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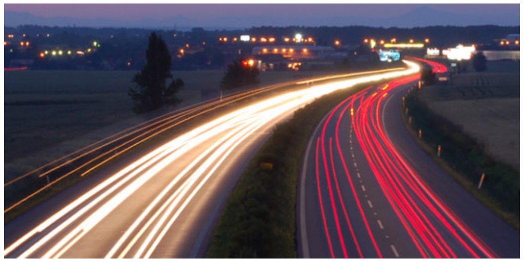
Large Graph Mining - Patterns, Explanations and Cascade Analysis

Christos Faloutsos

CMU

Roadmap

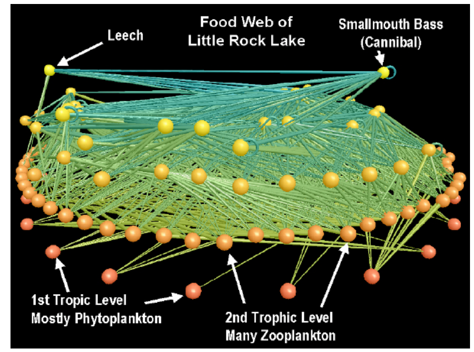


- Introduction – Motivation
  - Why study (big) graphs?
- Part#1: Patterns in graphs
- Part#2: Cascade analysis
- Conclusions

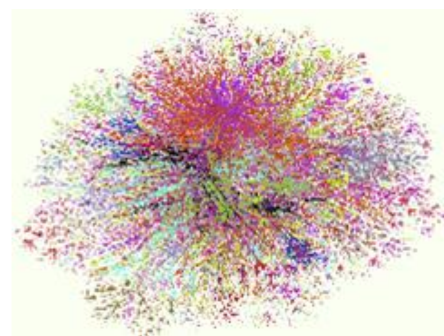
Graphs - why should we care?

>$10B revenue

>0.5B users



Food Web
[Martinez '91]



Internet Map
[lumeta.com]

Graphs - why should we care?

- web-log ('blog') news propagation
- computer network security: email/IP traffic and anomaly detection
- Recommendation systems
- ....

- Many-to-many db relationship -> graph

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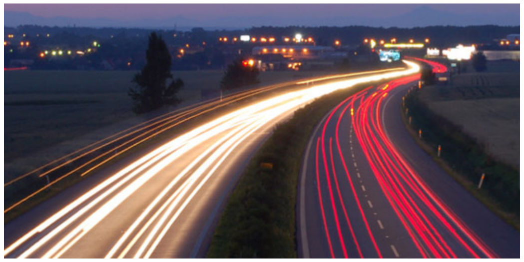
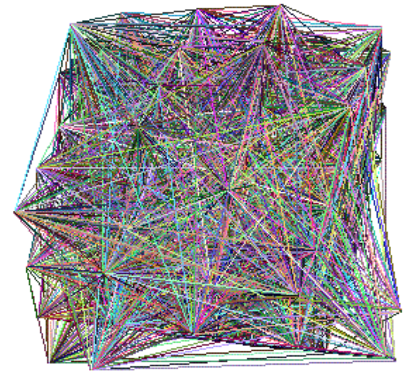
Roadmap



- Introduction – Motivation
- ➡ Part#1: Patterns in graphs
  - Static graphs
  - Time-evolving graphs
  - Why so many power-laws?
- Part#2: Cascade analysis
- Conclusions

Part 1: Patterns & Laws

Laws and patterns



- Q1: Are real graphs random?

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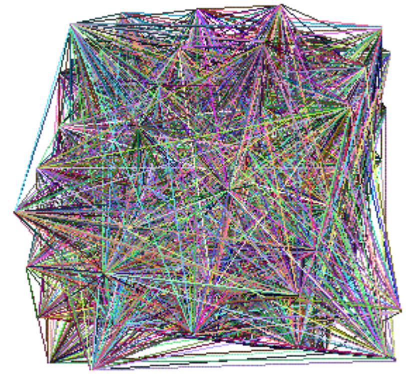


Laws and patterns

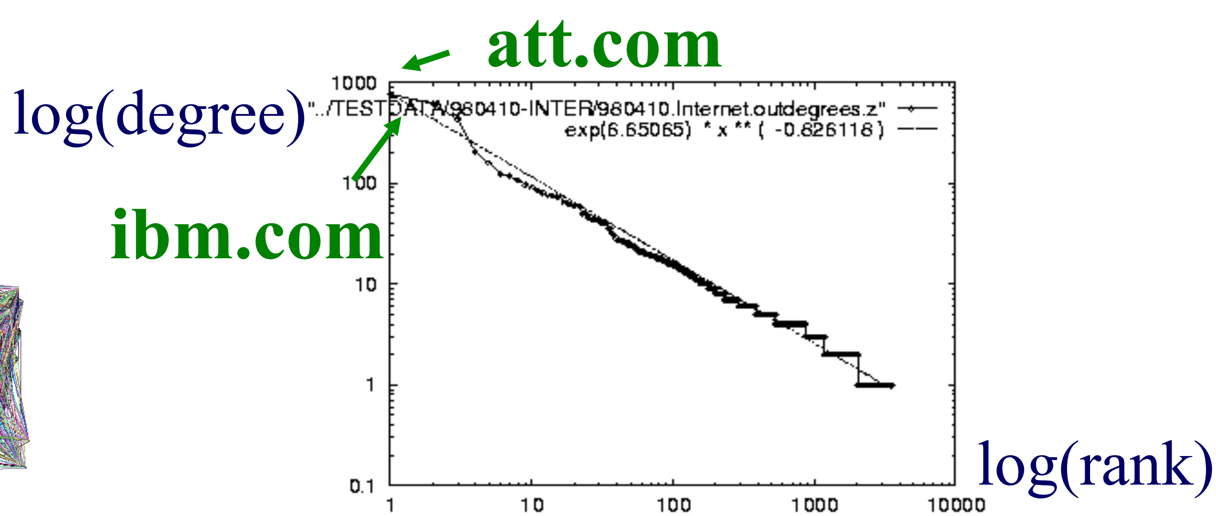
- Q1: Are real graphs random?
- A1: NO!!
  - Diameter
  - in- and out- degree distributions
  - other (surprising) patterns
- Q2: why ‘no good cuts’?
- A2: <self-similarity – stay tuned>

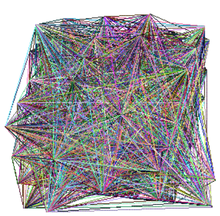
- So, let’s look at the data

Solution# S.1

- Power law in the degree distribution [Faloutsos x 3 SIGCOMM99; + Siganos]

internet domains





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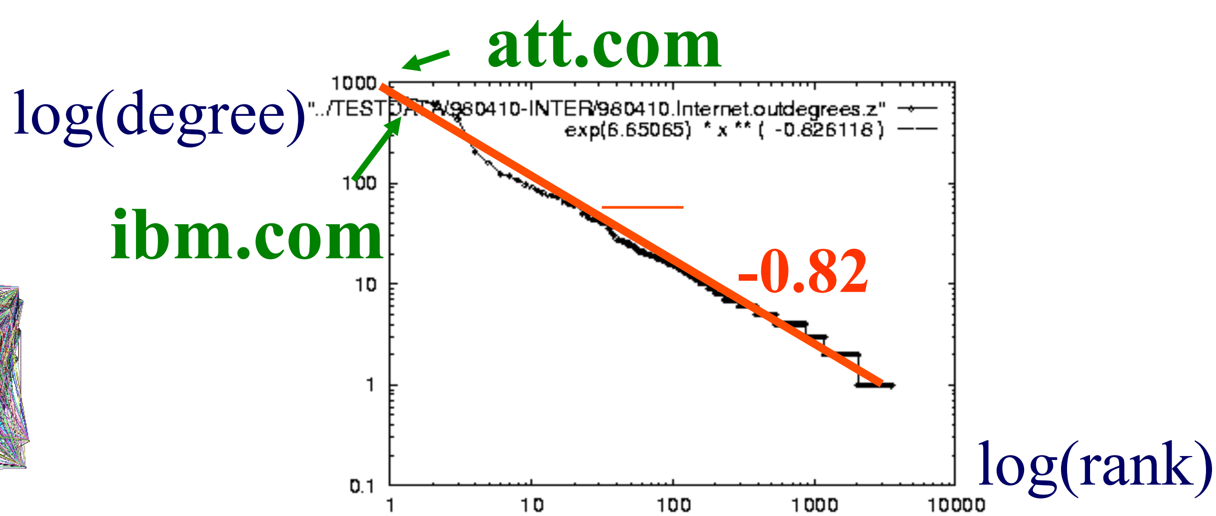

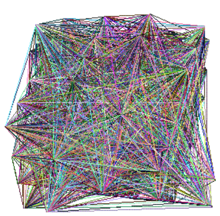

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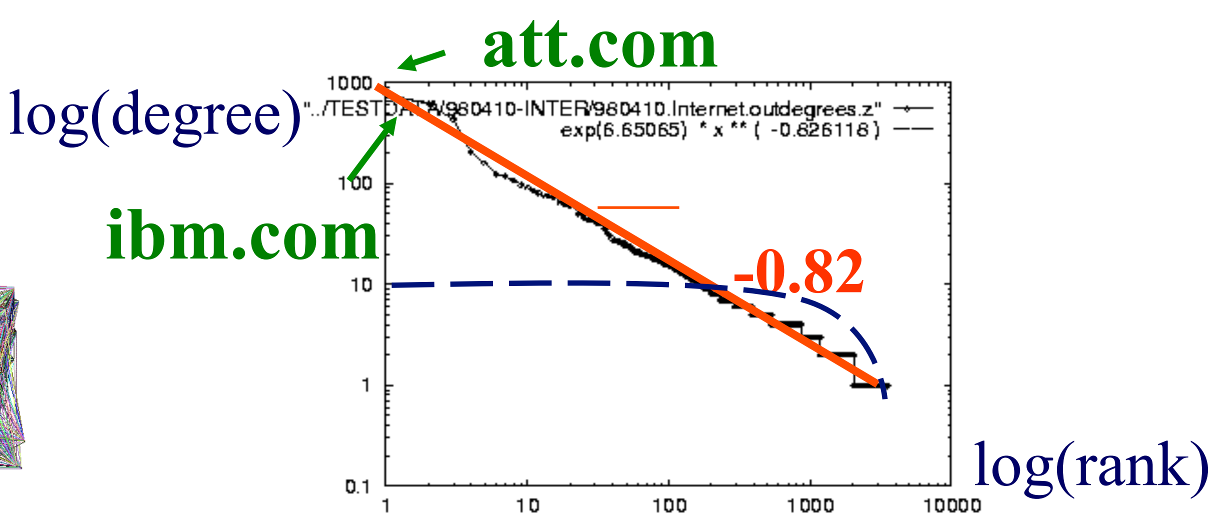
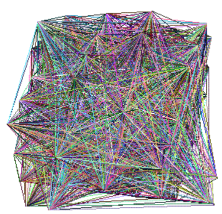
Solution# S.1

- Q: So what?

internet domains

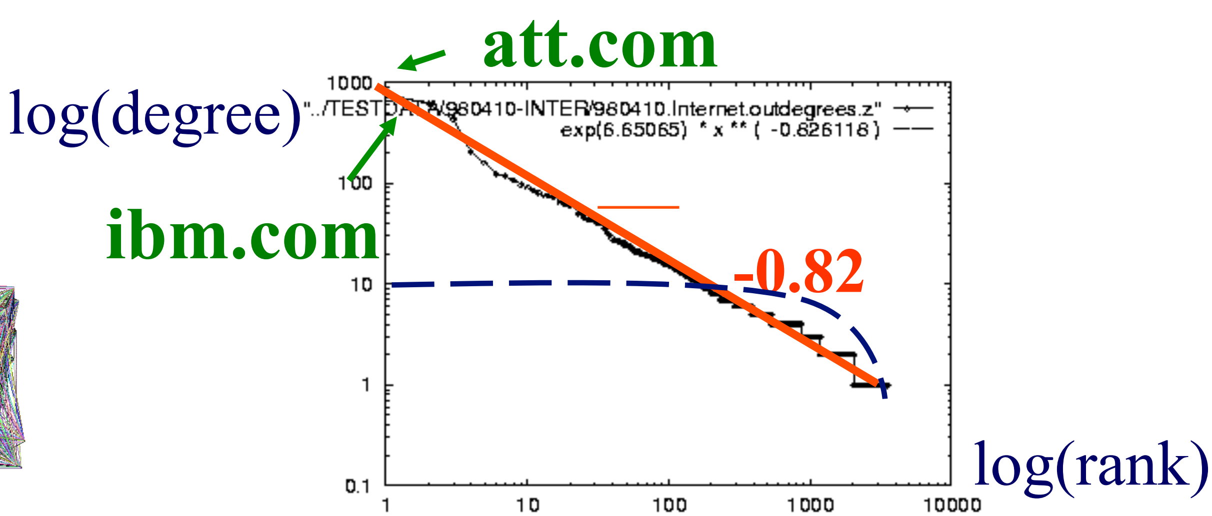


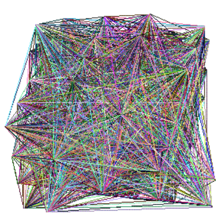


Solution# S.1

- Q: So what?
- A1: # of two-step-away pairs: = friends of friends (F.O.F.)

internet domains



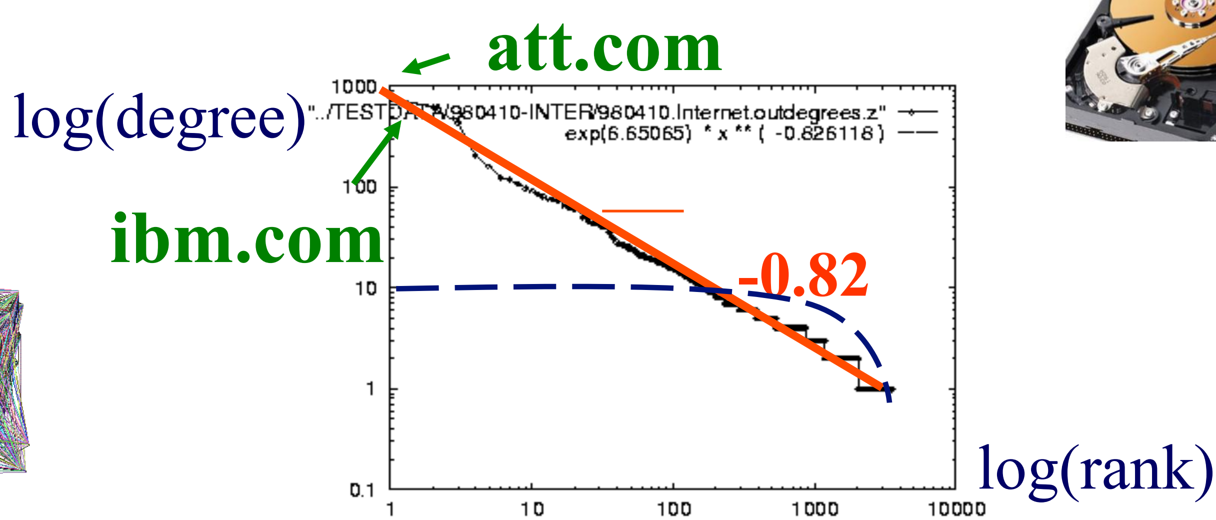


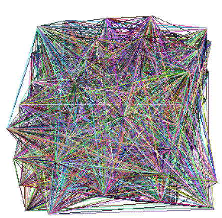
Solution# S.1

- Q: So what?
- A1: # of two-step-away pairs: 100^2 * N= 10 Trillion

= friends of friends (F.O.F.)

