Step-to-step Dynamic Programming Results

1 Sagittal Plane Only

We are exploring using dynamic programming to determine foot placement, first in a planar walker, and then in a simple 3D walker. The walker has massless legs, with all mass located at a point at the hips. The trajectory of the stance leg is given by the dynamics of a simple inverted pendulum, with a mass, length, and moment of inertia of 1. Gravity is $9.81 m/s^2$, and the integration time step is 1 millisecond.

We will use a Poincare section to define the state of our model. We place the section when the stance leg is vertical, so the hip is at its highest point (TOP). The state at TOP is just the angular velocity of the stance leg. We will consider angular velocities at TOP between 0 and 3 radians/second, as above 3 the foot lifts off the ground.

We will define two actions that are taken at TOP and held constant through the step cycle until the next TOP. The first action is to choose a hip angle in the sagittal or pitch direction (hip pitch angle), and thus a landing angle of the swing leg. The swing leg pitch angle is measured relative to the stance leg. The second action is a constant pitching torque at whatever ankle is the stance ankle (the same torque is used before and after the foot transition at impact). The torque is necessary to make up for energy losses during impacts. We will consider hip angles less than one radian and thus leg angles with respect to vertical of less than half a radian, to remain within the friction cone and avoid slipping on landing. We will limit foot/ankle torques to less than 1 Newton-meter in magnitude so that the center of pressure remains within a foot of length 20cm.

The criteria being optimized is a torque penalty of $T * \tau^2$ on the ankle torque τ, with T being the cycle duration (from TOP to TOP), a swing penalty of $(\phi/T^2)^2$ on the touchdown hip angle, with the T^2 relating the touchdown hip pitch angle ϕ to the acceleration necessary to move the swing leg to that angle in that time in a bang-bang trajectory, and a velocity penalty of $(2 * l * sin(\phi/2)/T - v_d)^2$ on the estimated velocity where l is the leg length, and v_d is the desired velocity. These penalties are evaluated at each occurance of TOP. The swing penalty is weighted by 0.1 relative to the other penalties. The desired velocity is 2, typically achieving an actual velocity of about 1 in terms of optimized performance. There is a discount factor of 0.99 per step, to allow the value function computation to converge.

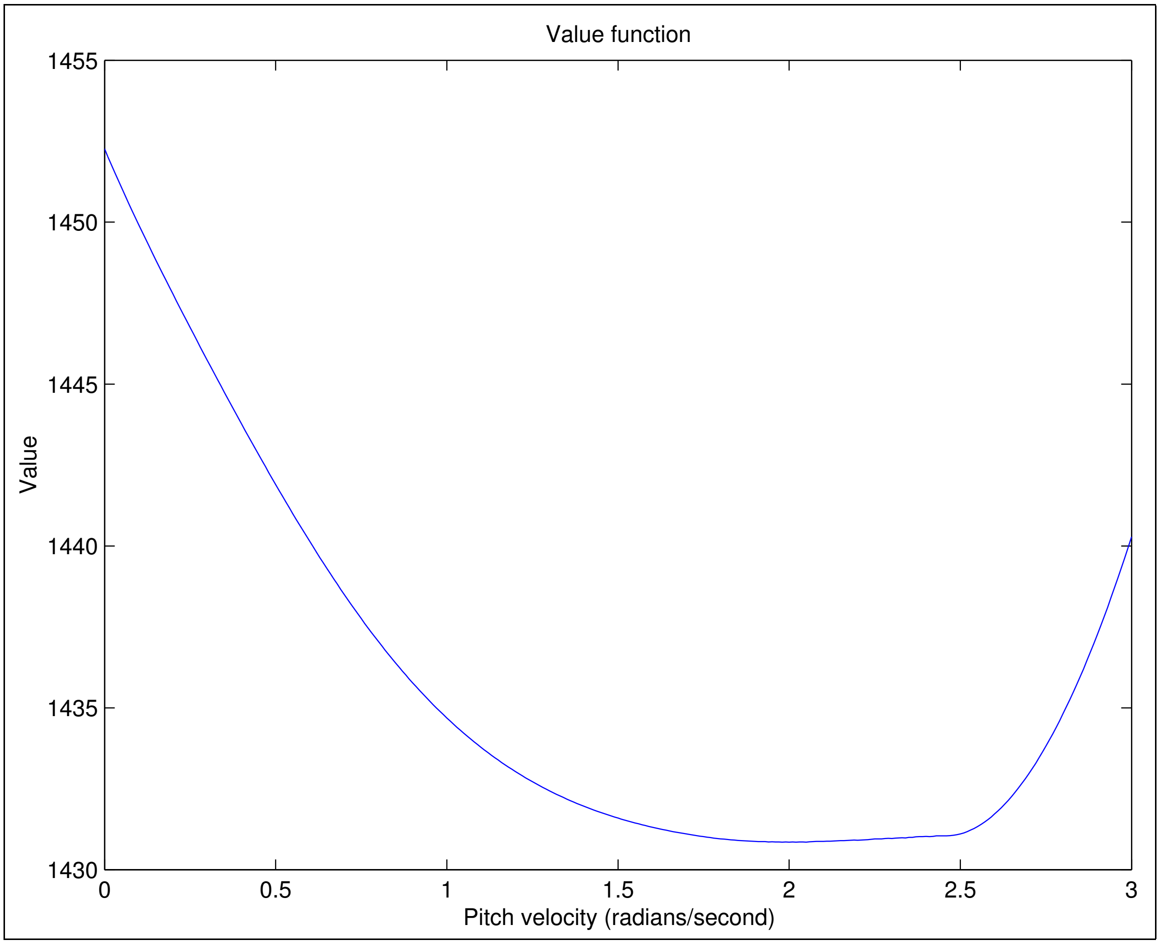

Figure 1: Value function for sagittal plane only. X axis is angular velocity at top, and Y axis is value.

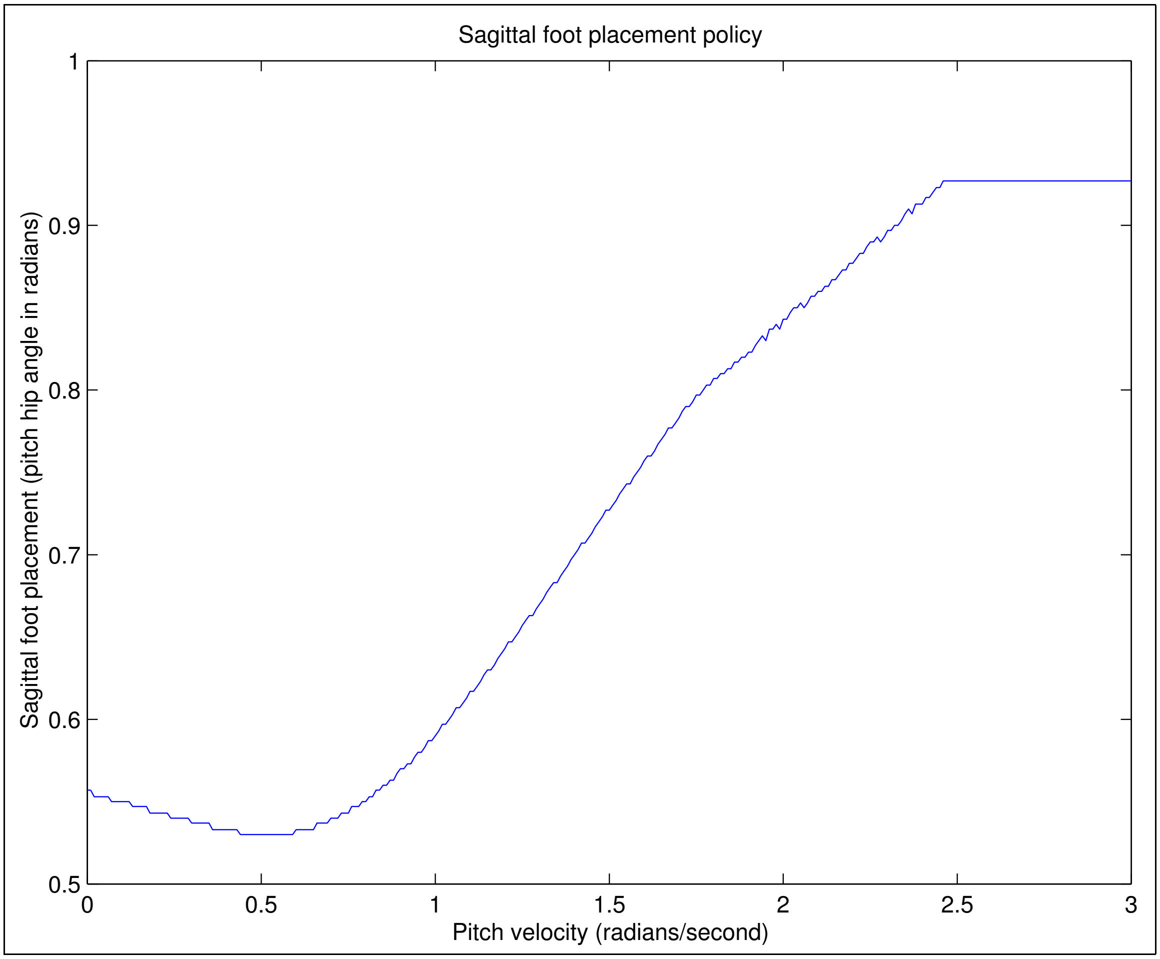

Figure 2: Touchdown hip angle for sagittal plane only. X axis is angular velocity at top, and Y axis is hip angle in the pitch direction.

