier $h_t : X \to \mathbb{R}$.
- Choose $\alpha_t \in \mathbb{R}$. **Magic (+ve)**
- Update:

$$D_{t+1}(i) = \frac{D_t(i)}{Z_t} \begin{cases} e^{-\alpha_t} & \text{if } y_i = h_t(x_i) \\ e^{\alpha_t} & \text{if } y_i \neq h_t(x_i) \end{cases}$$

$$= \frac{D_t(i)\exp(-\alpha_t y_i h_t(x_i))}{Z_t}$$

Increase weight if wrong on pt i
$y_i h_t(x_i) = -1 < 0$

where Z_t is a normalization factor

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$$Z_t = \sum_{i=1}^{m} D_t(i)\exp(-\alpha_t y_i h_t(x_i))$$

Weights for all pts must sum to 1
$\sum_t D_{t+1}(i) = 1$

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Output the final classifier:

$$H(x) = \text{sign}\left(\sum_{t=1}^{T} \alpha_t h_t(x)\right).$$

What α_t to choose for hypothesis h_t?

Weight Update Rule:

$$D_{t+1}(i) = \frac{D_t(i)\exp(-\alpha_t y_i h_t(x_i))}{Z_t}$$

$$\boxed{\alpha_t = \frac{1}{2}\ln\left(\frac{1-\epsilon_t}{\epsilon_t}\right)}$$

[Freund & Schapire'95]

Weighted training error

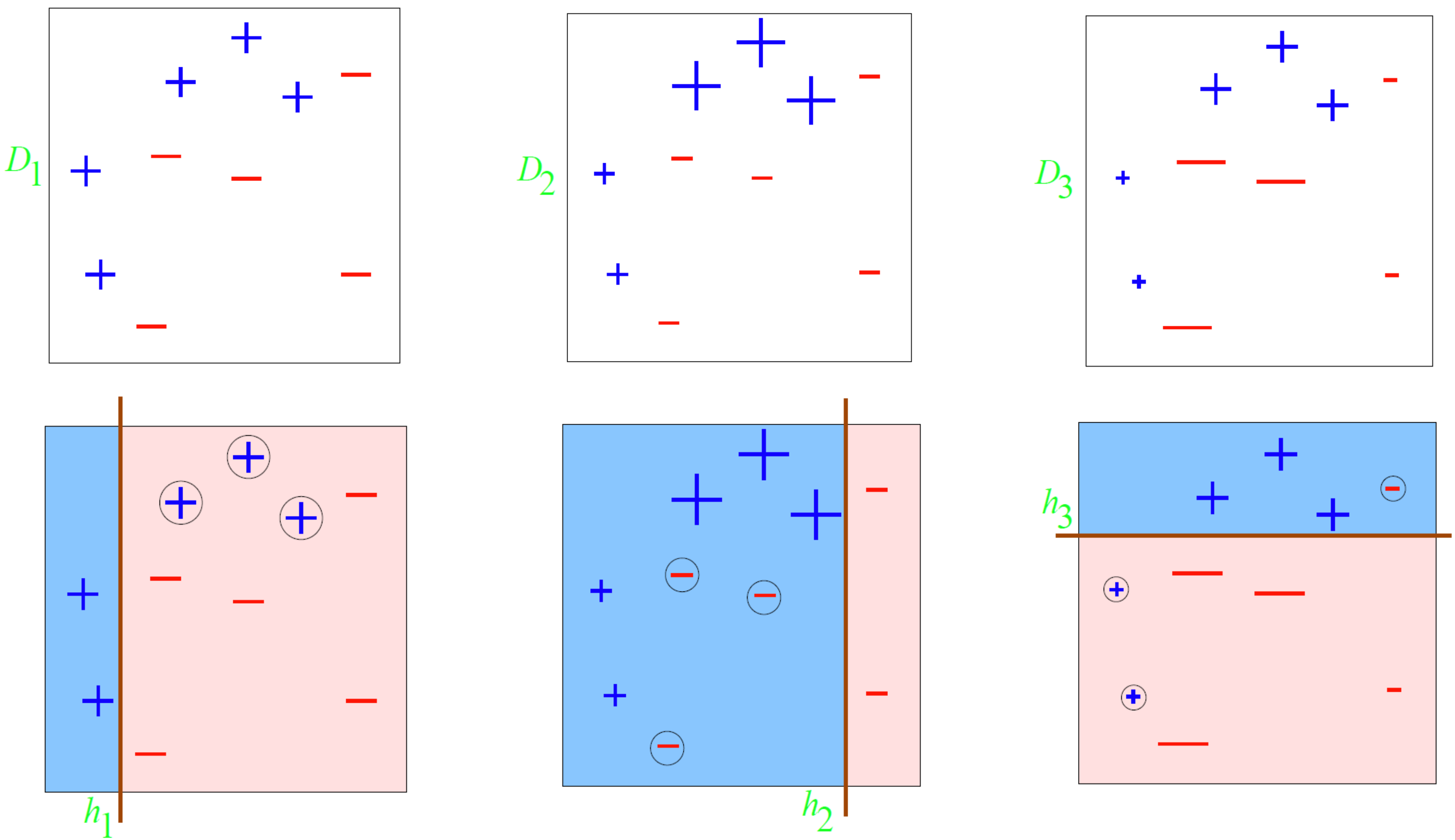
$$\epsilon_t = P_{i\sim D_t(i)}[h_t(\mathbf{x}^i) \neq y^i] = \sum_{i=1}^{m} D_t(i)\underbrace{\delta(h_t(x_i) \neq y_i)}_{\text{Does } h_t \text{ get } i^{th} \text{ point wrong}}$$

