Large Graph Mining

Christos Faloutsos CMU

Thank you!

- Ed Kao
- Lori Tsoulas
- Joan Meehan-Dion

Our goal:

Open source system for mining huge graphs:

PEGASUS project (PEta GrAph mining System)

• www.cs.cmu.edu/~pegasus



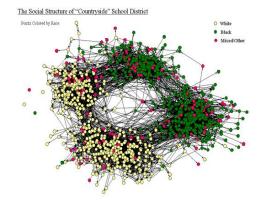
• code and papers

Outline

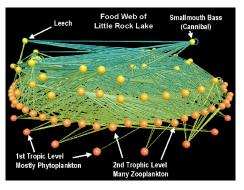
- Introduction Motivation
 - Problem#1: Patterns in graphs
 - Problem#2: Tools
 - Problem#3: Scalability
 - Conclusions

Graphs - why should we care?

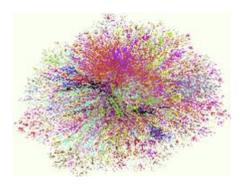




Friendship Network [Moody '01]



Food Web [Martinez '91]

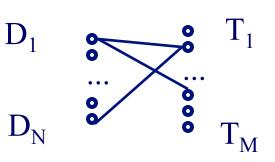


Internet Map [lumeta.com]

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Graphs - why should we care?

• IR: bi-partite graphs (doc-terms)



• web: hyper-text graph

• ... and more:

GraphEx'11

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Graphs - why should we care?

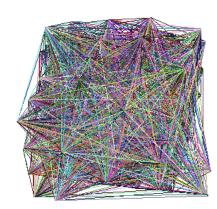
- 'viral' marketing
- web-log ('blog') news propagation
- computer network security: email/IP traffic and anomaly detection

•

Outline

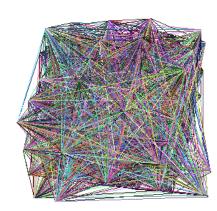
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Problem #1 - network and graph mining



- What does the Internet look like?
- What does FaceBook look like?
- What is 'normal'/'abnormal'?
- which patterns/laws hold?

Problem #1 - network and graph mining

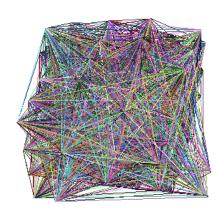


• What does the Internet look like?

- What does FaceBook look like?
- What is 'normal'/'abnormal'?
- which patterns/laws hold?
 - To spot anomalies (rarities), we have to discover patterns

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Problem #1 - network and graph mining



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- What does the Internet look like?
- What does FaceBook look like?
- What is 'normal'/'abnormal'?
- which patterns/laws hold?
 - To spot anomalies (rarities), we have to discover patterns
 - Large datasets reveal patterns/anomalies that may be invisible otherwise...

Graph mining

• Are real graphs random?

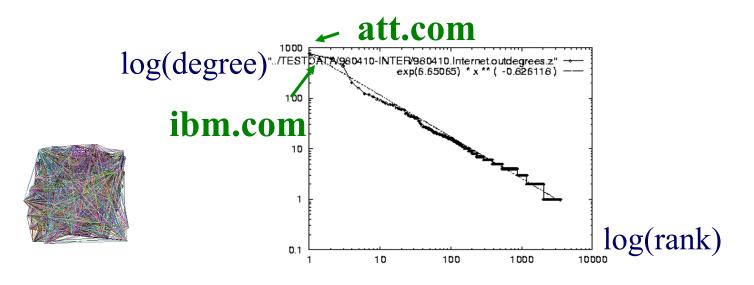
Laws and patterns

- Are real graphs random?
- A: NO!!
 - Diameter
 - in- and out- degree distributions
 - other (surprising) patterns
- So, let's look at the data

Solution# S.1

• Power law in the degree distribution [SIGCOMM99]

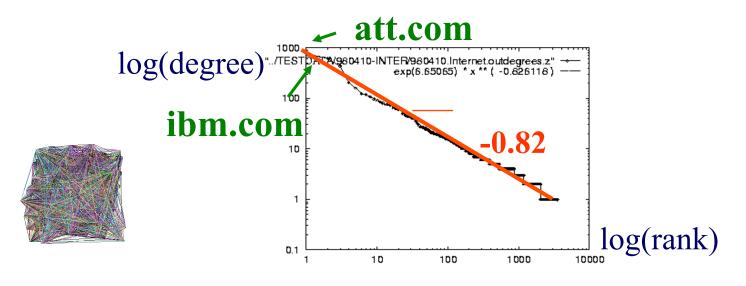
internet domains



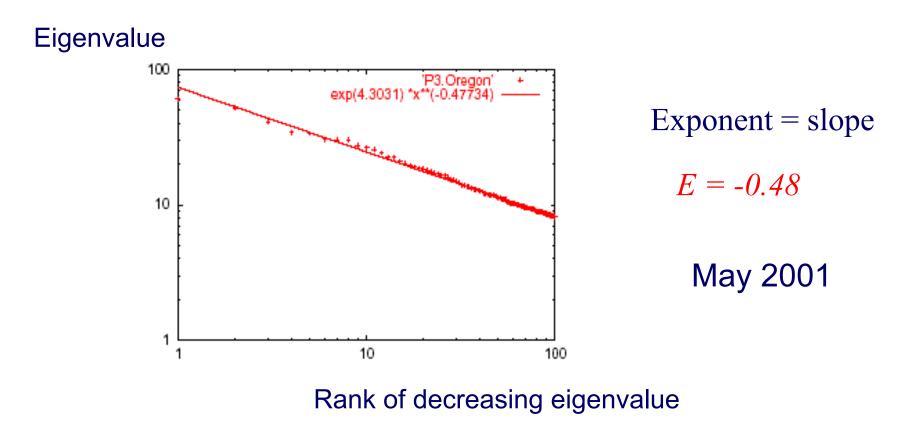
Solution# S.1

• Power law in the degree distribution [SIGCOMM99]

internet domains



Solution# S.2: Eigen Exponent E



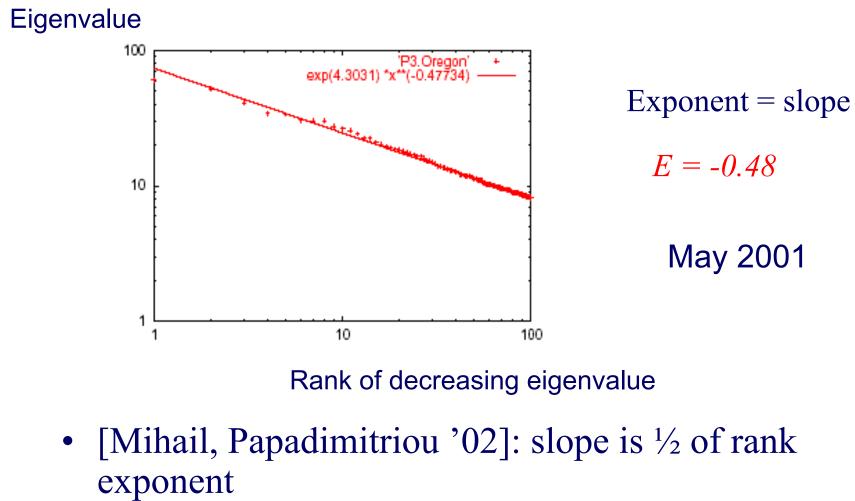
• A2: power law in the eigenvalues of the adjacency matrix

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Solution# S.2: Eigen Exponent E



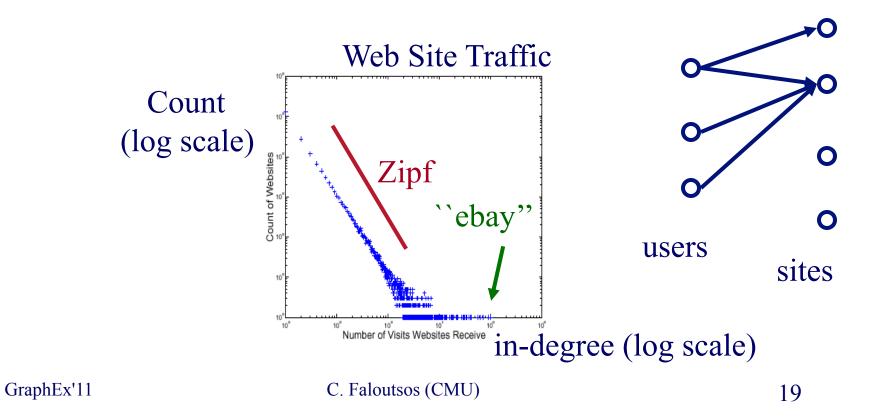
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But:

How about graphs from other domains?

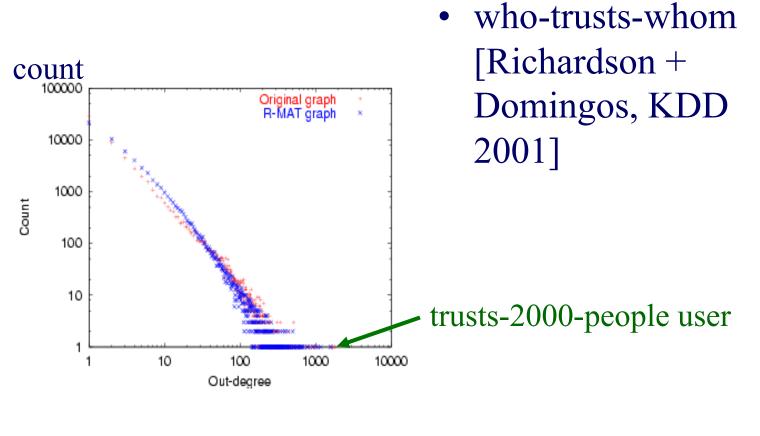
More power laws:

• web hit counts [w/ A. Montgomery]



0

epinions.com



(out) degree



And numerous more

- # of sexual contacts
- Income [Pareto] –'80-20 distribution'
- Duration of downloads [Bestavros+]
- Duration of UNIX jobs ('mice and elephants')
- Size of files of a user
- •
- 'Black swans'

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 - Static graphs
 - degree, diameter, eigen,
 - triangles
 - cliques
 - Weighted graphs
 - Time evolving graphs
- Problem#2: Tools

Solution# S.3: Triangle 'Laws'

• Real social networks have a lot of triangles

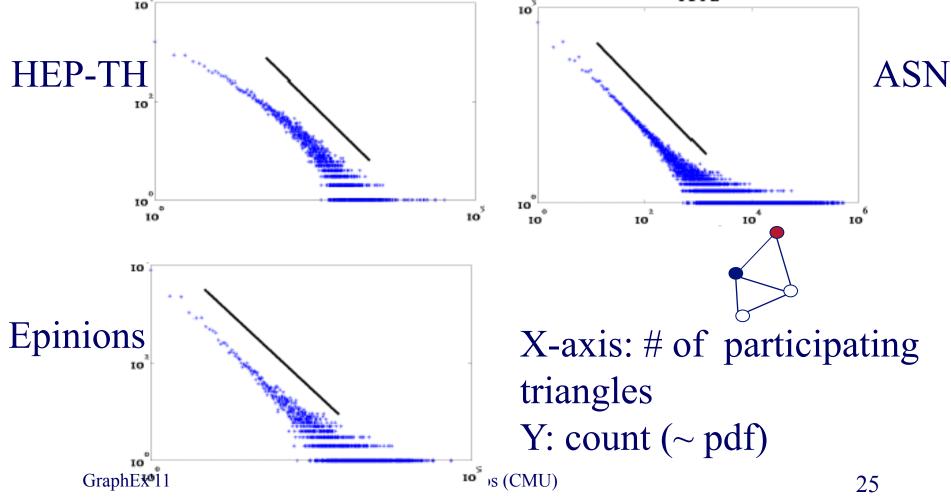
Solution# S.3: Triangle 'Laws'

- Real social networks have a lot of triangles

 Friends of friends are friends
- Any patterns?