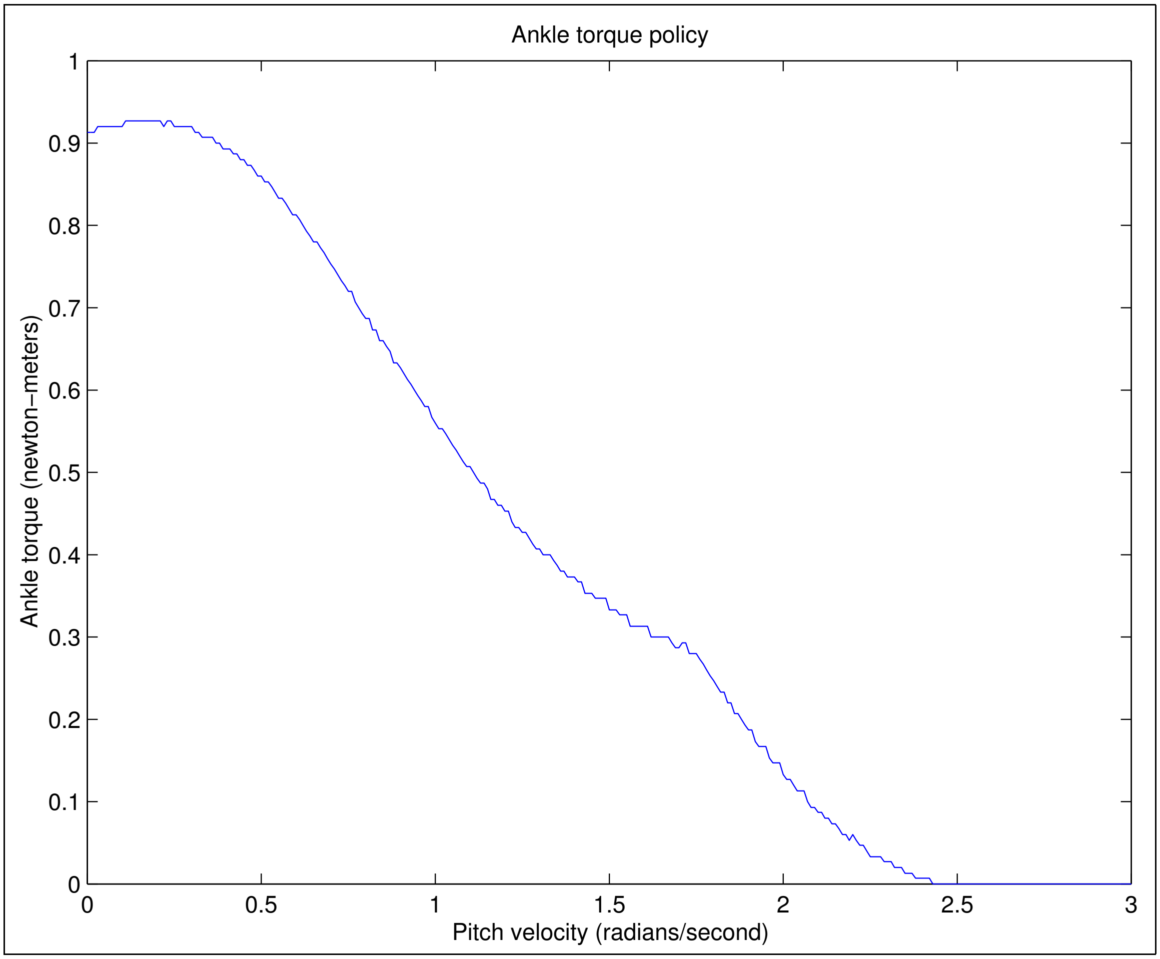

Figure 3: Ankle torque for sagittal plane only. X axis is angular velocity at top, and Y axis is ankle torque command.

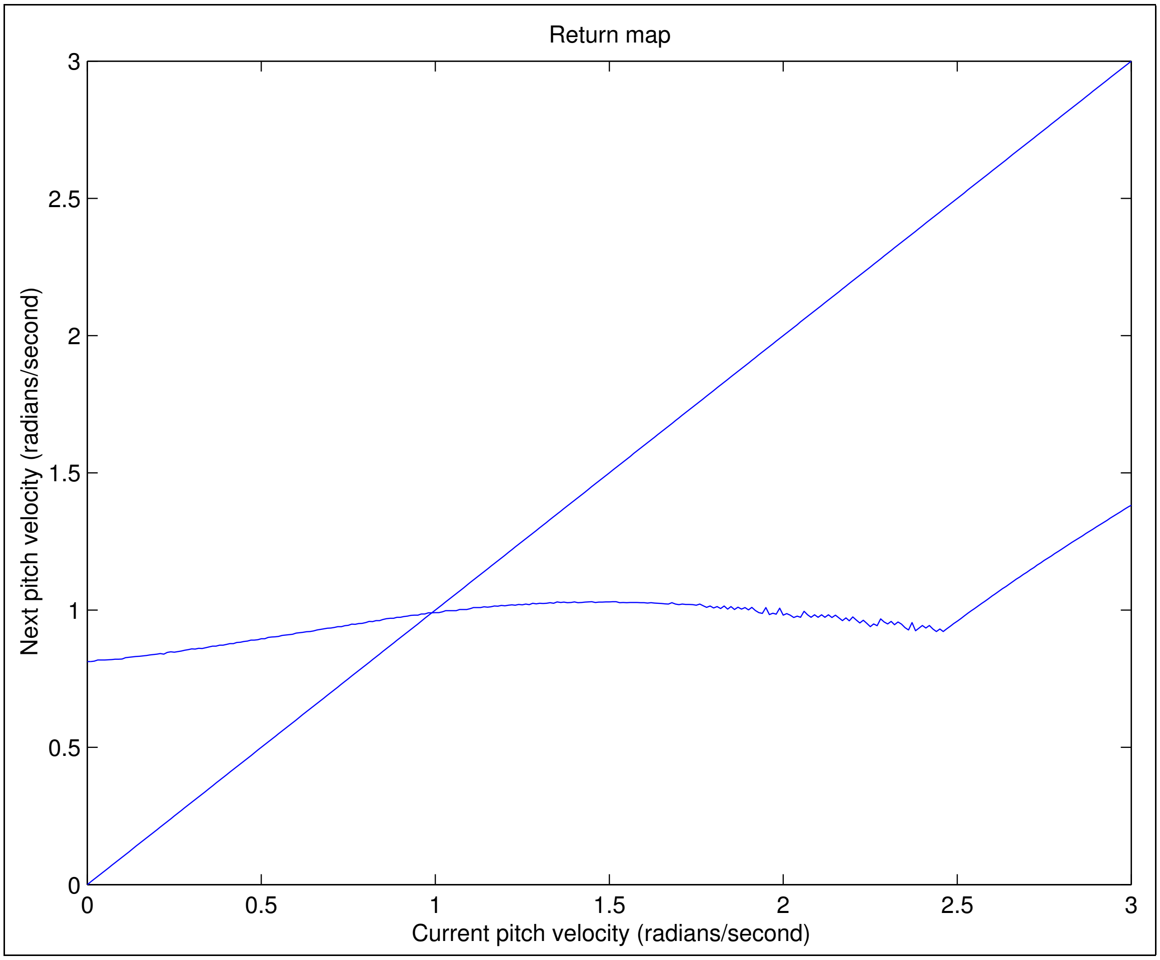

Figure 4: Return map for sagittal plane only. X axis is angular velocity at top, and Y axis is angular velocity at top on subsequent step with controller running. Diagonal line $x = y$ is for reference.