$\varepsilon_t = 0$ if h_t perfectly classifies all weighted data pts $\quad \alpha_t = \infty$

$\varepsilon_t = 1$ if h_t perfectly wrong => $-h_t$ perfectly right $\quad \alpha_t = -\infty$

$\varepsilon_t = 0.5 \quad \alpha_t = 0$

Boosting Example (Decision Stumps)



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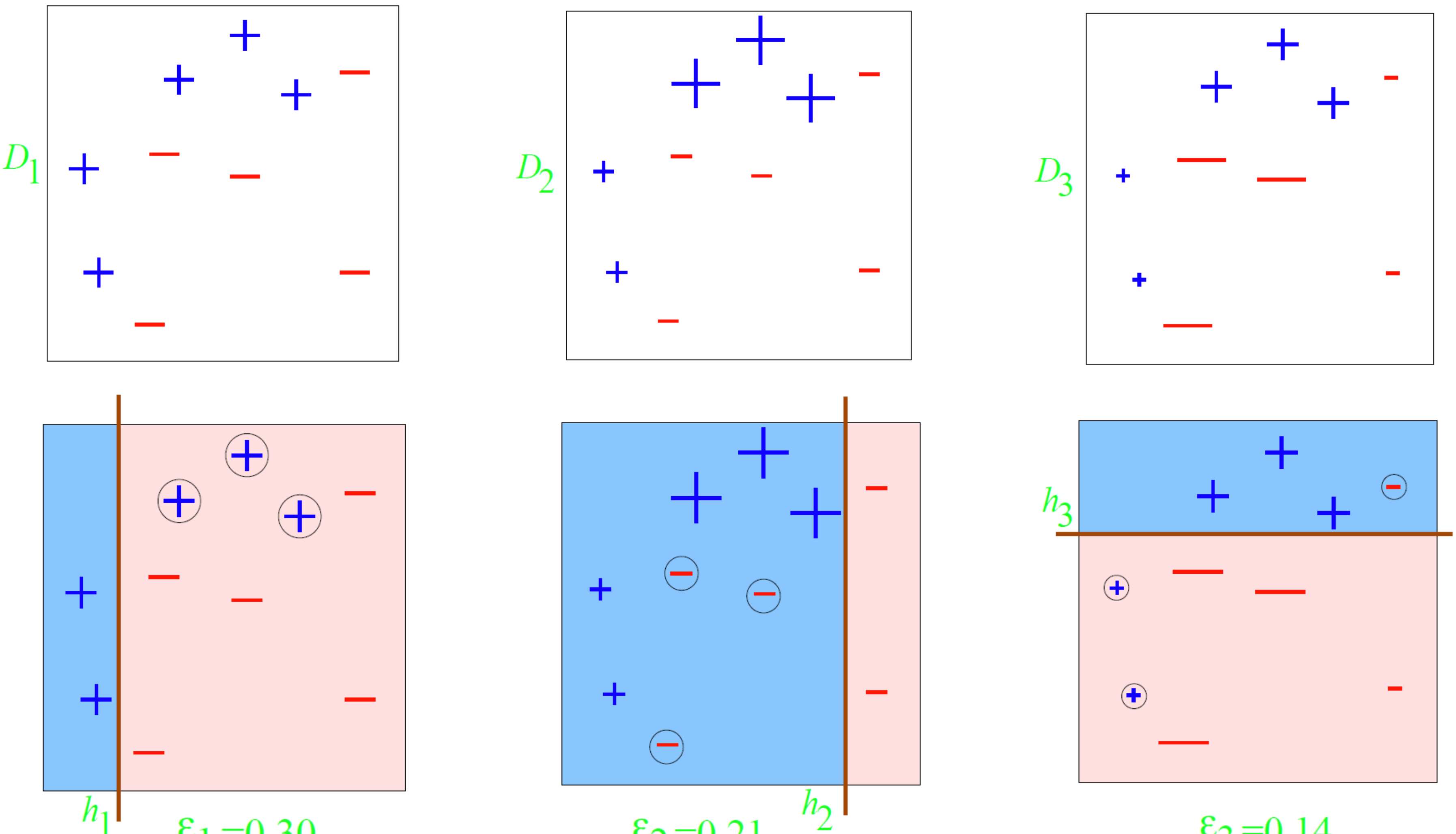

