

# **Mining Large Graphs and Tensors - Patterns, Tools and Discoveries.**

*Christos Faloutsos*

CMU

# Thank you!

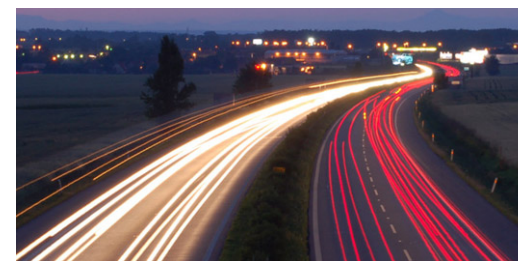


- Nikos Sidiropoulos
- Kuo-Chu Chang
- Zhi (Gerry) Tian



# Roadmap

- ➔ • Introduction – Motivation
  - Why ‘big data’
  - Why (big) graphs?
- Problem#1: Patterns in graphs
- Problem#2: Tools
- Conclusions



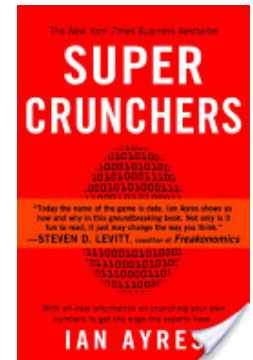
# Why 'big data'

- Why?
- What is the problem definition?

## Main message:

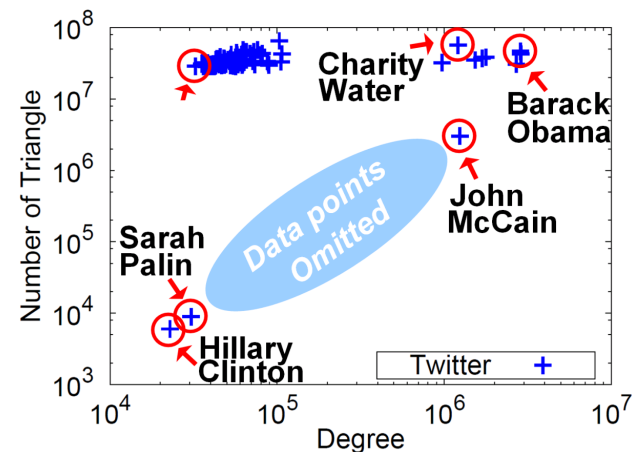
# Big data: often > experts

- ‘Super Crunchers’ *Why Thinking-By-Numbers is the New Way To Be Smart* by Ian Ayres, 2008



- Google won the machine translation competition 2005
- [http://www.itl.nist.gov/iad/mig//tests/mt/2005/doc/mt05eval\\_official\\_results\\_release\\_20050801\\_v3.html](http://www.itl.nist.gov/iad/mig//tests/mt/2005/doc/mt05eval_official_results_release_20050801_v3.html)

# Problem definition – big picture

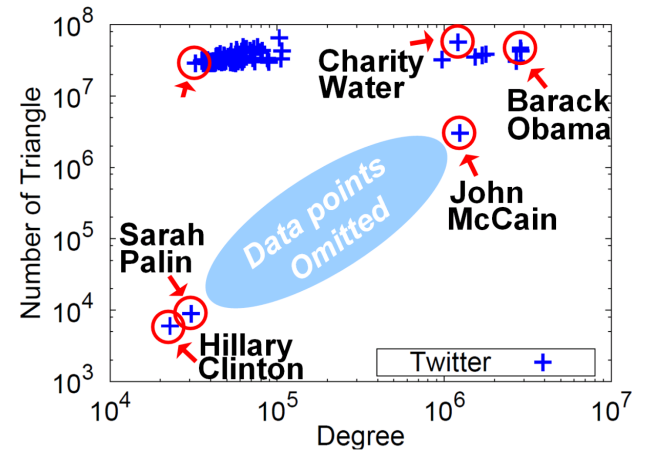


Tera/Peta-byte  
data

Analytics

Insights,  
outliers

# Problem definition – big picture



Tera/Peta-byte  
data

Analytics

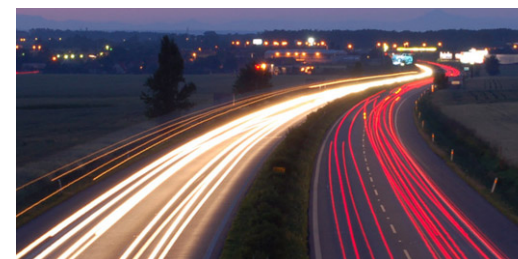
Insights,  
outliers



Main emphasis in this talk

# Roadmap

- Introduction – Motivation
  - Why ‘big data’
  - ➔ – Why (big) graphs?
- Problem#1: Patterns in graphs
- Problem#2: Tools
- Problem#3: Scalability
- Conclusions



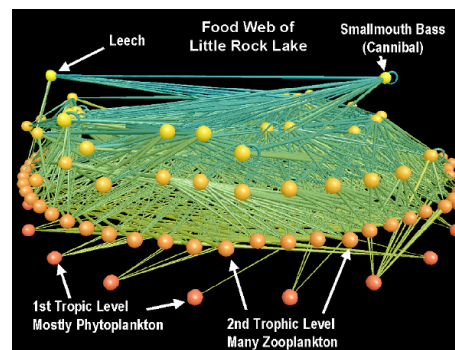


# Graphs - why should we care?

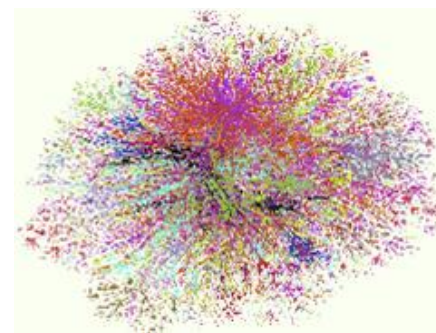


>\$10B revenue

>0.5B users



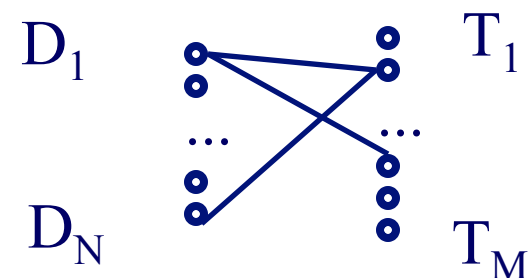
Food Web  
[Martinez '91]



Internet Map  
[lumeta.com]

# Graphs - why should we care?

- IR: bi-partite graphs (doc-terms)



- web: hyper-text graph

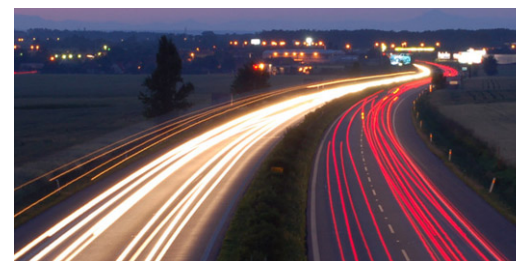
- ... and more:

# Graphs - why should we care?

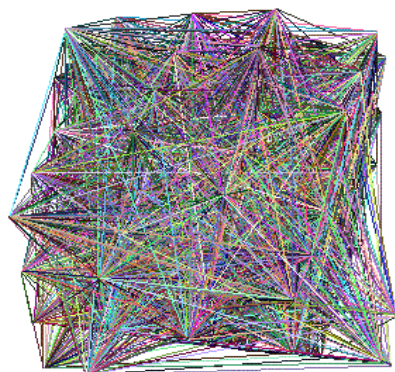
- web-log ('blog') news propagation
- computer network security: email/IP traffic and anomaly detection
- 'viral' marketing
- Supplier-supply business chains (-> instabilities)
- ....
- Subject-verb-object -> graph
- Many-to-many db relationship -> graph

## Outline

- Introduction – Motivation
- ➔ • Problem#1: Patterns in graphs
  - Static graphs
  - Time evolving graphs
- Problem#2: Tools
- Conclusions

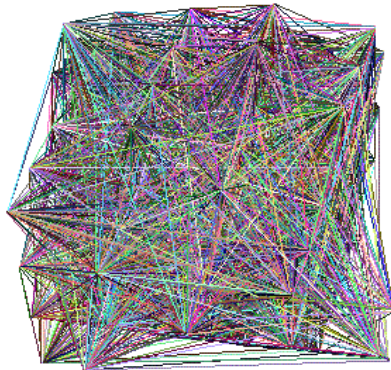


# Problem #1 - network and graph mining

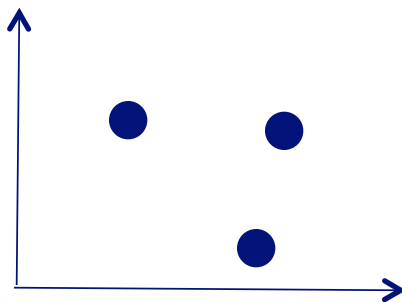


- What does the Internet look like?
- What does FaceBook look like?
- What is ‘normal’/‘abnormal’?
- which patterns/laws hold?

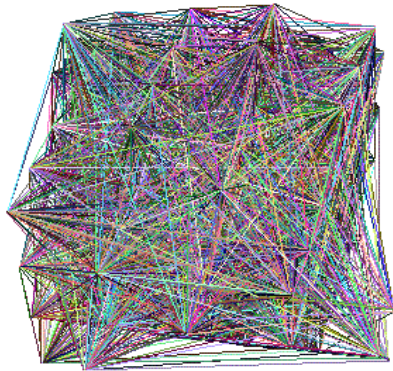
# Problem #1 - network and graph mining



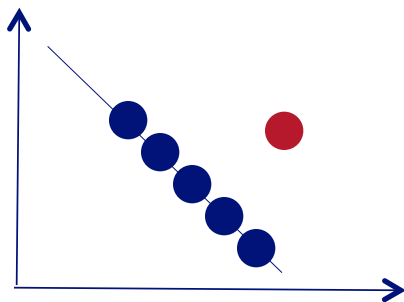
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  - To spot **anomalies** (rarities), we have to discover **patterns**



# Problem #1 - network and graph mining



- What does the Internet look like?
- What does FaceBook look like?
- What is ‘normal’/‘abnormal’?
- which patterns/laws hold?
  - To spot **anomalies** (rarities), we have to discover **patterns**
  - **Large** datasets reveal patterns/anomalies that may be invisible otherwise...



NSF, 3/2013

C. Faloutsos (CMU)

# Graph mining

- Are real graphs random?



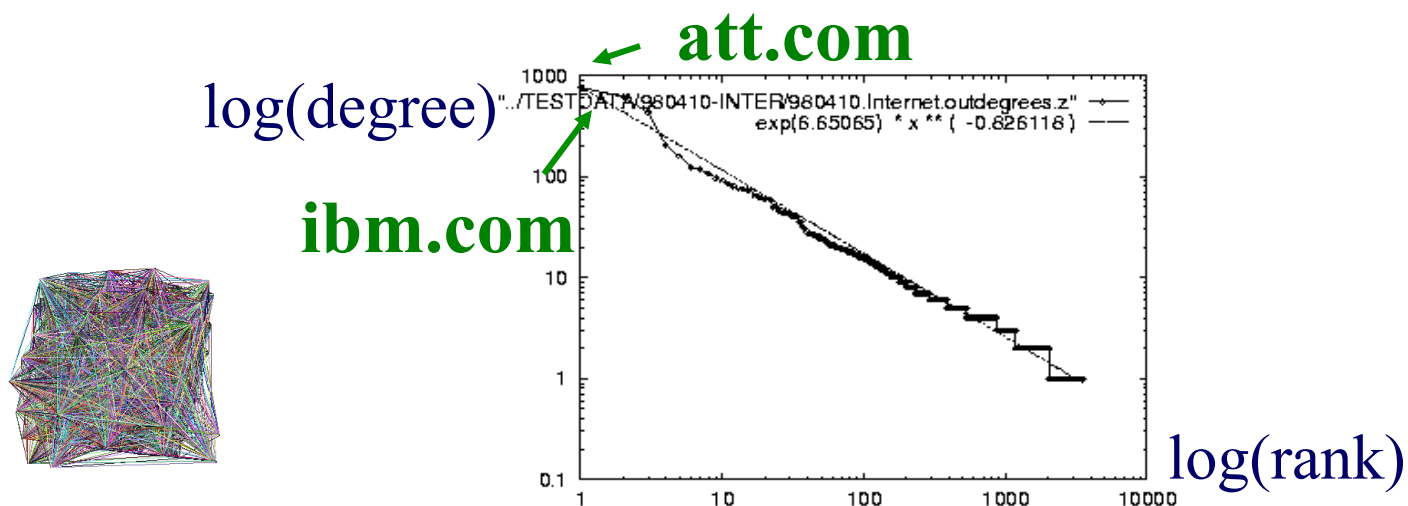
# Laws and patterns

- Are real graphs random?
- A: NO!!
  - Diameter
  - in- and out- degree distributions
  - other (surprising) patterns
- So, let's look at the data

# Solution# S.1

- Power law in the degree distribution [SIGCOMM99]