2 3D case

We study the 3D case to get some insight into lateral foot placement.

For the inverted pendulum walker, 3D walking has two directions, pitch (positive is forward) and roll (positive is to the right side). Since the body mass is concentrated at a point, yaw movement has no effect and is ignored. In this version of the inverted pendulum walker we represent body configuration as if there was a universal joint at the ankle with pitch first (inner gimball) and roll second (outer gimball). The swing leg has a similar arrangement at the hip, with pitch as the inner gimball and roll as the outer gimball.

The parameters of the model are the same. The walker has massless legs, with all mass located at a point at the hips. The trajectory of the stance leg is given by the dynamics of a simple inverted pendulum, with a mass, length, and moment of inertia of 1. Gravity is 9.81, and the integration time step is 1 millisecond.

We use a similar Poincare section to define the state of our model. We place the section when the stance leg has zero pitch. The state at TOP is given by the pitch velocity, the roll, and the roll velocity. We will consider pitch velocities at TOP between 0 and 3 radians/second, as above 3 the foot lifts off the ground. We consider roll angles less than 0.25 radians, and roll velocities less than 0.5 radians/second.

We add a roll hip angle (lateral foot placement) to the pitch hip angle (sagittal foot placement) and ankle torque actions. We will consider roll hip angles less than 0.5 radian, to avoid slipping on landing.

We add a swing penalty to the criteria of $(\phi_{roll}/T^2)^2$ on the touchdown hip roll angle that matches the swing penalty on the pitch hip angle.

SHOW PITCH ONLY RESULTS ARE THE SAME.

A useful set of coordinates for thinking about roll is to consider roll angles and angular velocities that will result in the inverted pendulum returning to vertical with no actuation. These are given by $velocity = -\alpha * position$ near the equilibrium where the natural behavior of the inverted pendulum is an unstable exponential divergence from the equilibrium. $\alpha = mass * length * g/moment - of - inertia$. In (roll,roll velocity) coordinates the direction $(1, -\alpha)$ takes no effort to return to the equilibrium, and the direction $(\alpha, 1)$ requires maximal cost, in some sense. We will refer to the no effort direction $(1, -\alpha)$ as the α direction.

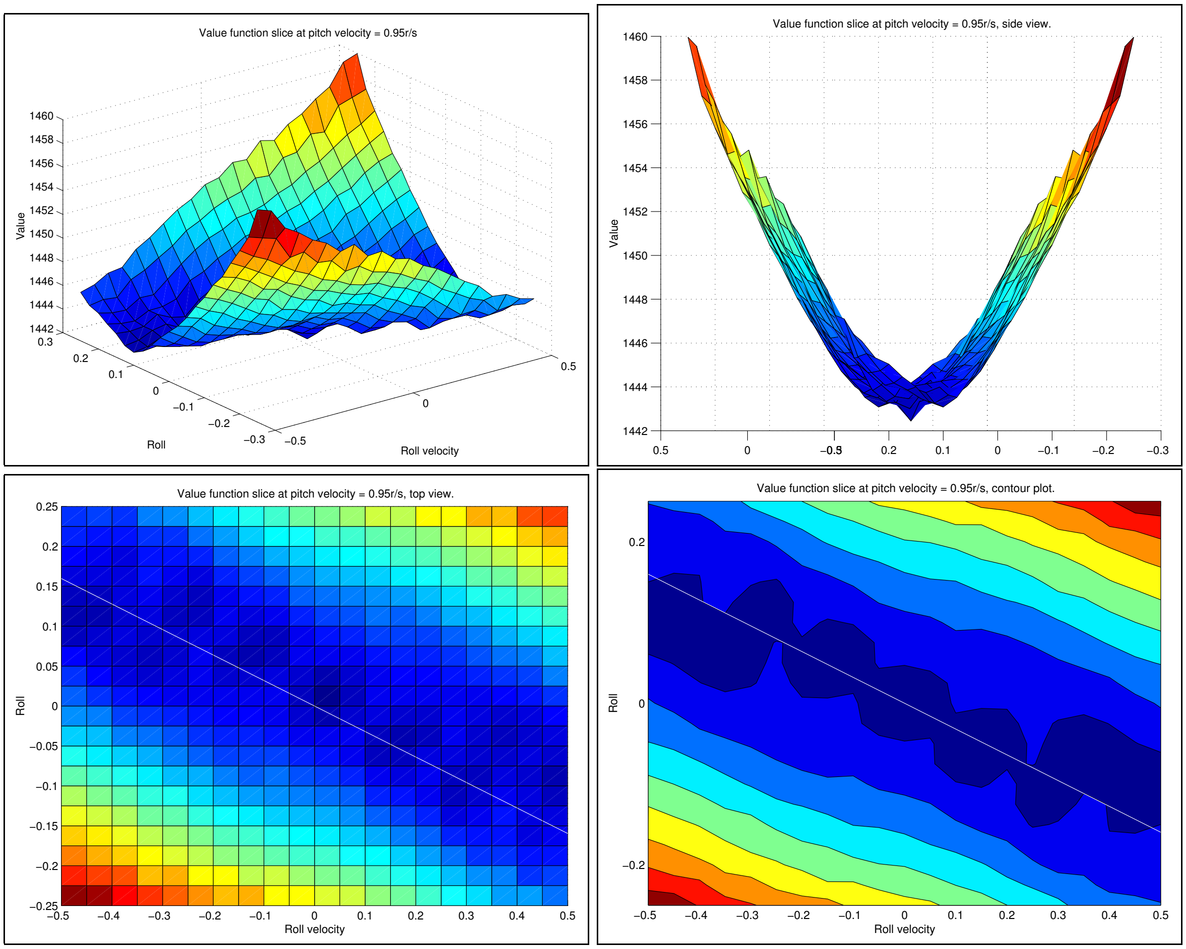

Figure 5: Value function slice at pitch velocity = 0.95r/s. Top left: 3d plot. Top right: 3d plot viewed along the α direction. Bottom left: top view. Bottom right: contour map. The diagonal line is the alpha direction in the bottom plots. We see a fairly simple quadratic value function in the direction perpendicular to the α direction.