$\varepsilon_1 = 0.30$
$\alpha_1 = 0.42$

$\varepsilon_2 = 0.21$
$\alpha_2 = 0.65$

$\varepsilon_3 = 0.14$
$\alpha_3 = 0.92$

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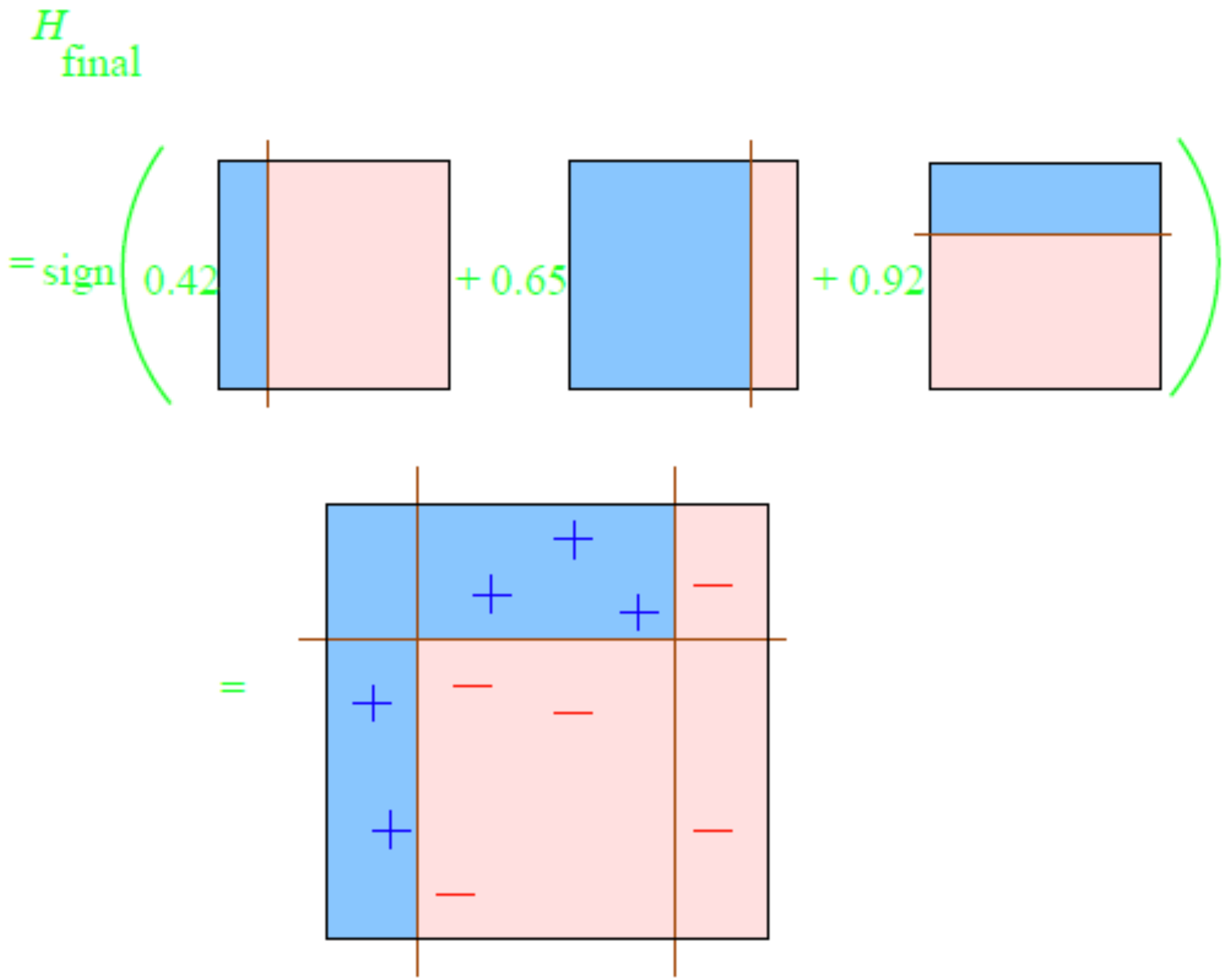
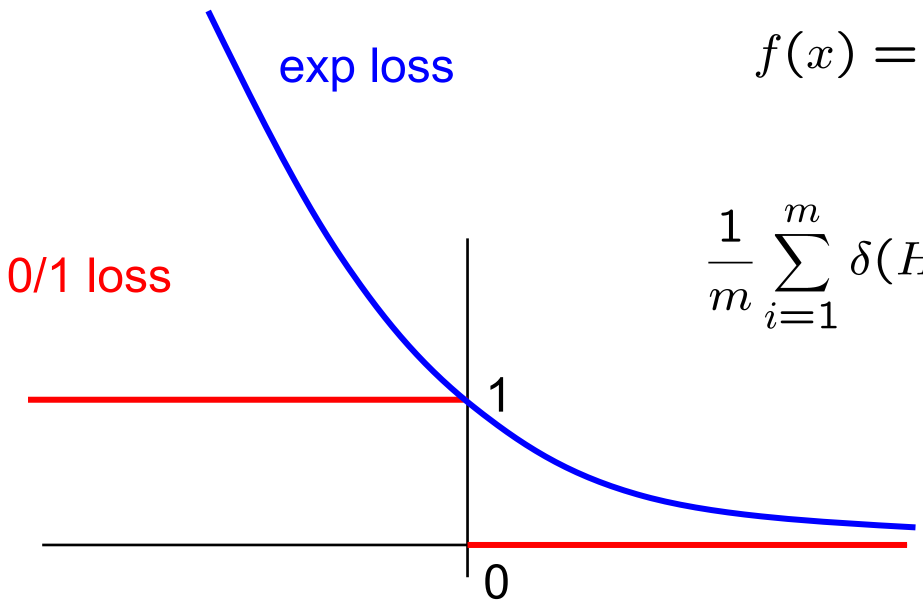


Analysis for Boosting

- Choice of α_t and hypothesis h_t obtained by coordinate descent on exp loss (convex upper bound on 0/1 loss)



$$f(x) = \sum_t \alpha_t h_t(x); H(x) = sign(f(x))$$

$$\frac{1}{m}\sum_{i=1}^{m} \delta(H(x_i) \neq y_i) \leq \frac{1}{m}\sum_{i=1}^{m} \exp(-y_i f(x_i))$$

0/1 loss

exp loss

Analysis for Boosting

Analysis reveals:

- If each weak learner h_t is slightly better than random guessing ($\varepsilon_t < 0.5$), then training error of AdaBoost decays exponentially fast in number of rounds T.