internet domains





Solution# S.1

- Q: So what?
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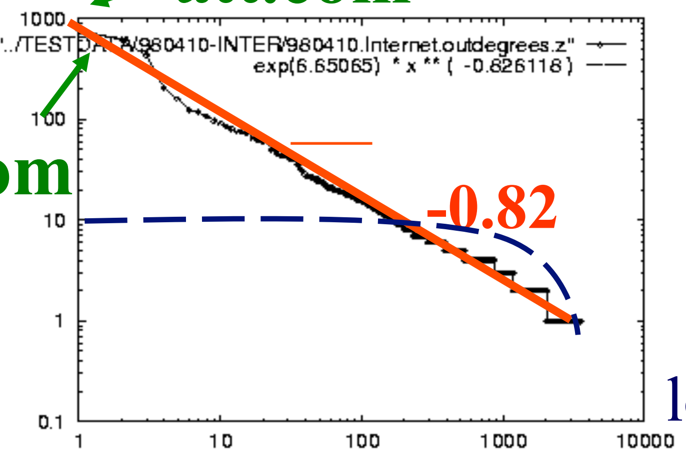
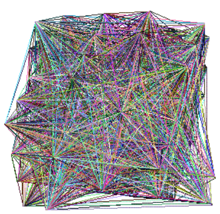




Gaussian trap

Solution# S.1

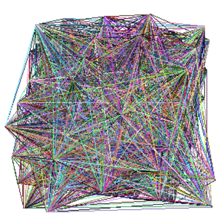
- Q: So what?
- A1: # of two-step-away pairs: O(d_max ^2) ~ 10M^2

= friends of friends (F.O.F.)

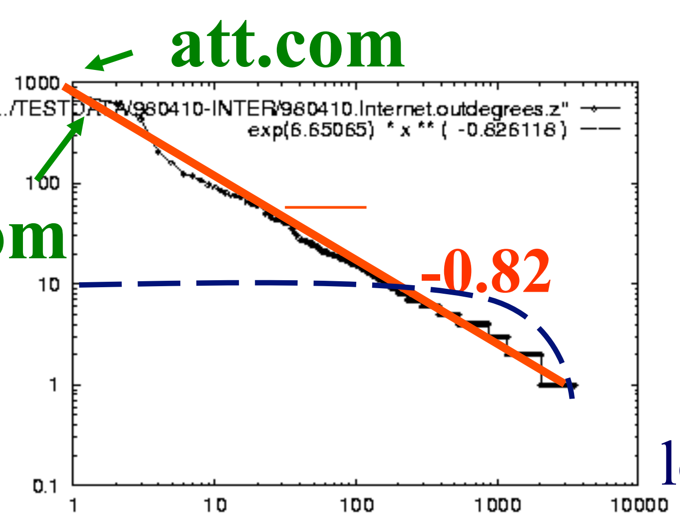
⇩

~0.8PB -> a data center(!)

internet domains



log(degree)



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Gaussian trap

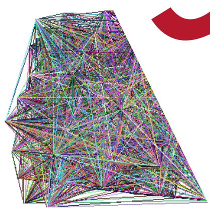
Solution# S.1

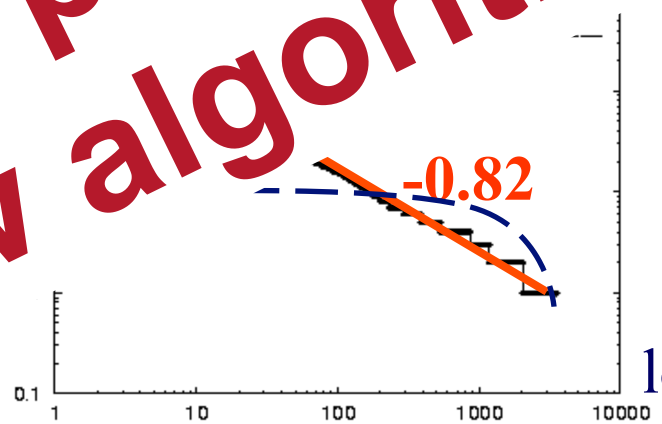
- Q: So what?
- A1: # of two-step-aw... ?) ~ 10M^2

inte...

~0.8PB ->
a data center(!)

Such patterns ->
New algorithms

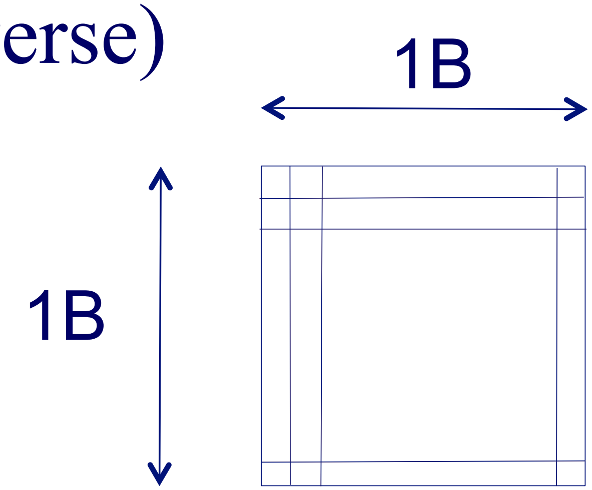




Observation – big-data:

- O(N^2) algorithms are ~intractable - N=1B

- N^2 seconds = 31B years (>2x age of universe)



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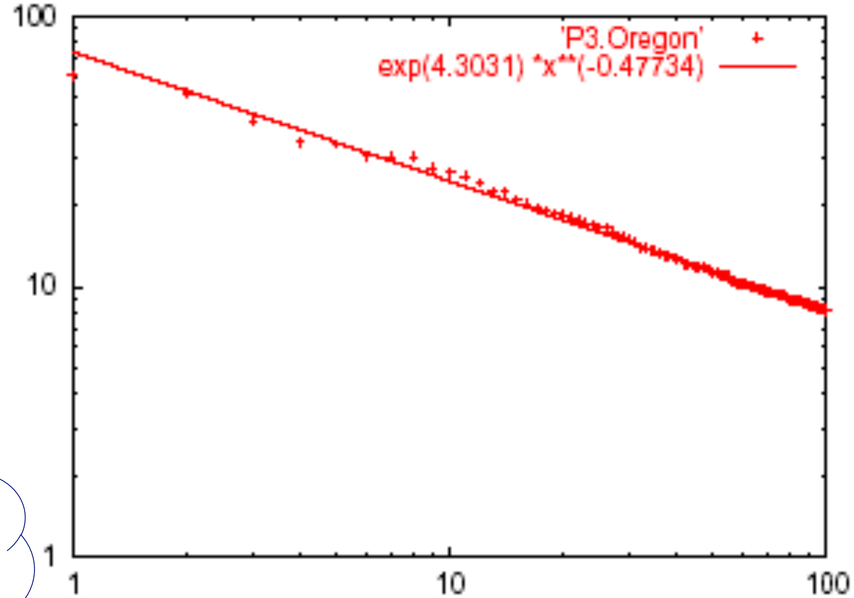
Solution# S.2: Eigen Exponent *E*

Eigenvalue



Rank of decreasing eigenvalue

Exponent = slope

E = -0.48

May 2001

$$A\, x = \lambda\, x$$

- A2: power law in the eigenvalues of the adjacency matrix

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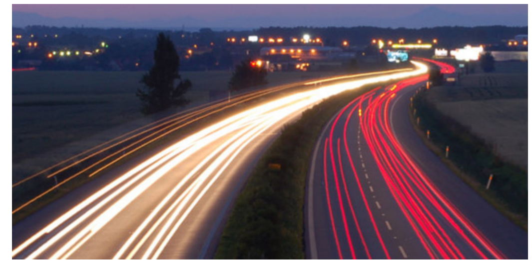
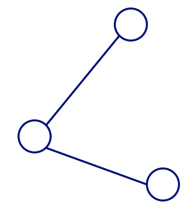
Roadmap



- Introduction – Motivation
- Problem#1: Patterns in graphs
  - Static graphs
    - degree, diameter, eigen,
    - ➡ Triangles
  - Time evolving graphs
- Problem#2: Tools

Solution# S.3: Triangle 'Laws'



- Real social networks have a lot of triangles

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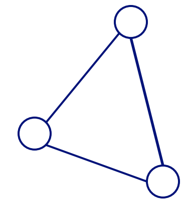
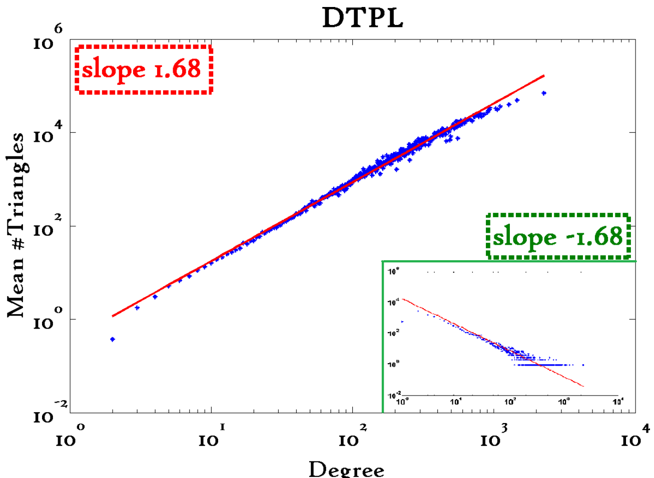
Solution# S.3: Triangle 'Laws'



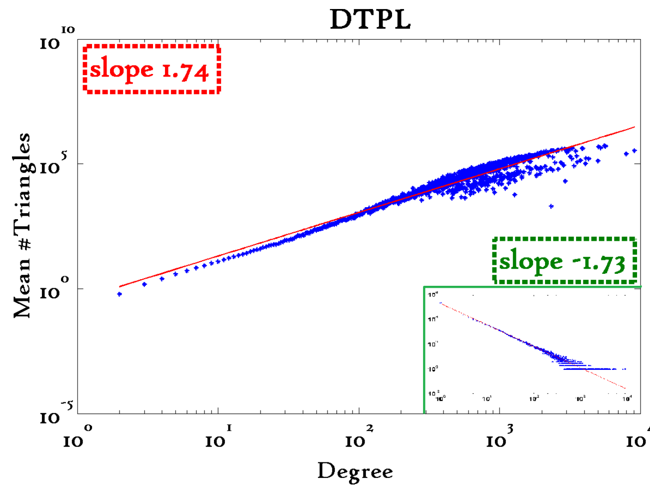
- Real social networks have a lot of triangles
  - Friends of friends are friends
- Any patterns?
  - 2x the friends, 2x the triangles ?

Triangle Law: #S.3
[Tsourakakis ICDM 2008]

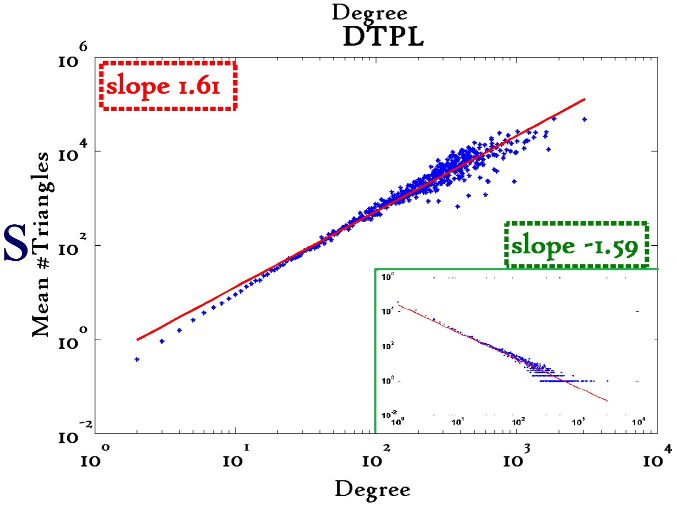
Reuters

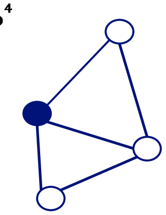


SN



Epinions





X-axis: degree

Y-axis: mean # triangles

n friends -> ~$n^{1.6}$ triangles

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details

Triangle Law: Computations
[Tsourakakis ICDM 2008]

But: triangles are expensive to compute
(3-way join; several approx. algos) – $O(d_{max}^2)$

Q: Can we do that quickly?

A:

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details

Triangle Law: Computations

[Tsourakakis ICDM 2008]

But: triangles are expensive to compute

(3-way join; several approx. algos) – $O(d_{max}{}^2)$

Q: Can we do that quickly?

A: Yes!