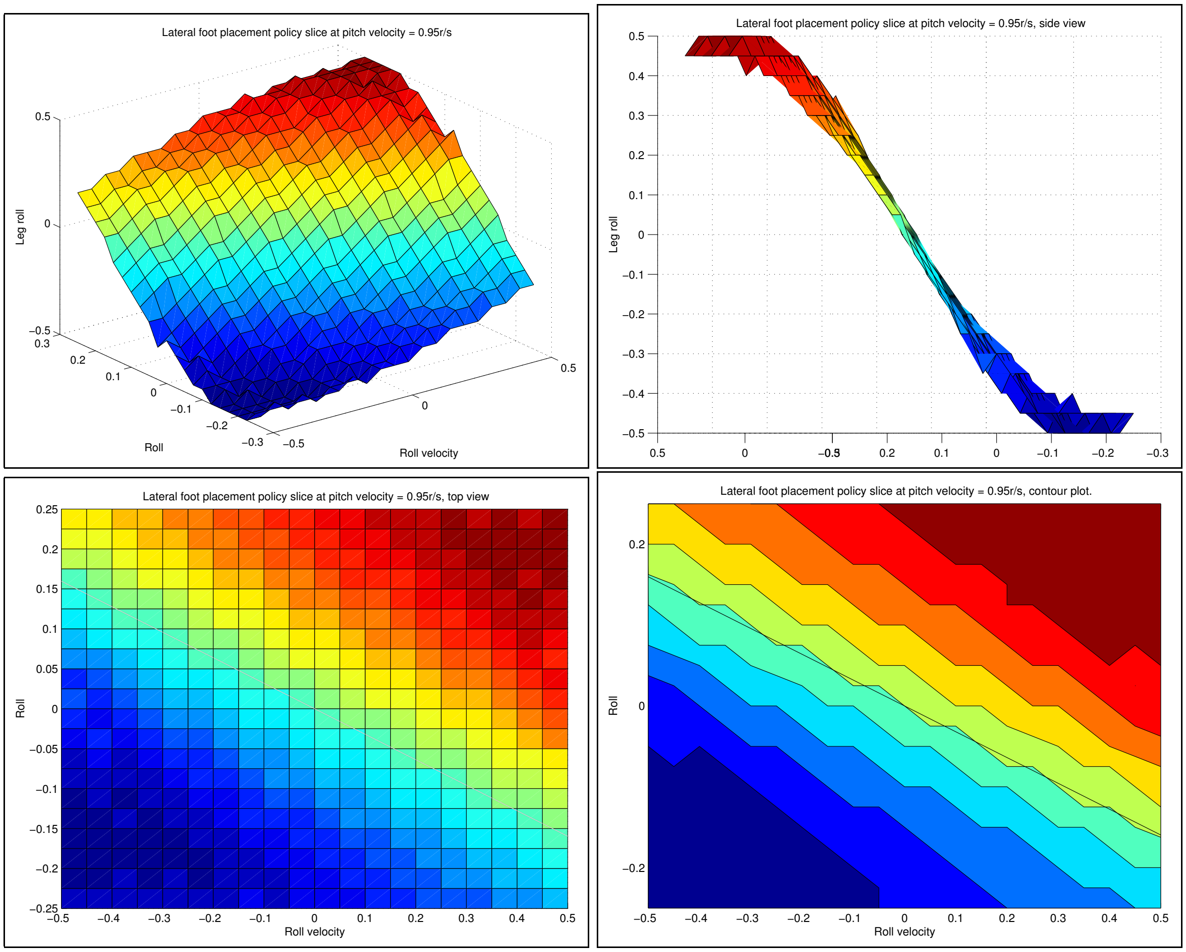

Figure 6: Lateral foot placement (hip angle in roll direction) policy slice at pitch velocity = 0.95r/s. Top left: 3d plot. Top right: 3d plot viewed along the α direction. Bottom left: top view. Bottom right: contour map. The diagonal line is the alpha direction in the bottom plots. We see that the lateral foot placement dependence on body roll and roll velocity is almost linear, at a fixed body pitch velocity.

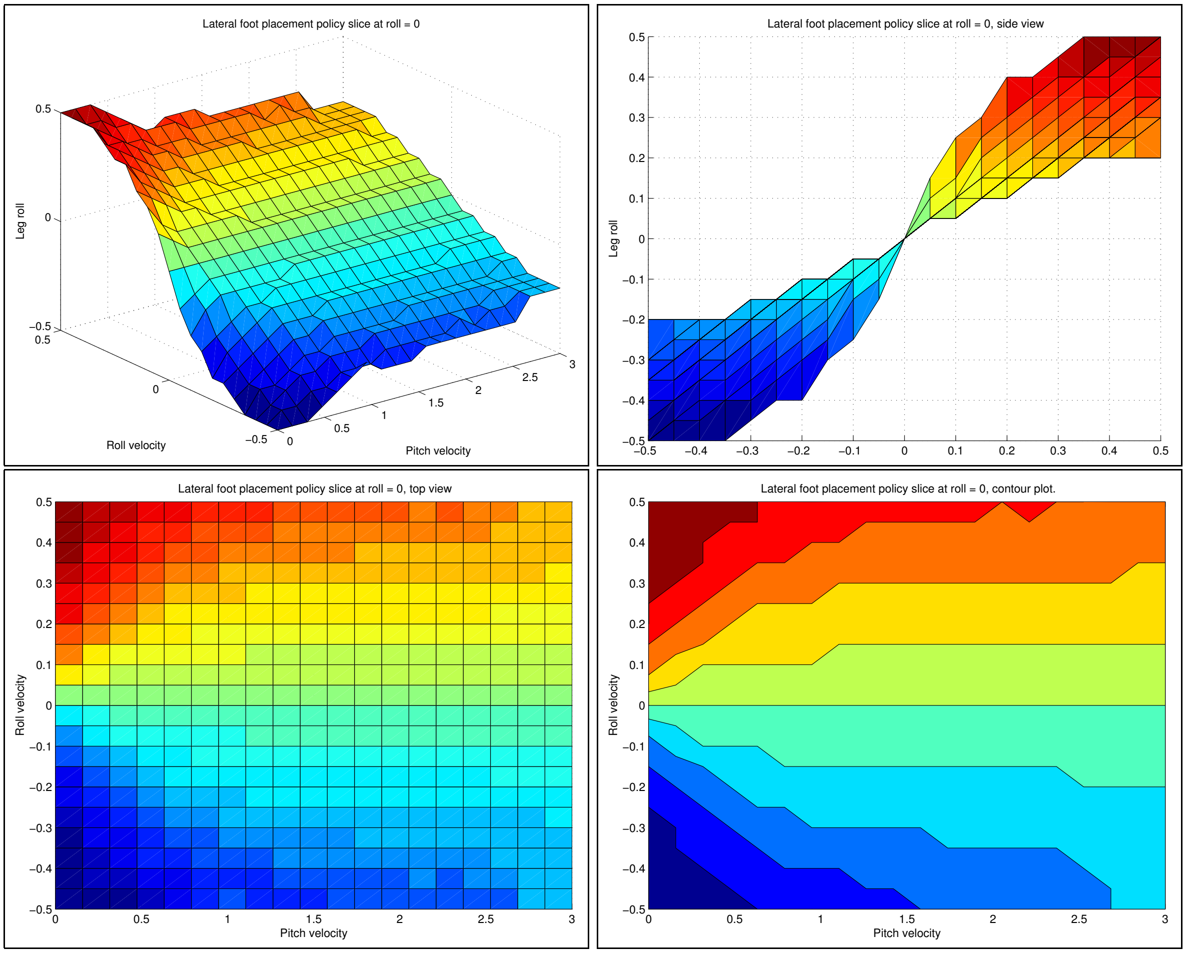

Figure 7: Lateral foot placement (hip angle in roll direction) policy slice at roll = 0. Top left: 3d plot. Top right: 3d plot viewed along the pitch velocity axis. Bottom left: top view. Bottom right: contour map. We see that the lateral foot placement dependence on body roll velocity is almost linear, at a fixed body pitch velocity. The “gain” of the lateral foot placement dependence on roll velocity increases for pitch velocities below 1r/s.