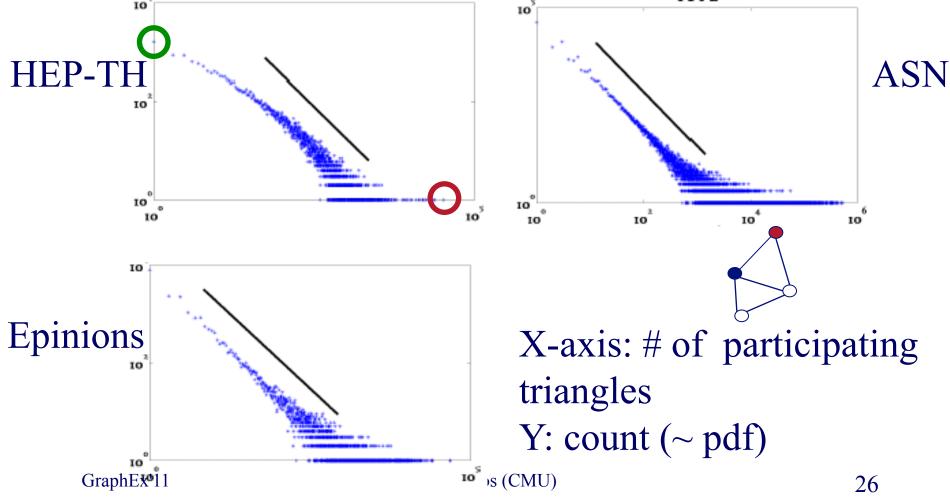
Triangle Law: #S.3 [Tsourakakis ICDM 2008]



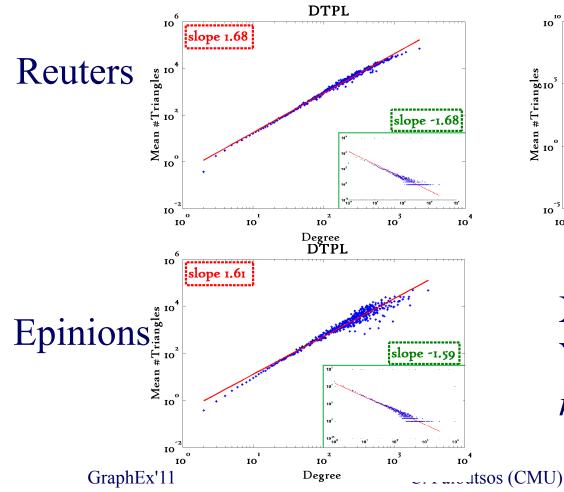


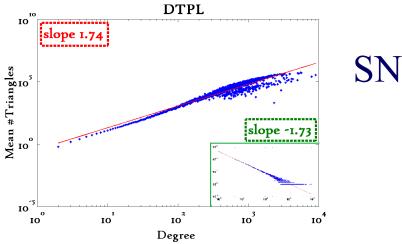
Triangle Law: #S.3 [Tsourakakis ICDM 2008]





Triangle Law: #S.4 [Tsourakakis ICDM 2008]





X-axis: degree Y-axis: mean # triangles *n* friends -> $\sim n^{1.6}$ triangles

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Triangle Law: Computations [Tsourakakis ICDM 2008]

But: triangles are expensive to compute (3-way join; several approx. algos) Q: Can we do that quickly?



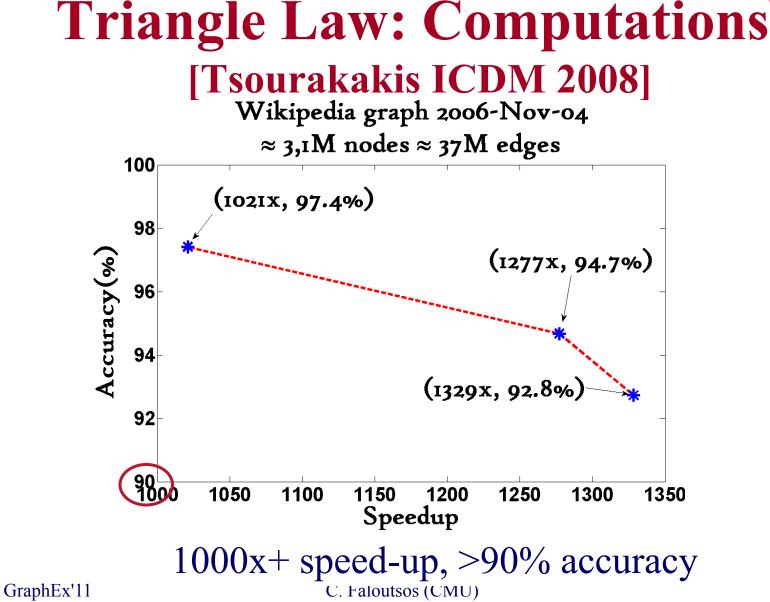
Triangle Law: Computations [Tsourakakis ICDM 2008]

But: triangles are expensive to compute (3-way join; several approx. algos)Q: Can we do that quickly?A: Yes!

#triangles = 1/6 Sum (λ_i^3) (and, because of skewness (S2), we only need the top few eigenvalues!

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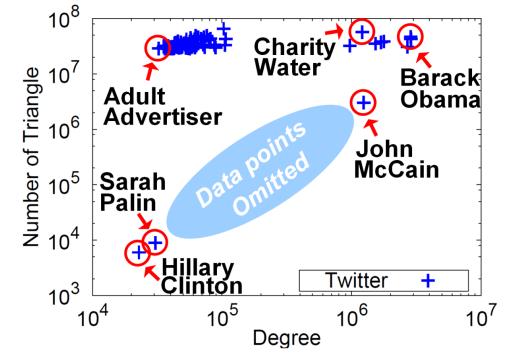
30

Triangle counting for large graphs?

Anomalous nodes in Twitter(~ 3 billion edges) [U Kang, Brendan Meeder, +, PAKDD'11]

GraphEx'11

Triangle counting for large graphs?



Anomalous nodes in Twitter(~ 3 billion edges) [U Kang, Brendan Meeder, +, PAKDD'11]

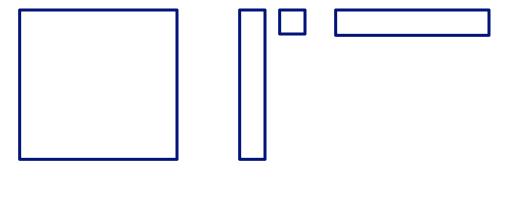
GraphEx'11



B. Aditya Prakash, Mukund Seshadri, Ashwin Sridharan, Sridhar Machiraju and Christos
Faloutsos: *EigenSpokes: Surprising Patterns and Scalable Community Chipping in Large Graphs*, PAKDD 2010, Hyderabad, India, 21-24 June 2010.

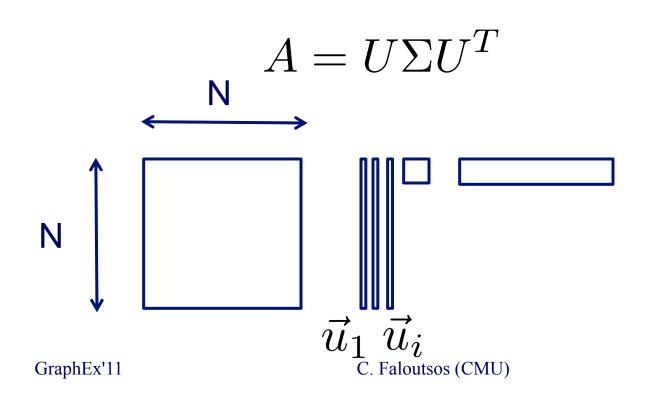
- Eigenvectors of adjacency matrix
 - equivalent to singular vectors (symmetric, undirected graph)

$$A = U\Sigma U^T$$



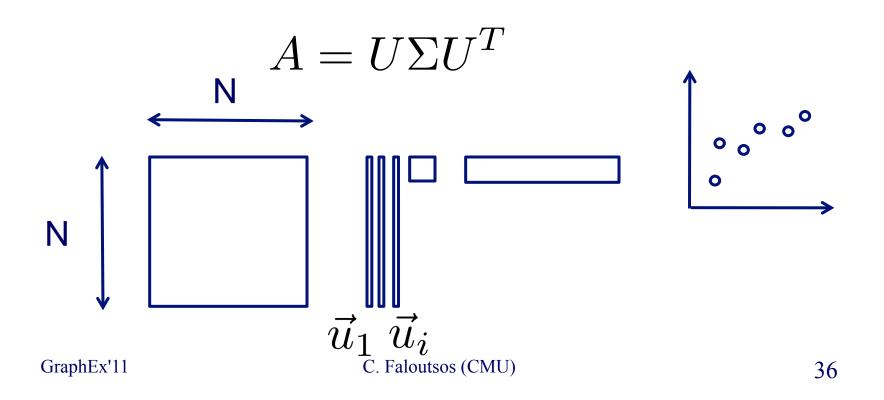


- Eigenvectors of adjacency matrix
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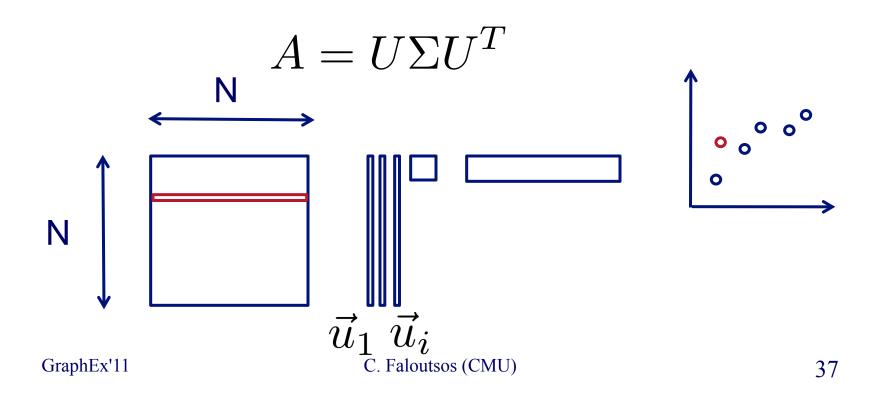


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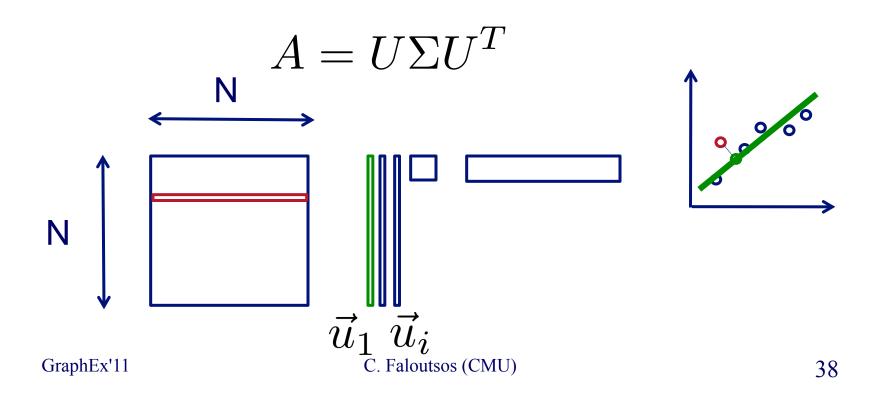


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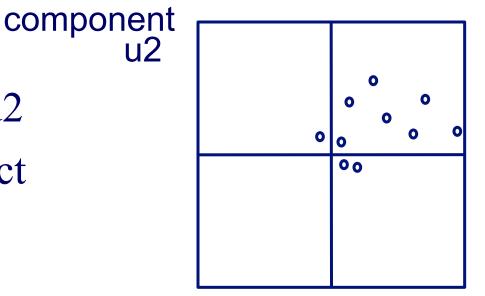


- Eigenvectors of adjacency matrix
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2nd Principal

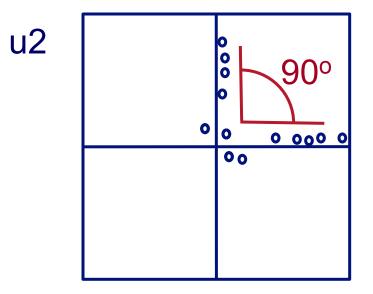
- EE plot:
- Scatter plot of scores of u1 vs u2
- One would expect
 - Many points @ origin
 - A few scattered
 ~randomly



u1 1st Principal component

- EE plot:
- Scatter plot of scores of u1 vs u2
- One would expect
 - Many points @ origin





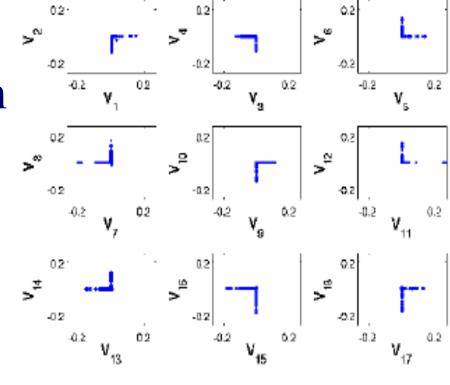
u1

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EigenSpokes - pervasiveness

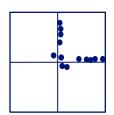
- Present in mobile social graph
 - across time and space

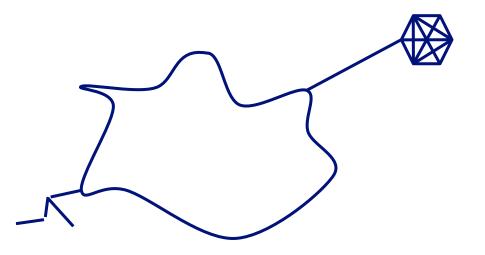




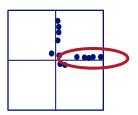
C. Faloutsos (CMU)