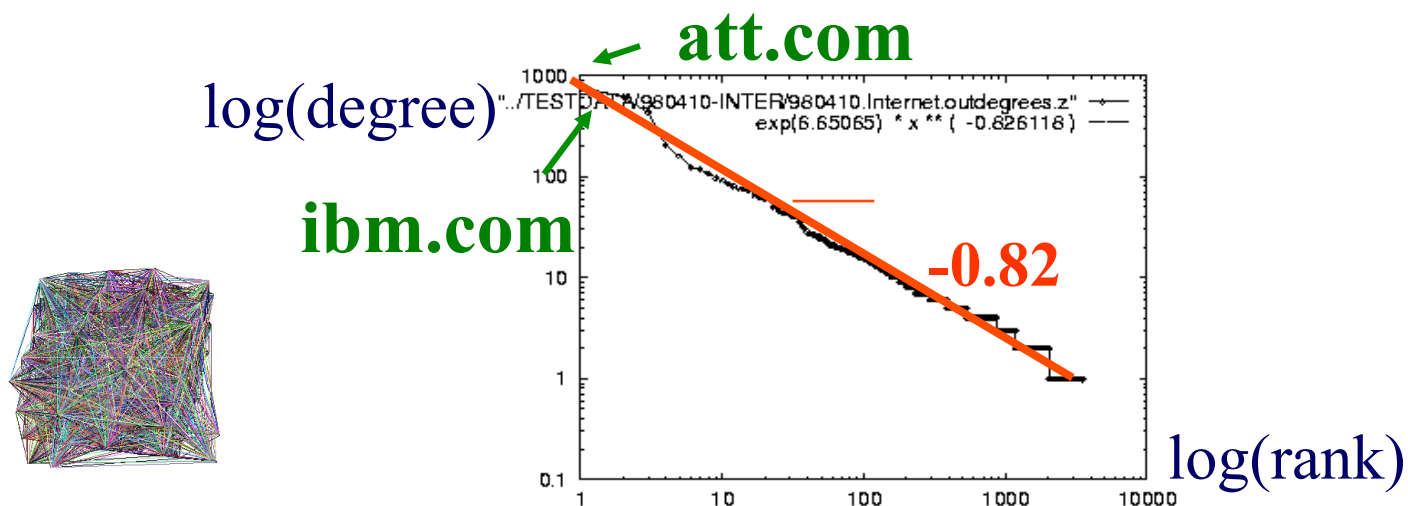
internet domains



# Solution# S.1

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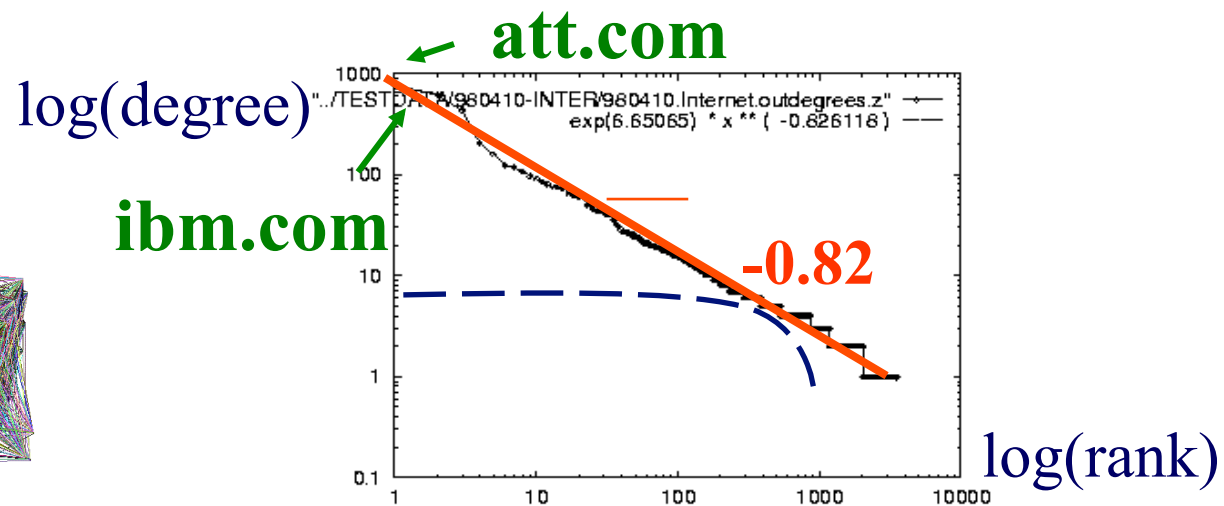
internet domains



# Solution# S.1

- Q: So what?

internet domains

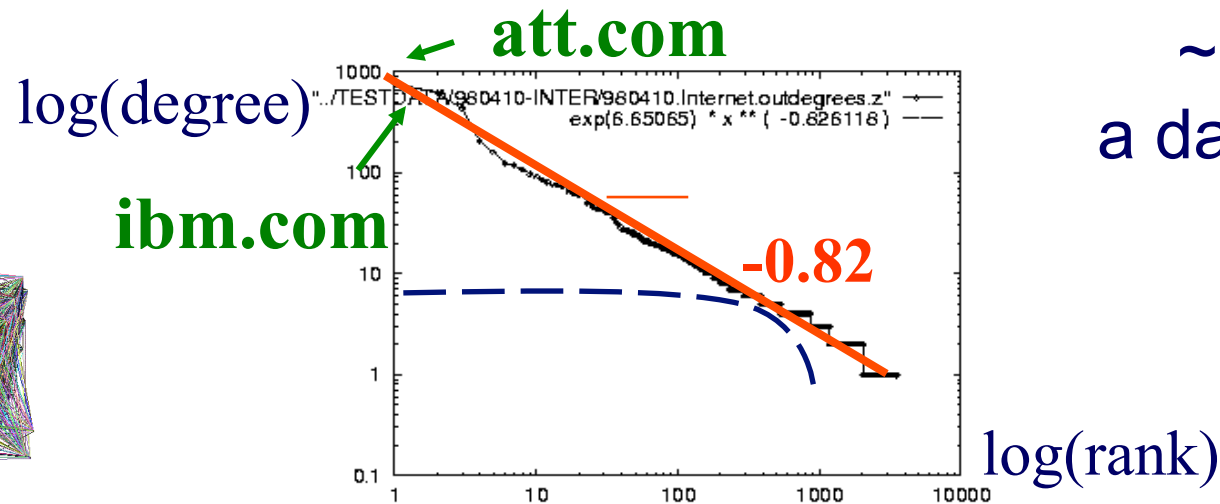
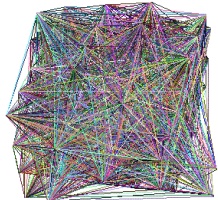


# Solution# S.1

- Q: So what?
- A1: # of two-step-away pairs:  $O(d_{\max}^2) \sim 10M^2$   
internet domains



$\sim 0.8\text{PB} \rightarrow$   
a data center(!)



# Solution# S.1

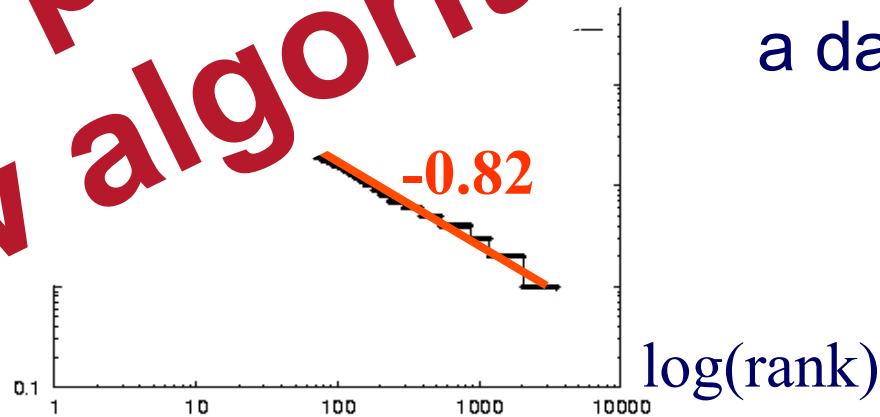
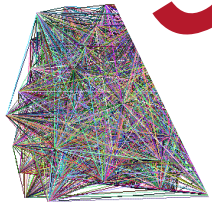
- Q: So what?
- A1: # of two-step-away inter

**Such patterns ->**  
**New algorithms**

? ) ~  $10M^2$

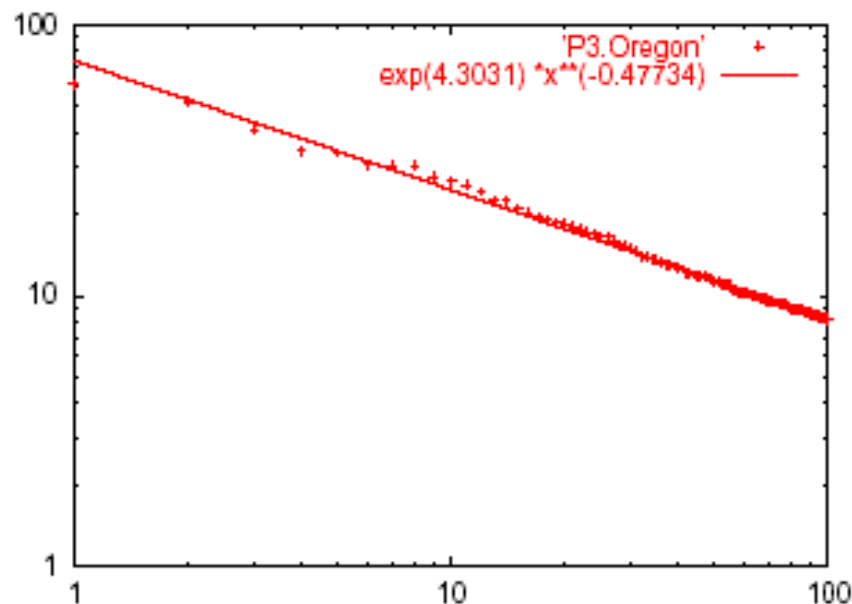


~0.8PB ->  
 a data center(!)



# Solution# S.2: Eigen Exponent $E$

Eigenvalue



Exponent = slope

$$E = -0.48$$

May 2001

Rank of decreasing eigenvalue

- A2: power law in the eigenvalues of the adjacency matrix

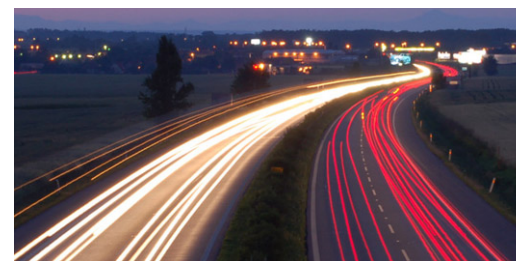
## Many more power laws

- # of sexual contacts
- Income [Pareto] – ‘80-20 distribution’
- Duration of downloads [Bestavros+]
- Duration of UNIX jobs (‘mice and elephants’)
- Size of files of a user
- ...
- ‘Black swans’

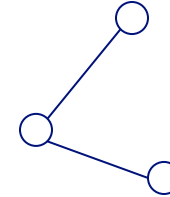


# Roadmap

- Introduction – Motivation
- Problem#1: Patterns in graphs
  - Static graphs
    - degree, diameter, eigen,
    - triangles
    - cliques
  - Weighted graphs
  - Time evolving graphs
- Problem#2: Tools

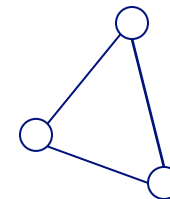


## Solution# S.3: Triangle ‘Laws’



- Real social networks have a lot of triangles

## Solution# S.3: Triangle ‘Laws’

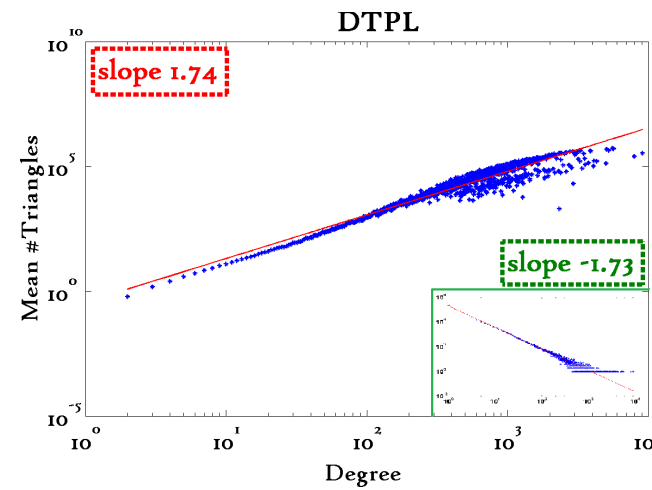
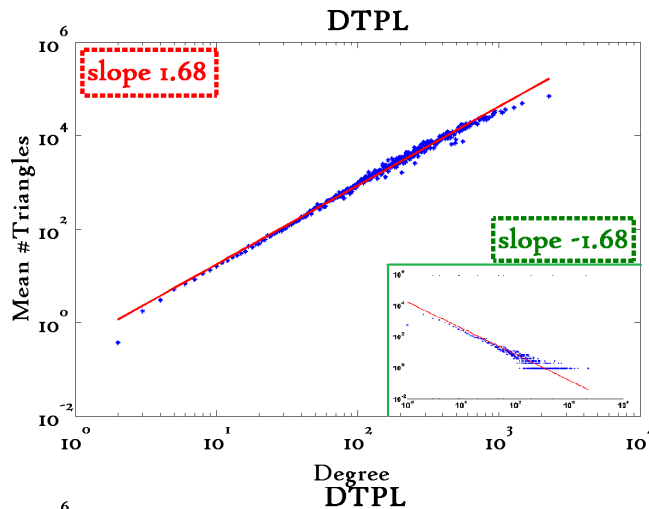


- Real social networks have a lot of triangles
  - Friends of friends are friends
- Any patterns?

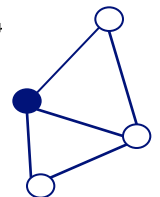
# Triangle Law: #S.3

## [Tsourakakis ICDM 2008]

Reuters

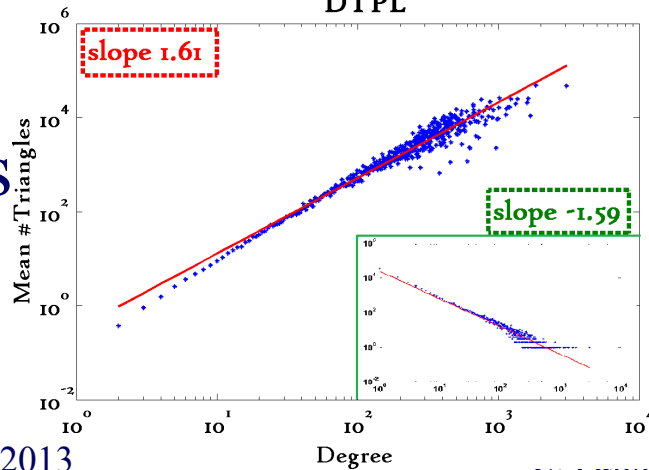


SN



X-axis: degree  
 Y-axis: mean # triangles  
 $n$  friends  $\rightarrow \sim n^{1.6}$  triangles

Epinions



# Triangle Law: Computations

[Tsourakakis ICDM 2008]

But: triangles are expensive to compute

(3-way join; several approx. algos) –  $O(d_{\max}^2)$

Q: Can we do that quickly?

A:

# Triangle Law: Computations

[Tsourakakis ICDM 2008]

But: triangles are expensive to compute

(3-way join; several approx. algos) –  $O(d_{\max}^2)$

Q: Can we do that quickly?

A: Yes!