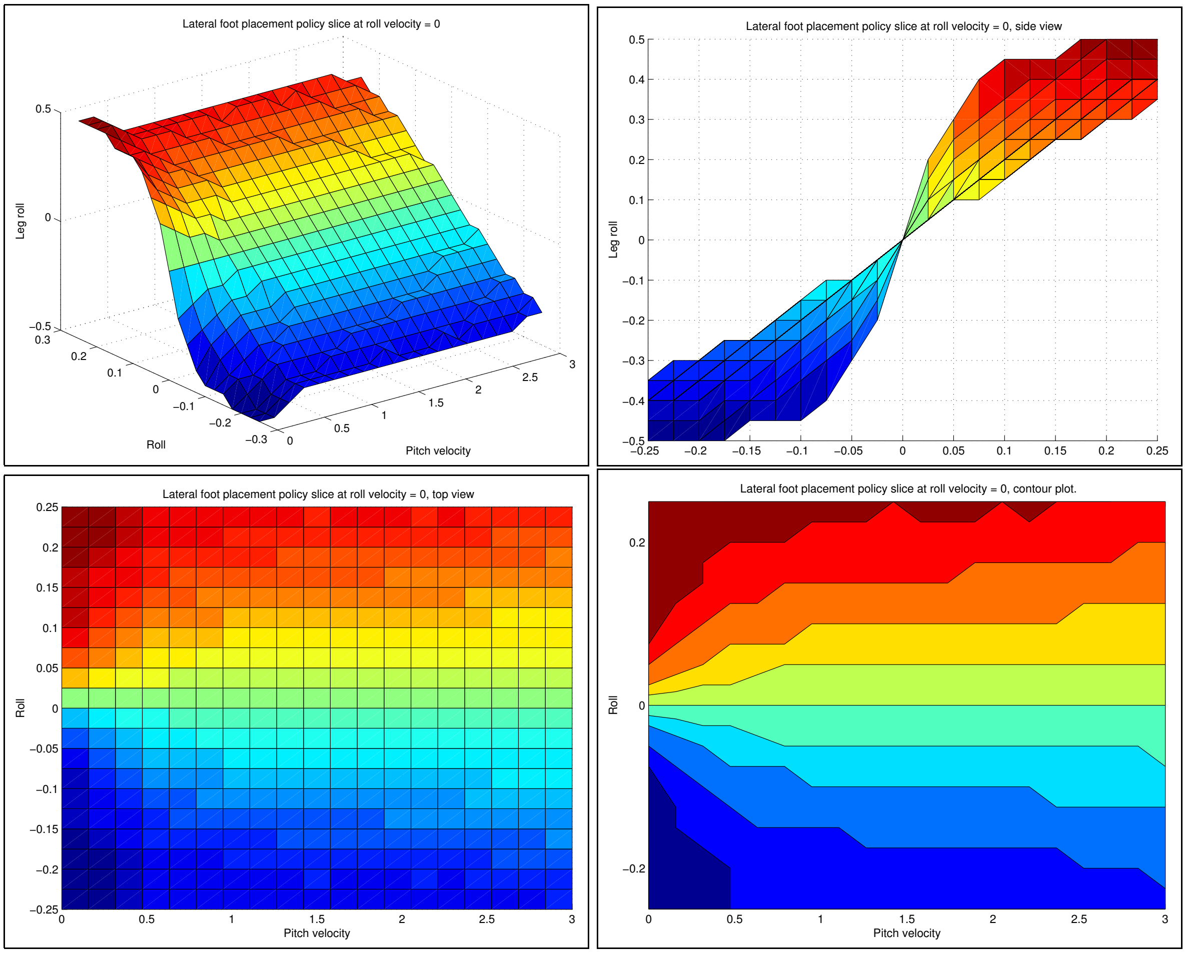

Figure 8: Lateral foot placement (hip angle in roll direction) policy slice at roll velocity = 0. Top left: 3d plot. Top right: 3d plot viewed along the pitch velocity axis. Bottom left: top view. Bottom right: contour map. We see that the lateral foot placement dependence on body roll is almost linear, at a fixed body pitch velocity. The "gain" of the lateral foot placement dependence on roll increases for pitch velocities below 1r/s.

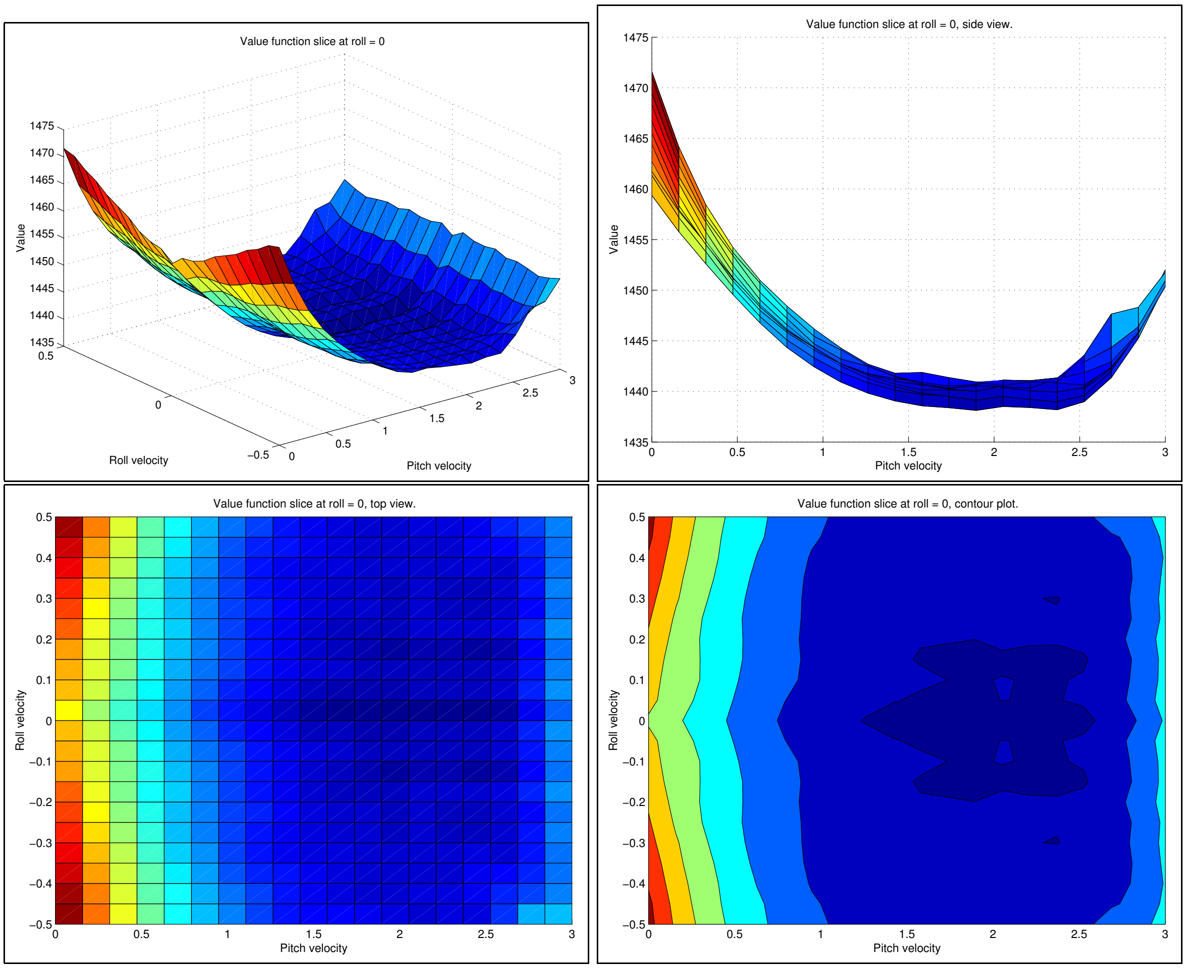

Figure 9: Value function slice at roll = 0. Top left: 3d plot. Top right: 3d plot viewed along the roll velocity axis. Bottom left: top view. Bottom right: contour map. We see a small dependence on roll velocity.

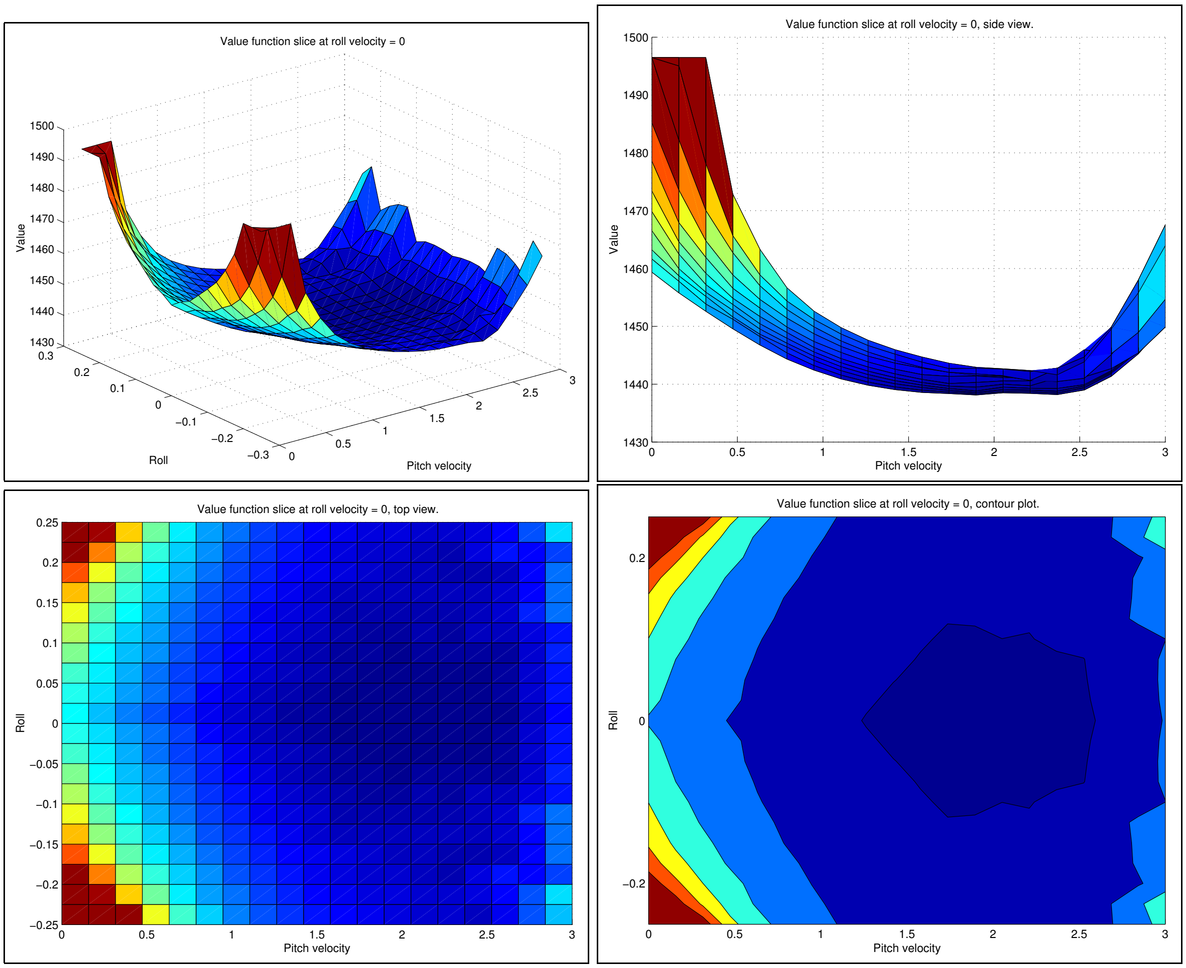

Figure 10: Value function slice at roll velocity = 0. Top left: 3d plot. Top right: 3d plot viewed along the roll axis. Bottom left: top view. Bottom right: contour map. We see a small dependence on roll.