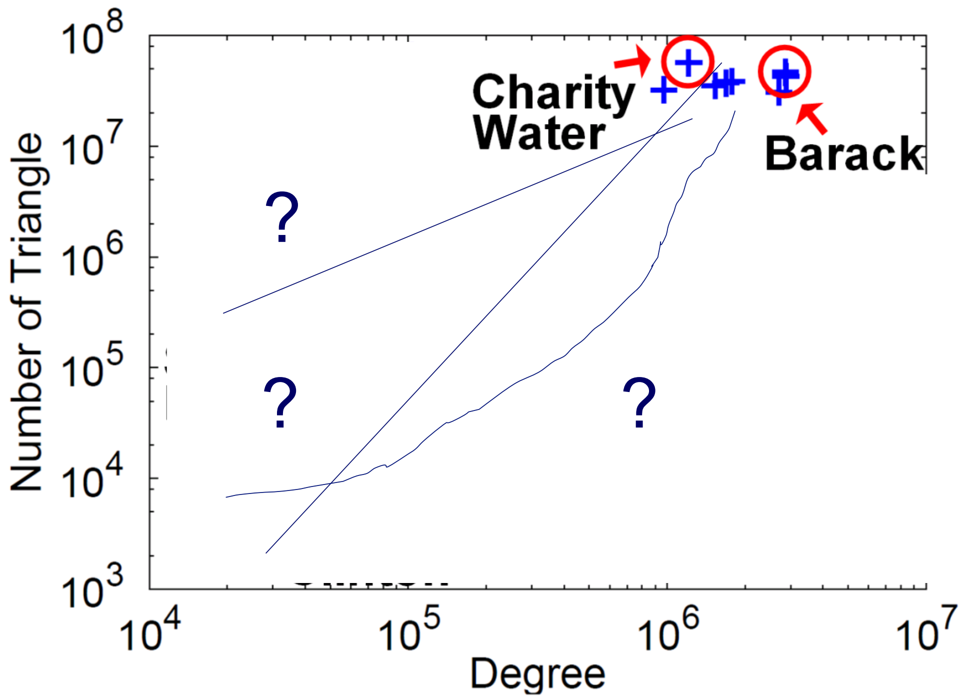
#triangles = 1/6 Sum (λ_i^3)

(and, because of skewness (S2) ,

we only need the top few eigenvalues! - O(E)

A x = λ **x**

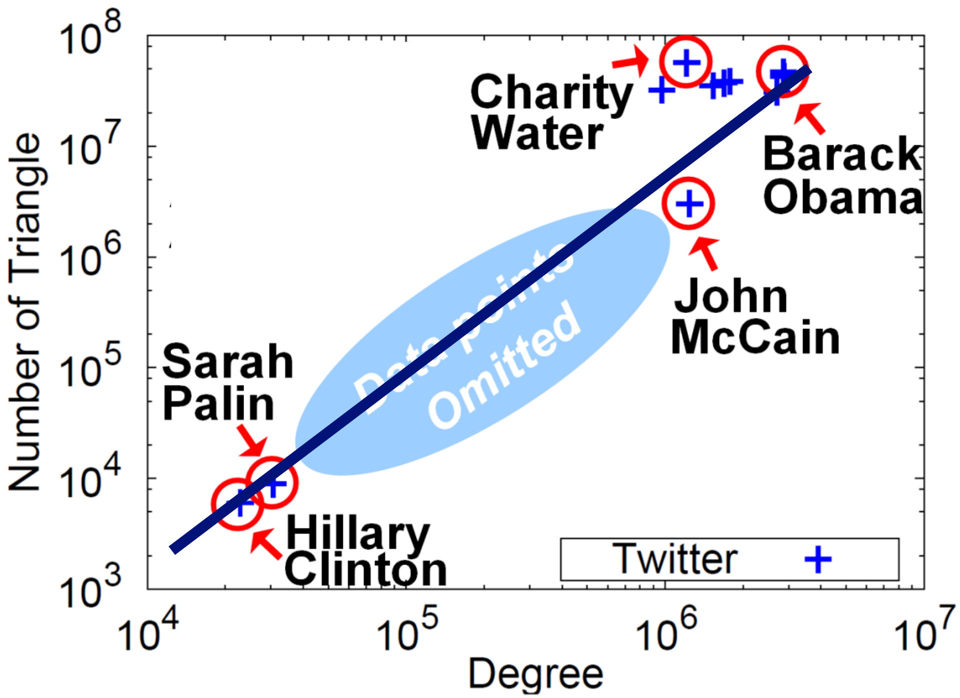
Triangle counting for large graphs?



Anomalous nodes in Twitter(~ 3 billion edges)

[U Kang, Brendan Meeder, +, PAKDD'11]

Triangle counting for large graphs?



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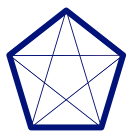

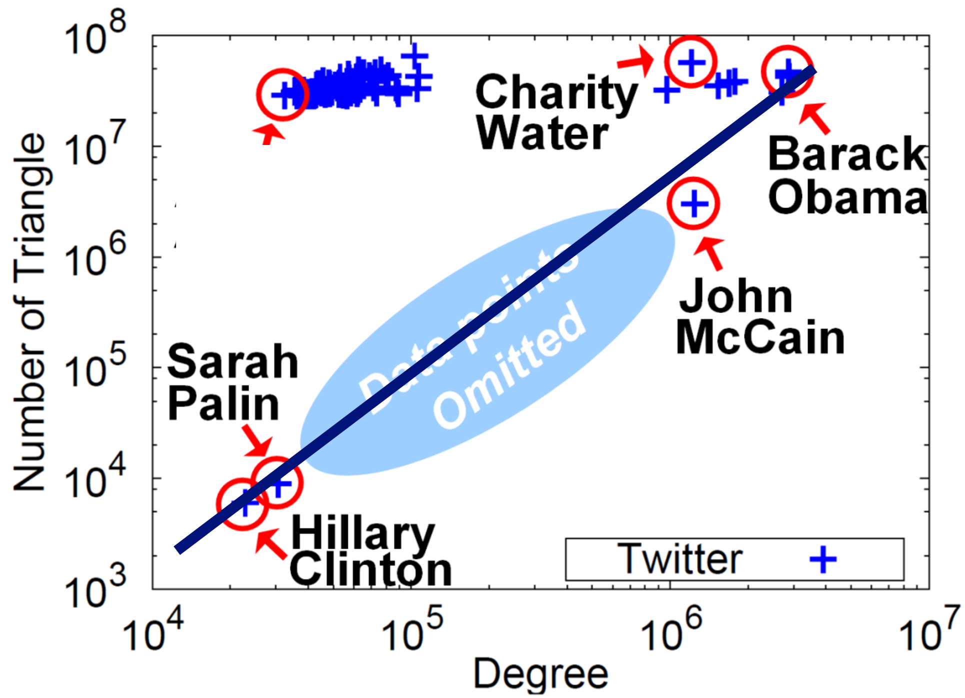

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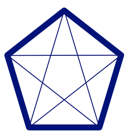

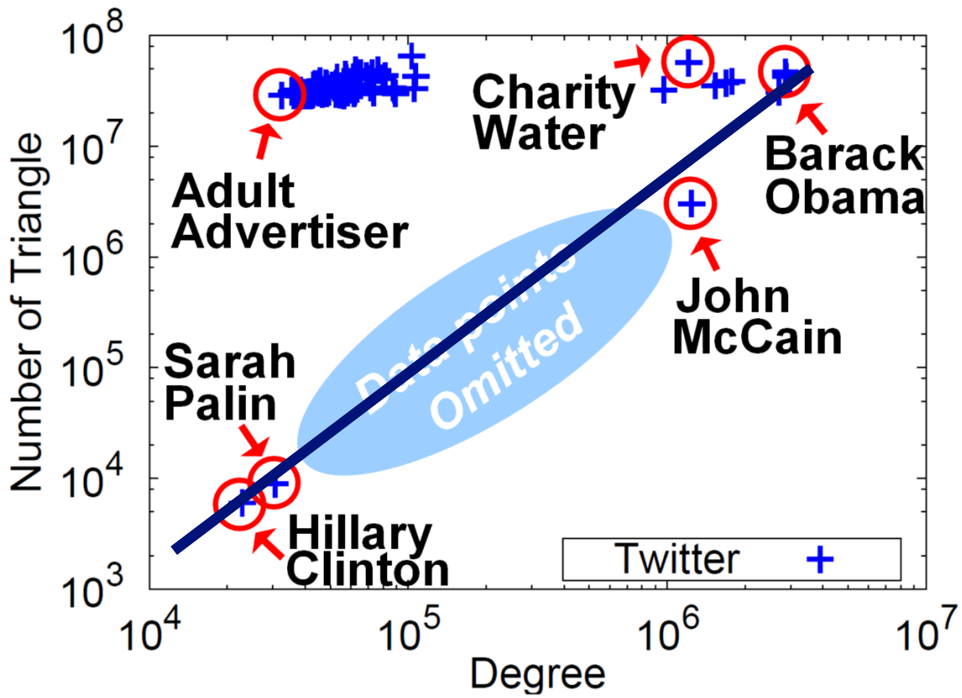

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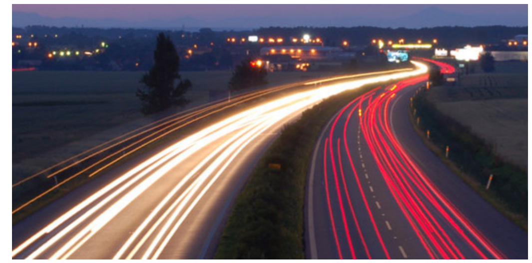
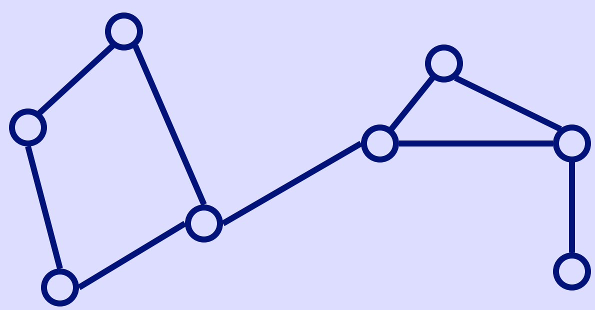
Roadmap



- Introduction – Motivation
- Part#1: Patterns in graphs
  - Static graphs
    - Power law degrees; eigenvalues; triangles
    - ➡ Anti-pattern: NO good cuts!
  - Time-evolving graphs
- ….
- Conclusions

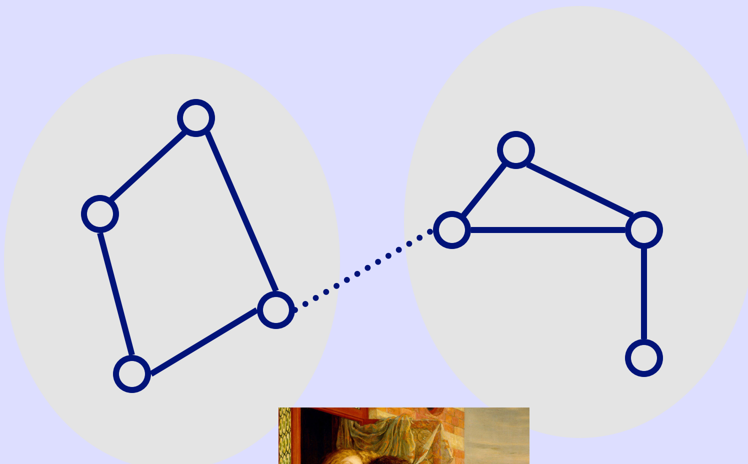
Background: Graph cut problem

- Given a graph, and *k*
- Break it into *k* (disjoint) communities

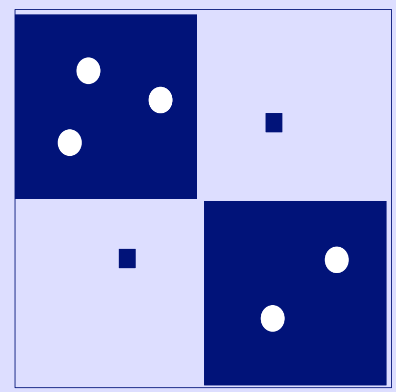


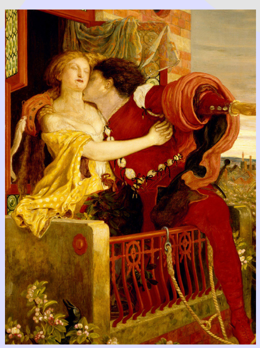
Graph cut problem

- Given a graph, and *k*
- Break it into *k* (disjoint) communities
- (assume: block diagonal = ‘cavemen’ graph)



$k = 2$





Many algo's for graph partitioning


- METIS [Karypis, Kumar +]
- 2nd eigenvector of Laplacian
- Modularity-based [Girwan+Newman]
- Max flow [Flake+]
- …
- …
- …

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Strange behavior of min cuts

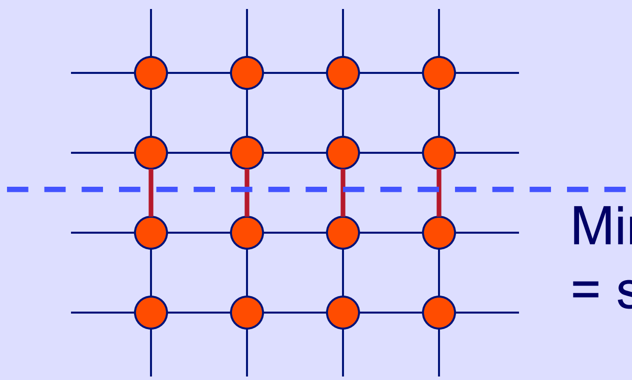
- Subtle details: next
  - Preliminaries: min-cut plots of ‘usual’ graphs

NetMine: New Mining Tools for Large Graphs, by D. Chakrabarti, Y. Zhan, D. Blandford, C. Faloutsos and G. Blelloch, in the SDM 2004 Workshop on Link Analysis, Counter-terrorism and Privacy

Statistical Properties of Community Structure in Large Social and Information Networks, J. Leskovec, K. Lang, A. Dasgupta, M. Mahoney. WWW 2008.

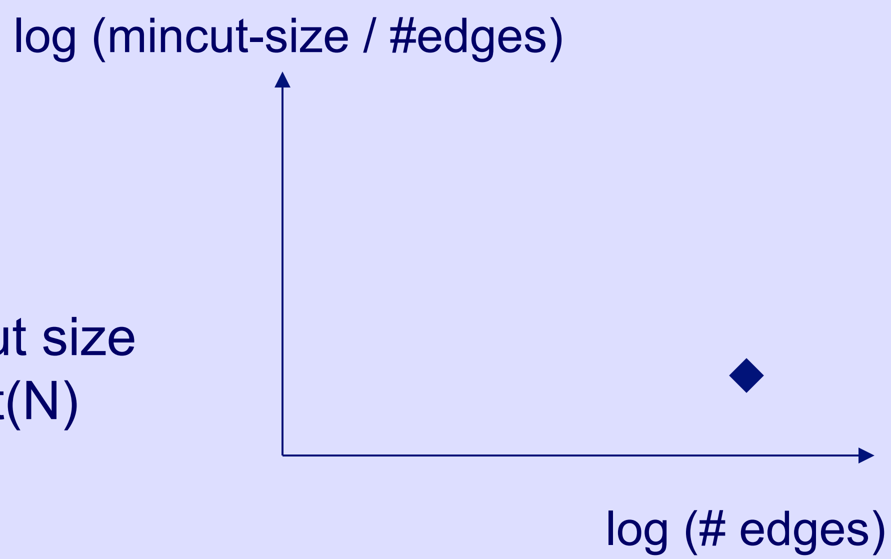
"Min-cut" plot

- Do min-cuts recursively.