**#triangles =  $1/6 \text{ Sum } (\lambda_i^3)$**

(and, because of skewness (S2) ,

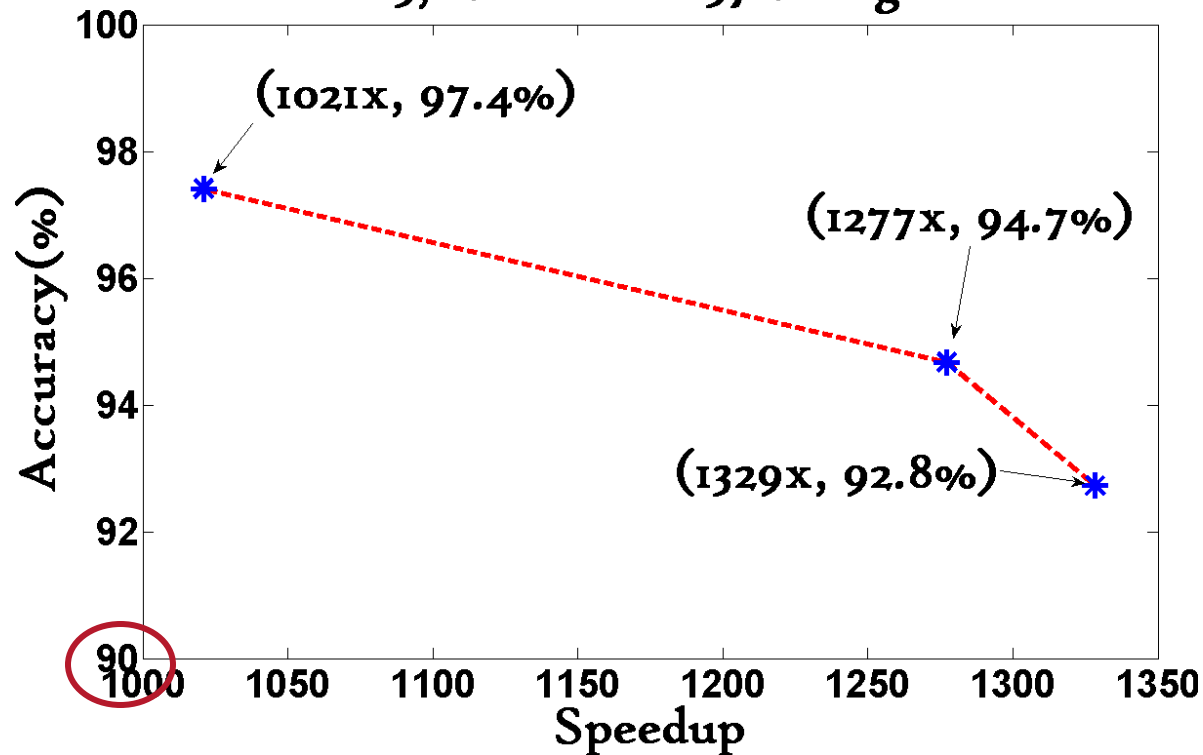
we only need the top few eigenvalues! -  $O(E)$

# Triangle Law: Computations

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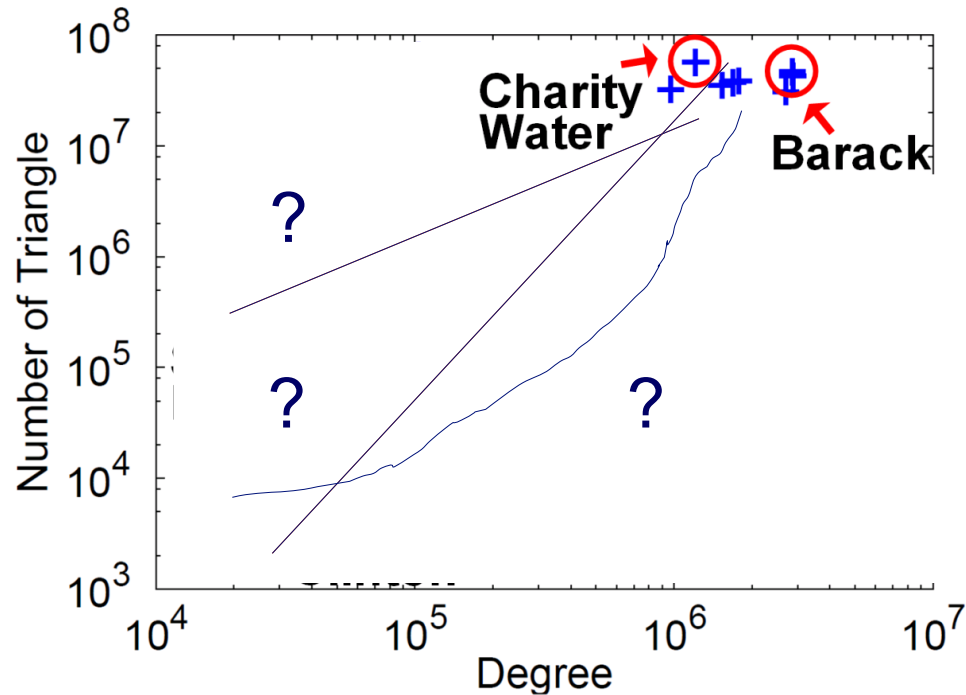
Wikipedia graph 2006-Nov-04

$\approx 3.1\text{M}$  nodes  $\approx 37\text{M}$  edges



1000x+ speed-up, >90% accuracy

# Triangle counting for large graphs?

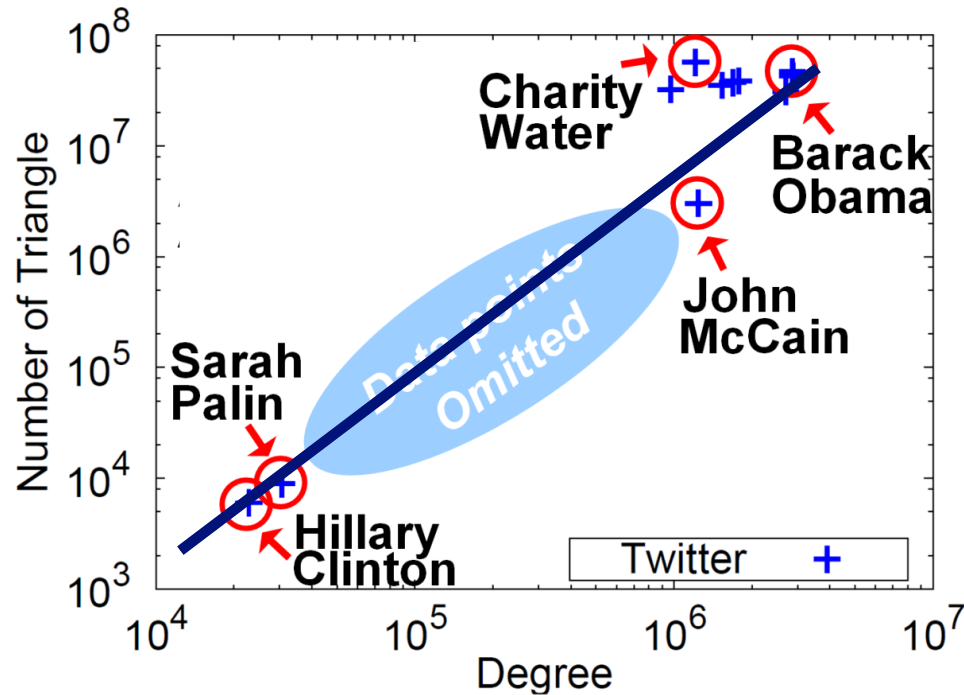


Anomalous nodes in Twitter (~ 3 billion edges)

[U Kang, Brendan Meeder, +, PAKDD'11]



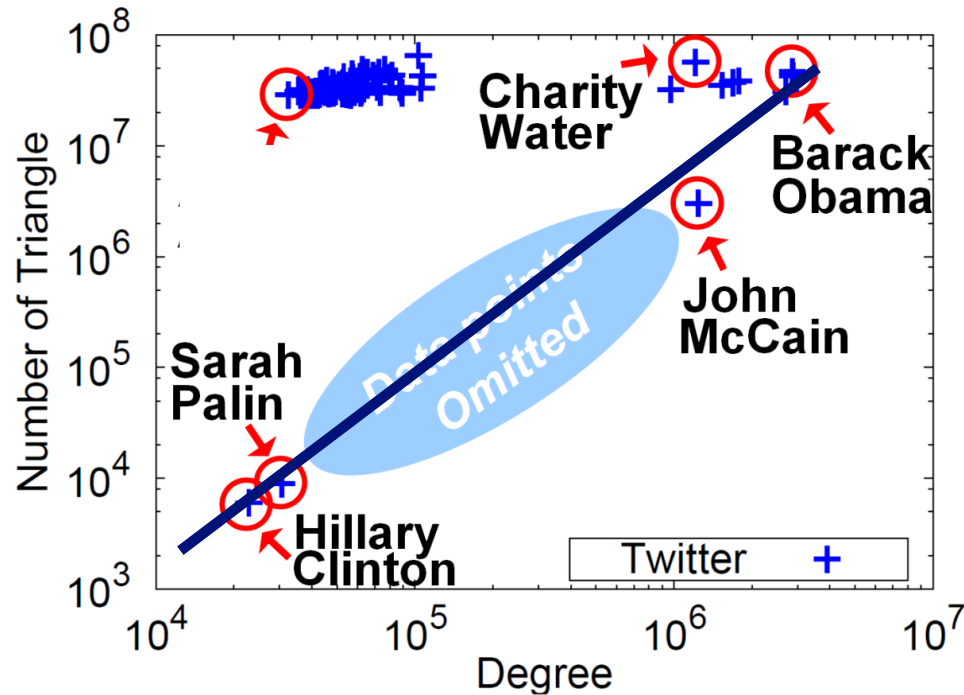
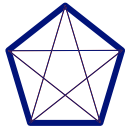
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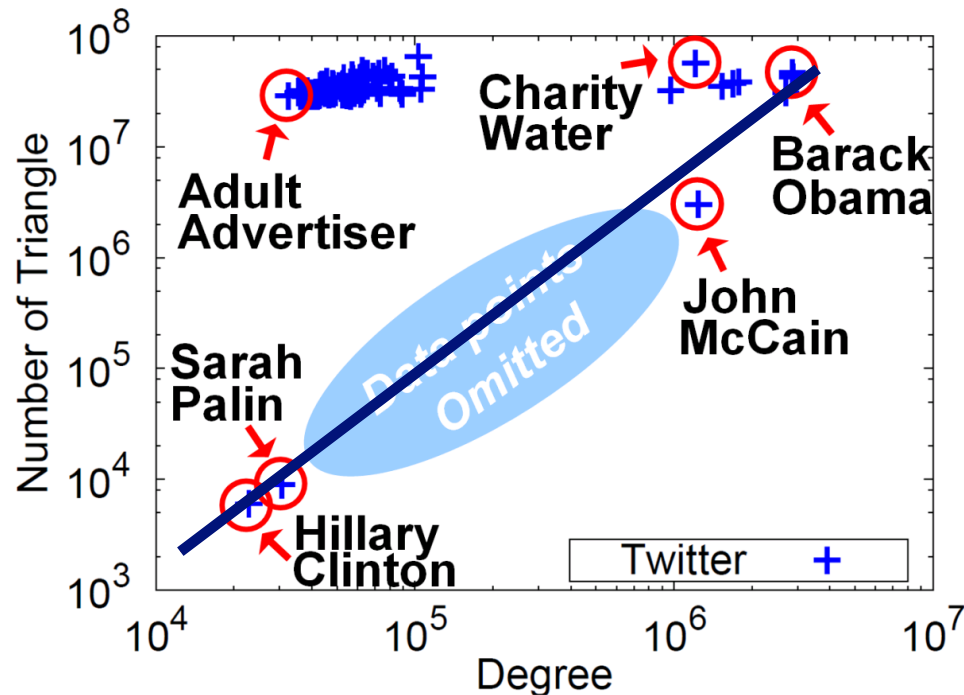
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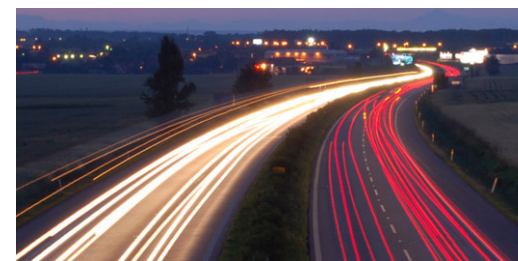


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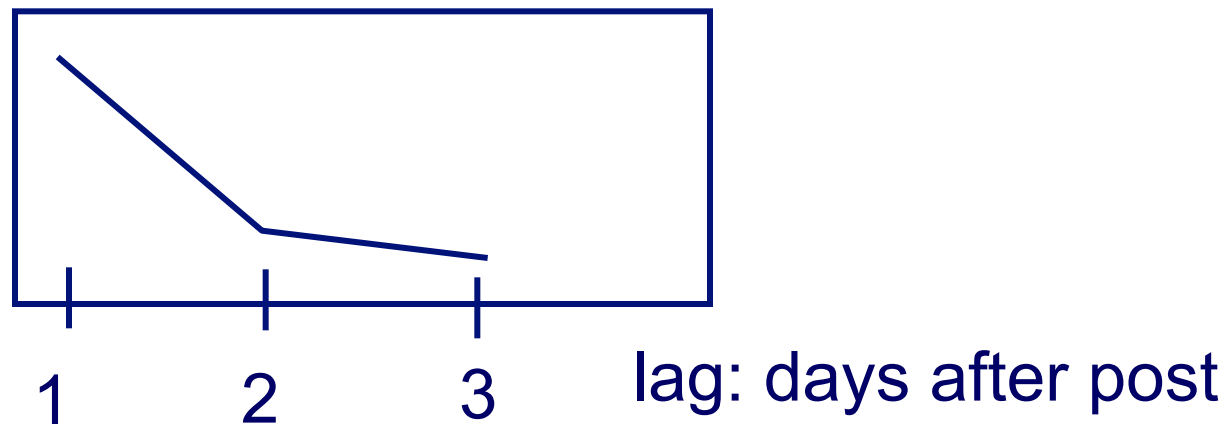
# Roadmap

- Introduction – Motivation
- Problem#1: Patterns in graphs
  - Static graphs
  - ➔ – Time evolving graphs
- Problem#2: Tools
- ...

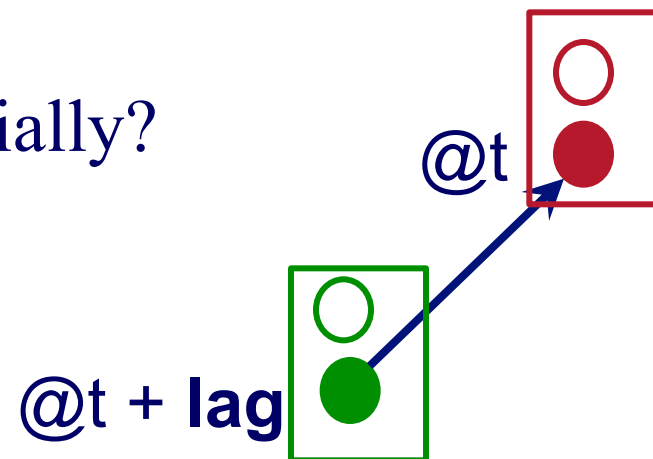


# T.1 : popularity over time

# in links

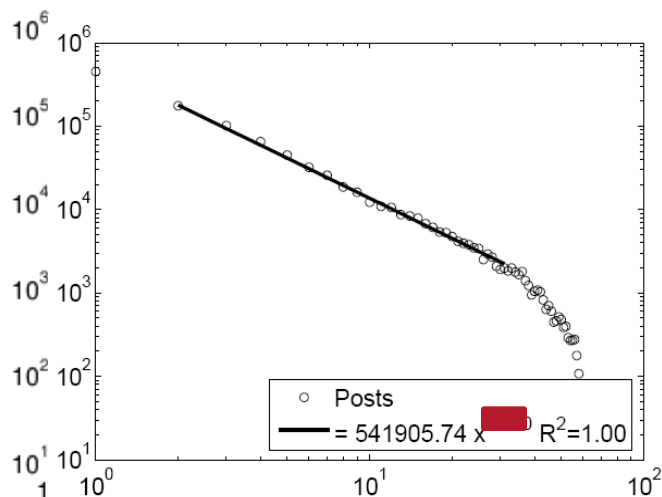


Post popularity drops-off – exponentially?