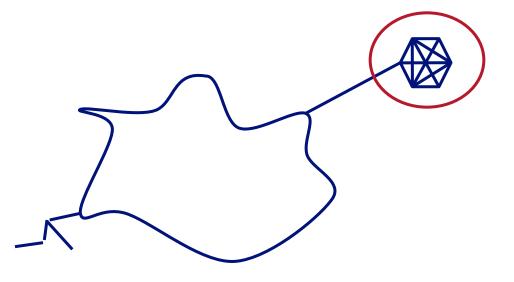
Near-cliques, or nearbipartite-cores, loosely connected



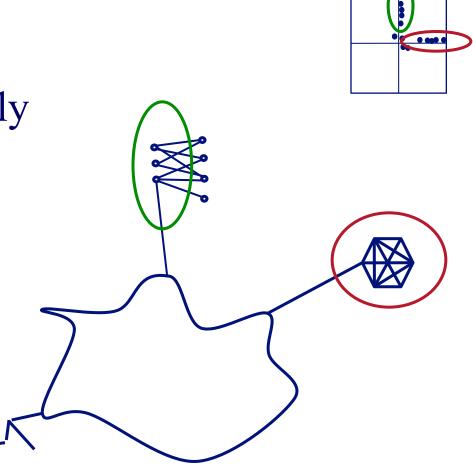


Near-cliques, or nearbipartite-cores, loosely connected





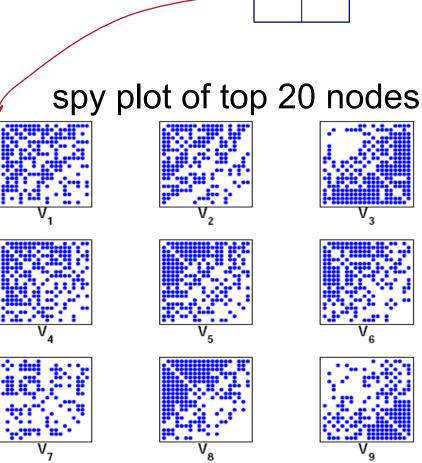
Near-cliques, or nearbipartite-cores, loosely connected



Near-cliques, or nearbipartite-cores, loosely connected

So what?

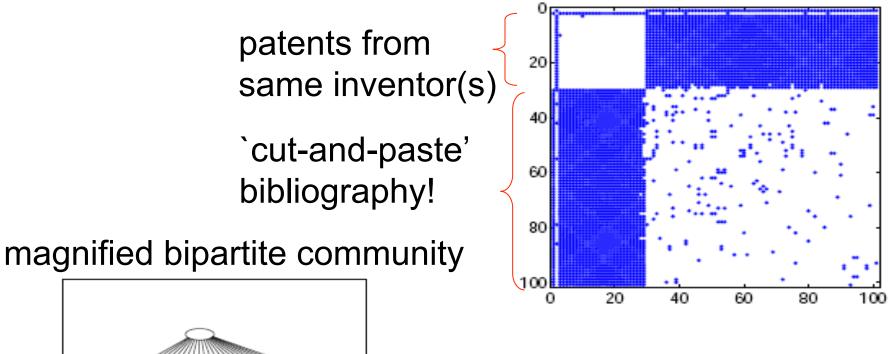
- Extract nodes with high scores
- high connectivity
- Good "communities"

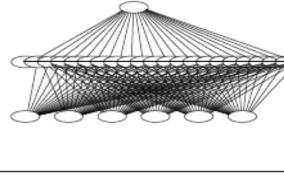


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Bipartite Communities!





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- Problem#2: Tools

Observations on weighted graphs?

• A: yes - even more 'laws'!



M. McGlohon, L. Akoglu, and C. Faloutsos Weighted Graphs and Disconnected Components: Patterns and a Generator. SIG-KDD 2008

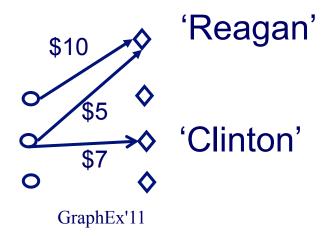
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Observation W.1: Fortification

Q: How do the weights of nodes relate to degree?

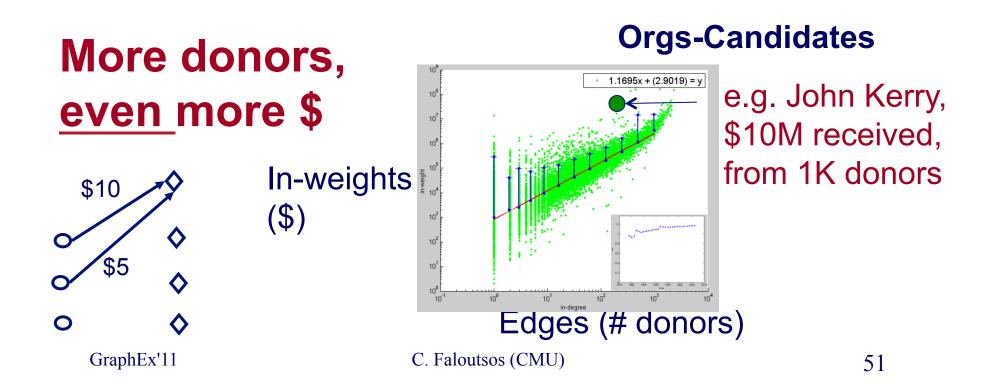
Observation W.1: Fortification

More donors, more \$?



Observation W.1: fortification: Snapshot Power Law

- Weight: super-linear on in-degree
- exponent 'iw': 1.01 < iw < 1.26



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Problem: Time evolution

 with Jure Leskovec (CMU -> Stanford)

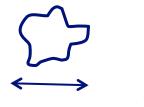


• and Jon Kleinberg (Cornell – sabb. @ CMU)



T.1 Evolution of the Diameter

- Prior work on Power Law graphs hints at **slowly growing diameter**:
 - diameter $\sim O(\log N)$
 - diameter $\sim O(\log \log N)$



• What is happening in real data?

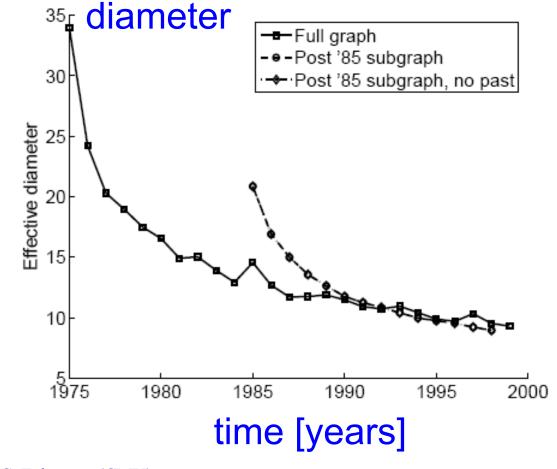
T.1 Evolution of the Diameter

- Prior work on Power Law graphs hints at slowly growing diameter:

 - $\text{ diameter} \sim (\ln n)$ $\text{ diameter} \sim O(\log n)$
- What is happening in real data?
- Diameter shrinks over time

T.1 Diameter – "Patents"

- Patent citation network
- 25 years of data
- @1999
 - 2.9 M nodes
 - 16.5 M edges



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T.2 Temporal Evolution of the Graphs

- N(t) ... nodes at time t
- E(t) ... edges at time t
- Suppose that

N(t+1) = 2 * N(t)

• Q: what is your guess for E(t+1) =? 2 * E(t)

T.2 Temporal Evolution of the Graphs

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- E(t) ... edges at time t
- Suppose that

N(t+1) = 2 * N(t)

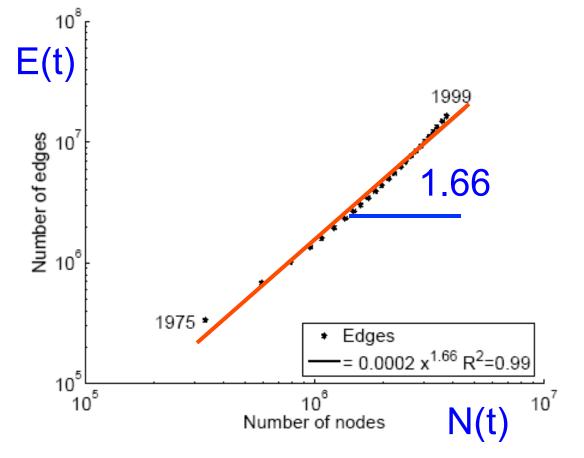
- Q: what is your guess for E(t+1) * E(t)
- A: over-doubled!

– But obeying the ``Densification Power Law''

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T.2 Densification – Patent Citations

- Citations among patents granted
- @1999
 - 2.9 M nodes
 - 16.5 M edges
- Each year is a datapoint



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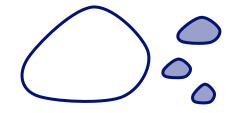
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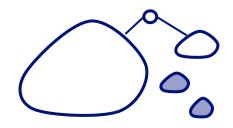
More on Time-evolving graphs

M. McGlohon, L. Akoglu, and C. Faloutsos Weighted Graphs and Disconnected Components: Patterns and a Generator. SIG-KDD 2008

- *Q: How do NLCC's emerge and join with the GCC?*
- (``NLCC'' = non-largest conn. components)
- -Do they continue to grow in size?
- or do they shrink?
- or stabilize?



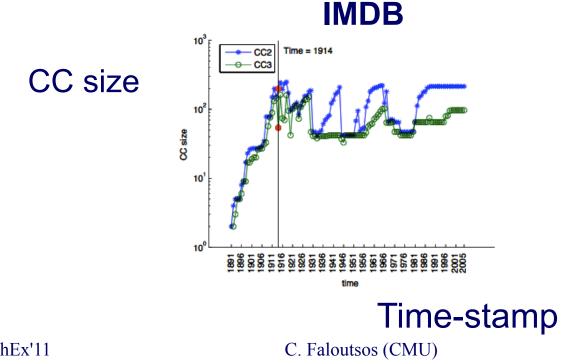
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Q: How do NLCC's emerge and join with the GCC?

(``NLCC'' = non-largest conn. components)
YES - Do they continue to grow in size?
YES - or do they shrink?
YES - or stabilize?

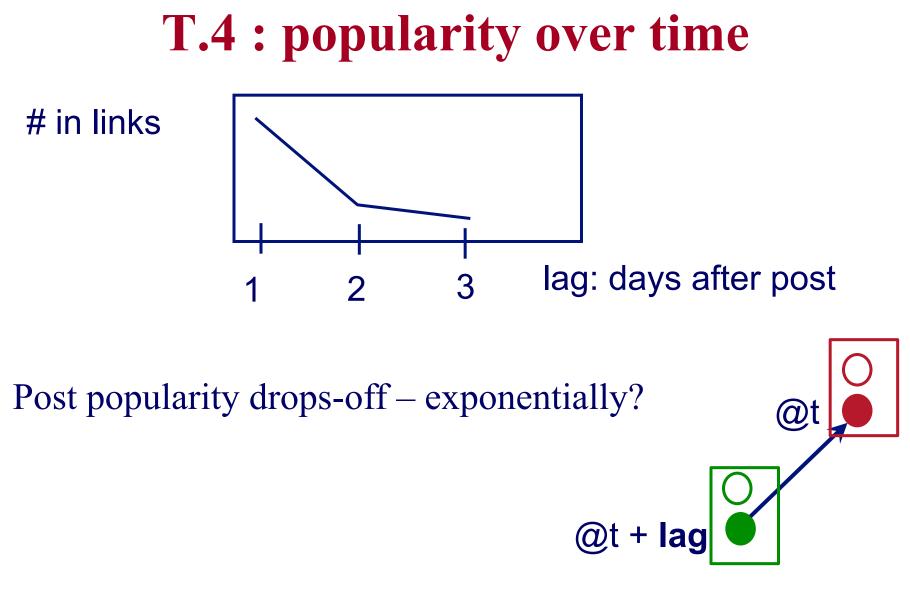
• After the gelling point, the GCC takes off, but NLCC's remain ~constant (actually, oscillate).