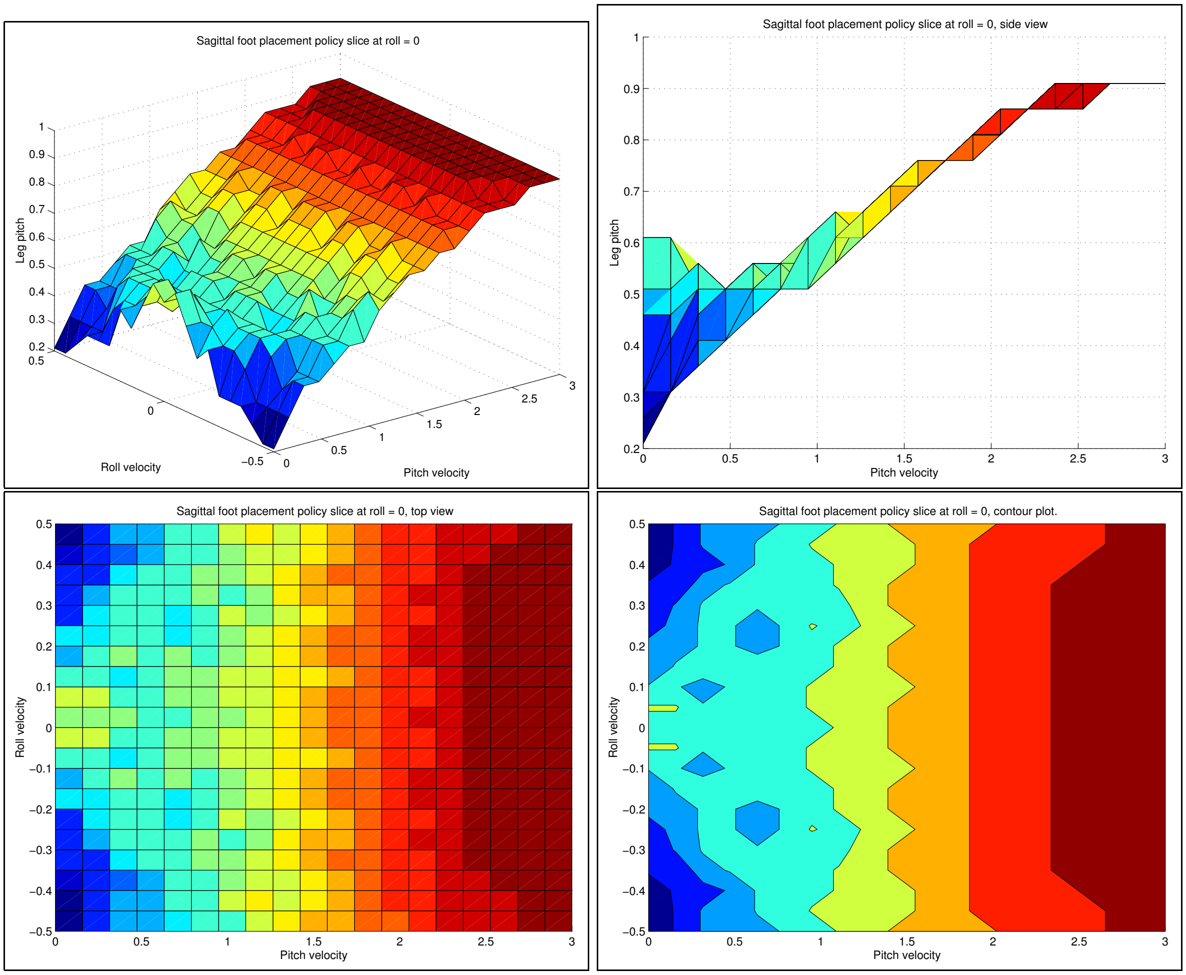

Figure 11: Sagittal foot placement (hip pitch angle) policy slice at roll = 0. Top left: 3d plot. Top right: 3d plot viewed along the roll velocity axis. Bottom left: top view. Bottom right: contour map. We see a small quadratic dependence on roll velocity at low pitch velocities.

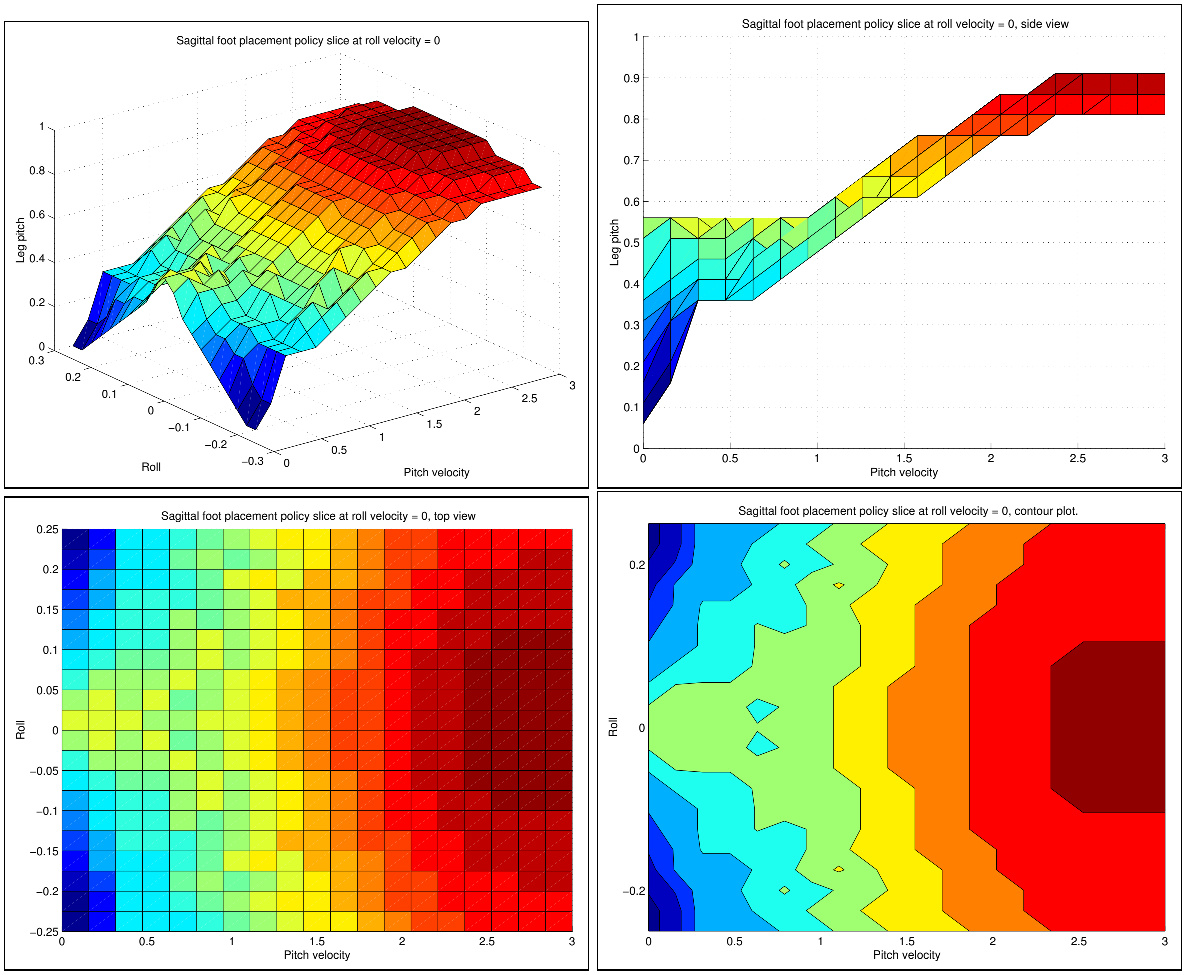

Figure 12: Sagittal foot placement (hip pitch angle) policy slice at roll velocity = 0. Top left: 3d plot. Top right: 3d plot viewed along the roll axis. Bottom left: top view. Bottom right: contour map. We see a small quadratic dependence on roll at low pitch velocities.