Mincut size
= sqrt(N)

N nodes



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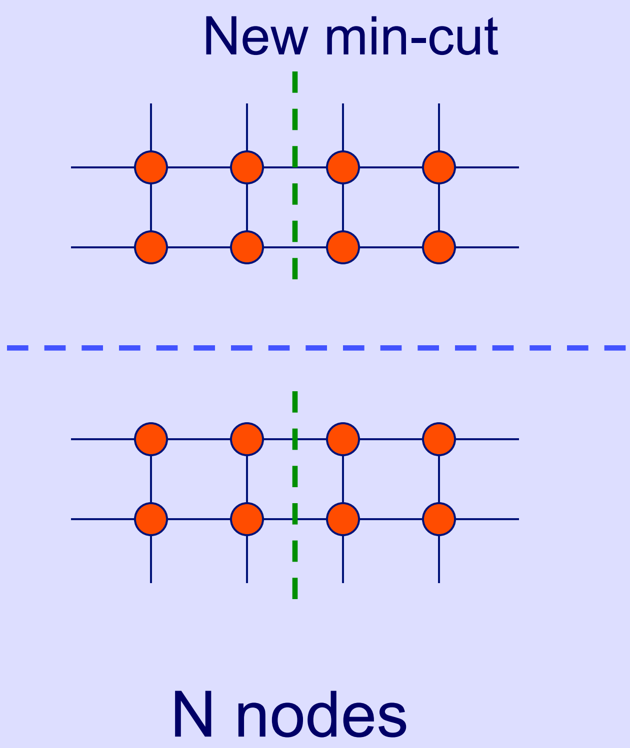

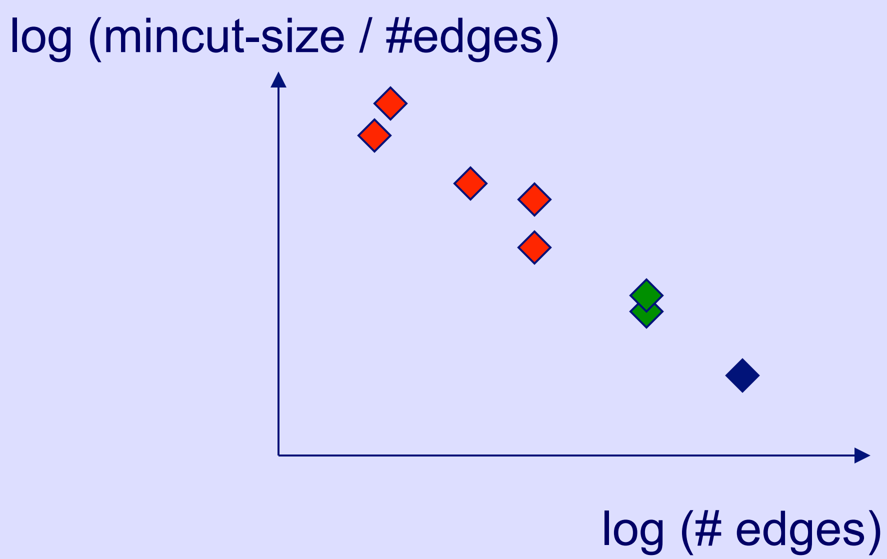

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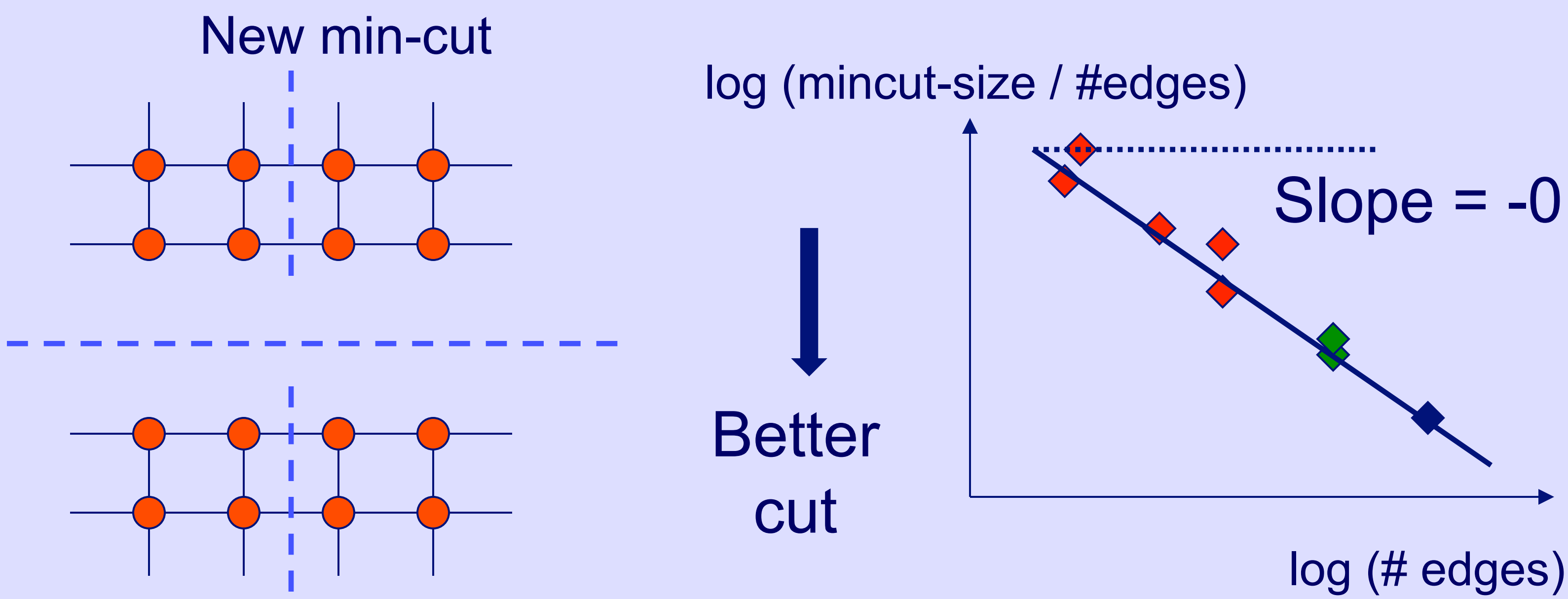
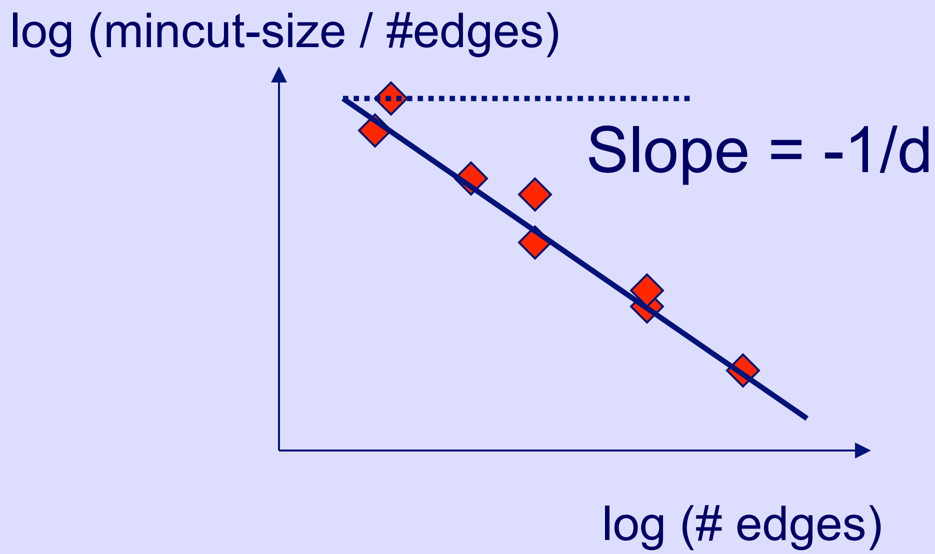
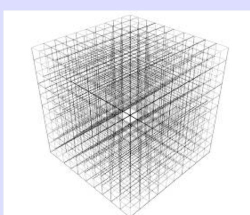


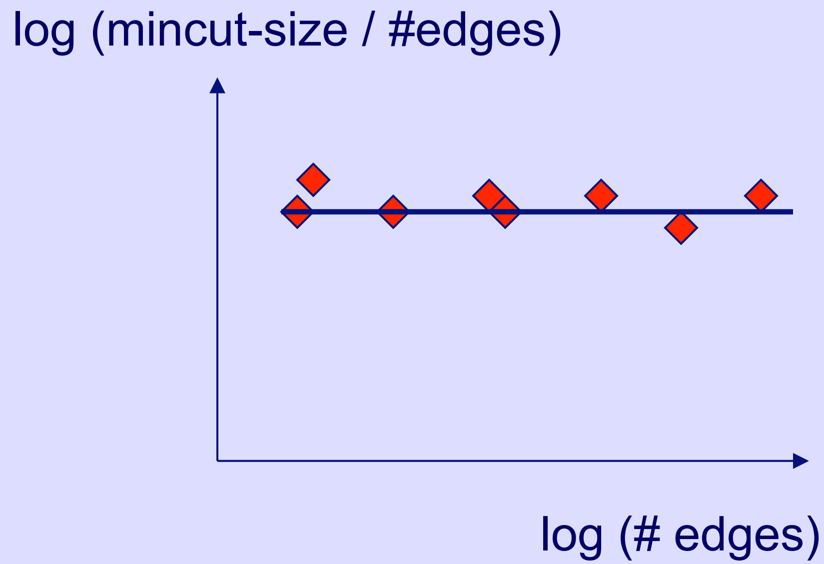
N nodes

“Min-cut” plot



For a d-dimensional grid, the slope is -1/d





For a random graph

(and clique),

the slope is 0

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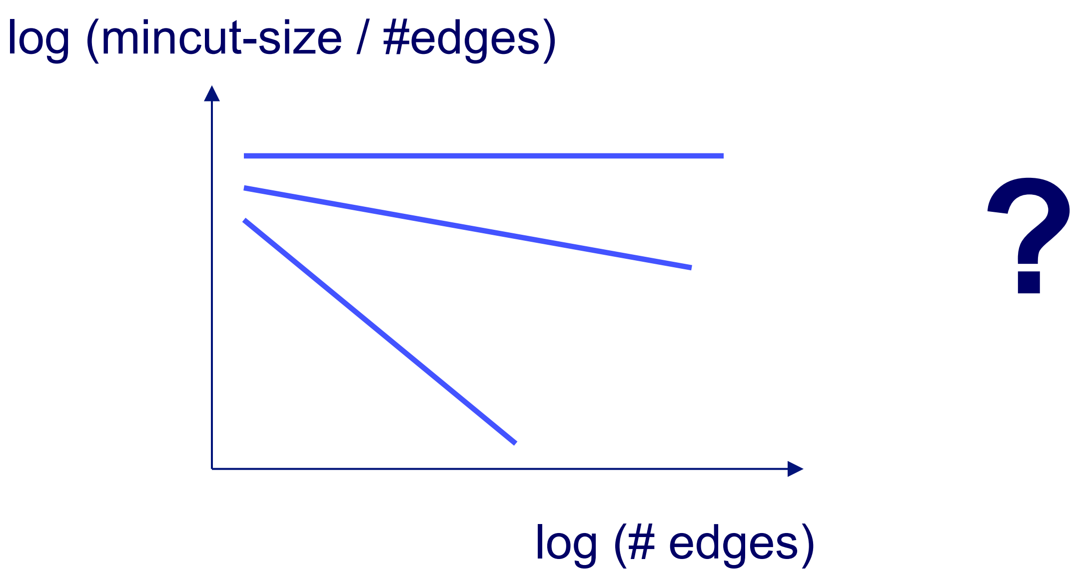
Experiments

- Datasets:
  - Google Web Graph: 916,428 nodes and 5,105,039 edges
  - Lucent Router Graph: Undirected graph of network routers from www.isi.edu/scan/mercator/maps.html; 112,969 nodes and 181,639 edges
  - User → Website Clickstream Graph: 222,704 nodes and 952,580 edges

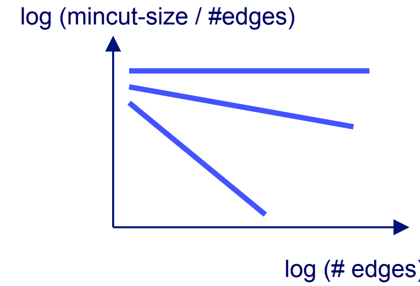
NetMine: New Mining Tools for Large Graphs, by D. Chakrabarti, Y. Zhan, D. Blandford, C. Faloutsos and G. Blelloch, in the SDM 2004 Workshop on Link Analysis, Counter-terrorism and Privacy

“Min-cut” plot

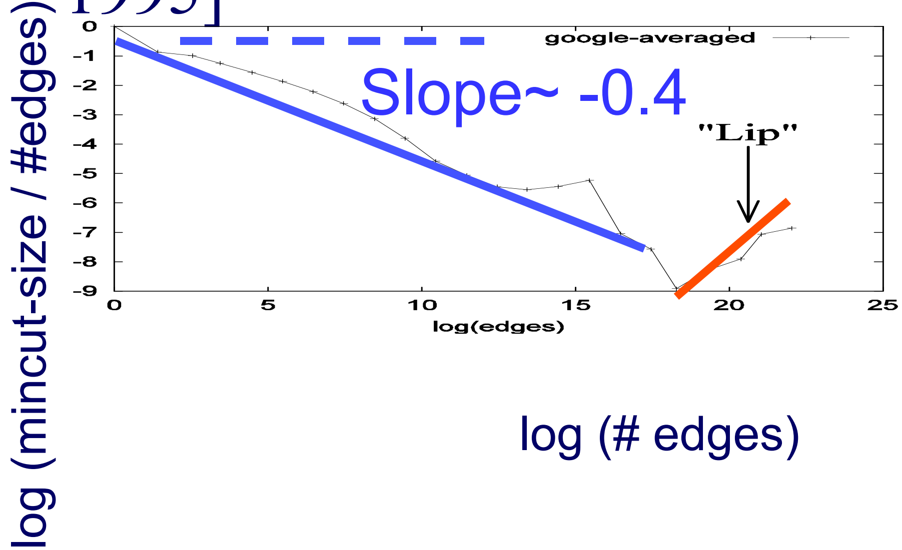
- What does it look like for a real-world graph?