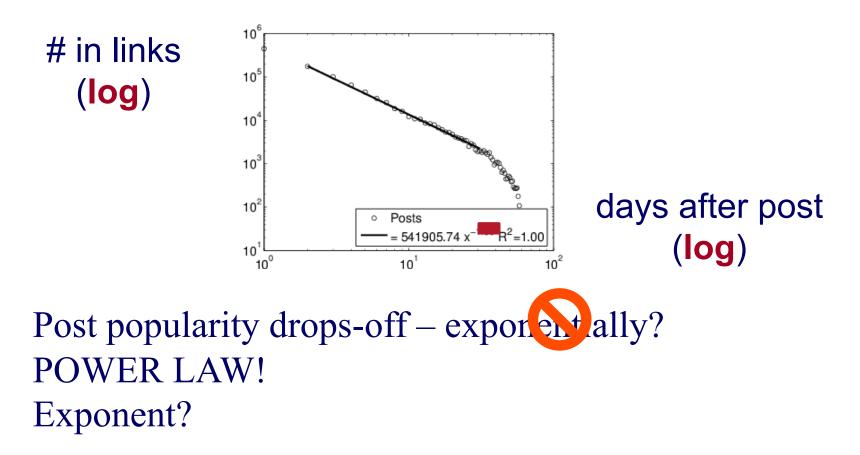


Timing for Blogs

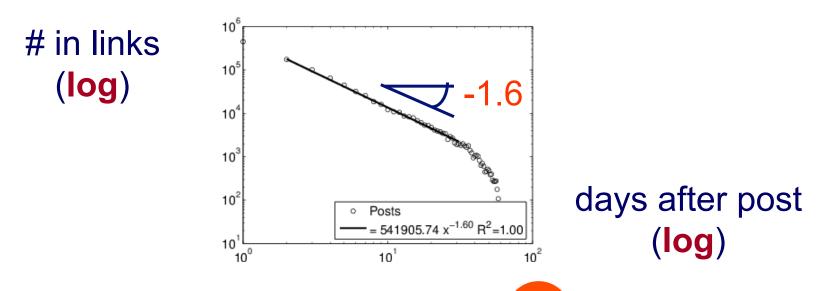
- with Mary McGlohon (CMU->Google)
- Jure Leskovec (CMU->Stanford)
- Natalie Glance (now at Google)
- Mat Hurst (now at MSR)
 [SDM'07]



T.4 : popularity over time

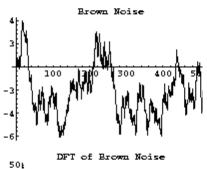


T.4 : popularity over time



Post popularity drops-off – exporent ally? POWER LAW! Exponent? -1.6

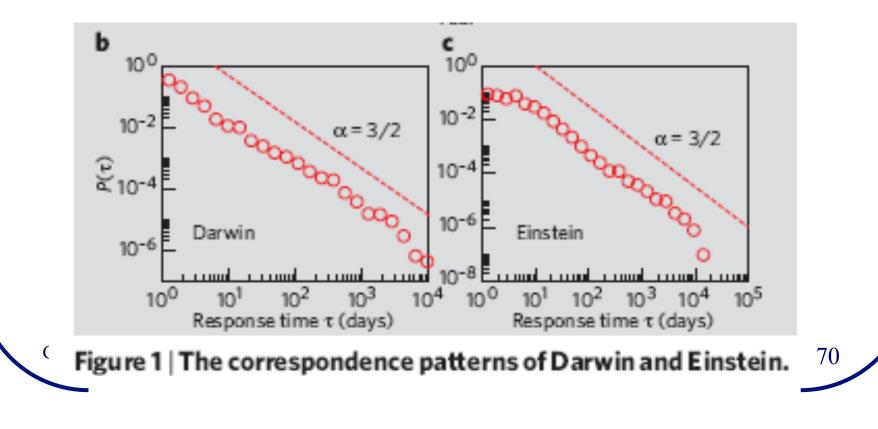
- close to -1.5: Barabasi's stack model
- and like the zero-crossings of a random walk GraphEx'11 C. Faloutsos (CMU)



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-1.5 slope

J. G. Oliveira & A.-L. Barabási Human Dynamics: The Correspondence Patterns of Darwin and Einstein. *Nature* **437**, 1251 (2005). [PDF]



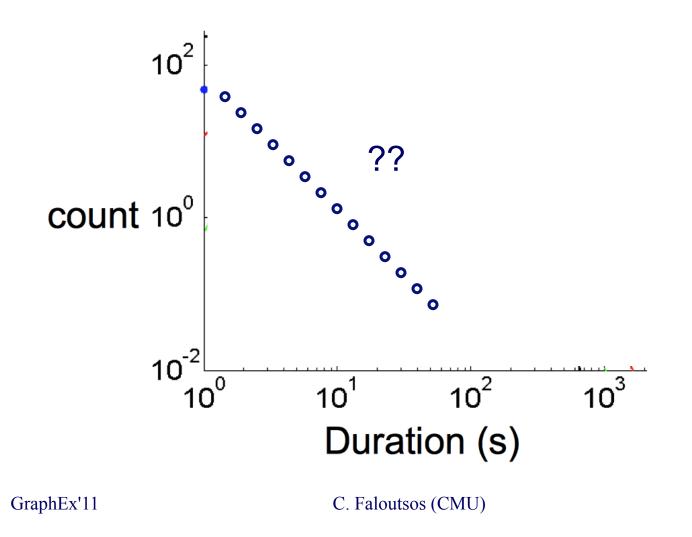
T.5: duration of phonecalls

Surprising Patterns for the Call Duration Distribution of Mobile Phone Users

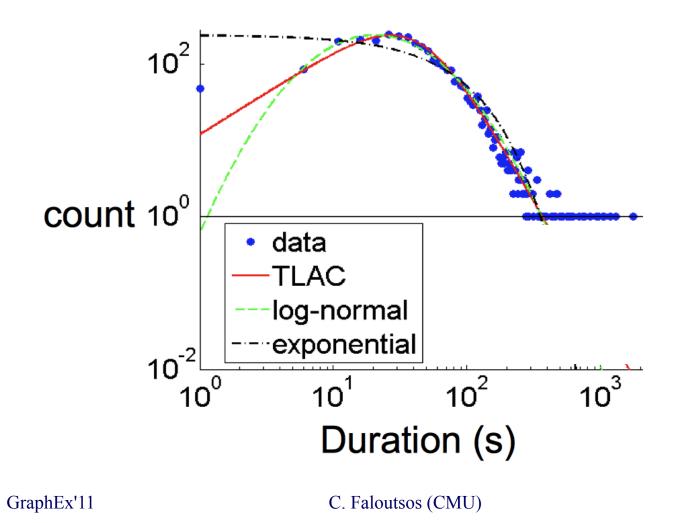


Pedro O. S. Vaz de Melo, LemanAkoglu, Christos Faloutsos, AntonioA. F. LoureiroPKDD 2010

Probably, power law (?)

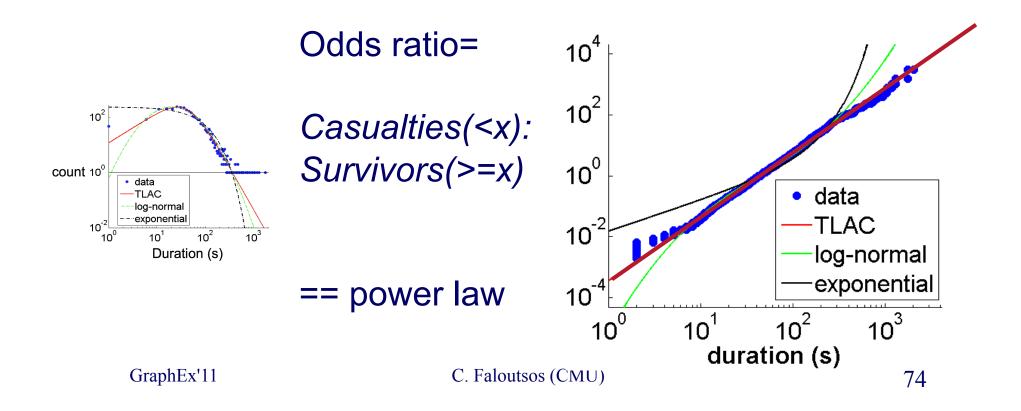


No Power Law (yet)



'TLaC: Lazy Contractor'

- The longer a task (phonecall) has taken,
- The even longer it will take



Data Description

- Data from a private mobile operator of a large city
 - 4 months of data
 - 3.1 million users
 - more than 1 billion phone records
- Over 96% of 'talkative' users obeyed a TLAC distribution ('talkative': >30 calls)
- Rest 4%: ~

GraphEx'11

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 - OddBall (anomaly detection)
 - Belief Propagation
 - Immunization
- Problem#3: Scalability
- Conclusions

teasers

OddBall: Spotting Anomalies in Weighted Graphs





Leman Akoglu, Mary McGlohon, Christos Faloutsos

> Carnegie Mellon University School of Computer Science

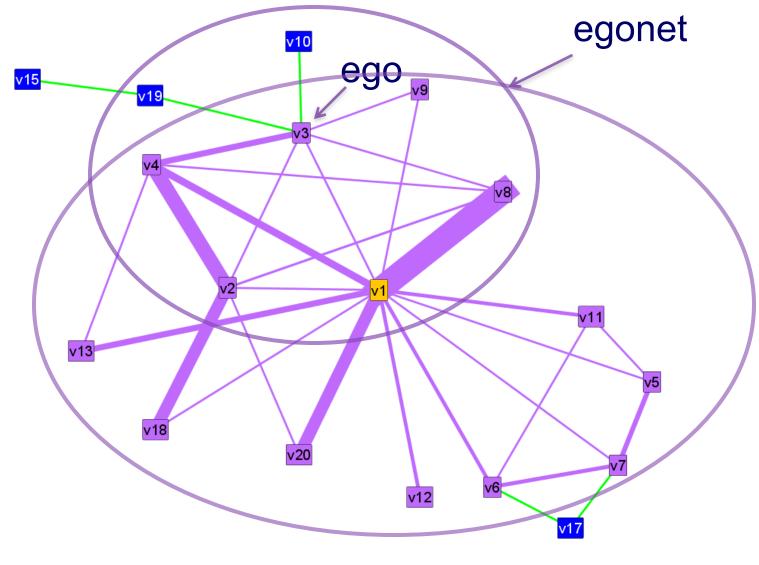
PAKDD 2010, Hyderabad, India

Main idea

For each node,

- extract 'ego-net' (=1-step-away neighbors)
- Extract features (#edges, total weight, etc etc)
- Compare with the rest of the population

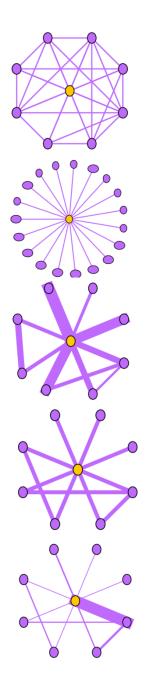
What is an egonet?



Carnegie Mellon

Selected Features

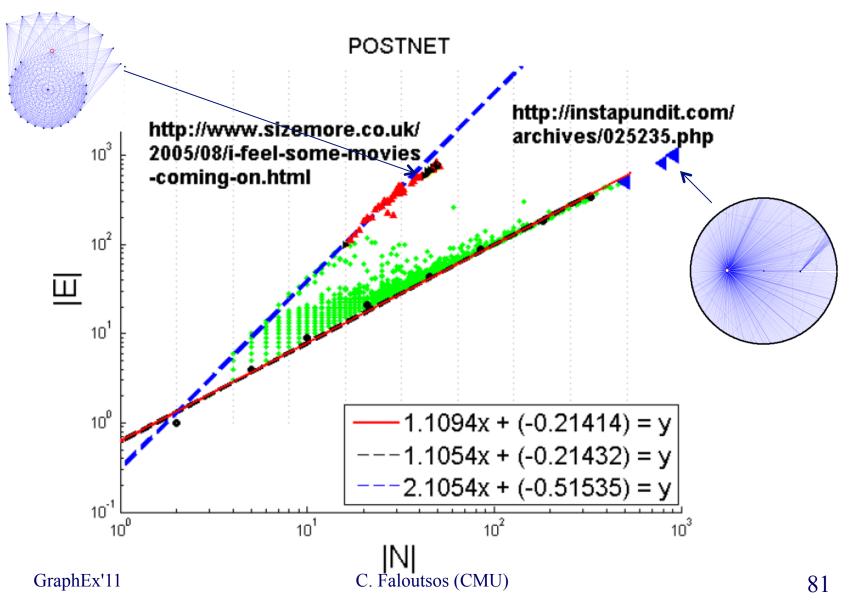
- N_i : number of neighbors (degree) of ego i
- E_i : number of edges in egonet i
- W_i : total weight of egonet *i*
- $\lambda_{w,i}$: principal eigenvalue of the weighted adjacency matrix of egonet *I*



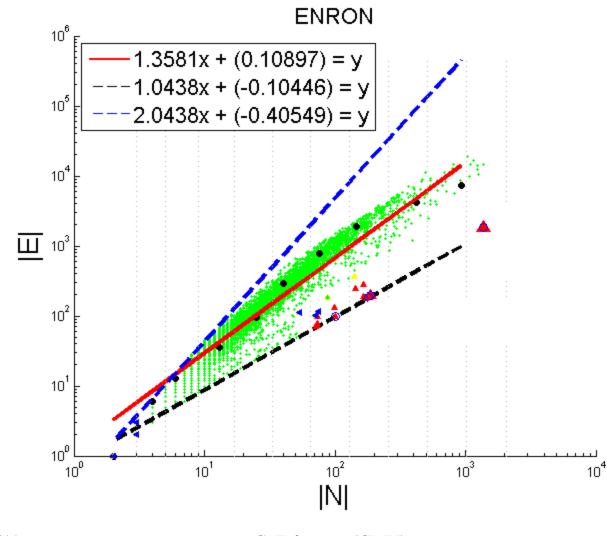
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Near-Clique/Star