# T.1 : popularity over time

# in links  
(log)

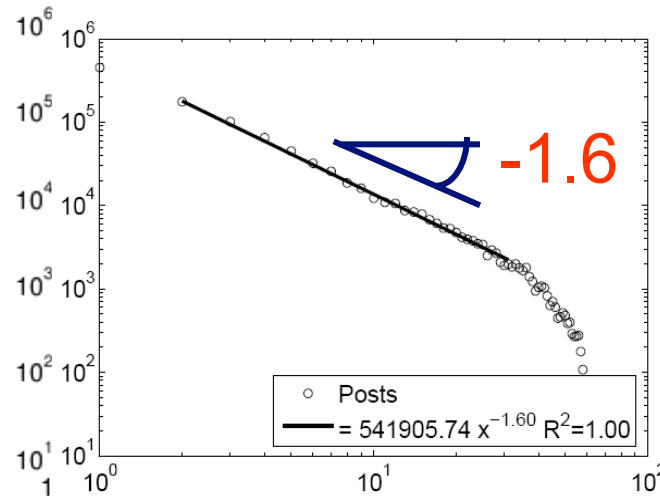


days after post  
(log)

Post popularity drops-off – exponentially?  
POWER LAW!  
Exponent?

# T.1 : popularity over time

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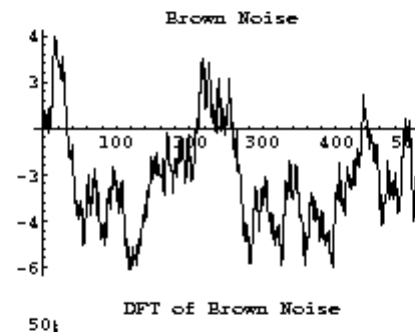
days after post  
(log)

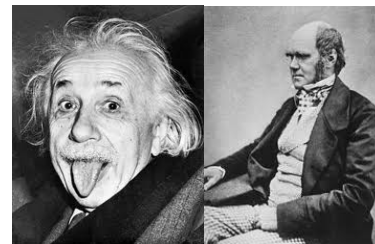
Post popularity drops-off – exponentially?

POWER LAW!

Exponent? -1.6

- close to -1.5: Barabasi's stack model
- and like the zero-crossings of a random walk

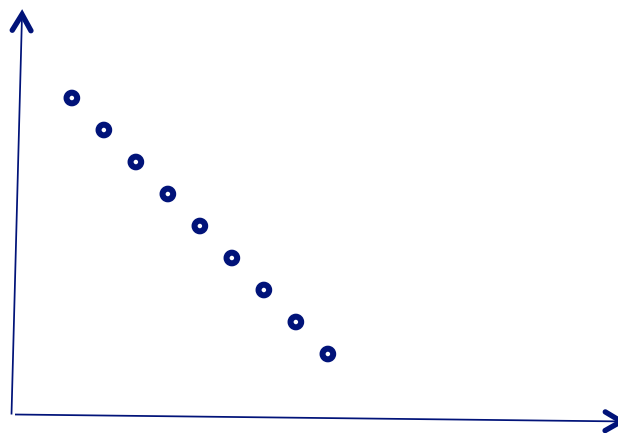




**-1.5 slope**

J. G. Oliveira & A.-L. Barabási Human Dynamics: The Correspondence Patterns of Darwin and Einstein. *Nature* **437**, 1251 (2005) . [[PDF](#)]

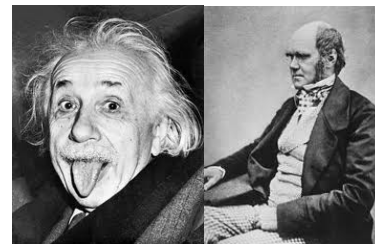
Prob(RT > x)  
(log)



Response time (log)



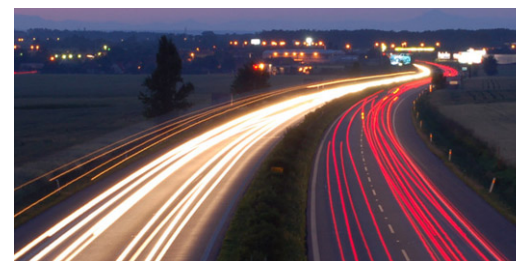
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# Roadmap

- Introduction – Motivation
- Problem#1: Patterns in graphs
- Problem#2: Tools
  - (Belief Propagation)
  - ➔ – Tensors
  - Spike analysis
- Conclusions



# GigaTensor: Scaling Tensor Analysis Up By 100 Times – Algorithms and Discoveries

**U  
Kang**



NSF, 3/2013

**Evangelos  
Papalexakis**



**Abhay  
Harpale**

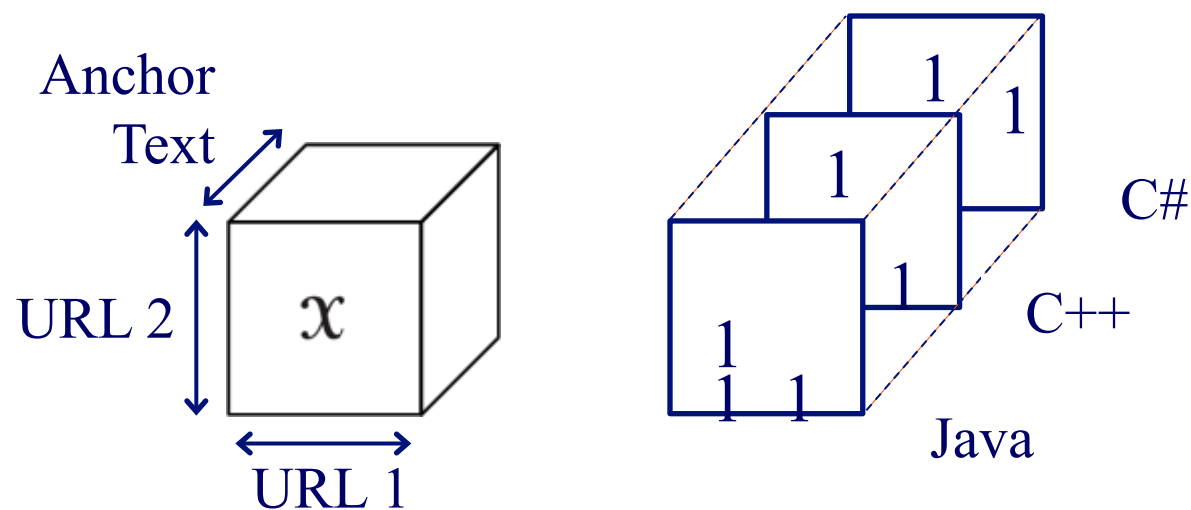
**Christos  
Faloutsos**

**KDD'12**

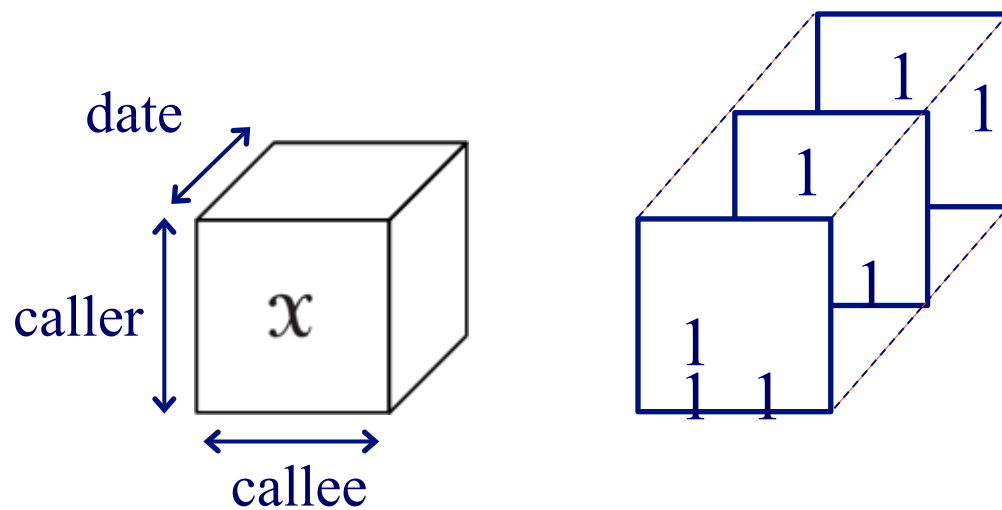
C. Faloutsos (CMU)

# Background: Tensor

- Tensors (=multi-dimensional arrays) are everywhere
  - Hyperlinks & anchor text [Kolda+,05]



# Time evolving graphs: Tensors

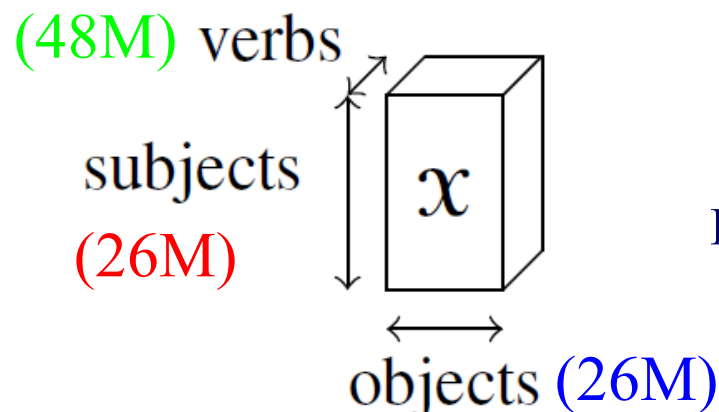


# Background: Tensor

- Tensors (=multi-dimensional arrays) are everywhere
  - Sensor stream (time, location, type)
  - Predicates (subject, verb, object) in knowledge base

“Eric Clapton plays  
guitar”

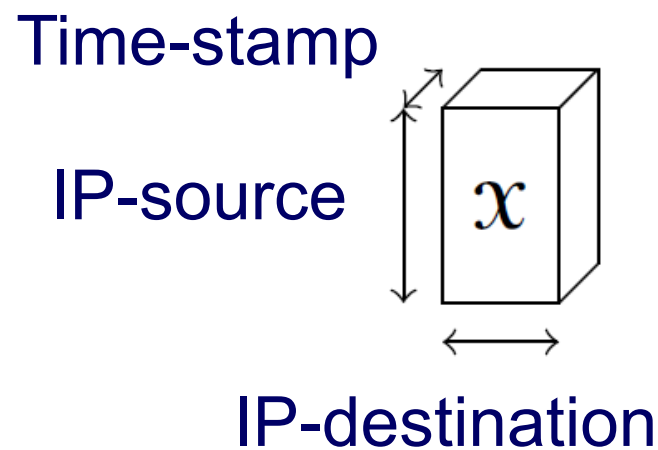
“Barrack Obama is  
the president of  
U.S.”



NELL (Never Ending  
Language Learner) data  
Nonzeros = 144M

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- Tensors (=multi-dimensional arrays) are everywhere
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*Anomaly  
Detection in  
Computer  
networks*

# all I learned on tensors: from



Nikos Sidiropoulos  
UMN

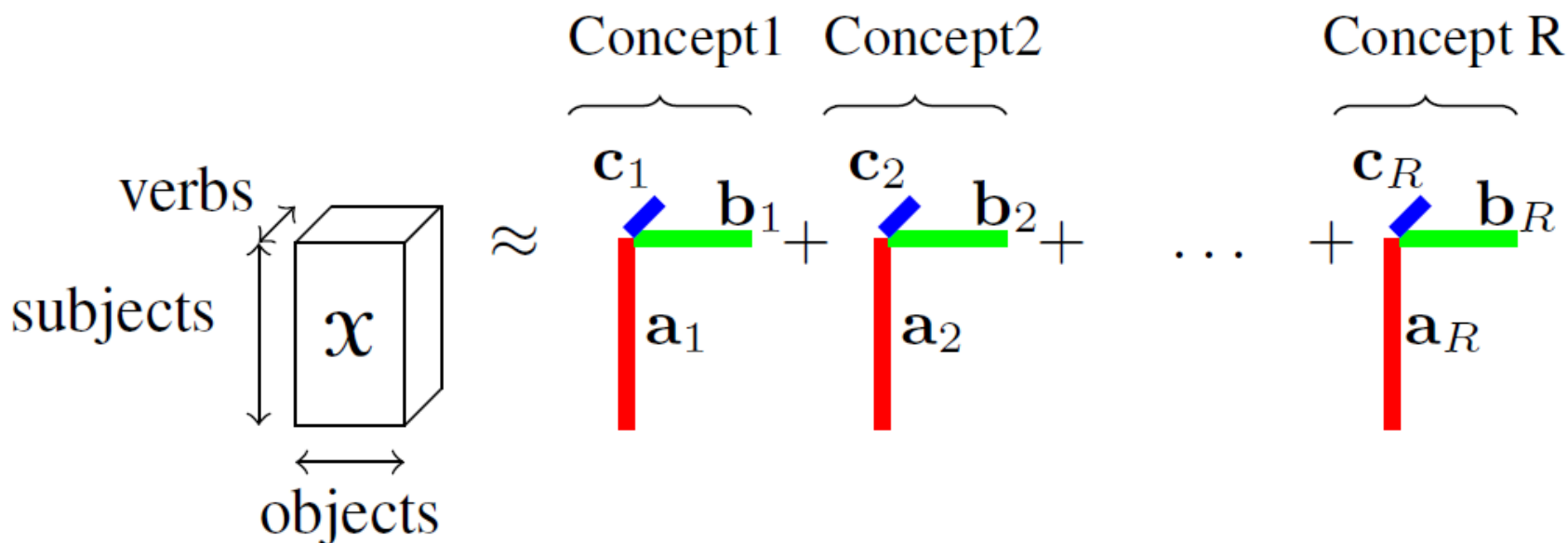


Tamara Kolda,  
Sandia Labs  
(tensor toolbox)