Experiments



- Used the METIS algorithm [Karypis, Kumar, 1995]



- Google Web graph
- Values along the y-axis are averaged
- "lip" for large # edges
- Slope of -0.4, corresponds to a 2.5-dimensional grid!

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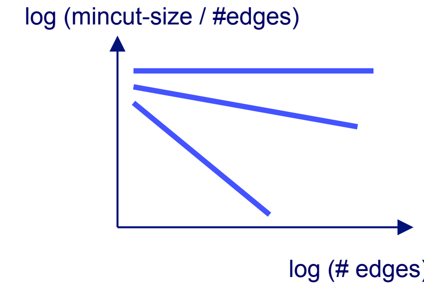
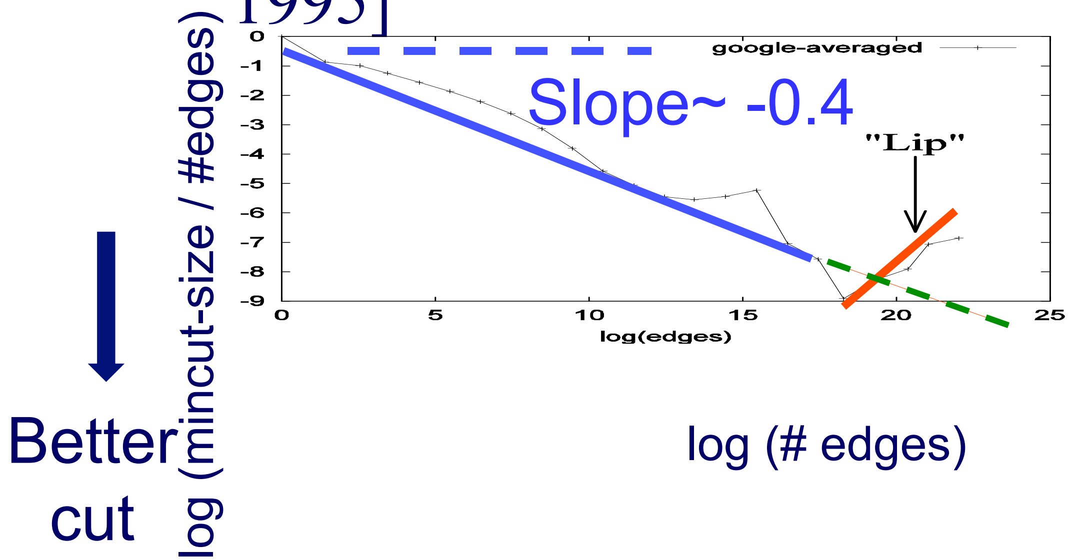


Experiments

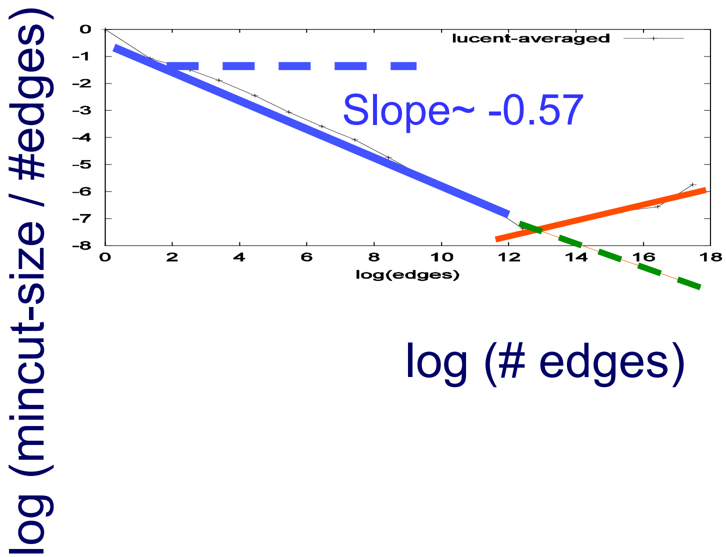
- Used the METIS algorithm [Karypis, Kumar, 1995]

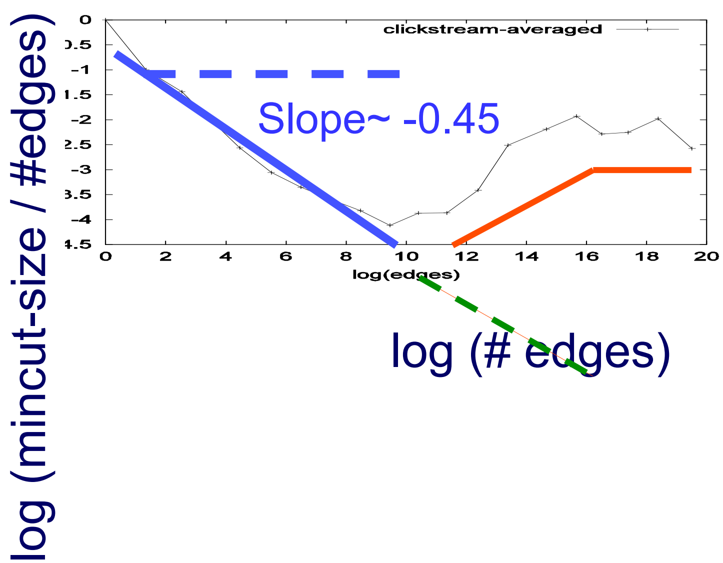


- Google Web graph
- Values along the y-axis are averaged
- "lip" for large # edges
- Slope of -0.4, corresponds to a 2.5-dimensional grid!

Experiments

- Same results for other graphs too…





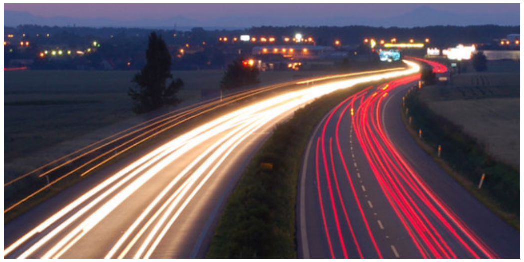
Lucent Router graph

Clickstream graph

Why no good cuts?

- Answer: self-similarity (few foils later)

Roadmap



- Introduction – Motivation
- Part#1: Patterns in graphs
  - Static graphs
  - ➡ Time-evolving graphs
  - Why so many power-laws?
- Part#2: Cascade analysis
- Conclusions

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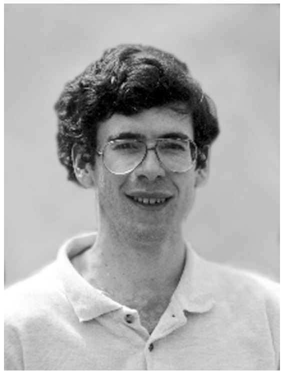
Problem: Time evolution

- with Jure Leskovec (CMU -> Stanford)

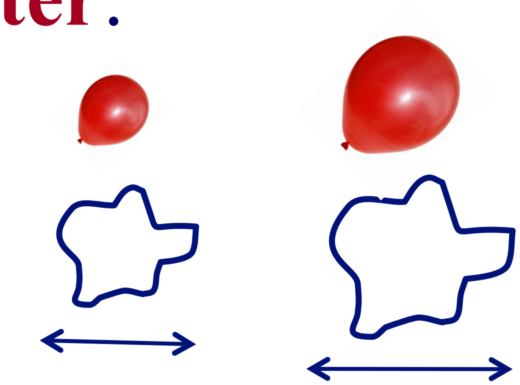
- and Jon Kleinberg (Cornell – sabb. @ CMU)



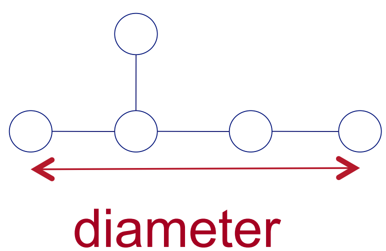
Jure Leskovec, Jon Kleinberg and Christos Faloutsos: *Graphs over Time: Densification Laws, Shrinking Diameters and Possible Explanations*, KDD 2005

T.1 Evolution of the Diameter

- Prior work on Power Law graphs hints at **slowly growing diameter**:
  - [diameter ~ O($N^{1/3}$)]
  - diameter ~ O(log N)
  - diameter ~ O(log log N)



- What is happening in real data?



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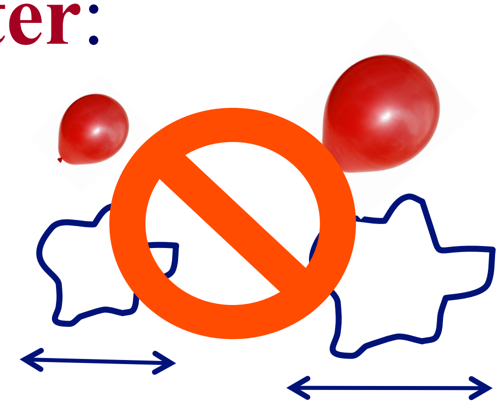
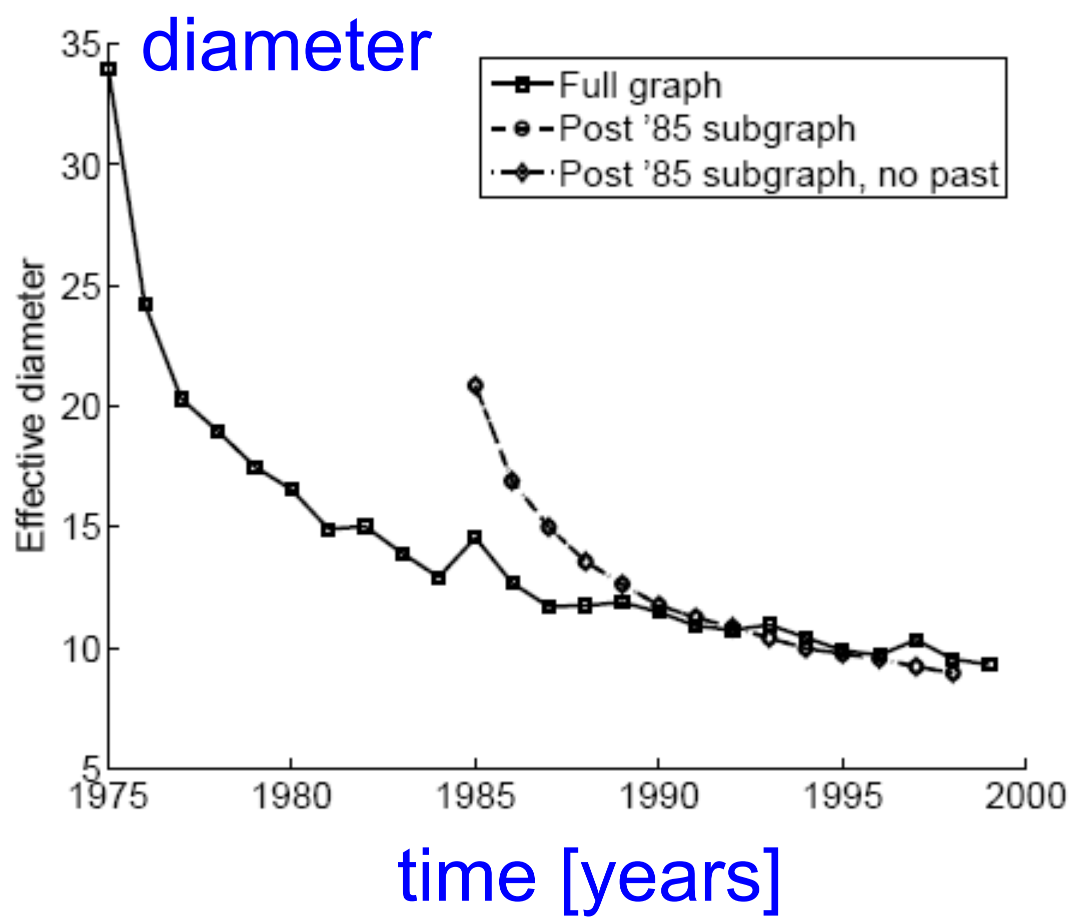


- What is happening in real data?
- Diameter **shrinks** over time

T.1 Diameter – “Patents”

- Patent citation network
- 25 years of data
- @1999
  - 2.9 M nodes
  - 16.5 M edges



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T.2 Temporal Evolution of the Graphs

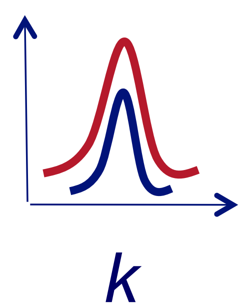
- N(t) … nodes at time t
- E(t) … edges at time t
- Suppose that

  N(t+1) = 2 * N(t)

- Q: what is your guess for

  E(t+1) =? 2 * E(t)

Say, *k* friends on average



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  - But obeying the ``Densification Power Law''

Gaussian trap

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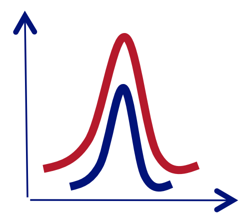