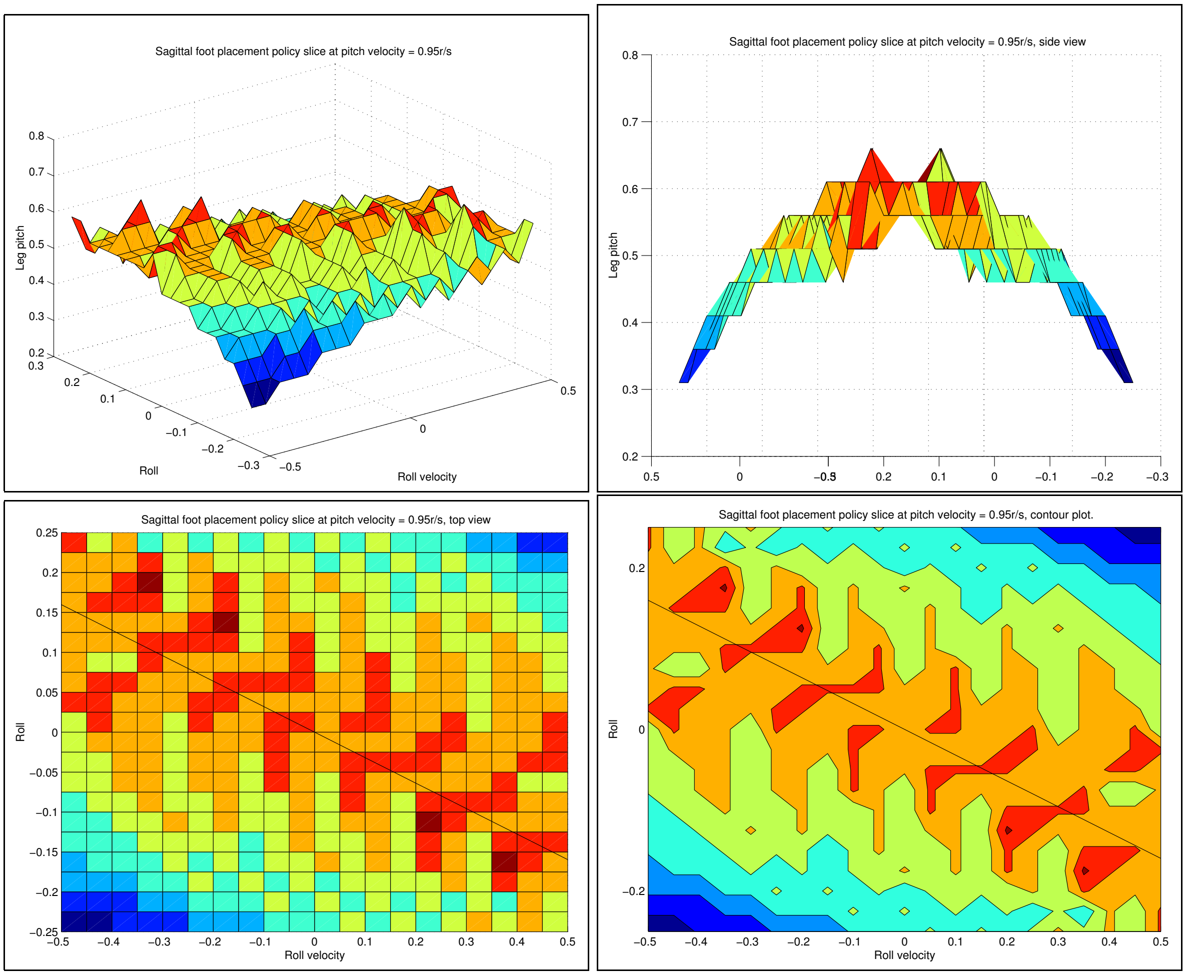

Figure 13: Sagittal foot placement (hip angle in pitch direction) policy slice at pitch velocity = 0.95r/s. Top left: 3d plot. Top right: 3d plot viewed along the α direction. Bottom: top view. Bottom right: contour map. The diagonal line is the alpha direction in the bottom plots. We see that the sagittal foot placement dependence on body roll and roll velocity is quadratic in the direction perpendicular to the alpha direction.

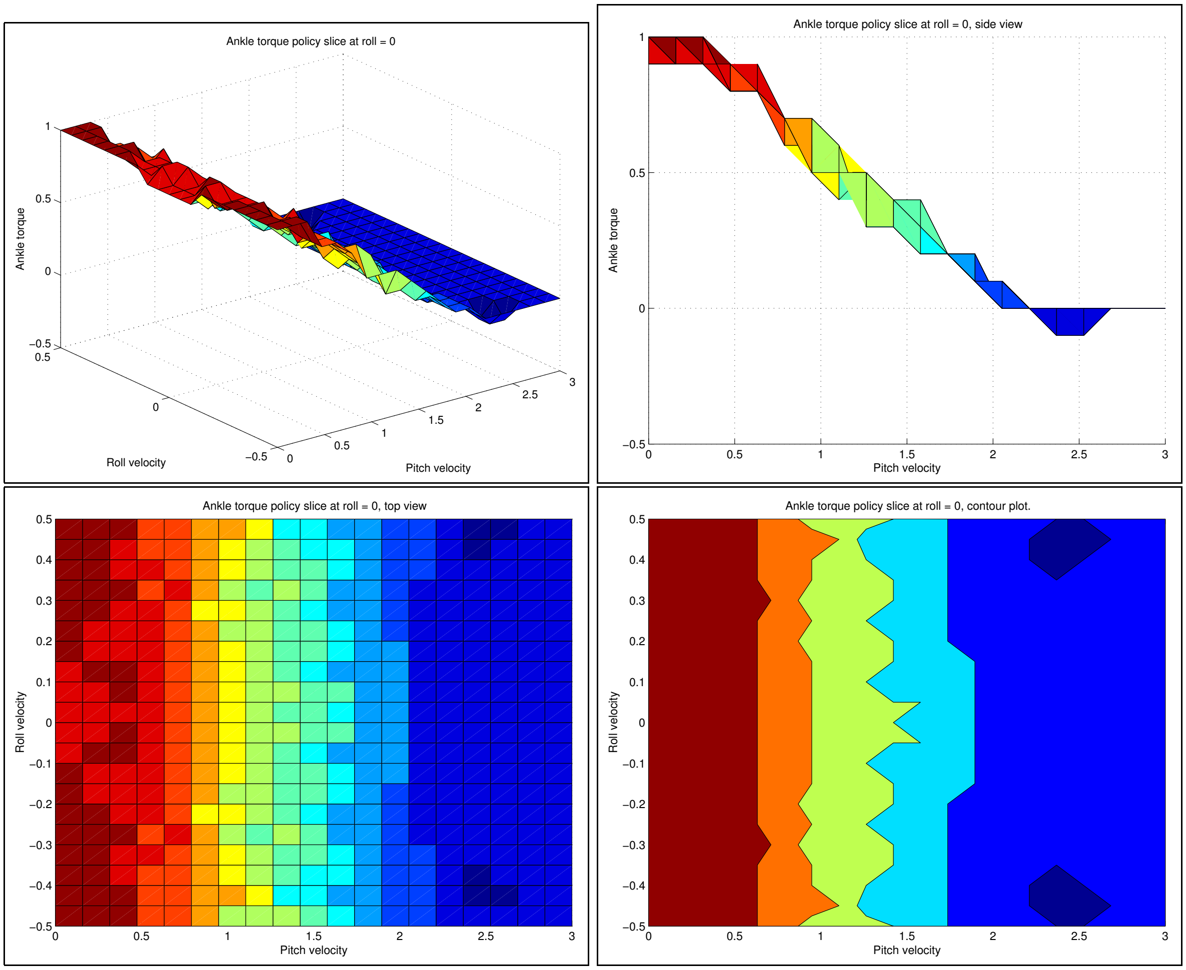

Figure 14: Ankle torque policy slice at roll = 0. Top left: 3d plot. Top right: 3d plot viewed along the roll velocity axis. Bottom left: top view. Bottom right: contour map. We don't see much dependence on roll velocity.