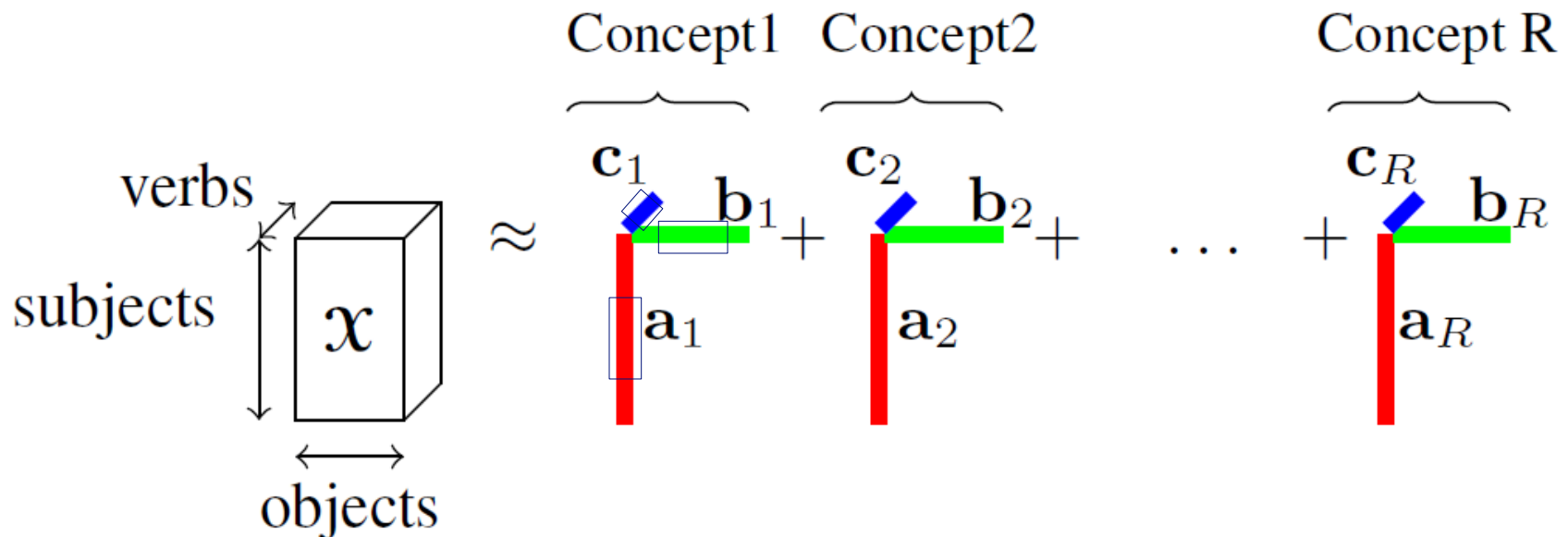
# Problem Definition

- How to decompose a billion-scale tensor?
  - Corresponds to SVD in 2D case



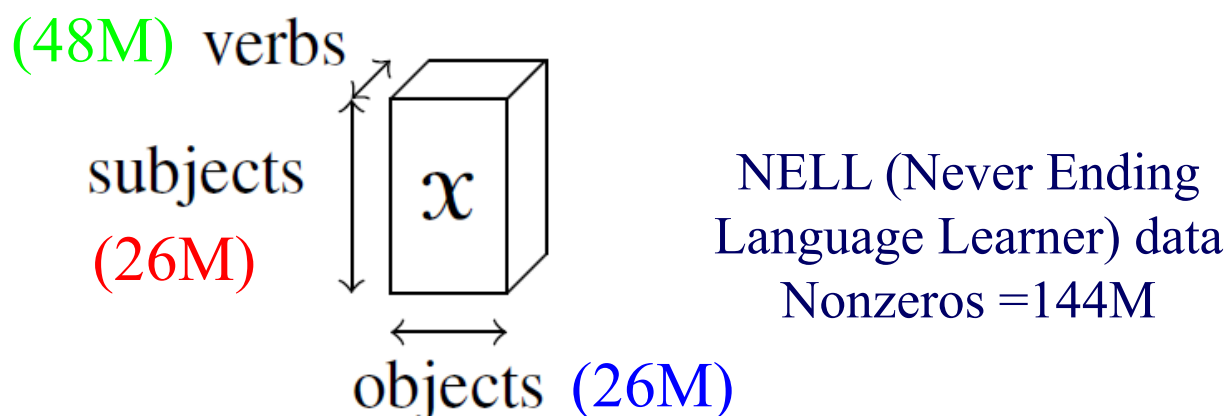
# Problem Definition

- How to decompose a billion-scale tensor?
  - Corresponds to SVD in 2D case = soft clustering



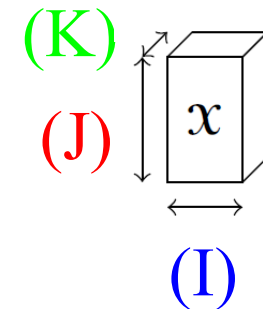
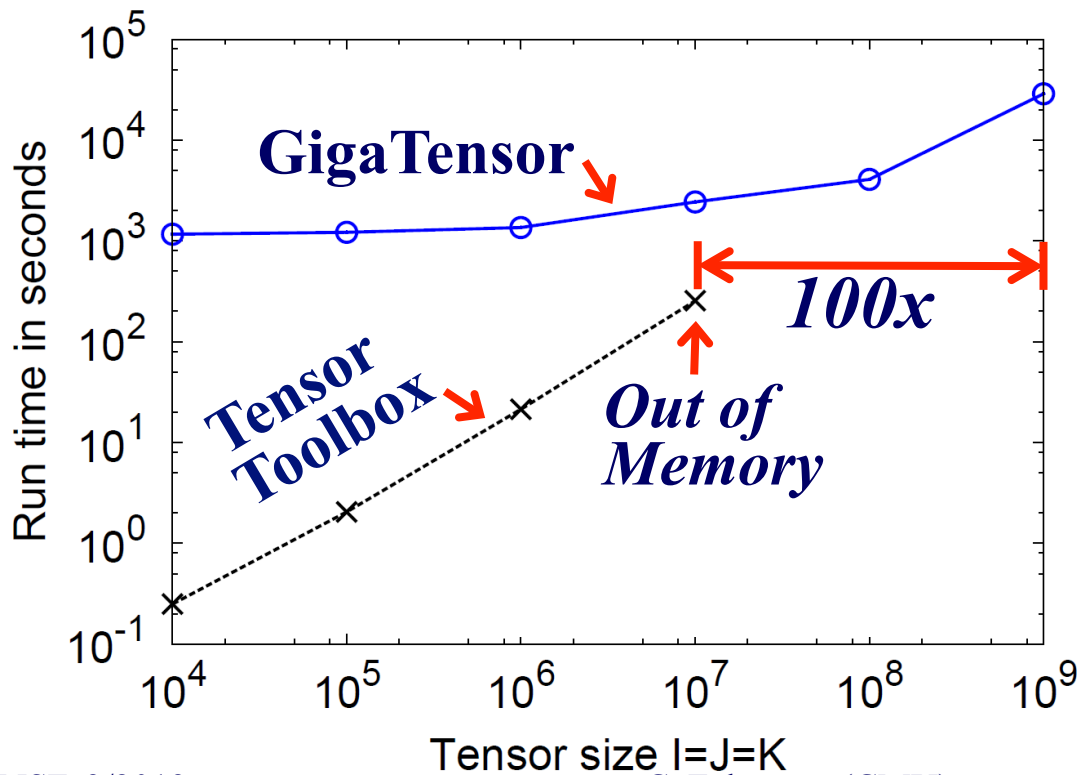
# Problem Definition

- ❑ Q1: Dominant concepts/topics?
- ❑ Q2: Find synonyms to a given noun phrase?
- ❑ (and how to scale up:  $|\text{data}| > \text{RAM}$ )



# Experiments

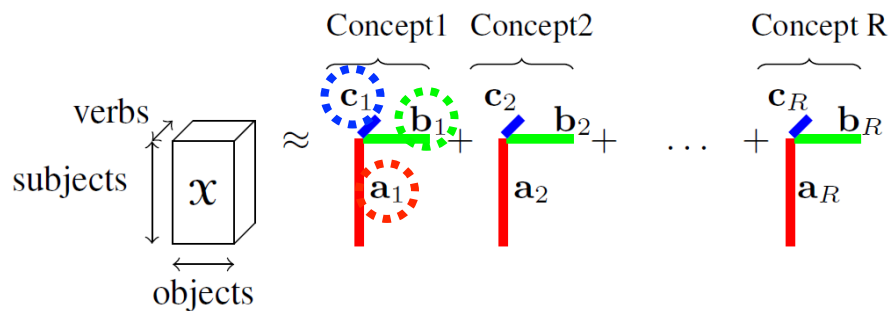
- GigaTensor solves *100x* larger problem



Number of  
nonzero  
=  $I / 50$

# A1: Concept Discovery

- Concept Discovery in Knowledge Base



Noun Phrase 1	Noun Phrase 2	Context
<b>Concept 1: "Web Protocol"</b>		
internet	protocol	'np1' 'stream' 'np2'
file	software	'np1' 'marketing' 'np2'
data	suite	'np1' 'dating' 'np2'
<b>Concept 2: "Credit Cards"</b>		
credit	information	'np1' 'card' 'np2'
Credit	debt	'np1' 'report' 'np2'
library	number	'np1' 'cards' 'np2'
<b>Concept 3: "Health System"</b>		
health	provider	'np1' 'care' 'np2'
child	providers	'np' 'insurance' 'np2'
home	system	'np1' 'service' 'np2'
<b>Concept 4: "Family Life"</b>		
life	rest	'np2' 'of' 'my' 'np1'
family	part	'np2' 'of' 'his' 'np1'
body	years	'np2' 'of' 'her' 'np1'

# A1: Concept Discovery

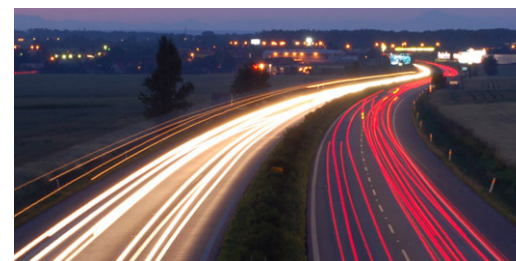
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credit	information	'np1' 'card' 'np2'
Credit	debt	'np1' 'report' 'np2'
library	number	'np1' 'cards' 'np2'
<b>Concept 3: "Health System"</b>		
health	provider	'np1' 'care' 'np2'
child	providers	'np' 'insurance' 'np2'
home	system	'np1' 'service' 'np2'

## A2: Synonym Discovery

<b>(Given)</b> <b>Noun Phrase</b>	<b>(Discovered)</b> <b>Potential Synonyms</b>
pollutants	dioxin, sulfur dioxide, greenhouse gases, particulates, nitrogen oxide, air pollutants, cholesterol
disabilities	infections, dizziness, injuries, diseases, drowsiness, stiffness, injuries
vodafone	verizon, comcast
Christian history	European history, American history, Islamic history, history
disbelief	dismay, disgust, astonishment

# Roadmap

- Introduction – Motivation
- Problem#1: Patterns in graphs
- Problem#2: Tools
  - Belief propagation
  - Tensors
  - ➔ – Spike analysis
  - Graph summarization
- Conclusions



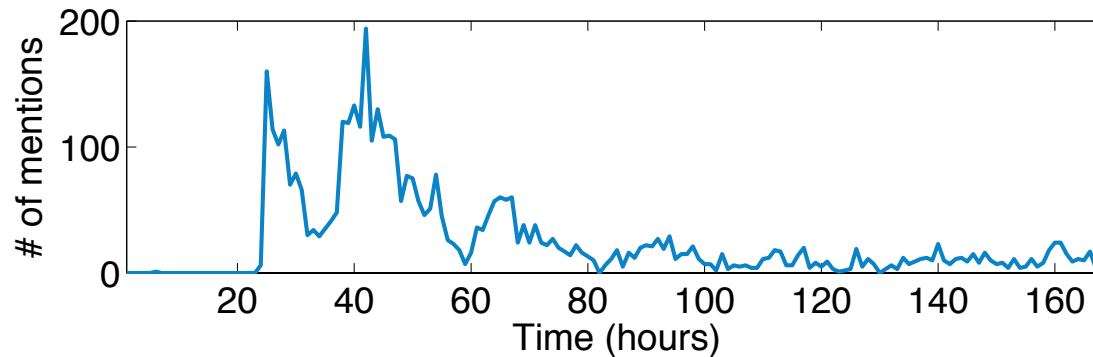


# Rise and fall patterns in social media

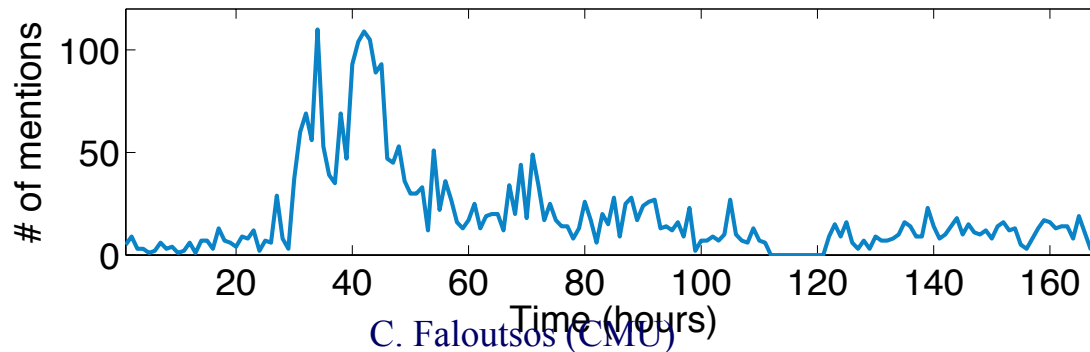
- Meme (# of mentions in blogs)

- short phrases Sourced from U.S. politics in 2008

“you can put lipstick on a pig”

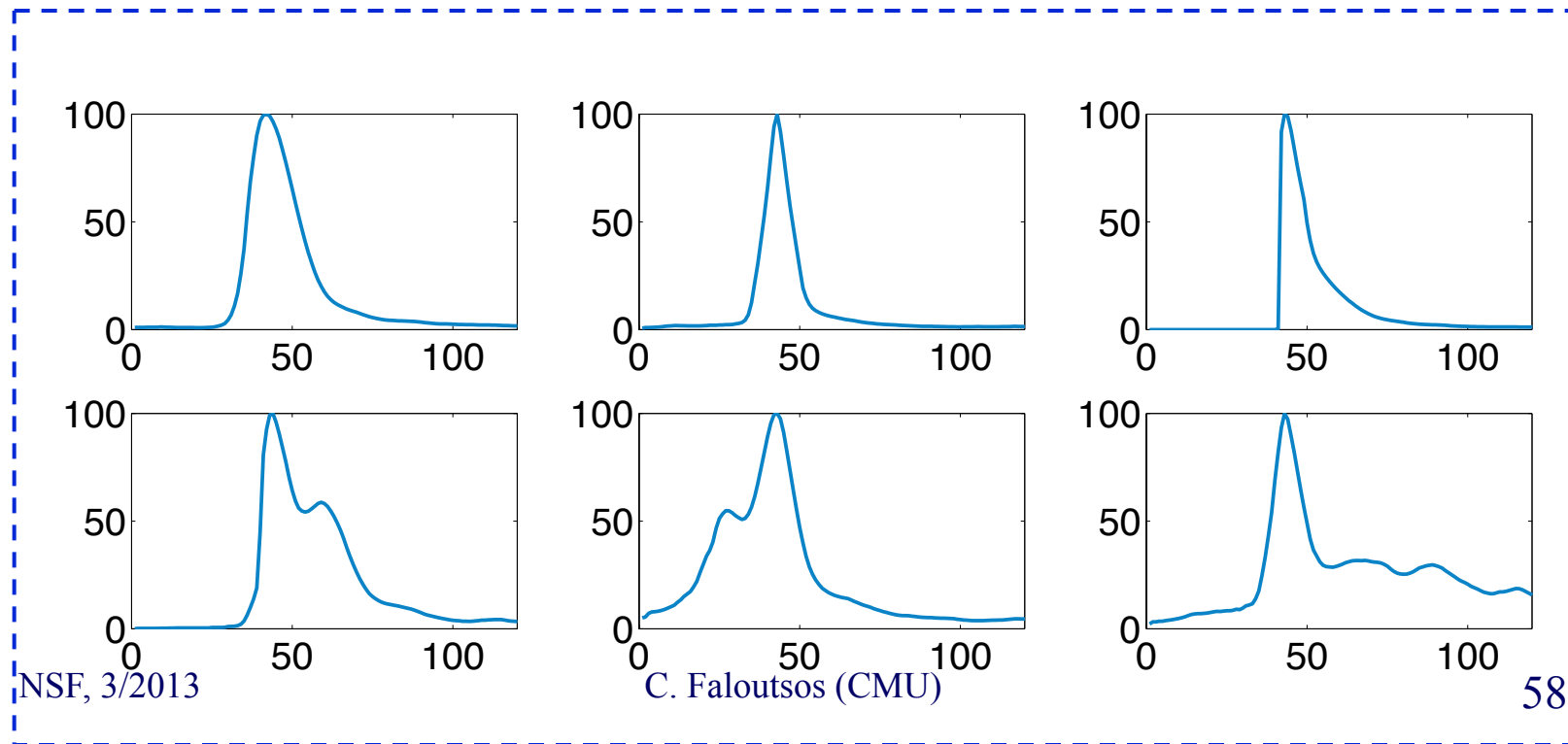


“yes we can”