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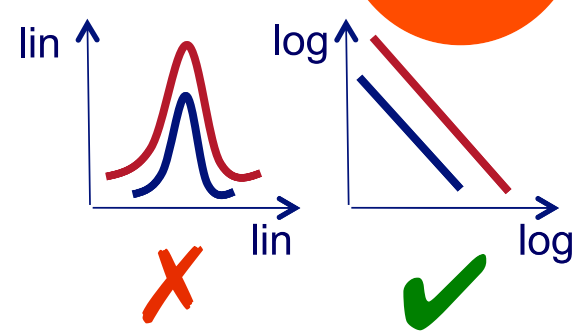
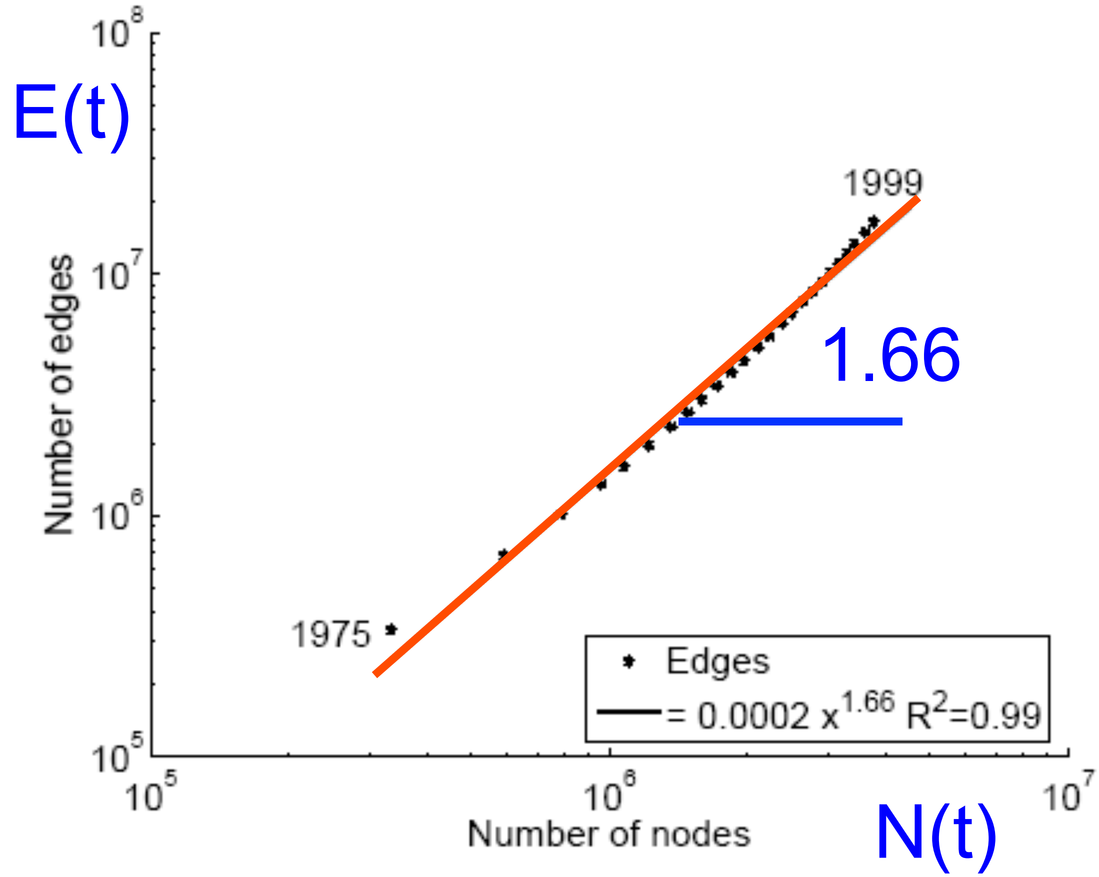


T.2 Densification – Patent Citations

- Citations among patents granted
- @1999
  - 2.9 M nodes
  - 16.5 M edges
- Each year is a datapoint



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MORE Graph Patterns

	Unweighted	Weighted
Static	**L01.** Power-law degree distribution [Faloutsos et al. `99, Kleinberg et al. `99, Chakrabarti et al. `04, Newman `04] **L02.** Triangle Power Law (TPL) [Tsourakakis `08] **L03.** Eigenvalue Power Law (EPL) [Siganos et al. `03] **L04.** Community structure [Flake et al. `02, Girvan and Newman `02]	**L10.** Snapshot Power Law (SPL) [McGlohon et al. `08]
Dynamic	**L05.** Densification Power Law (DPL) [Leskovec et al. `05] **L06.** Small and shrinking diameter [Albert and Barabási `99, Leskovec et al. `05] **L07.** Constant size 2$^{nd}$ and 3$^{rd}$ connected components [McGlohon et al. `08] **L08.** Principal Eigenvalue Power Law (λ_1PL) [Akoglu et al. `08] **L09.** Bursty/self-similar edge/weight additions [Gomez and Santonja `98, Gribble et al. `98, Crovella and	**L11.** Weight Power Law (WPL) [McGlohon et al. `08]

RTG: A Recursive Realistic Graph Generator using Random Typing Leman Akoglu and Christos Faloutsos. *PKDD*'09.

MORE Graph Patterns

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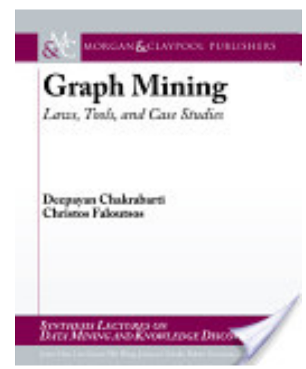
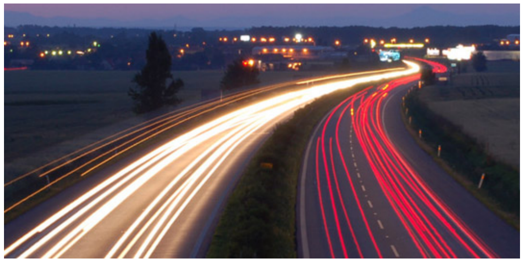
- Mary McGlohon, Leman Akoglu, Christos Faloutsos. *Statistical Properties of Social Networks.* in "Social Network Data Analytics" (Ed.: Charu Aggarwal)



- Deepayan Chakrabarti and Christos Faloutsos, *Graph Mining: Laws, Tools, and Case Studies* Oct. 2012, Morgan Claypool.

Roadmap



- Introduction – Motivation
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  - …
  - ➡ Why so many power-laws?
  - Why no 'good cuts'?
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- Conclusions

2 Questions, one answer

- Q1: why so many power laws
- Q2: why no ‘good cuts’?

possible

2 Questions, one answer

- Q1: why so many power laws
- Q2: why no 'good cuts'?
- A: Self-similarity = fractals = 'RMAT' ~ 'Kronecker graphs'

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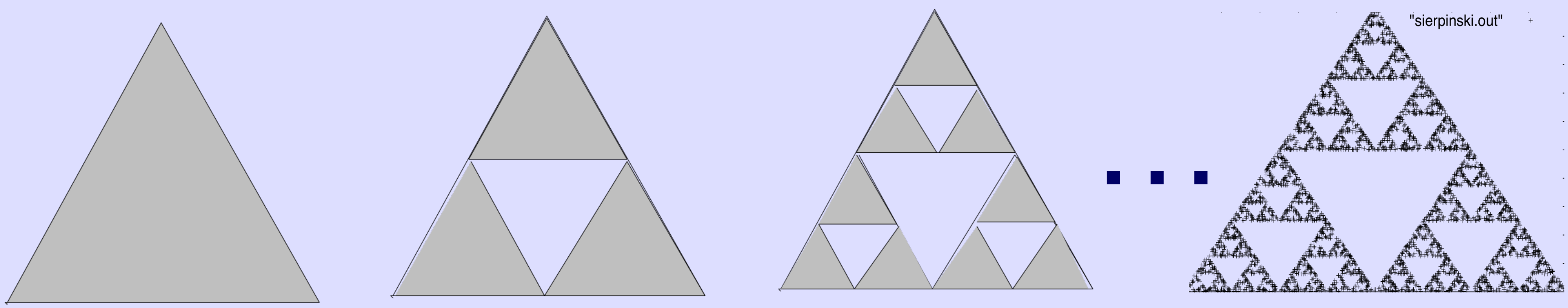
20'' intro to fractals

- Remove the middle triangle; repeat
- -> Sierpinski triangle
- (Bonus question - dimensionality?
  - >1 (inf. perimeter – $(4/3)^\infty$)
  - <2 (zero area – $(3/4)^\infty$)



20'' intro to fractals

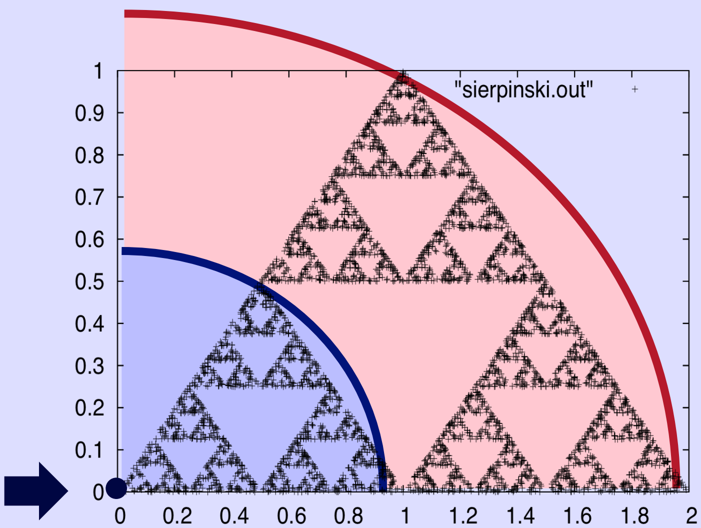
Self-similarity -> no char. scale

-> power laws, eg:

2x the radius,

3x the #neighbors nn(r)

$$nn(r) = C\ r^{\log 3/\log 2}$$



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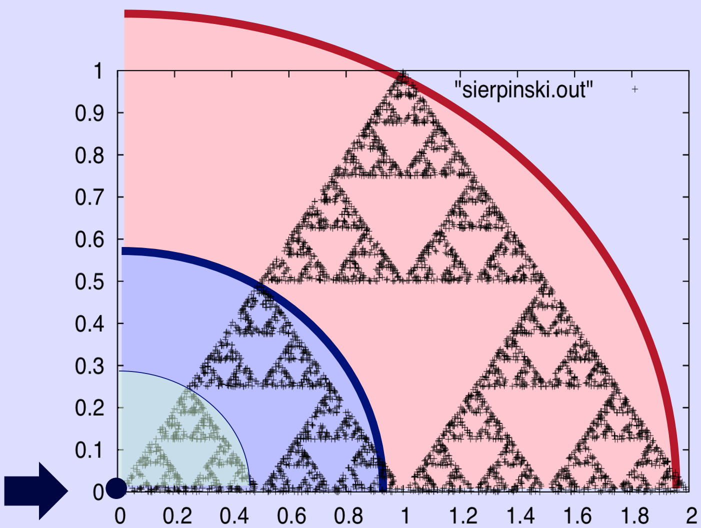

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Self-similarity -> no char. scale

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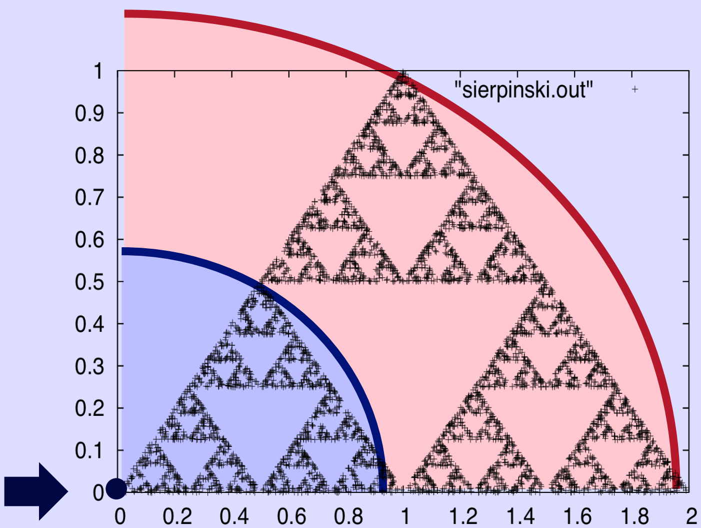
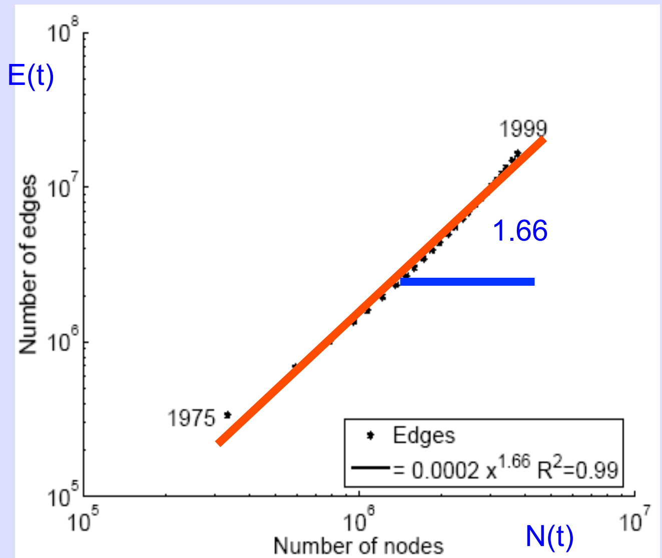
$nn = C\ r^{\log 3/\log 2}$

Reminder:
Densification P.L.
(2x nodes, ~3x edges)





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Self-similarity -> no char. scale

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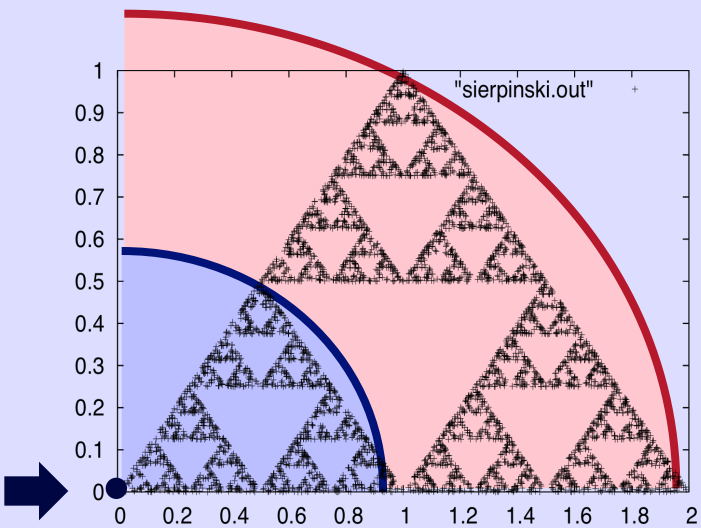
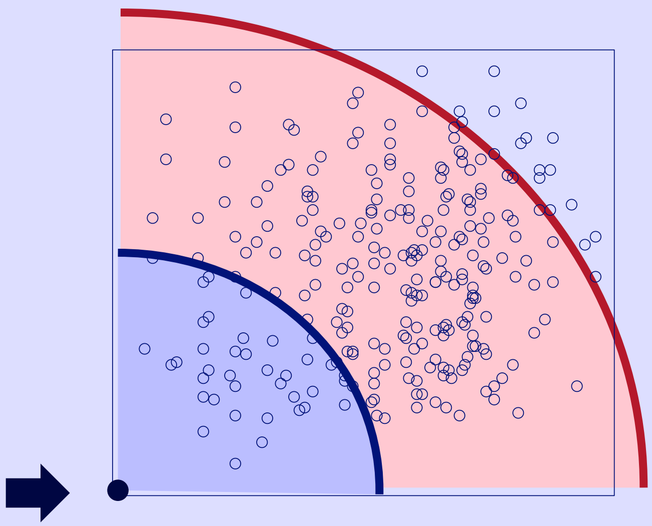
$$nn = C\ r^{\log 3/\log 2}$$