Near-Clique/Star

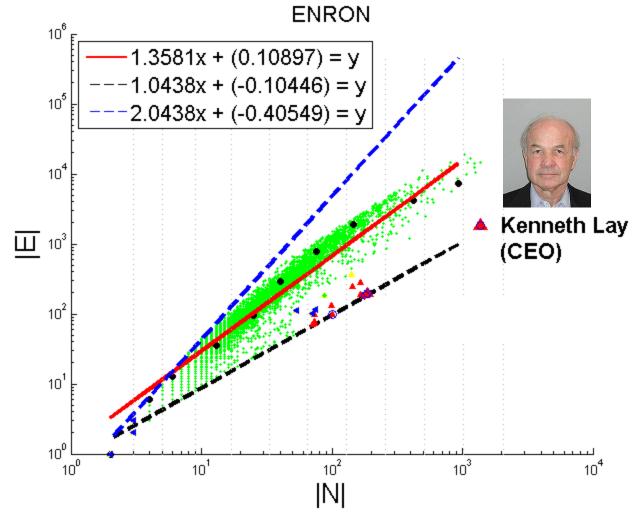


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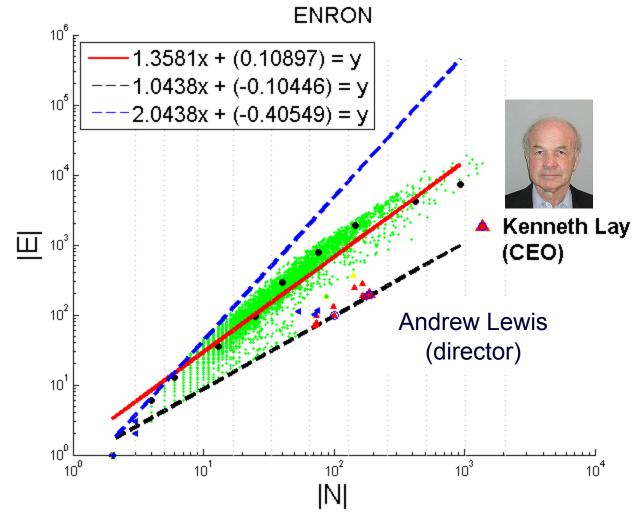
Near-Clique/Star





C. Faloutsos (CMU)

Near-Clique/Star



C. Faloutsos (CMU)

Outline

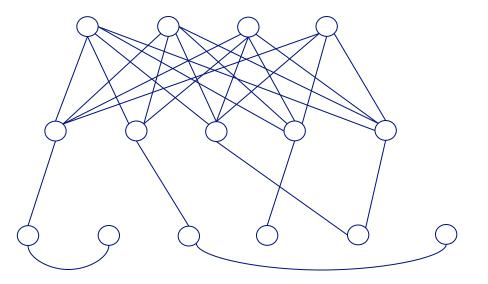
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E-bay Fraud detection

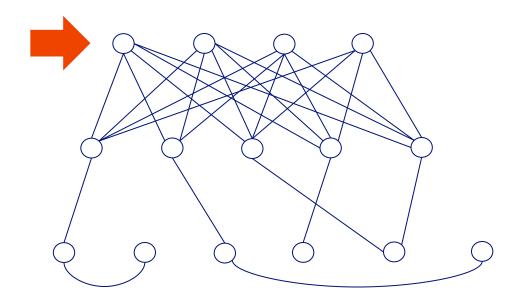




w/ Polo Chau & Shashank Pandit, CMU [www'07]

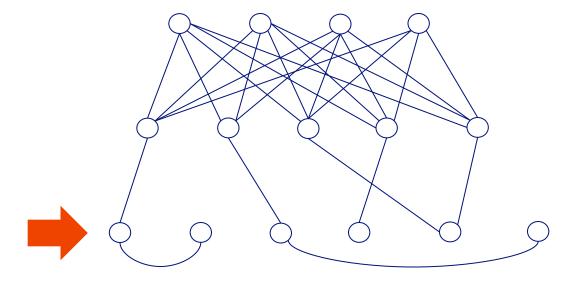


E-bay Fraud detection



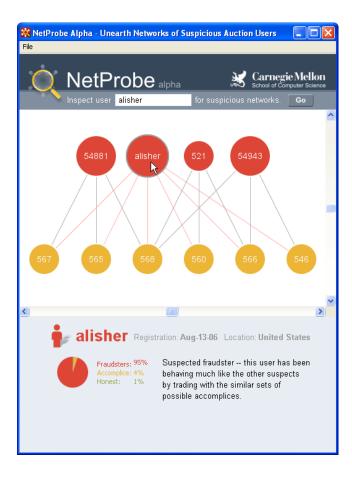
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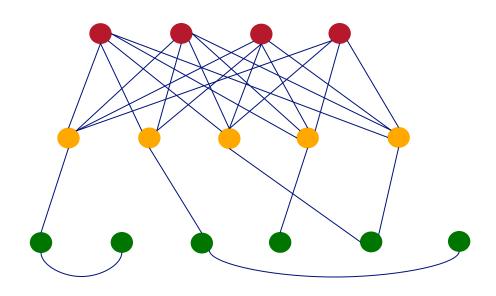
E-bay Fraud detection



GraphEx'11

E-bay Fraud detection - NetProbe





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Popular press



The Washington Post Los Angeles Times

And less desirable attention:

• E-mail from 'Belgium police' ('copy of your code?')

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Outline

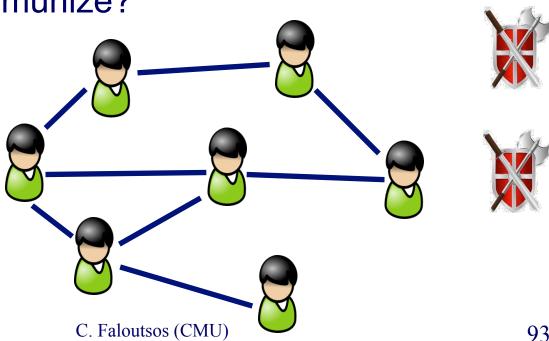
- Introduction Motivation
- Problem#1: Patterns in graphs
- Problem#2: Tools
 - OddBall (anomaly detection)
 - Belief propagation
 - Immunization
- Problem#3: Scalability -PEGASUS
- Conclusions

Immunization and epidemic thresholds

- Q1: which nodes to immunize?
- Q2: will a virus vanish, or will it create an epidemic?

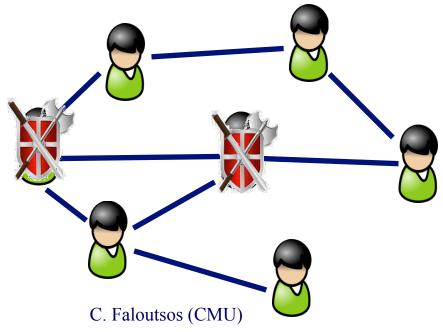
•Given

•a network, •k vaccines, and •the virus details •Which nodes to immunize?



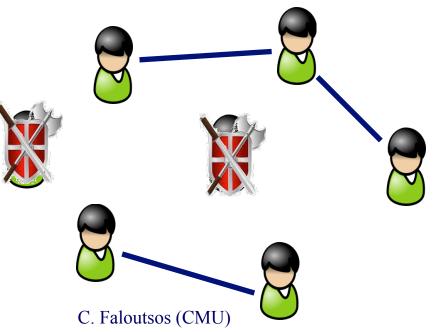
•Given

a network,
k vaccines, and
the virus details
Which nodes to immunize?



•Given

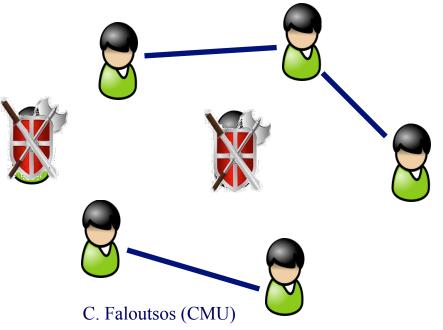
a network,
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Which nodes to immunize?



•Given

a network,
k vaccines, and
the virus details
Which nodes to immunize?

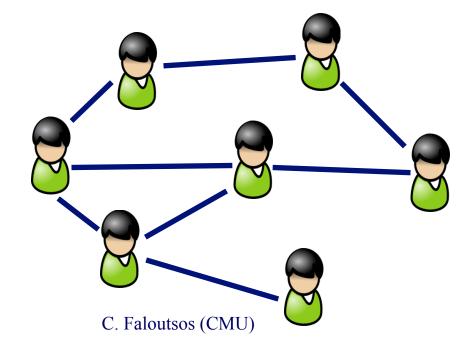
A: immunize the ones that maximally raise the `epidemic threshold' [Tong+, ICDM'10]



Q2: will a virus take over?

- Flu-like virus (no immunity, 'SIS')
- Mumps (life-time immunity, 'SIR')
- Pertussis (finite-length immunity, 'SIRS')

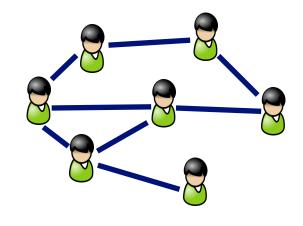
β: attack prob δ: heal prob



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- Flu-like virus (no immunity, 'SIS')
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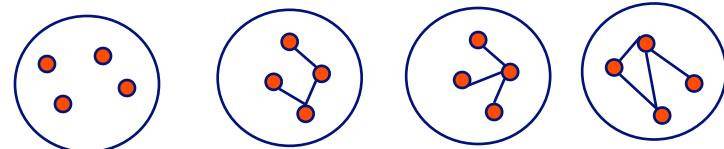
- β: attack prob δ: heal prob
- A: depends on connectivity (avg degree? Max degree? variance? Something else? _{GraphEx'11} C. Faloutsos (CMU)



Epidemic threshold $\boldsymbol{\tau}$

What should τ depend on?

- avg. degree? and/or highest degree?
- and/or variance of degree?
- and/or third moment of degree?
- and/or diameter?



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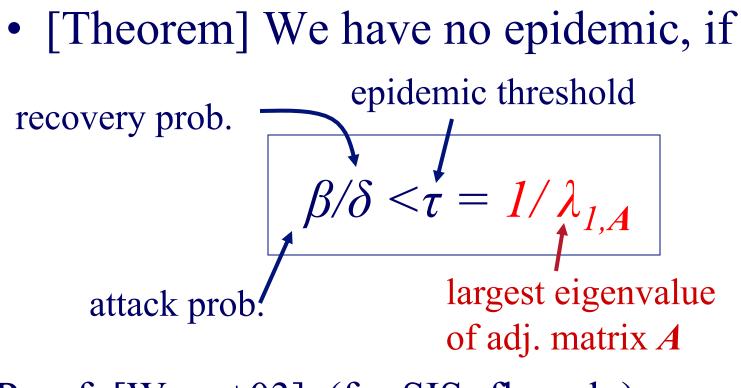
Epidemic threshold

• [Theorem] We have no epidemic, if

$$\beta/\delta < \tau = 1/\lambda_{l,A}$$

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Epidemic threshold



Proof: [Wang+03] (for SIS=flu only)

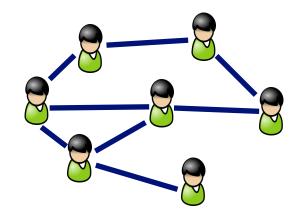
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A2: will a virus take over?