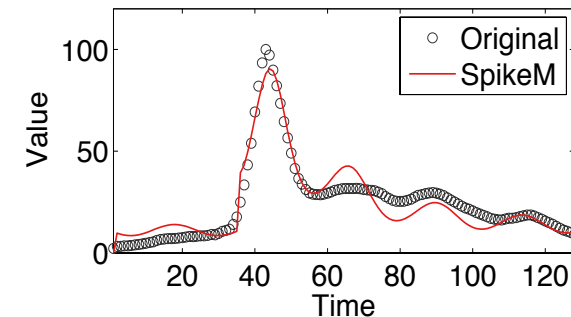
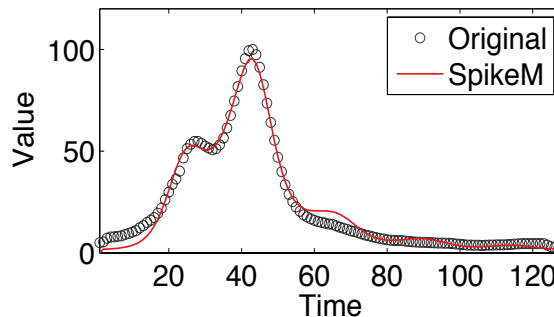
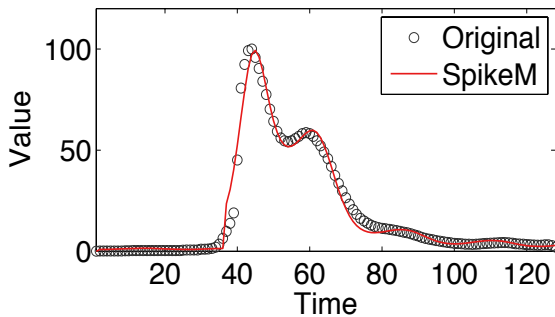
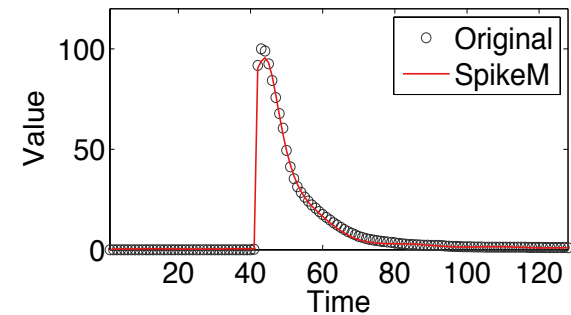
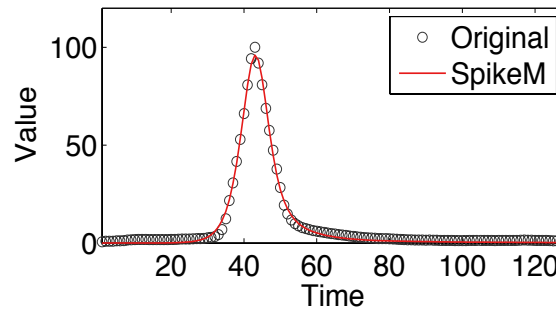
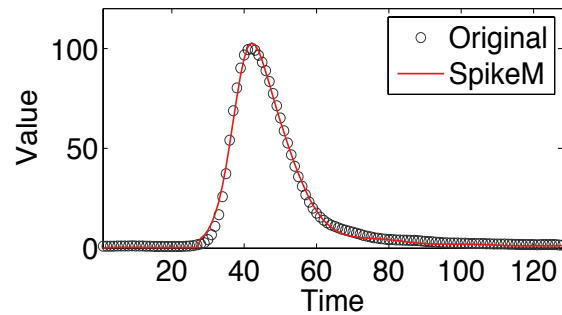
## Rise and fall patterns in social media

- Can we find a unifying model, which includes these patterns?
  - **four** classes on YouTube [Crane et al. '08]
  - **six** classes on Meme [Yang et al. '11]



# Rise and fall patterns in social media

- Answer: YES!

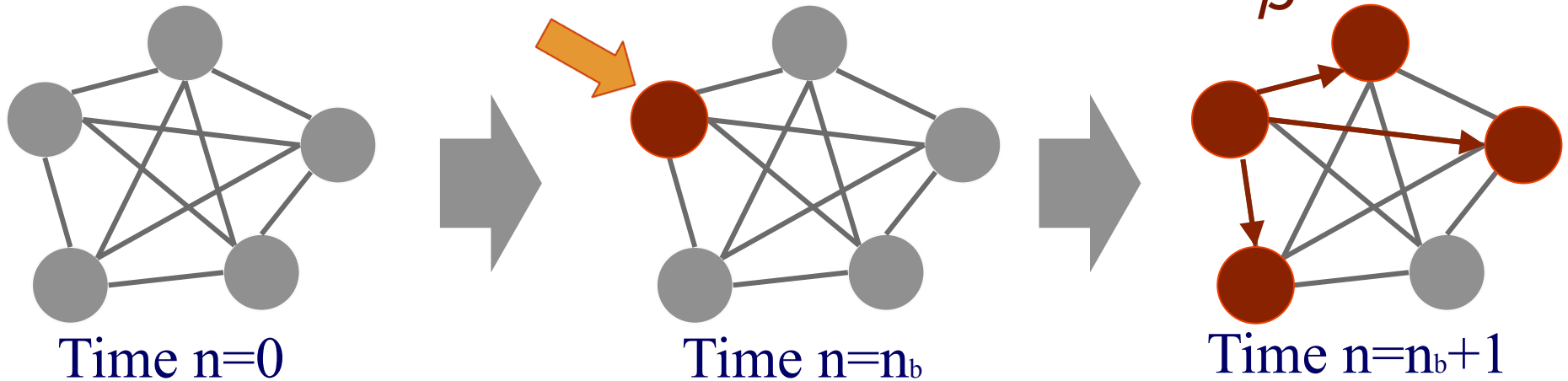


- We can represent **all** patterns by single model

In Matsubara+ SIGKDD 2012

# Main idea - SpikeM

- 1. **Un-informed** bloggers (uninformed about rumor)
- 2. **External shock** at time  $n_b$  (e.g, breaking news)
- 3. **Infection** (word-of-mouth)



Infectiveness of a blog-post at age  $n$ :

- $\beta$             - Strength of infection (quality of news)
- $f(n)$         - Decay function

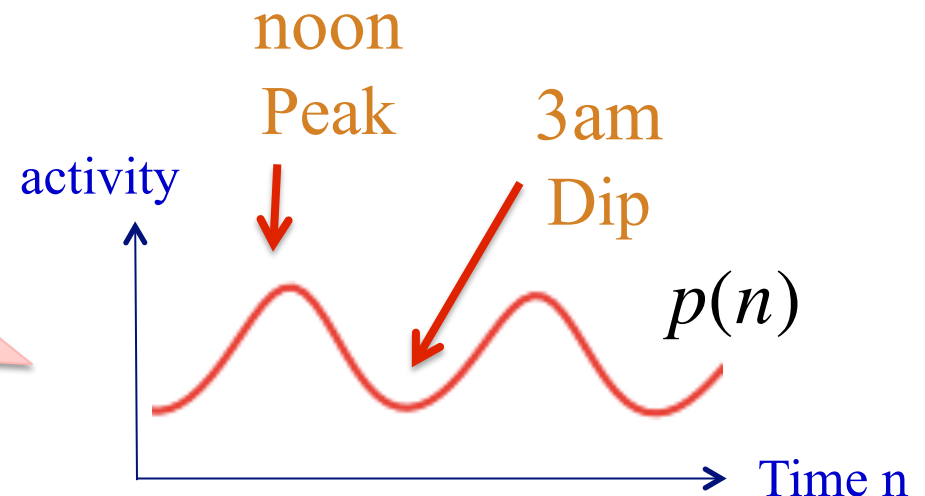


# SpikeM - with periodicity

- Full equation of SpikeM

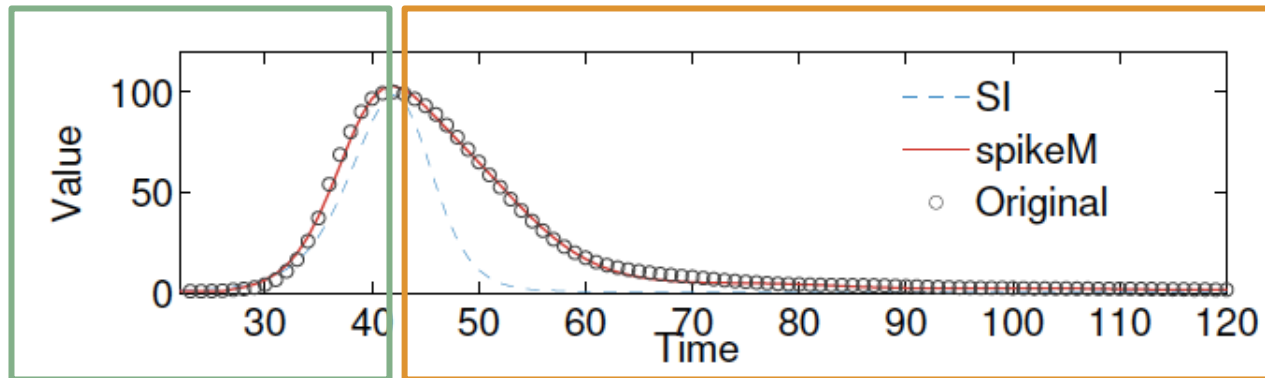
$$\Delta B(n+1) = \underbrace{p(n+1)}_{\text{Periodicity}} \cdot \left[ U(n) \cdot \sum_{t=n_b}^n (\Delta B(t) + S(t)) \cdot f(n+1-t) + \varepsilon \right]$$

Bloggers change their activity over time (e.g., daily, weekly, yearly)