2x the radius,

4x neighbors

$$nn = C\ r^{\log 4/\log 2} = C\ r^{2}$$





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20'' intro to fractals

Self-similarity -> no char. scale

-> power laws, eg:

2x the radius,

3x the #neighbors

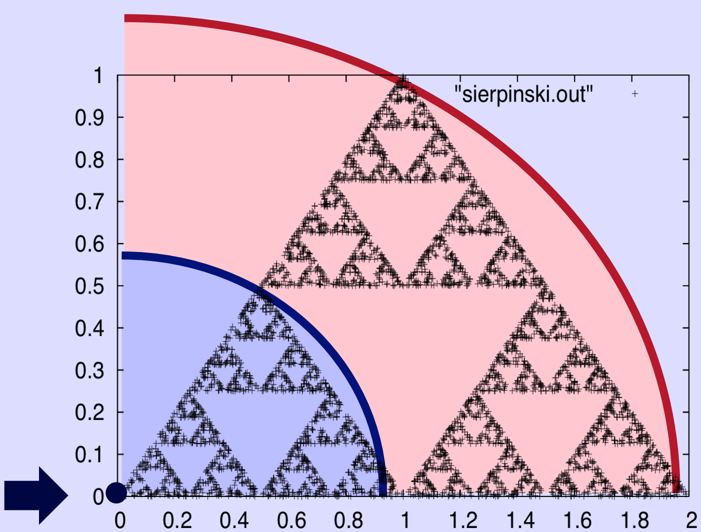
$nn = C\, r^{\log 3/\log 2}$ =1.58

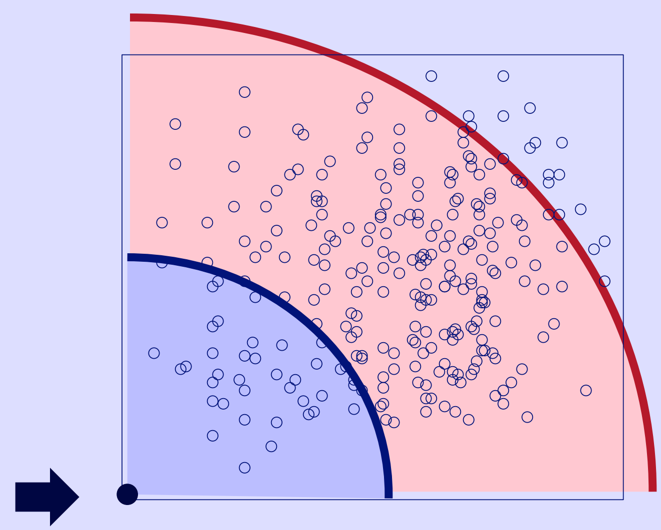
2x the radius,

4x neighbors

$nn = C\, r^{\log 4/\log 2} = C\, r^{2}$

Fractal dim.





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Fractal dim.

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How does self-similarity help in graphs?

- A: RMAT/Kronecker generators
  - With self-similarity, we get all power-laws, automatically,
  - And small/shrinking diameter
  - And `no good cuts'

R-MAT: A Recursive Model for Graph Mining,
by D. Chakrabarti, Y. Zhan and C. Faloutsos,
SDM 2004, Orlando, Florida, USA

Realistic, Mathematically Tractable Graph Generation and Evolution, Using Kronecker Multiplication,
by J. Leskovec, D. Chakrabarti, J. Kleinberg,
and C. Faloutsos, in PKDD 2005, Porto, Portugal

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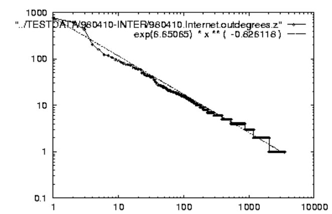
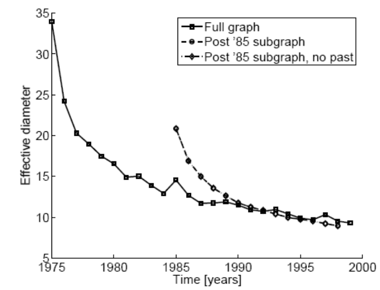
Graph gen.: Problem dfn

- Given a growing graph with count of nodes N_1, N_2, …
- Generate a realistic sequence of graphs that will obey all the patterns
  - Static Patterns
    - S1 Power Law Degree Distribution
    - S2 Power Law eigenvalue and eigenvector distribution
    - Small Diameter
  - Dynamic Patterns
    - T2 Growth Power Law (2x nodes; 3x edges)
    - T1 Shrinking/Stabilizing Diameters





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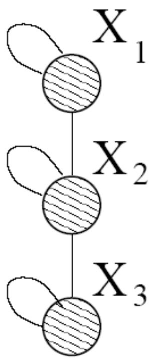
Kronecker Graphs

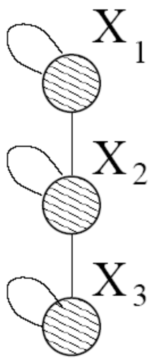


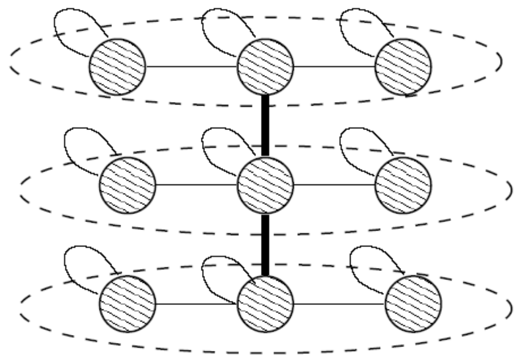
1	1	0
1	1	1
0	1	1

G_1

Adjacency matrix

Kronecker Graphs





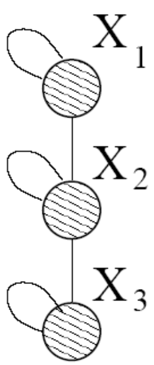
Intermediate stage

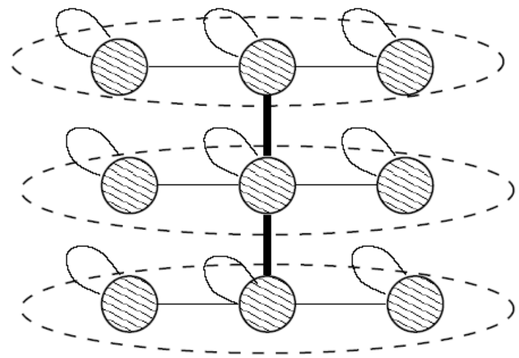
1	1	0
1	1	1
0	1	1

G_1

Adjacency matrix

Kronecker Graphs





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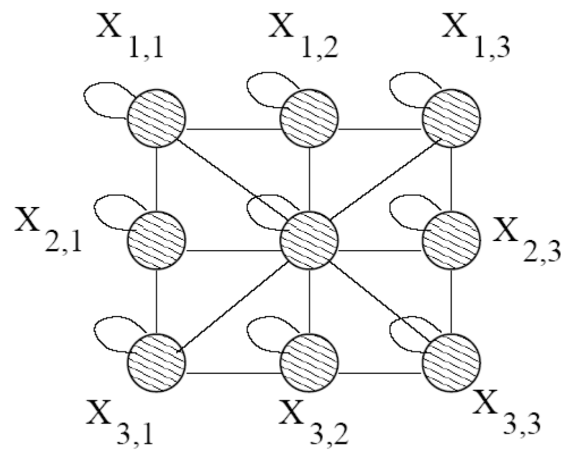


1	1	0
1	1	1
0	1	1

G_1

Adjacency matrix

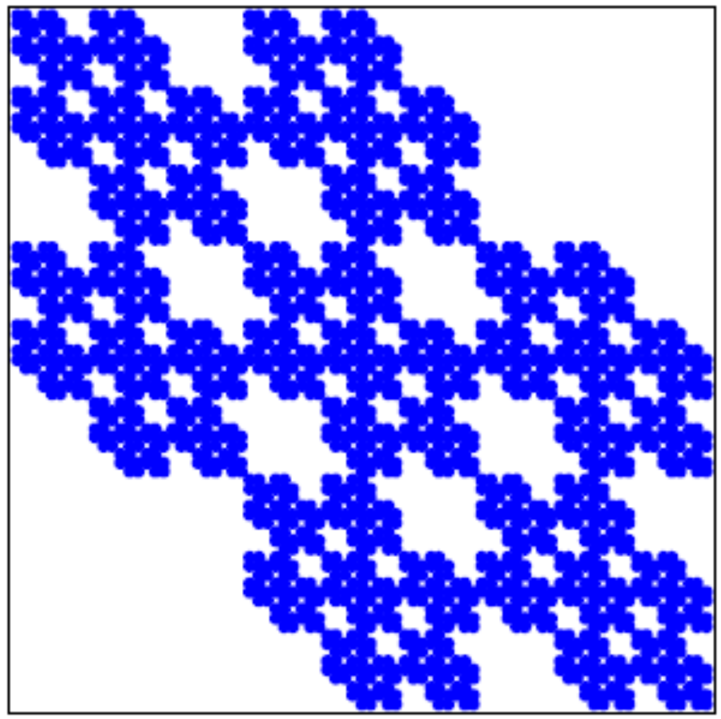
G_1	G_1	0
G_1	G_1	G_1
0	G_1	G_1

$G_2 = G_1 \otimes G_1$

Adjacency matrix

Kronecker Graphs

- Continuing multiplying with G_1 we obtain G_4 and so on …



G_4 adjacency matrix

Kronecker Graphs

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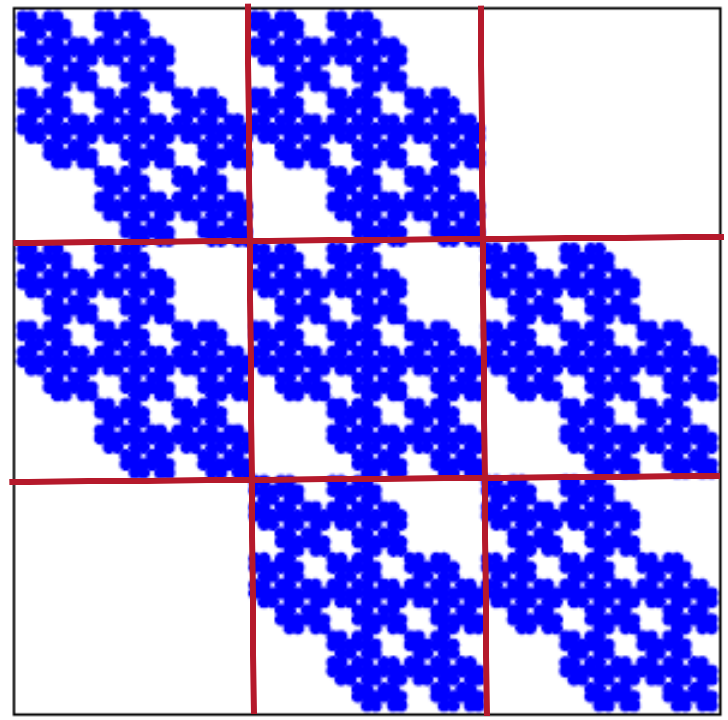

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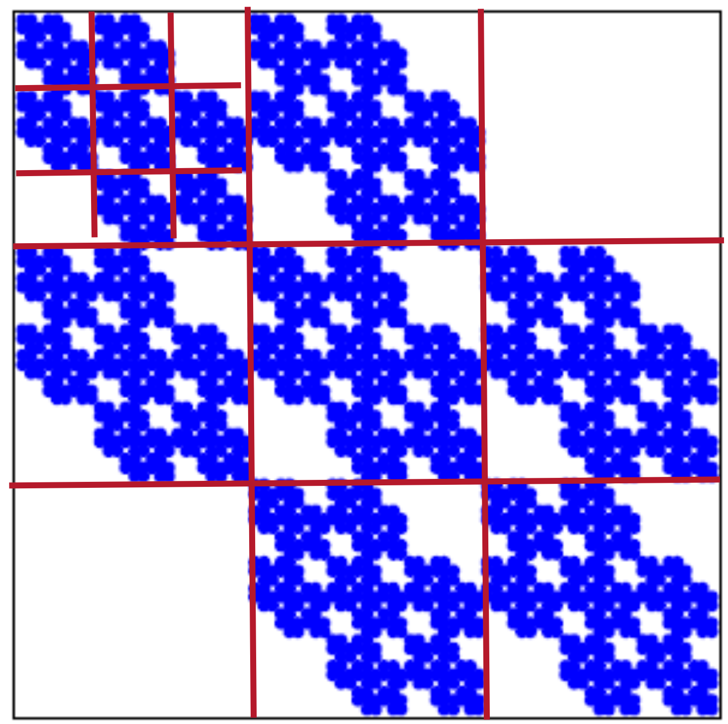

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Kronecker Graphs

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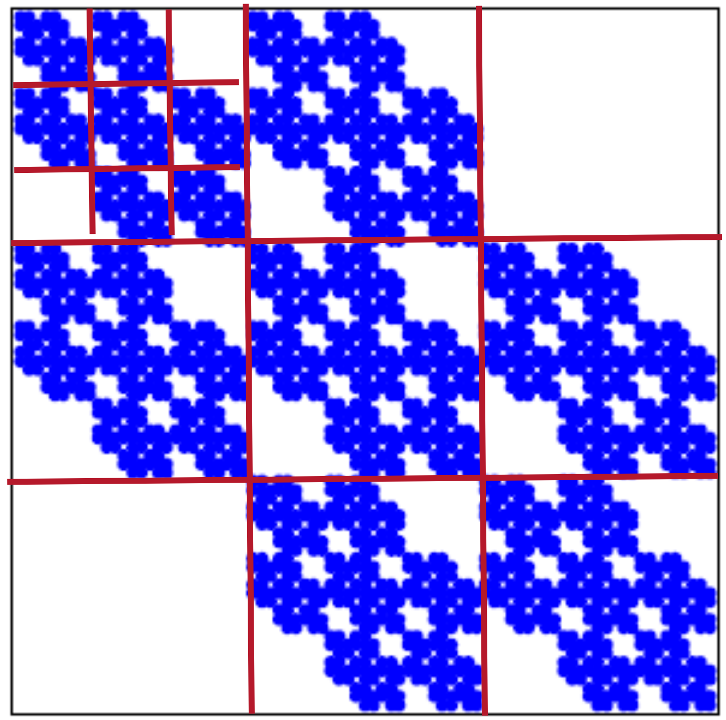
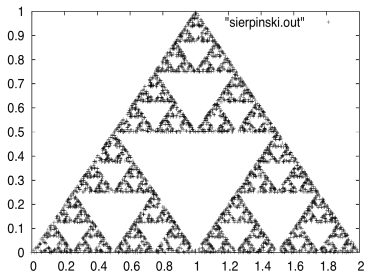
Holes within holes; Communities within communities



G_4 adjacency matrix



Properties:

- We can PROVE that
  - Degree distribution is multinomial ~ power law
  - new – Diameter: constant
  - Eigenvalue distribution: multinomial
  - First eigenvector: multinomial

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Problem Definition

- Given a growing graph with nodes N_1, N_2, …
- Generate a realistic sequence of graphs that will obey all the patterns
  - Static Patterns
    - ✓ Power Law Degree Distribution
    - ✓ Power Law eigenvalue and eigenvector distribution
    - ✓ Small Diameter
  - Dynamic Patterns
    - ✓ Growth Power Law
    - ✓ Shrinking/Stabilizing Diameters
- First generator for which we can **prove** all these properties

Impact: Graph500

- Based on RMAT (= 2x2 Kronecker)
- Standard for graph benchmarks
- http://www.graph500.org/
- Competitions 2x year, with all major entities: LLNL, Argonne, ITC-U. Tokyo, Riken, ORNL, Sandia, PSC, …

To iterate is human, to recurse is divine

R-MAT: A Recursive Model for Graph Mining,
by D. Chakrabarti, Y. Zhan and C. Faloutsos,
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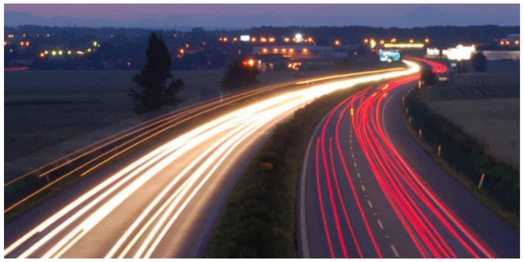
Roadmap



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A: real graphs ->
self similar ->
power laws

Q2: Why 'no good cuts'?