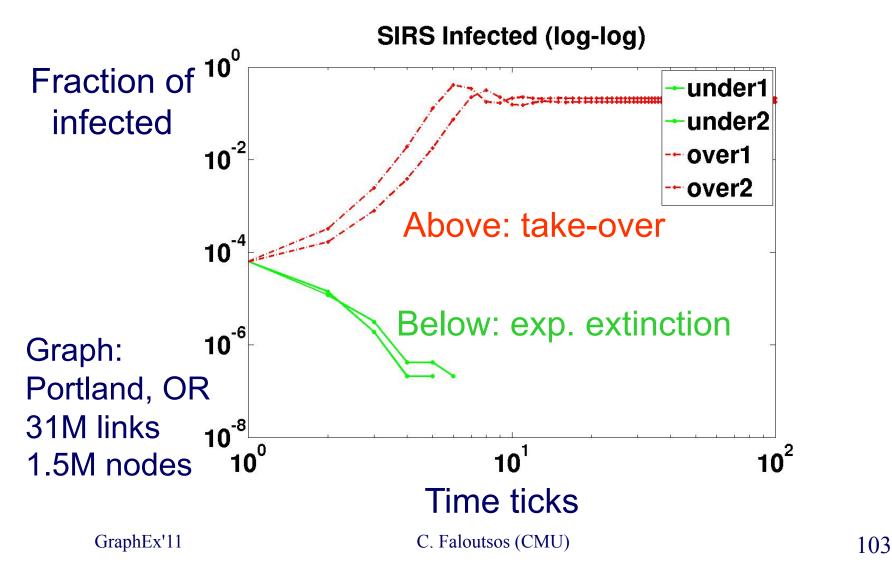
- For **all** typical virus propagation models (flu, mumps, pertussis, HIV, etc)
- The only connectivity measure that matters, is

 $1/\lambda_1$

the first eigenvalue of the
 adj. matrix
[Prakash+, '10, arxiv]



A2: will a virus take over?



Outline

- Introduction Motivation
- Problem#1: Patterns in graphs
- Problem#2: Tools
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 - Belief propagation
 - Immunization
- Problem#3: Scalability -PEGASUS
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Scalability



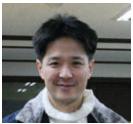
- Google: > 450,000 processors in clusters of ~2000 processors each [Barroso, Dean, Hölzle, "Web Search for a Planet: The Google Cluster Architecture" IEEE Micro 2003]
- Yahoo: 5Pb of data [Fayyad, KDD'07]
- Problem: machine failures, on a daily basis
- How to parallelize data mining tasks, then?
- A: map/reduce hadoop (open-source clone) http://hadoop.apache.org/



Outline – Algorithms & results

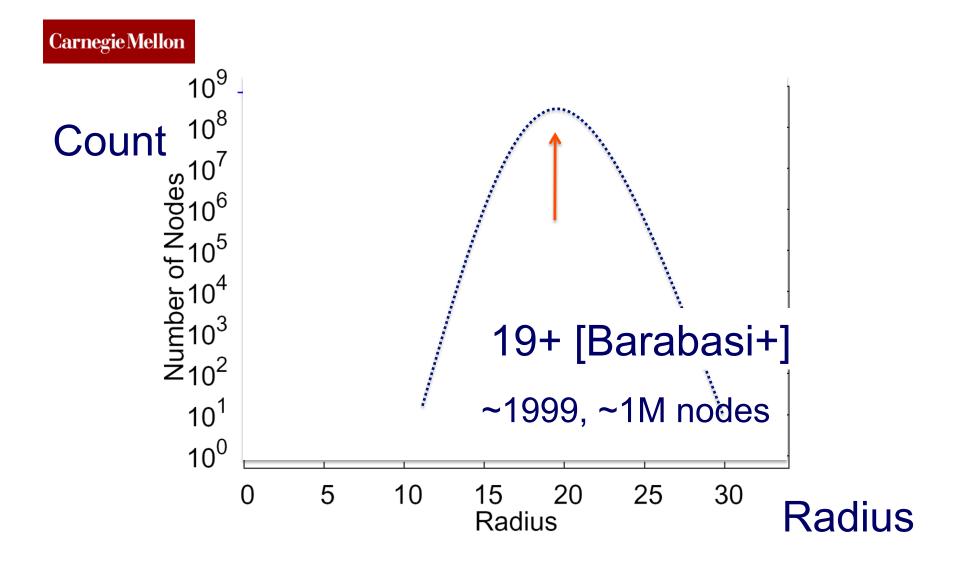
	Centralized	Hadoop/ PEGASUS
Degree Distr.	old	old
Pagerank	old	old
Diameter/ANF	old	HERE
Conn. Comp	old	HERE
Triangles	done	HERE
Visualization	started	

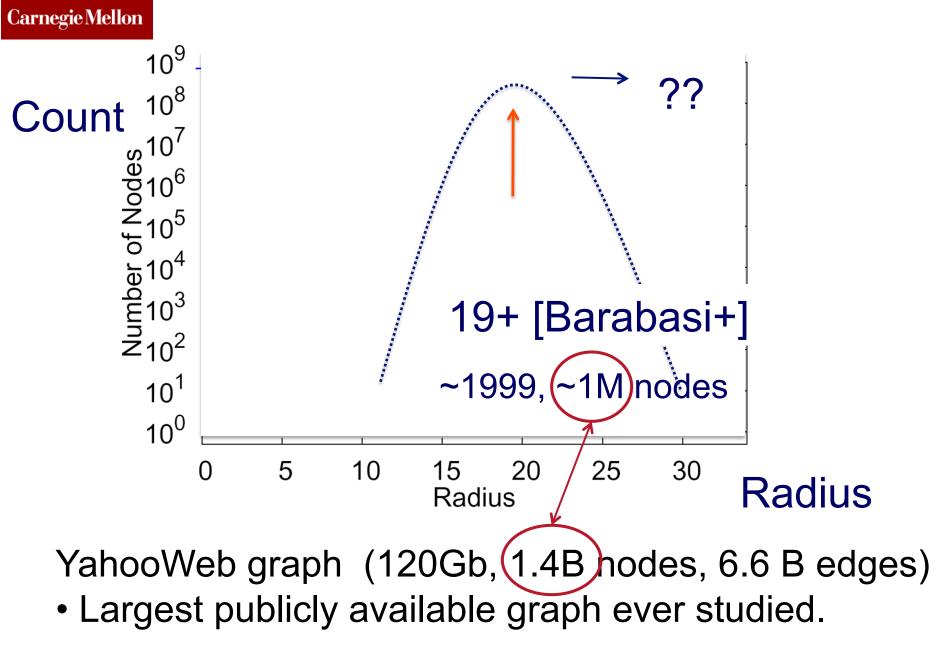
GraphEx'11



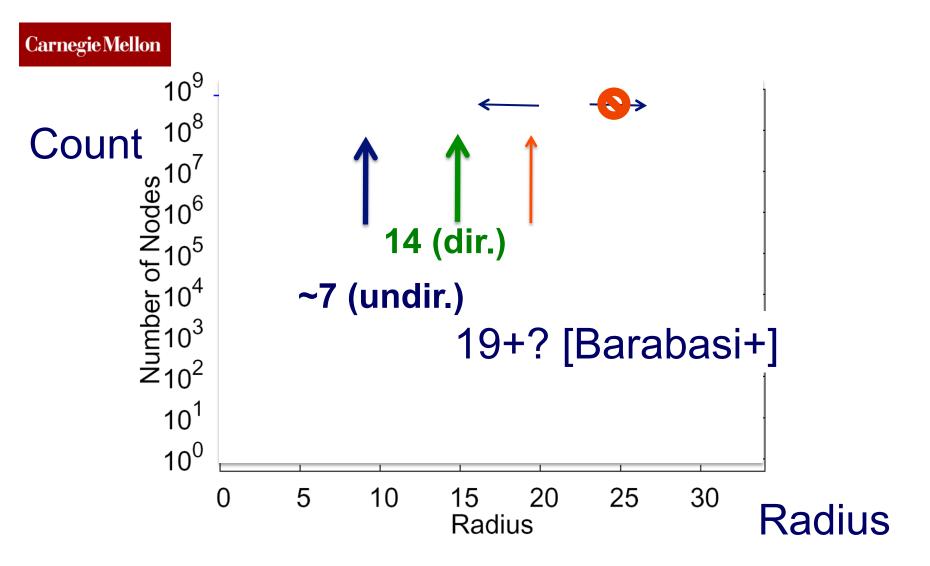
HADI for diameter estimation

- Radius Plots for Mining Tera-byte Scale Graphs U Kang, Charalampos Tsourakakis, Ana Paula Appel, Christos Faloutsos, Jure Leskovec, SDM'10
- Naively: diameter needs O(N**2) space and up to O(N**3) time – prohibitive (N~1B)
- Our HADI: linear on E (~10B)
 - Near-linear scalability wrt # machines
 - Several optimizations -> 5x faster

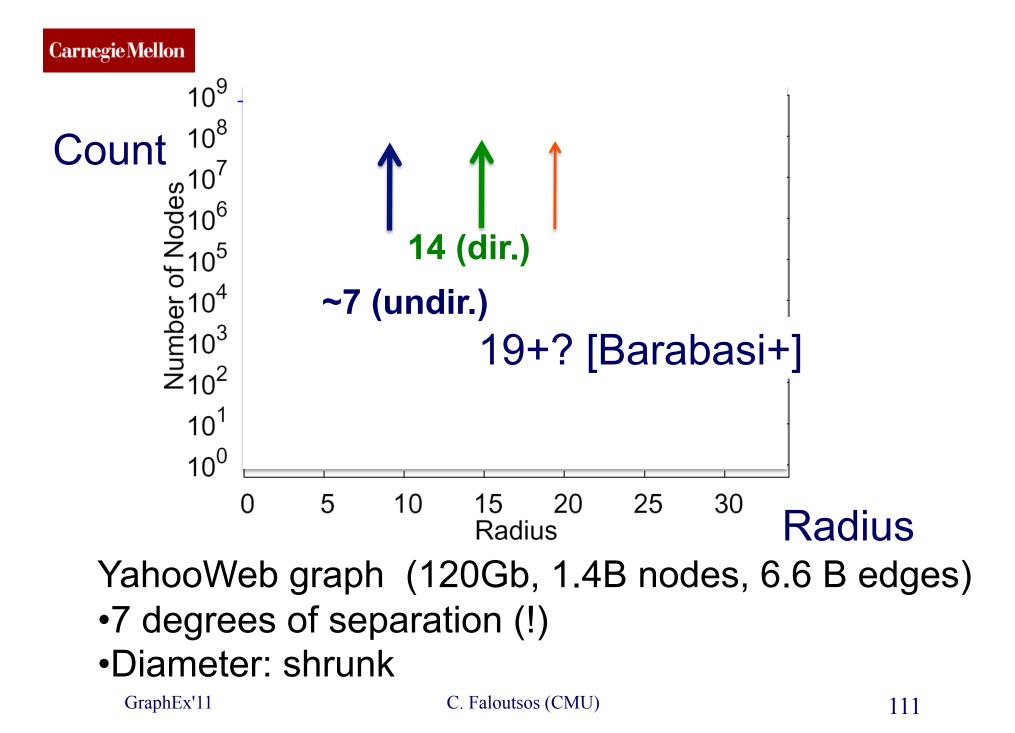


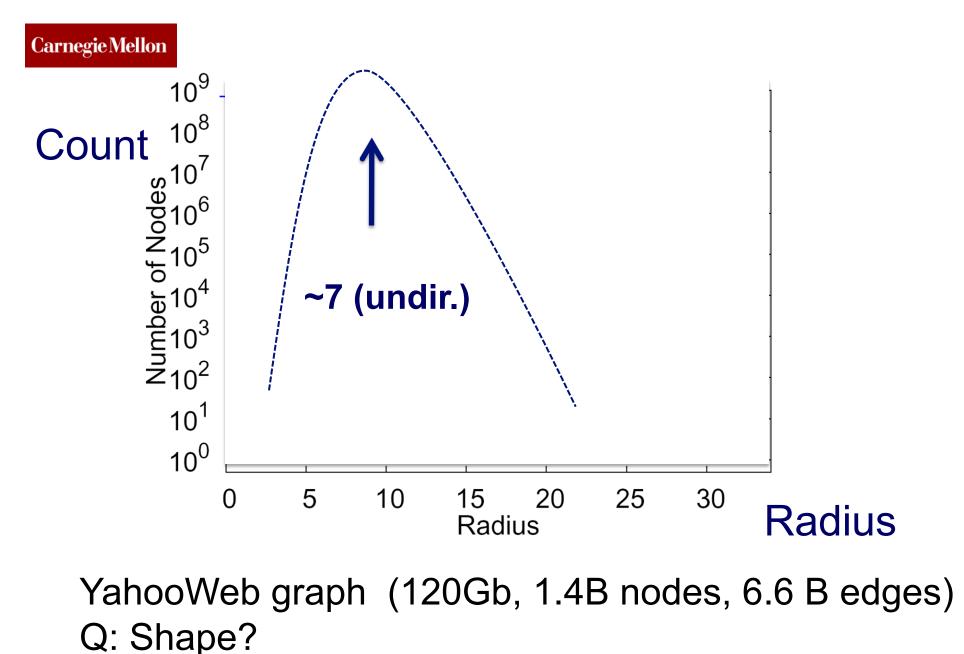


GraphEx'11



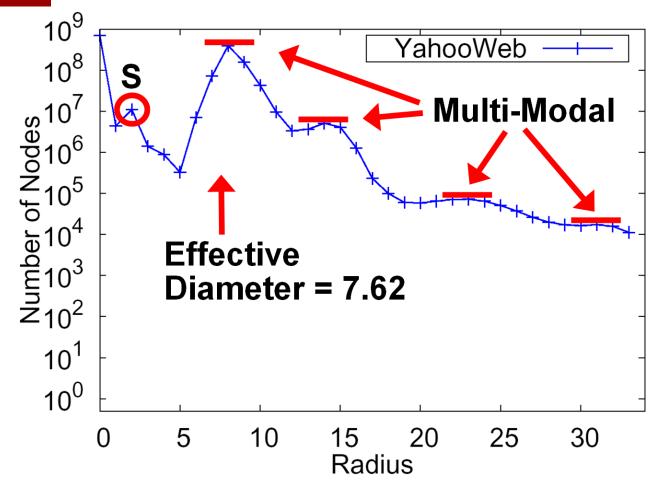
• Largest publicly available graph ever studied.