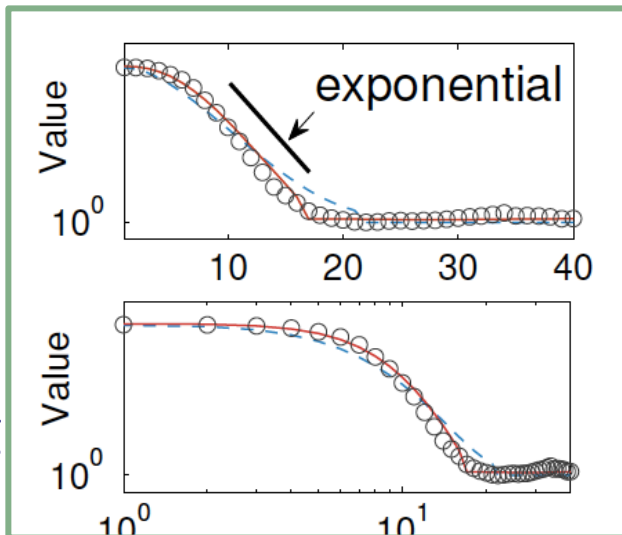


# Details

- Analysis – exponential rise and power-law fall



Lin-log



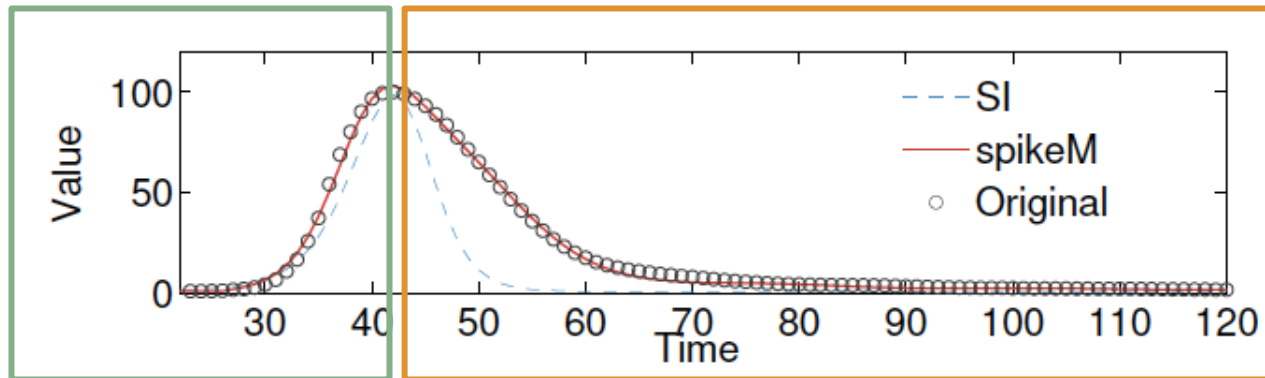
Log-log

## Rise-part

SI -> exponential  
SpikeM -> exponential

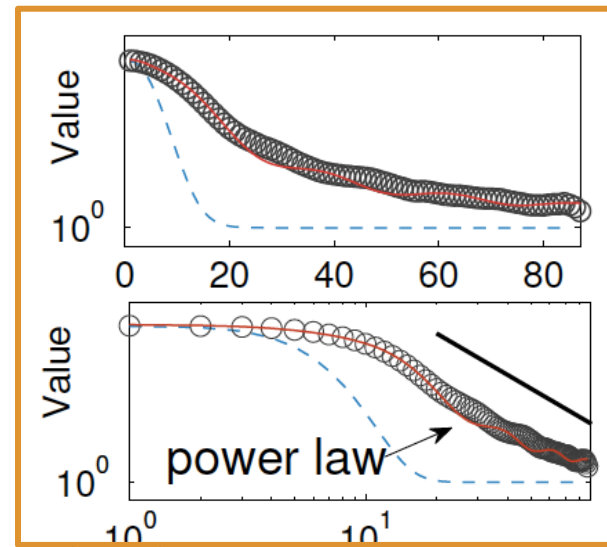
# Details

- Analysis – exponential rise and power-law fall



Fall-part

✗ SI -> exponential  
 SpikeM -> power law



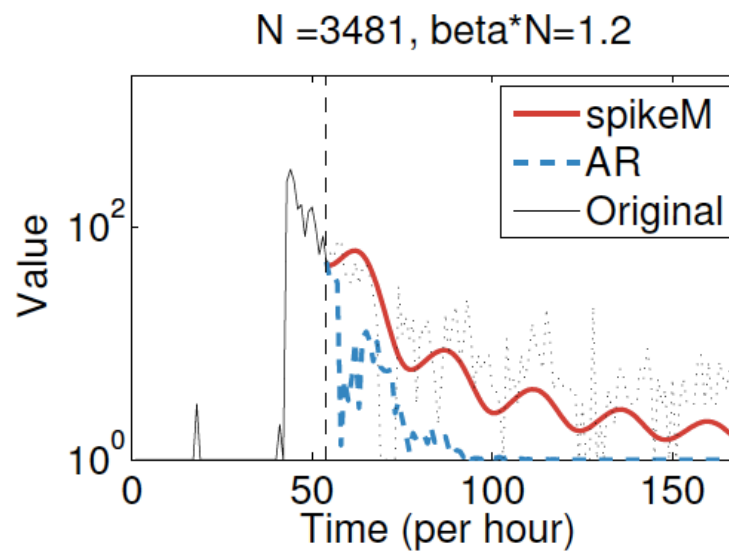
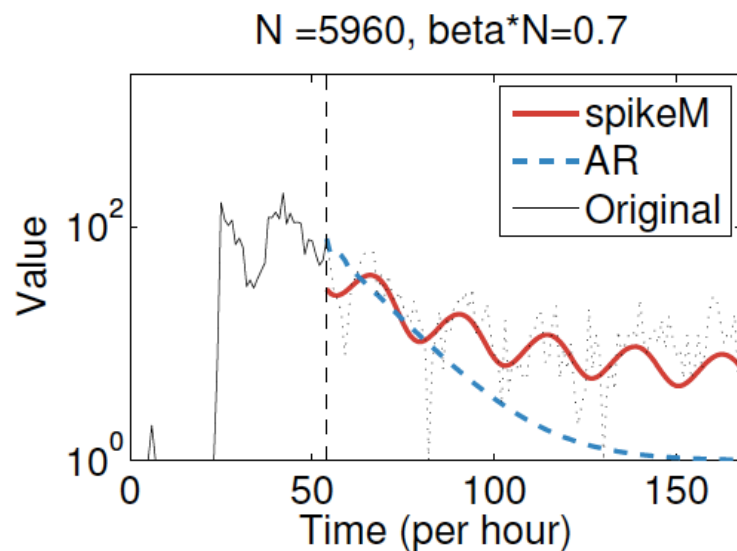
Lin-log

Log-log

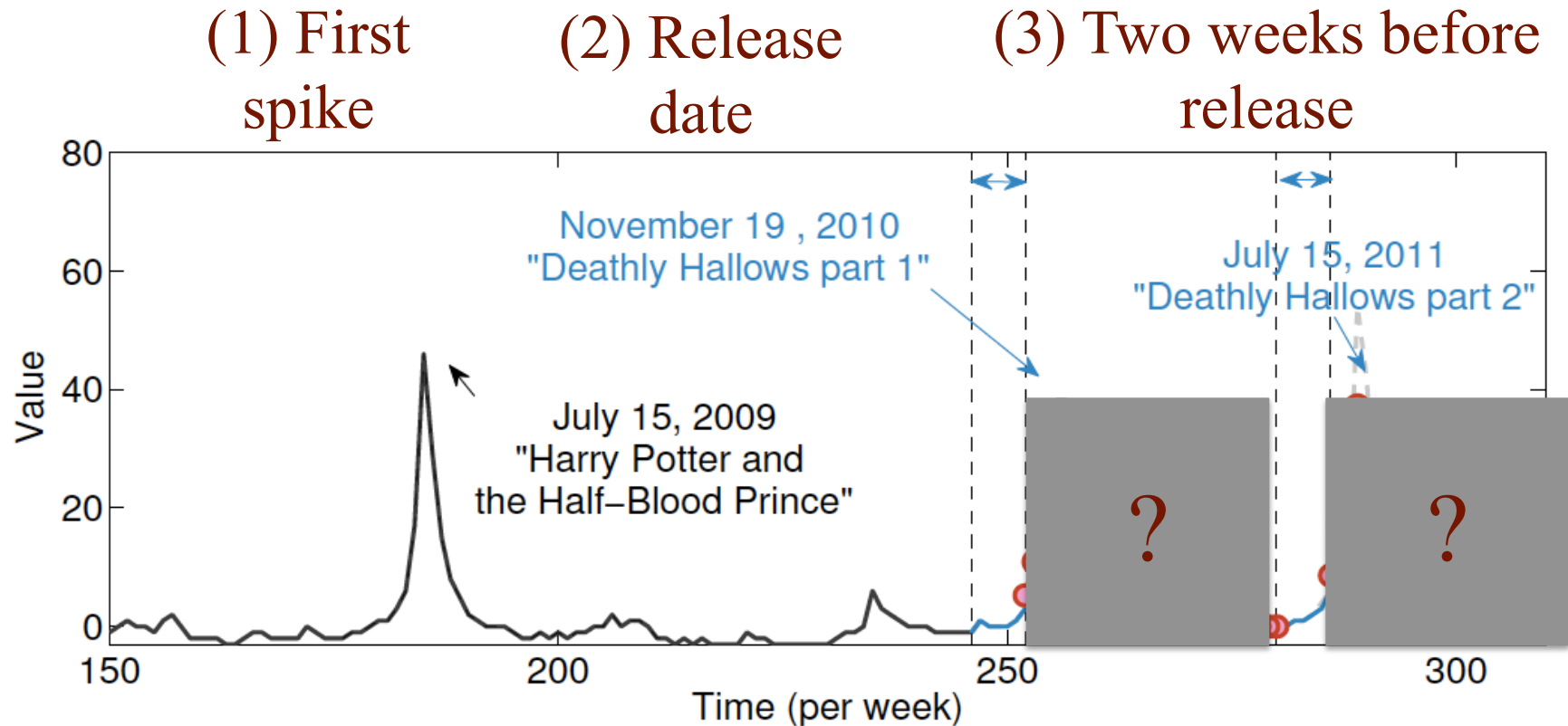


# Tail-part forecasts

- **SpikeM** can capture tail part

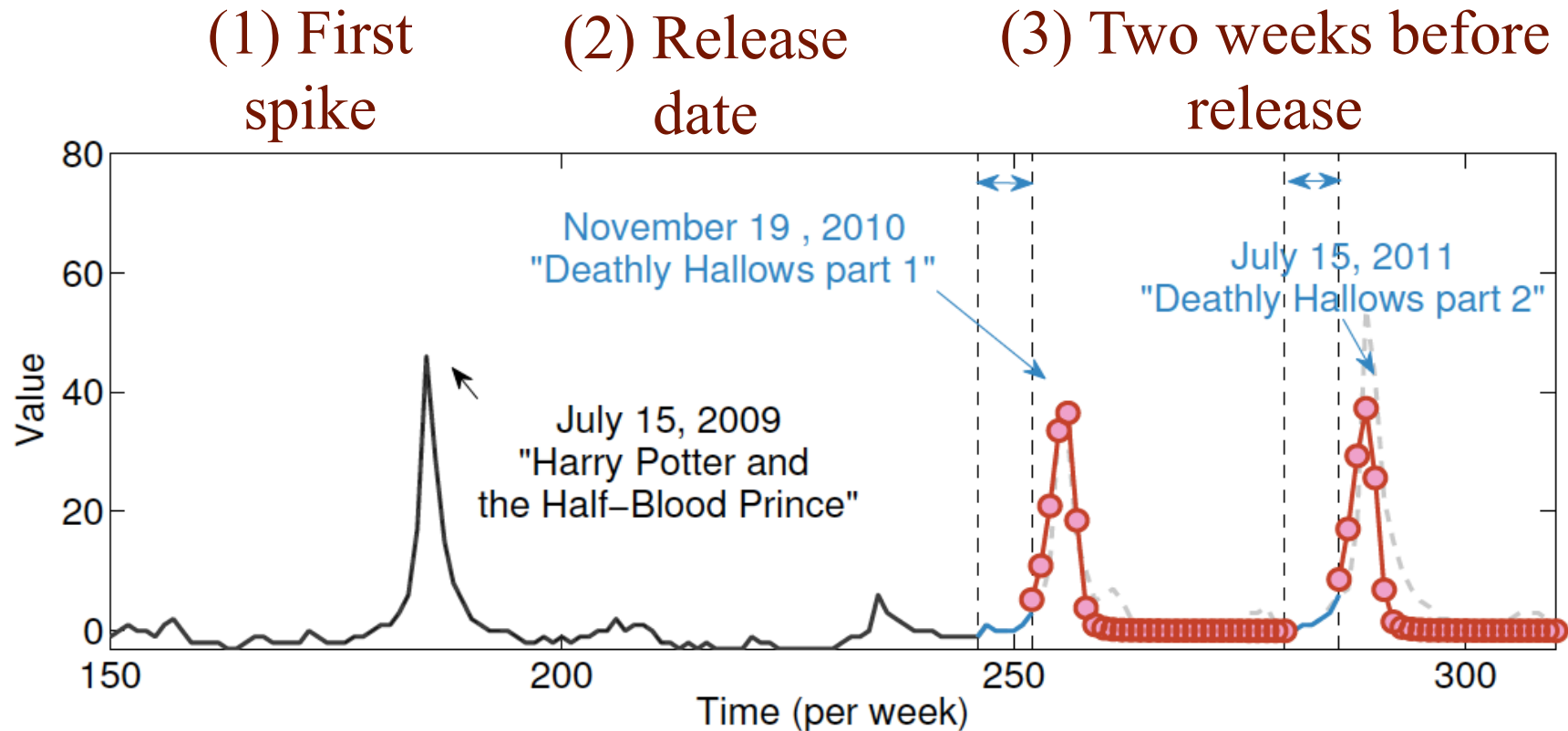


# “What-if” forecasting



- e.g., given (1) first spike,  
 (2) release date of two sequel movies  
 (3) access volume before the release date

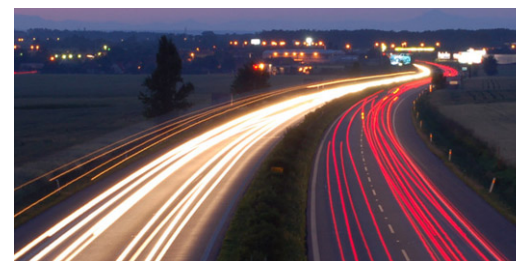
# “What-if” forecasting



SpikeM can forecast upcoming spikes

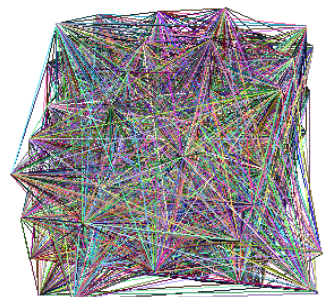
# Roadmap

- Introduction – Motivation
- Problem#1: Patterns in graphs
- Problem#2: Tools
  - Belief Propagation
  - Tensors
  - Spike analysis
  - ➔ – Graph understanding (through MDL)
- Conclusions

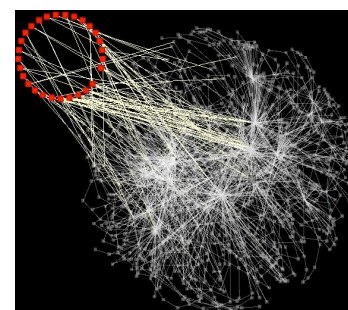
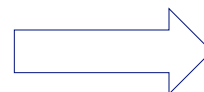


# Summarizing Graphs

**Goal:**

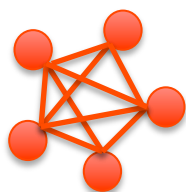


??



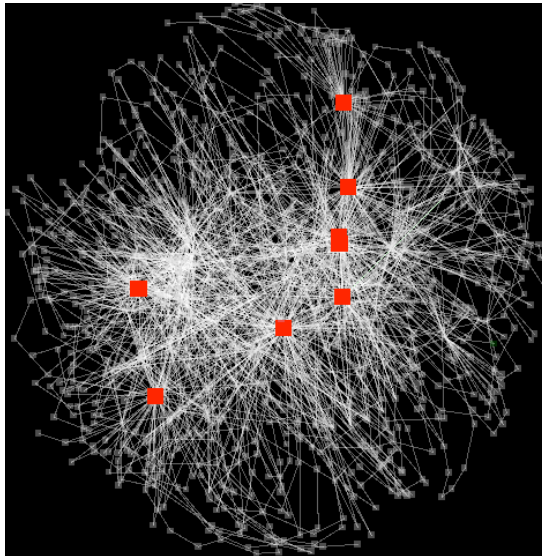
**Main Idea: MDL + 'syllables' :**

**star, clique, chain, bi-partite core**

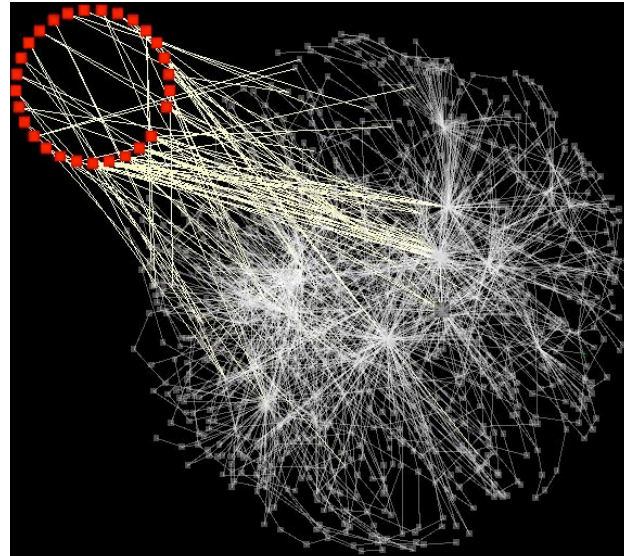


Koutra, Kang, Vreeken, et al, (subm.)