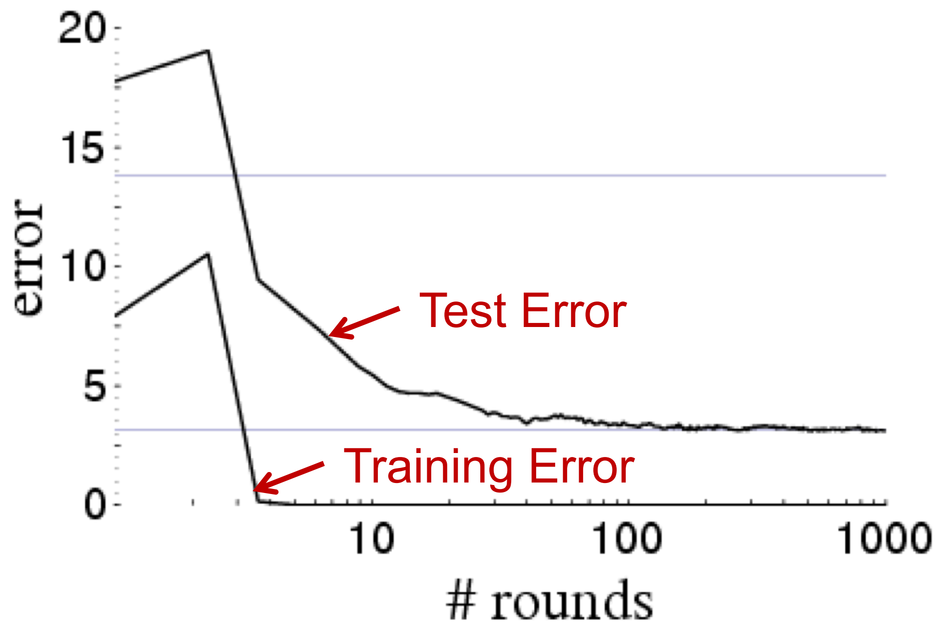
$$\frac{1}{m}\sum_{i=1}^{m}\delta(H(x_i) \neq y_i) \leq \exp\left(-2\sum_{t=1}^{T}(1/2-\epsilon_t)^2\right)$$

Training Error

What about test error?