• GraphEx'11

Carnegie Mellon

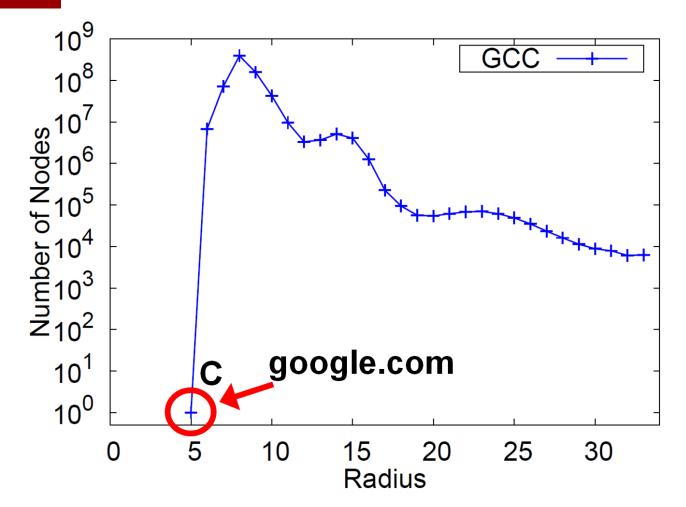


YahooWeb graph (120Gb, 1.4B nodes, 6.6 B edges)

- effective diameter: surprisingly small.
- Multi-modality (?!)

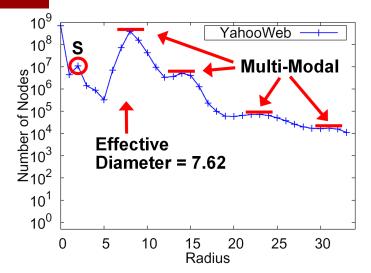
GraphEx'11

Carnegie Mellon



Radius Plot of GCC of YahooWeb.

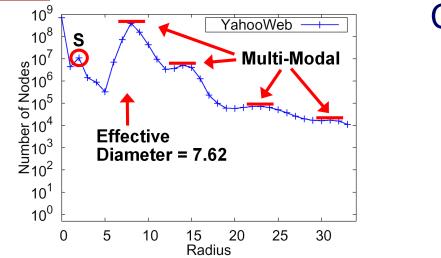


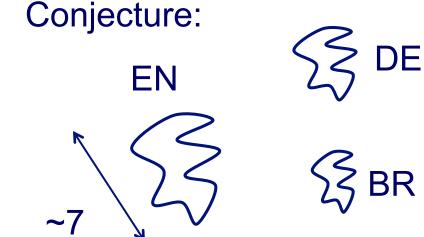


- effective diameter: surprisingly small.
- Multi-modality: probably mixture of cores .

GraphEx'11



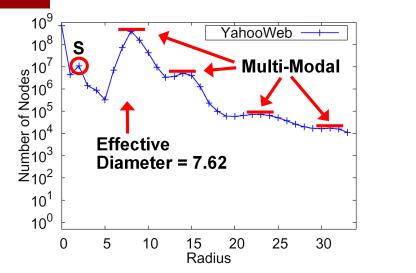


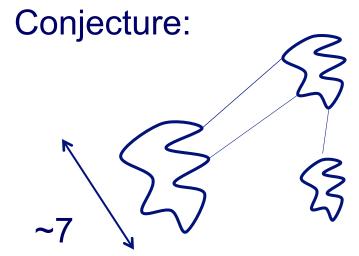


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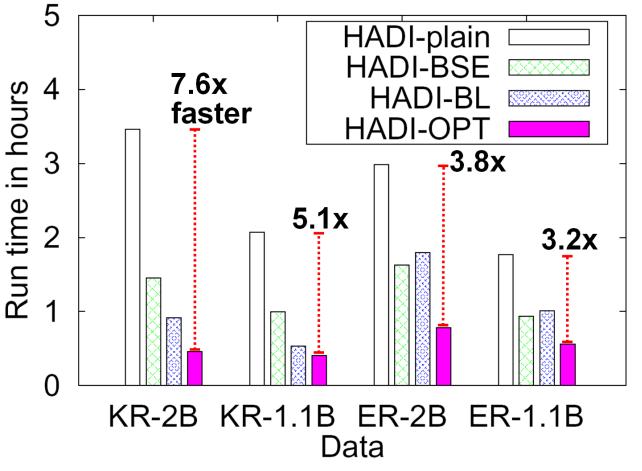




- effective diameter: surprisingly small.
- Multi-modality: probably mixture of cores .

GraphEx'11





Running time - Kronecker and Erdos-Renyi Graphs with billions edges.

Outline – Algorithms & results

	Centralized	Hadoop/ PEGASUS
Degree Distr.	old	old
Pagerank	old	old
Diameter/ANF	old	HERE
Conn. Comp	old	HERE
Triangles		HERE
Visualization	started	

GraphEx'11

Generalized Iterated Matrix Vector Multiplication (GIMV)

<u>PEGASUS: A Peta-Scale Graph Mining</u> <u>System - Implementation and Observations</u>. U Kang, Charalampos E. Tsourakakis, and Christos Faloutsos. (ICDM) 2009, Miami, Florida, USA. Best Application Paper (runner-up).



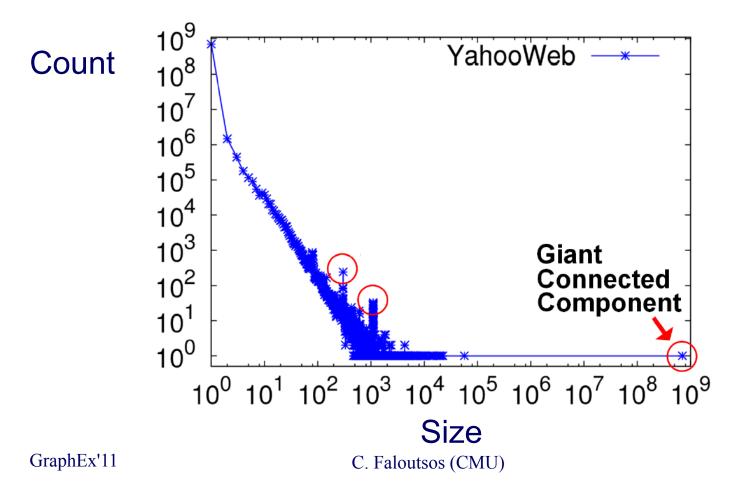
Generalized Iterated Matrix details Vector Multiplication (GIMV)



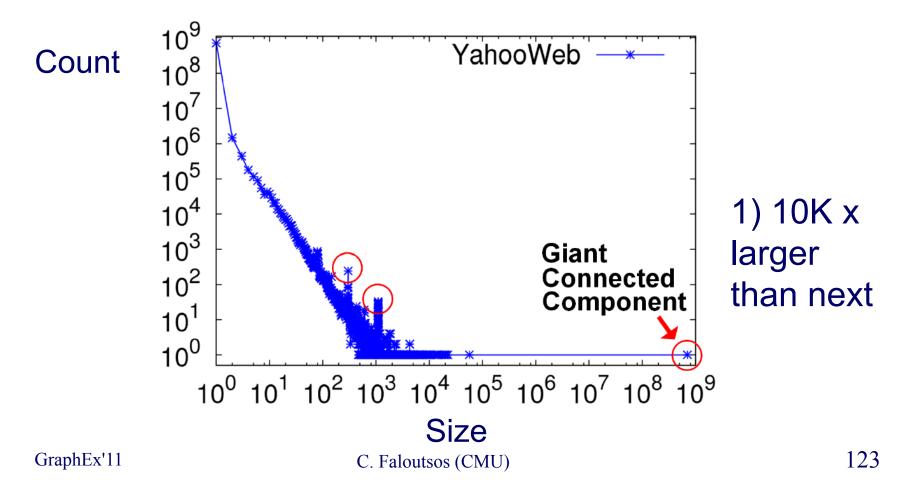
- proximity (RWR)
- Diameter
- Connected components
- (eigenvectors,
- Belief Prop.
- ...)

Matrix – vector Multiplication (iterated)

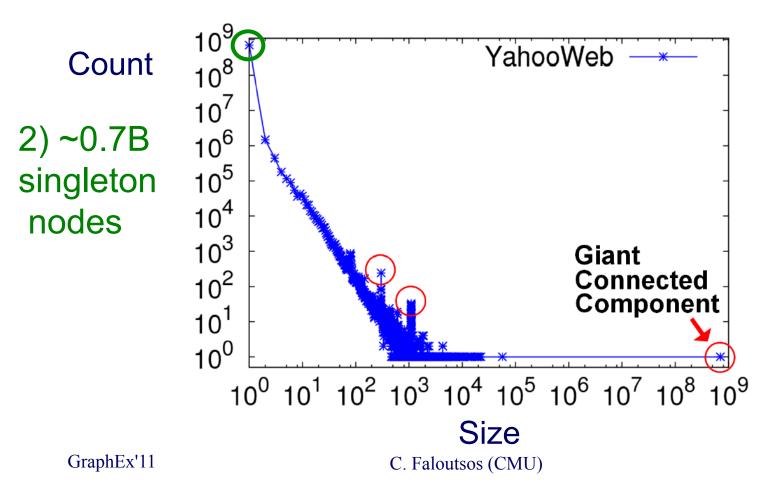
• Connected Components – 4 observations:



• Connected Components

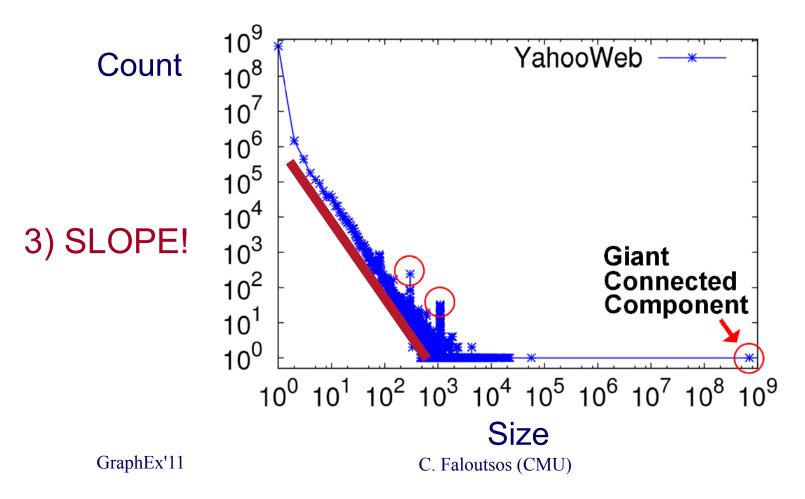


• Connected Components

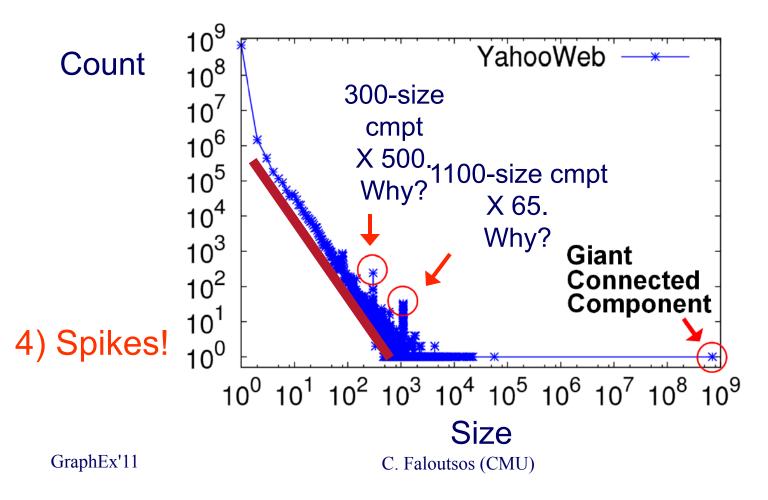


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• Connected Components

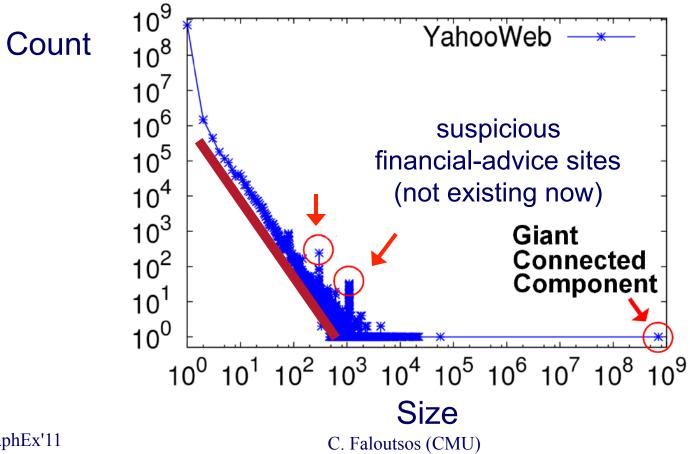


• Connected Components



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Connected Components

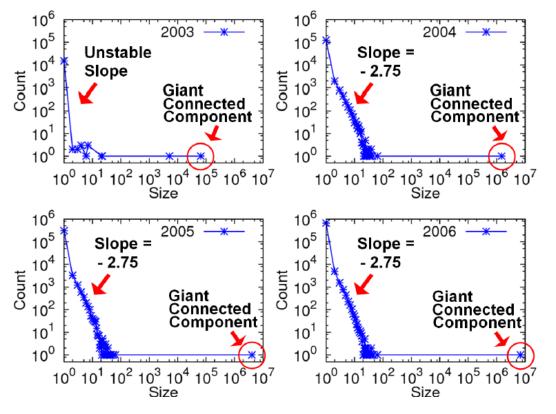


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GIM-V At Work

- Connected Components over Time
- LinkedIn: 7.5M nodes and 58M edges

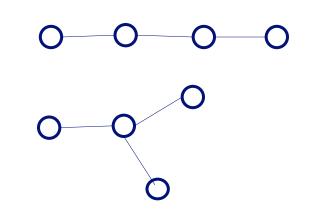


Stable tail slope after the gelling point

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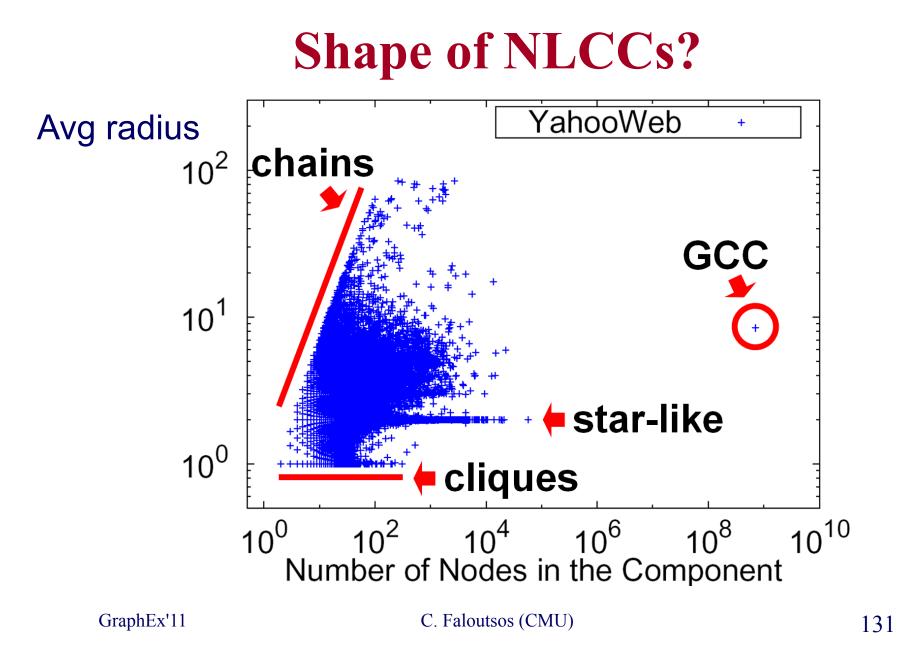
What do NLCC's look like?

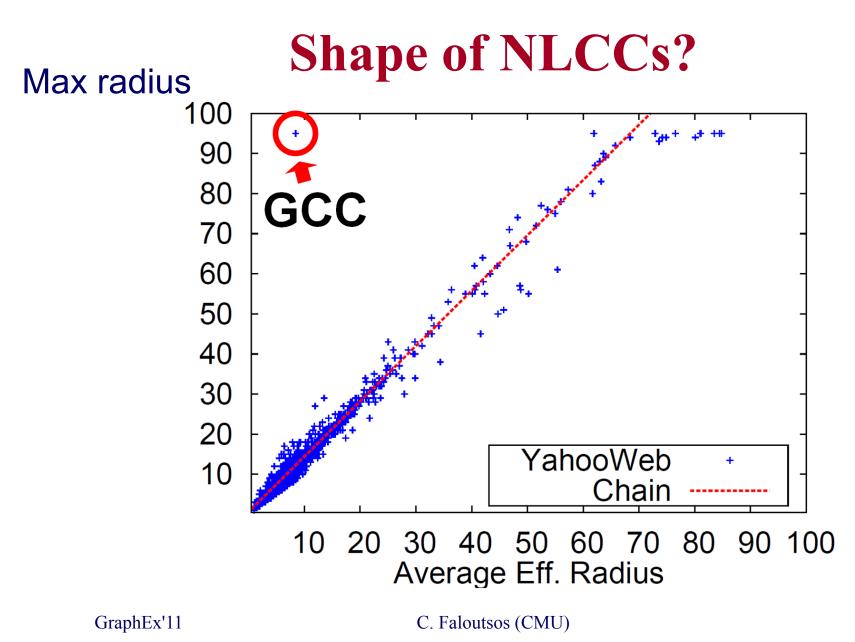
- Chains?
- Stars?
- General trees?
- Cliques?
- Miniature versions of the GCC?
- Something else?



Answer:

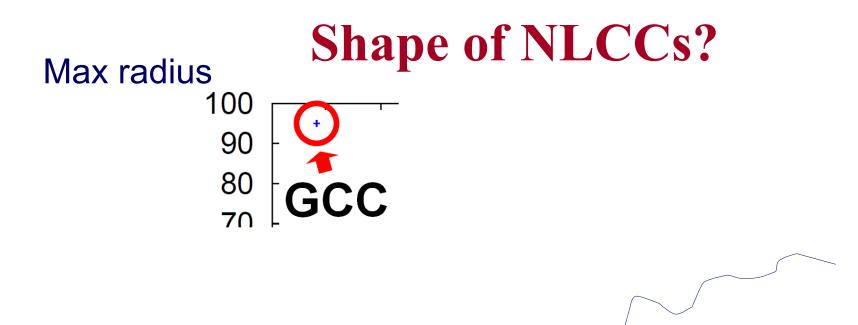
- A mixture
- Mostly, chain-like (but a bit 'thicker')
- Patterns on the Connected Components of Terabyte-Scale Graphs. U Kang, Mary McGlohon, Leman Akoglu, and Christos Faloutsos. ICDM 2010, Sydney, Australia.





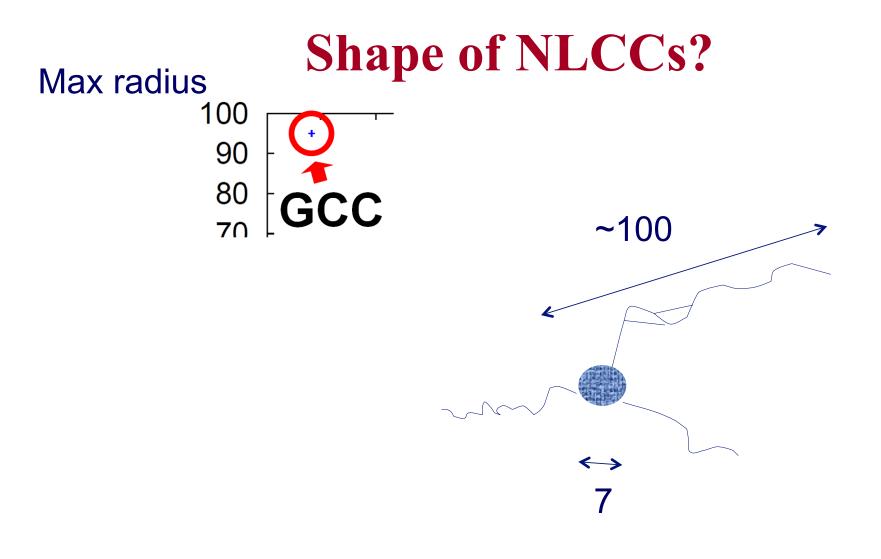






GraphEx'11





Outline

- Introduction Motivation
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OVERALL CONCLUSIONS – low level:

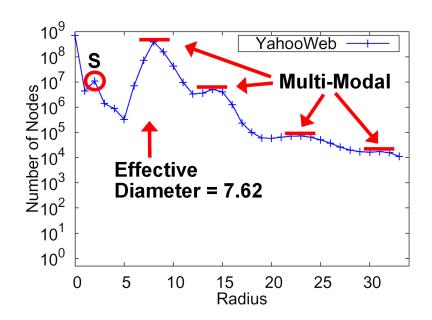
- Several new **patterns** (fortification, triangle-laws, conn. components, etc)
- New tools:

anomaly detection (OddBall), belief propagation, immunization

• Scalability: PEGASUS / hadoop

OVERALL CONCLUSIONS – high level

• **BIG DATA: Large** datasets reveal patterns/ outliers that are invisible otherwise



(Topics not mentioned here)

- Anomalies; fraud detection
- Immunization; epidemic thresholds
- Generators (Rmat/Kronecker, 'random typing'; agent-based)
- Time-evolving graphs (tensors, wavelets)
- Community detection (MDL)

- Leman Akoglu, Christos Faloutsos: *RTG: A Recursive Realistic Graph Generator Using Random Typing*. ECML/PKDD (1) 2009: 13-28
- Deepayan Chakrabarti, Christos Faloutsos: *Graph mining: Laws, generators, and algorithms*. ACM Comput. Surv. 38(1): (2006)

- Deepayan Chakrabarti, Yang Wang, Chenxi Wang, Jure Leskovec, Christos Faloutsos: *Epidemic thresholds in real networks*. ACM Trans. Inf. Syst. Secur. 10(4): (2008)
- Deepayan Chakrabarti, Jure Leskovec, Christos Faloutsos, Samuel Madden, Carlos Guestrin, Michalis Faloutsos: *Information Survival Threshold in Sensor and P2P Networks*. INFOCOM 2007: 1316-1324

 Christos Faloutsos, Tamara G. Kolda, Jimeng Sun: Mining large graphs and streams using matrix and tensor tools. Tutorial, SIGMOD Conference 2007: 1174

 T. G. Kolda and J. Sun. Scalable Tensor Decompositions for Multi-aspect Data Mining. In: ICDM 2008, pp. 363-372, December 2008.

- Jure Leskovec, Jon Kleinberg and Christos Faloutsos *Graphs over Time: Densification Laws, Shrinking Diameters and Possible Explanations*, KDD 2005 (Best Research paper award).
- Jure Leskovec, Deepayan Chakrabarti, Jon M. Kleinberg, Christos Faloutsos: *Realistic, Mathematically Tractable Graph Generation and Evolution, Using Kronecker Multiplication.* PKDD 2005: 133-145

- Jimeng Sun, Yinglian Xie, Hui Zhang, Christos Faloutsos. Less is More: Compact Matrix Decomposition for Large Sparse Graphs, SDM, Minneapolis, Minnesota, Apr 2007.
- Jimeng Sun, Spiros Papadimitriou, Philip S. Yu, and Christos Faloutsos, *GraphScope: Parameterfree Mining of Large Time-evolving Graphs* ACM SIGKDD Conference, San Jose, CA, August 2007



 Jimeng Sun, Dacheng Tao, Christos Faloutsos: *Beyond streams and graphs: dynamic tensor analysis*. KDD 2006: 374-383

- Hanghang Tong, Christos Faloutsos, and Jia-Yu Pan, *Fast Random Walk with Restart and Its Applications*, ICDM 2006, Hong Kong.
- Hanghang Tong, Christos Faloutsos, *Center-Piece Subgraphs: Problem Definition and Fast Solutions*, KDD 2006, Philadelphia, PA

 Hanghang Tong, Christos Faloutsos, Brian Gallagher, Tina Eliassi-Rad: Fast best-effort pattern matching in large attributed graphs. KDD 2007: 737-746

Project info

www.cs.cmu.edu/~pegasus



Chau, Polo



Koutra, Danae





Prakash,

Aditya



Akoglu, Leman Kang, U

, U

McGlohon, Mary

Tong, Hanghang

Thanks to: NSF IIS-0705359, IIS-0534205, CTA-INARC; Yahoo (M45), LLNL, IBM, SPRINT, Google, INTEL, HP, iLab