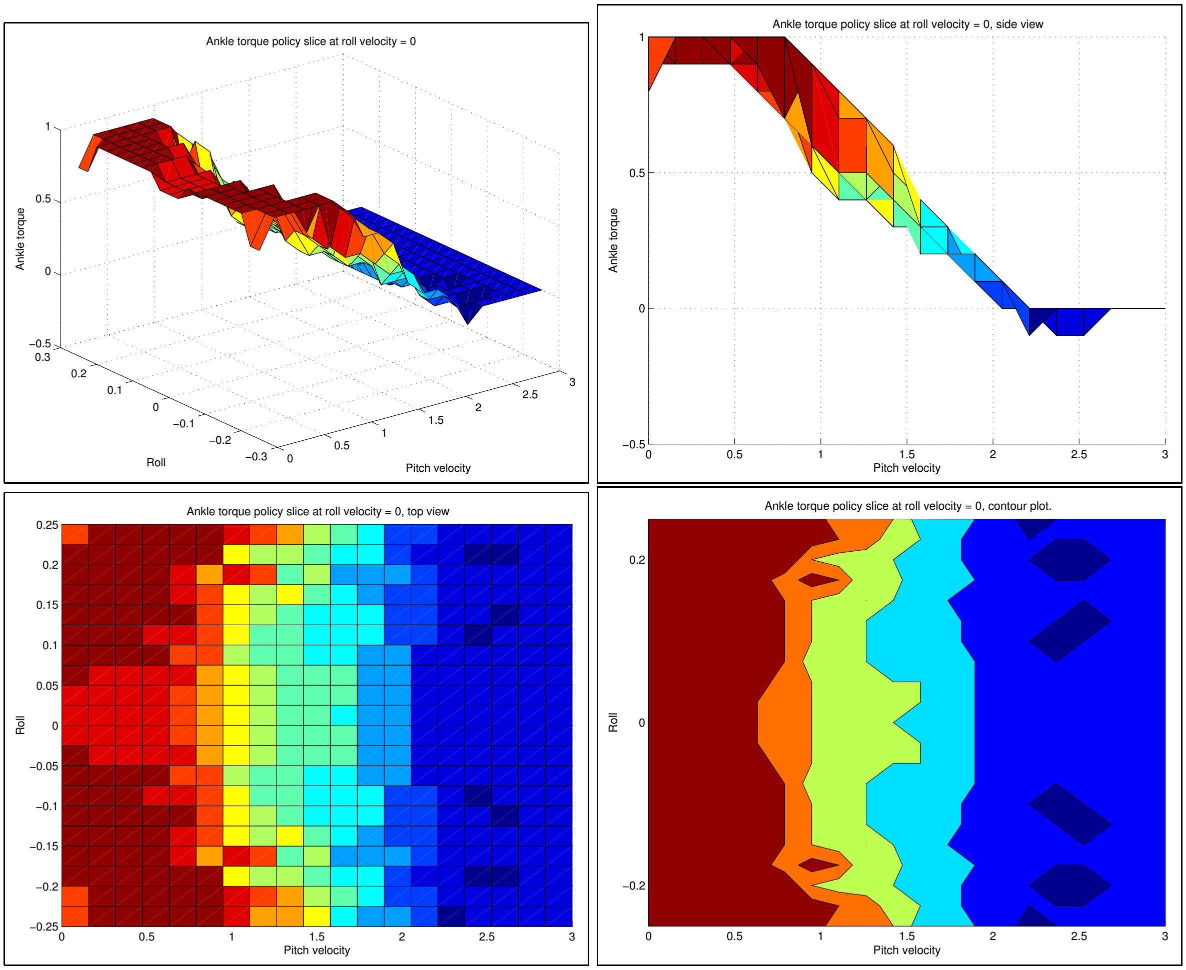

Figure 15: Ankle torque policy slice at roll velocity = 0. Top left: 3d plot. Top right: 3d plot viewed along the roll axis. Bottom left: top view. Bottom right: contour map. We don't see much dependence on roll.

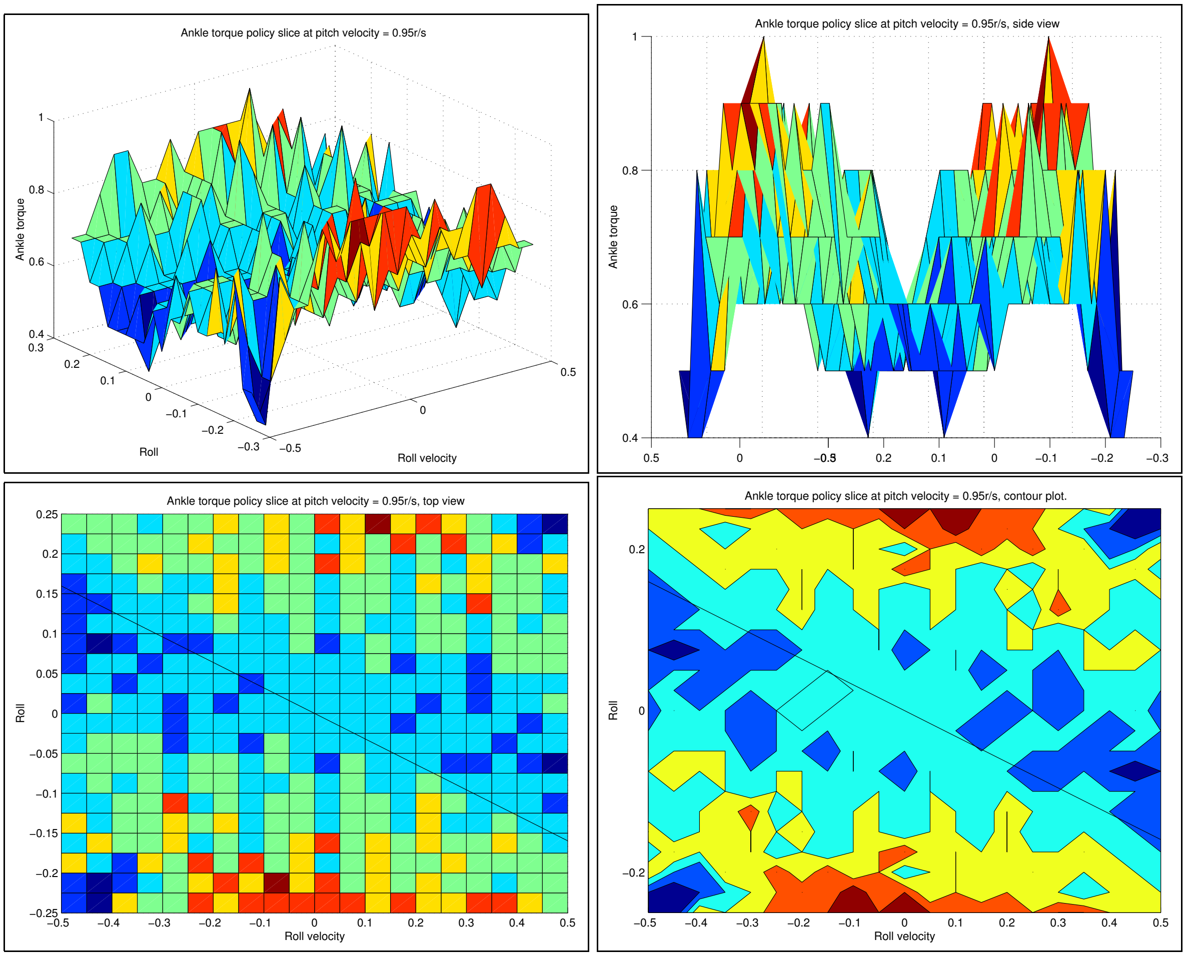

Figure 16: Ankle torque policy slice at pitch velocity = 0.95r/s. Top left: 3d plot. Top right: 3d plot viewed along the alpha direction. Bottom: top view. Bottom right: contour map. The diagonal line is the alpha direction in the bottom plots. We see that the ankle torque dependence on body roll and roll velocity is quadratic in the direction perpendicular to the α direction. with some possible edge effects.

3 Future Work

This analysis has some simplifications and missing elements:

- No leg dynamics, so no effect of leg swing on body.
- Add energy through ankle torque. could have considered a variety of ankle torque strategies (functions of time or state). Could have considered pushoff. Could have done this on ramp and eliminated one action. Or considered fixed pushoff or ankle torque strategy.
- No double support period.
- No left or right legs or hip spacing. Each leg can swing either way in the roll direction.
- No yaw dynamics. Since the body is a point mass, there is no notion of a facing direction, or having the robot turn due to dynamics.
- No consideration of uncertainty in planning. Vary ground height, landing location.

Compare to Kuo IJRR 99 lateral foot placement paper.
Get better agreement between pr, prd, rrd (u0, u1).