Boosting results – Digit recognition

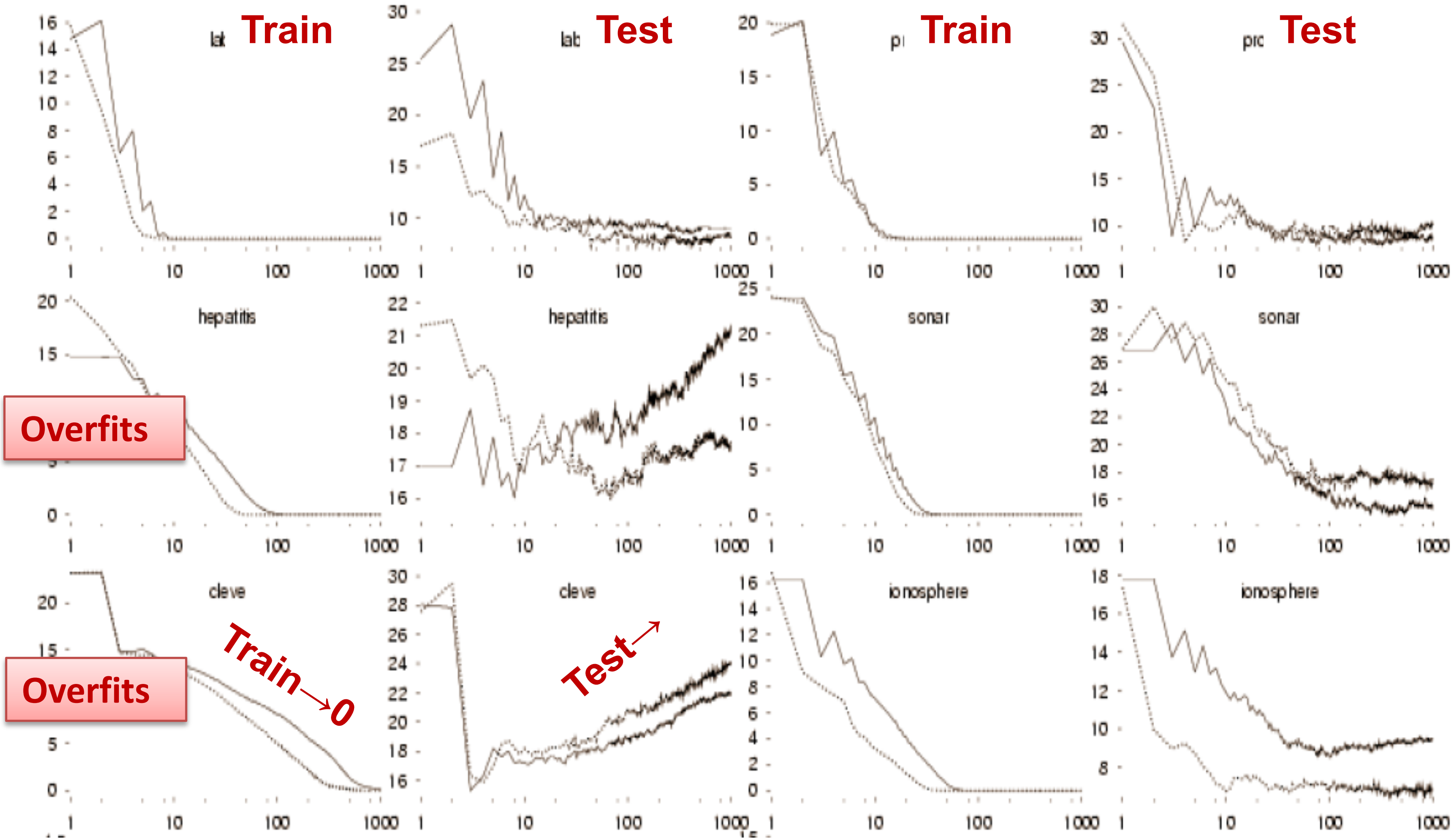
[Schapire, 1989]



- Boosting often,
  - Robust to overfitting
  - Test set error decreases even after training error is zero

- If margin between classes is large, subsequent weak learners agree and hence more rounds does not necessarily imply that final classifier is getting more complex.

AdaBoost and AdaBoost.MH on Train (left) and Test (right) data from Irvine repository. [Schapire and Singer, ML 1999]



Boosting can overfit if margin between classes is too small (high label noise) or weak learners are too complex.

Boosting and Logistic Regression

Logistic regression:

- Minimize log loss

$$\sum_{i=1}^{m} \ln(1 + \exp(-y_i f(x_i)))$$

- Define

$$f(x) = \sum_j w_j x_j$$

where x_j predefined features

(linear classifier)

- Jointly optimize over all weights $w_0, w_1, w_2 \ldots$

Boosting:

- Minimize exp loss

$$\sum_{i=1}^{m} \exp(-y_i f(x_i))$$

- Define

$$f(x) = \sum_t \alpha_t h_t(x)$$

where $h_t(x)$ defined dynamically to fit data

(not a linear classifier)

- Weights α_t learned per iteration t incrementally

Boosting Summary

- Combine weak classifiers to obtain strong classifier
  - Weak classifier – slightly better than random on training data
  - Resulting very strong classifier – can eventually provide zero training error
- AdaBoost algorithm
- Boosting v. Logistic Regression
  - Similar loss functions
  - Single optimization (LR) v. Incrementally improving classification (B)
- Most popular application of Boosting:
  - Boosted decision stumps!
  - Very simple to implement, very effective classifier