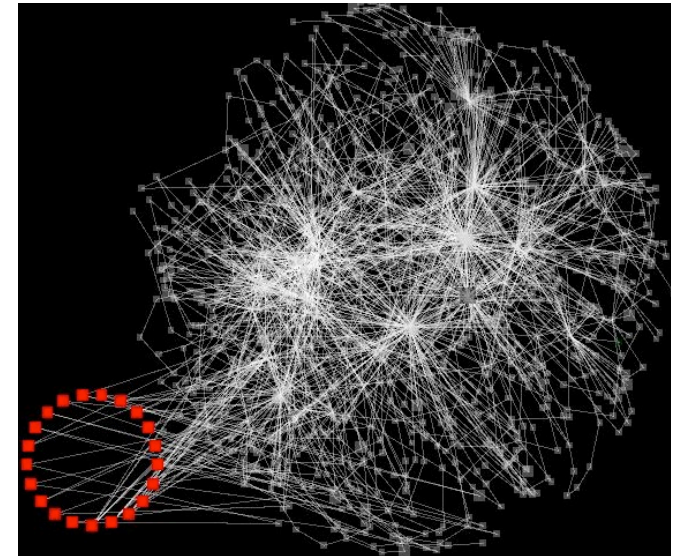
# Summarizing Wiki-controversy



*top-8 stars:  
admins, bots*



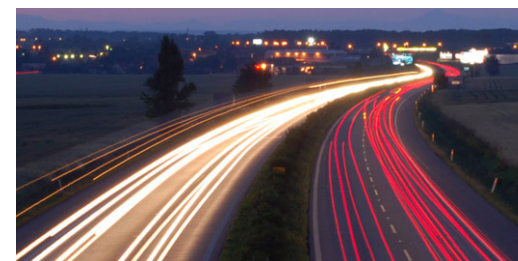
*top-1 and top-2 bipartite cores: edit wars.  
**Left:** warring factions ('Kiev' vs 'Kyev')  
**Right:** between vandals*





# Roadmap

- Introduction – Motivation
- Problem#1: Patterns in graphs
- Problem#2: Tools
- ➔ • Conclusions



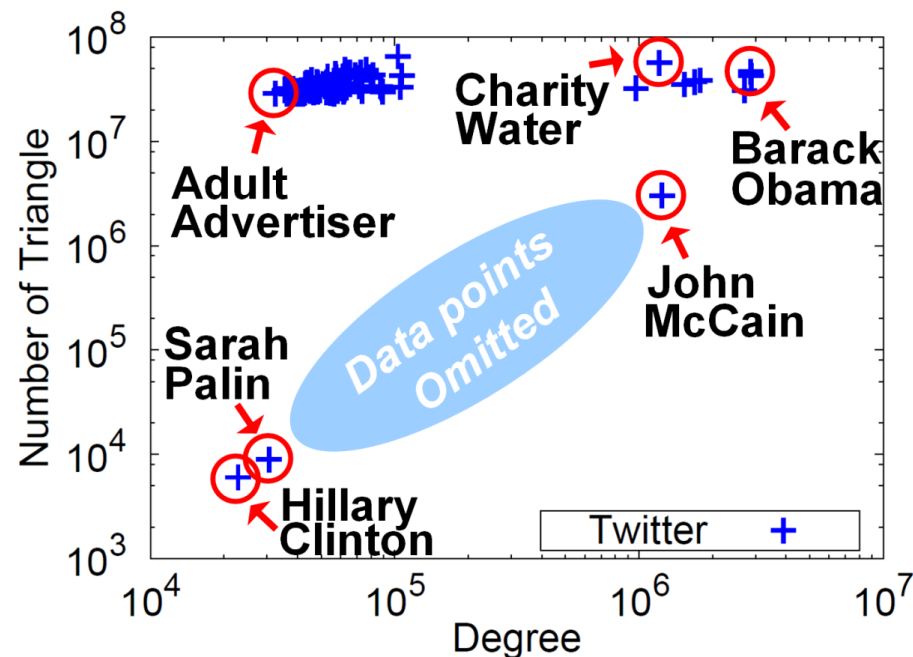
# OVERALL CONCLUSIONS – low level:

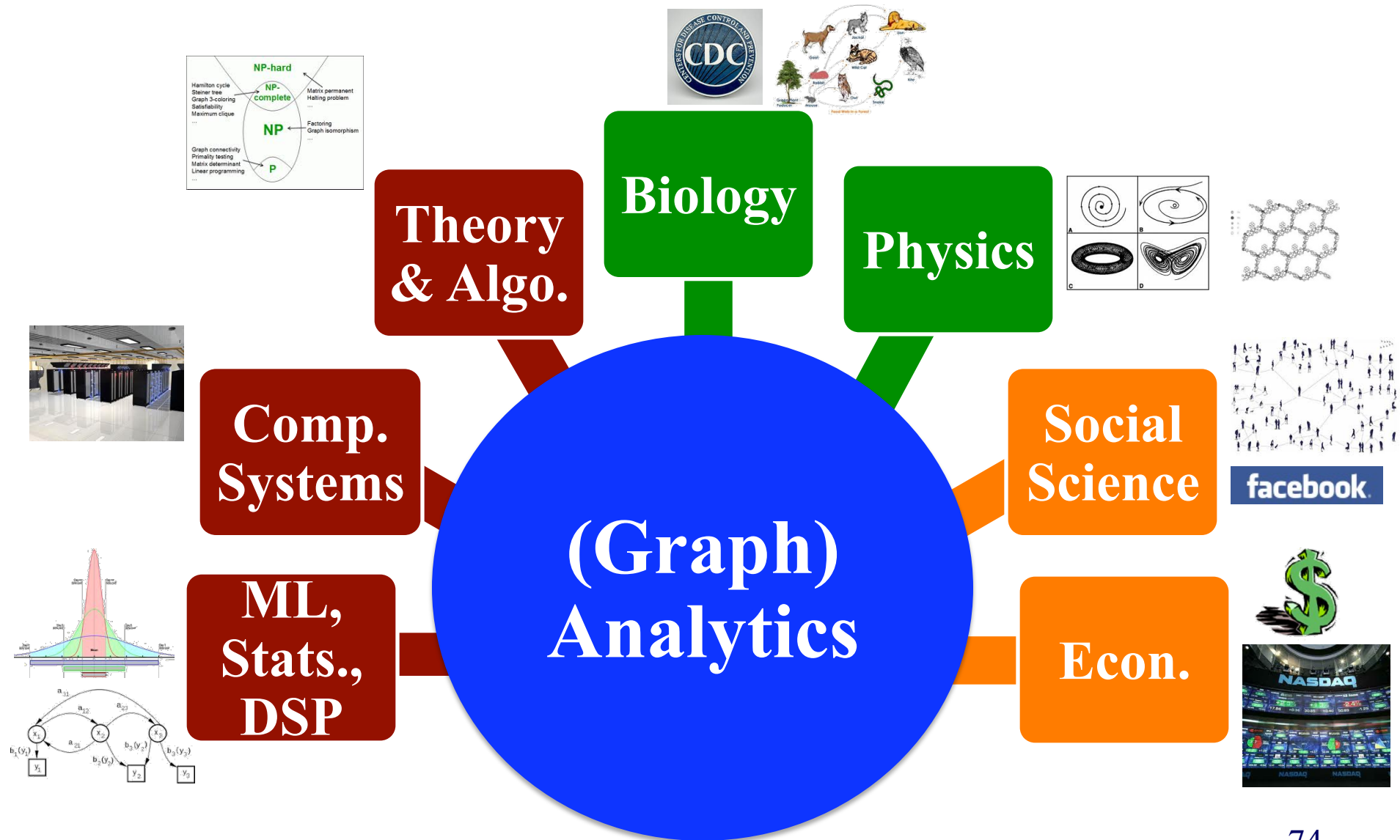
- Several new **patterns** (power laws, triangle-laws, etc)
- New **tools**:
  - belief propagation, gigaTensor, etc
- **Scalability**: PEGASUS / hadoop

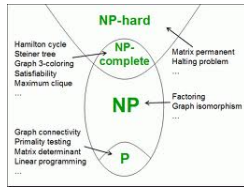


# OVERALL CONCLUSIONS – high level

- **BIG DATA:** Large datasets reveal patterns/outliers that are invisible otherwise







Theory  
& Algo

Biology



Computer

**Cross-disciplinarity:  
A must**

Physical  
Science



facebook

Analytics

Econ.

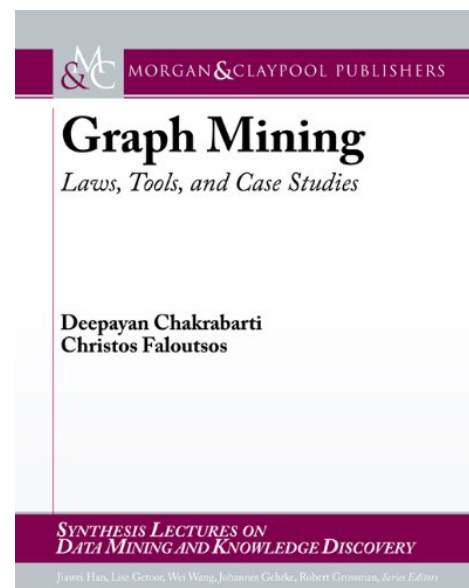


## References

- Leman Akoglu, Christos Faloutsos: *RTG: A Recursive Realistic Graph Generator Using Random Typing*. ECML/PKDD (1) 2009: 13-28
- Deepayan Chakrabarti, Christos Faloutsos: *Graph mining: Laws, generators, and algorithms*. ACM Comput. Surv. 38(1): (2006)

# References

- D. Chakrabarti, C. Faloutsos: *Graph Mining – Laws, Tools and Case Studies*, Morgan Claypool 2012
- <http://www.morganclaypool.com/doi/abs/10.2200/S00449ED1V01Y201209DMK006>



## References

- Deepayan Chakrabarti, Yang Wang, Chenxi Wang, Jure Leskovec, Christos Faloutsos: *Epidemic thresholds in real networks*. ACM Trans. Inf. Syst. Secur. 10(4): (2008)
- Deepayan Chakrabarti, Jure Leskovec, Christos Faloutsos, Samuel Madden, Carlos Guestrin, Michalis Faloutsos: *Information Survival Threshold in Sensor and P2P Networks*. INFOCOM 2007: 1316-1324

# References

- Christos Faloutsos, Tamara G. Kolda, Jimeng Sun: *Mining large graphs and streams using matrix and tensor tools*. Tutorial, SIGMOD Conference 2007: 1174

## References

- T. G. Kolda and J. Sun. *Scalable Tensor Decompositions for Multi-aspect Data Mining*. In: ICDM 2008, pp. 363-372, December 2008.



# References

- Jure Leskovec, Jon Kleinberg and Christos Faloutsos  
*Graphs over Time: Densification Laws, Shrinking Diameters and Possible Explanations*, KDD 2005  
(Best Research paper award).
- Jure Leskovec, Deepayan Chakrabarti, Jon M. Kleinberg, Christos Faloutsos: *Realistic, Mathematically Tractable Graph Generation and Evolution, Using Kronecker Multiplication*. PKDD 2005: 133-145

## References

- Yasuko Matsubara, Yasushi Sakurai, B. Aditya Prakash, Lei Li, Christos Faloutsos, "*Rise and Fall Patterns of Information Diffusion: Model and Implications*", KDD'12, pp. 6-14, Beijing, China, August 2012

# References

- Jimeng Sun, Yinglian Xie, Hui Zhang, Christos Faloutsos. *Less is More: Compact Matrix Decomposition for Large Sparse Graphs*, SDM, Minneapolis, Minnesota, Apr 2007.
- Jimeng Sun, Spiros Papadimitriou, Philip S. Yu, and Christos Faloutsos, *GraphScope: Parameter-free Mining of Large Time-evolving Graphs* ACM SIGKDD Conference, San Jose, CA, August 2007

## References

- Jimeng Sun, Dacheng Tao, Christos Faloutsos: *Beyond streams and graphs: dynamic tensor analysis*. KDD 2006: 374-383

## References

- Hanghang Tong, Christos Faloutsos, and Jia-Yu Pan, *Fast Random Walk with Restart and Its Applications*, ICDM 2006, Hong Kong.
- Hanghang Tong, Christos Faloutsos, *Center-Piece Subgraphs: Problem Definition and Fast Solutions*, KDD 2006, Philadelphia, PA

## References

- Hanghang Tong, Christos Faloutsos, Brian Gallagher, Tina Eliassi-Rad: *Fast best-effort pattern matching in large attributed graphs*. KDD 2007: 737-746
- (*Best paper award, CIKM'12*) Hanghang Tong, B. Aditya Prakash, Tina Eliassi-Rad, Michalis Faloutsos and Christos Faloutsos [Gelling, and Melting, Large Graphs by Edge Manipulation](#), Maui, Hawaii, USA, Oct. 2012.

## References

- Hanghang Tong, Spiros Papadimitriou, Christos Faloutsos, Philip S. Yu, Tina Eliassi-Rad: Gateway finder in large graphs: problem definitions and fast solutions. *Inf. Retr.* 15(3-4): 391-411 (2012)

# Project info & 'thanks'

[www.cs.cmu.edu/~pegasus](http://www.cs.cmu.edu/~pegasus)



Thanks to: NSF IIS-0705359, IIS-0534205,  
CTA-INARC; Yahoo (M45), LLNL, IBM, SPRINT,  
Google, INTEL, HP, iLab



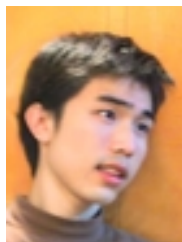
# Cast



Akoglu,  
Leman



Beutel,  
Alex



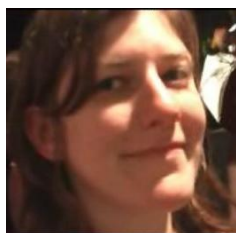
Chau,  
Polo



Kang, U



Koutra,  
Danai



McGlohon,  
Mary



Prakash,  
Aditya



Papalexakis,  
Vagelis



Tong,  
Hanghang

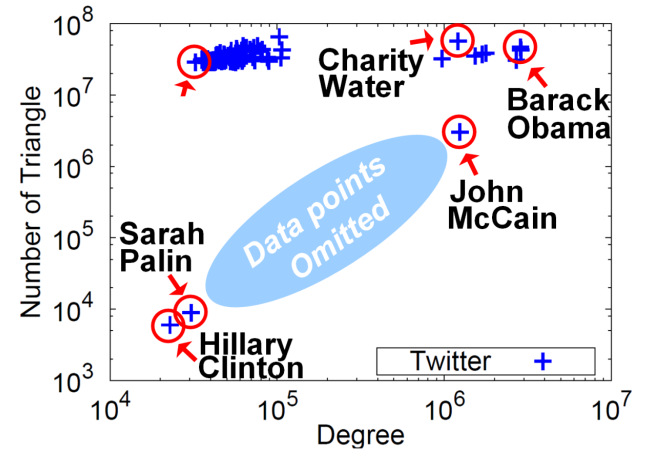
# Take-home message



Tera/Peta-byte  
data



Analytics



Insights,  
outliers

Big data reveal **insights** that would be invisible otherwise (even to **experts**)