- A: self-similarity
  - Communities within communities within communities …

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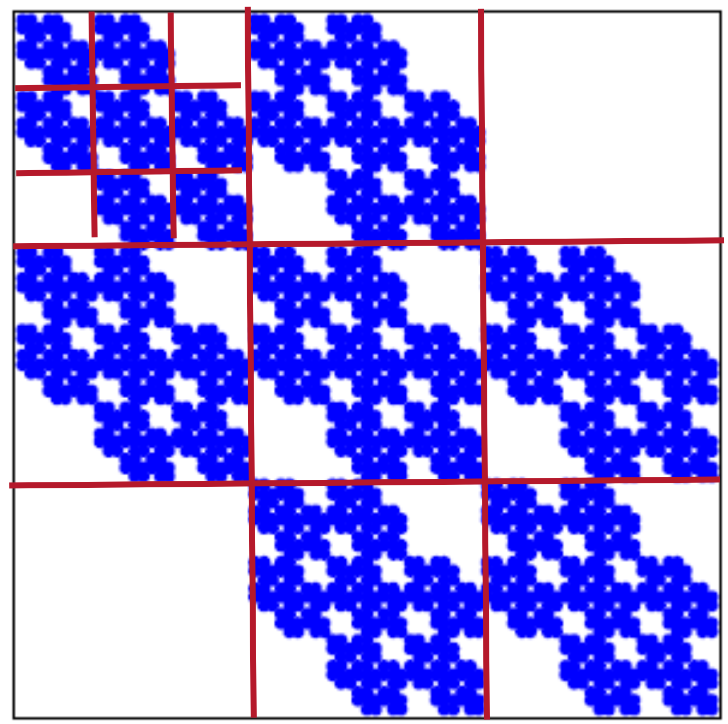
REMINDER

Kronecker Product – a Graph

- Continuing multiplying with G_1 we obtain G_4 and so on …



G_4 adjacency matrix

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REMINDER

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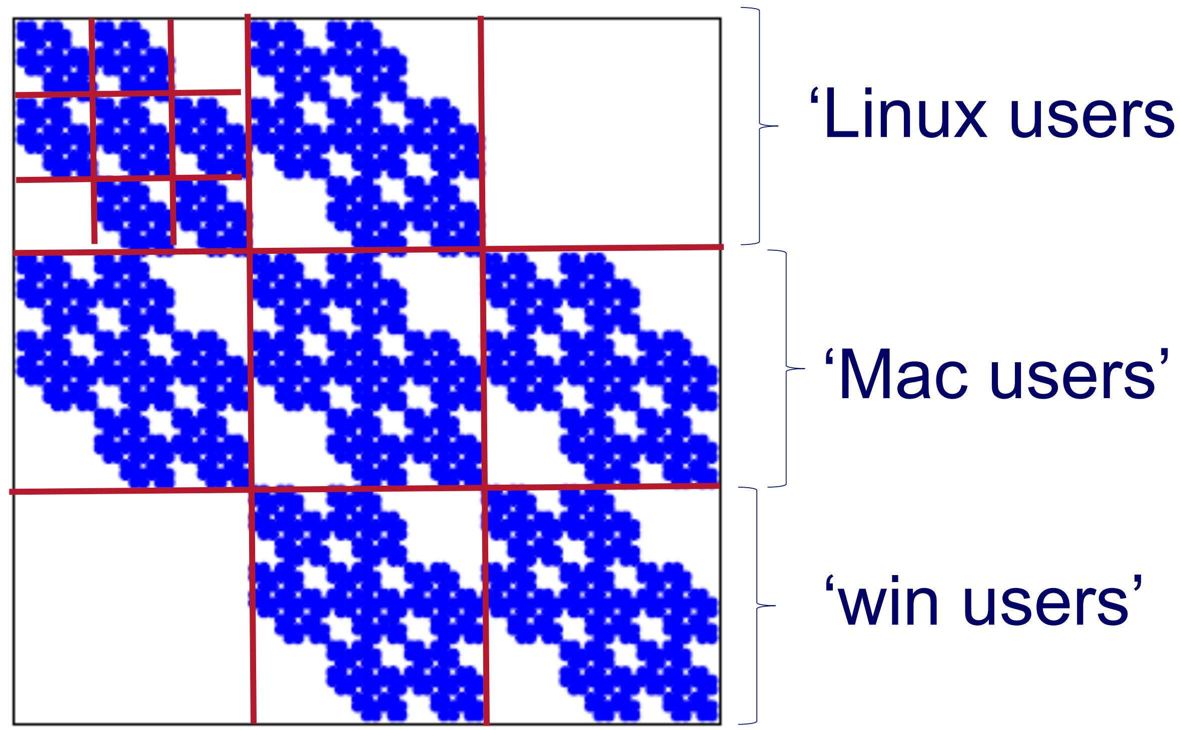

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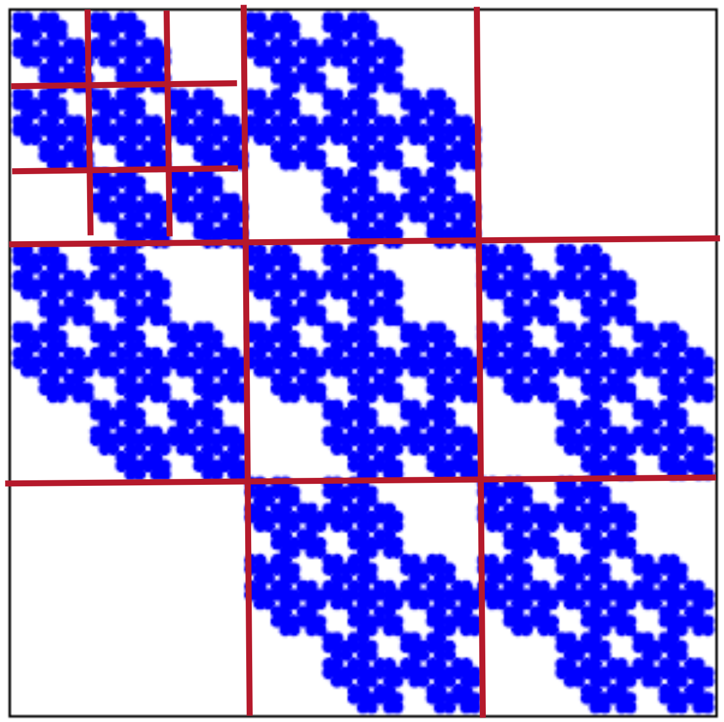

How many Communities?
3?
9?
27?

G_4 adjacency matrix

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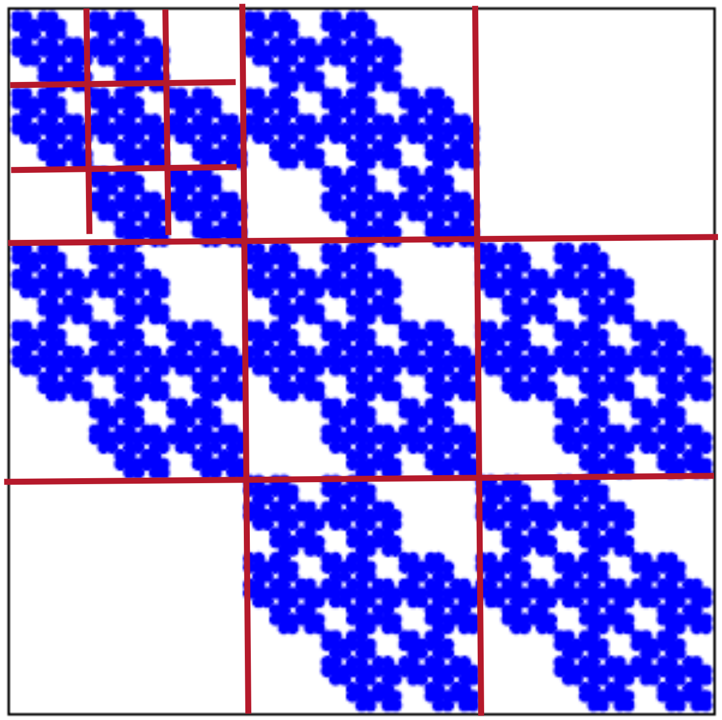
REMINDER

Kronecker Product – a Graph

- Continuing multiplying with G_1 we obtain G_4 and so on …

Communities within communities within communities …

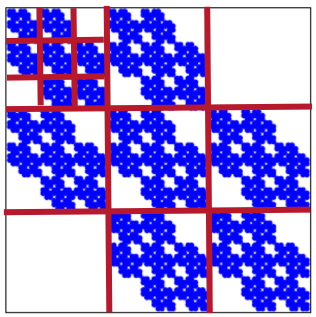


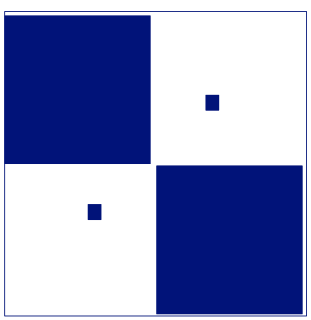
G_4 adjacency matrix

How many Communities?
3?
9?
27?

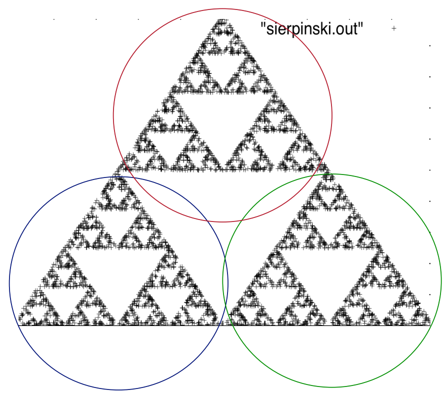
A: one – but not a typical, block-like community…

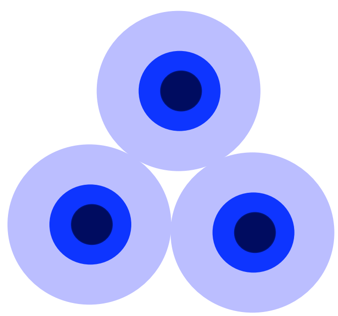
Communities?



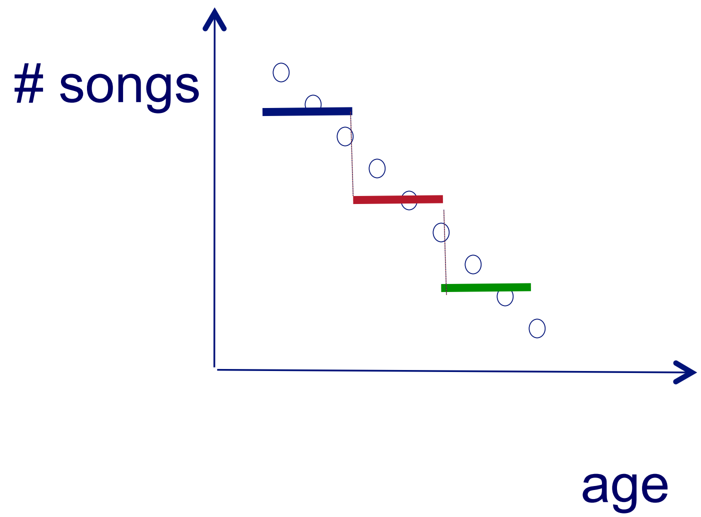


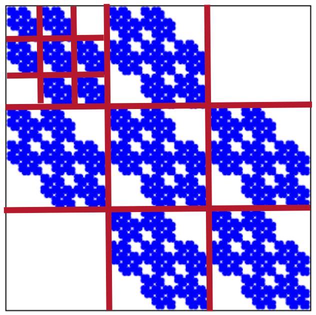
(Gaussian) Clusters?

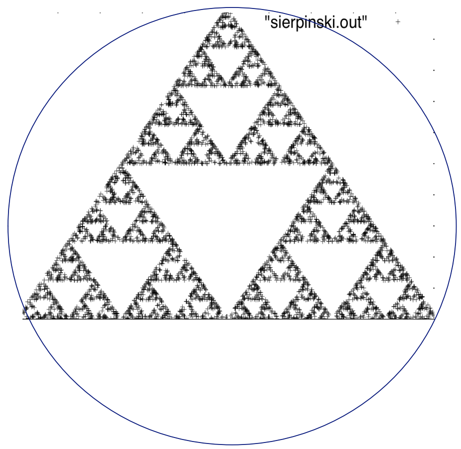


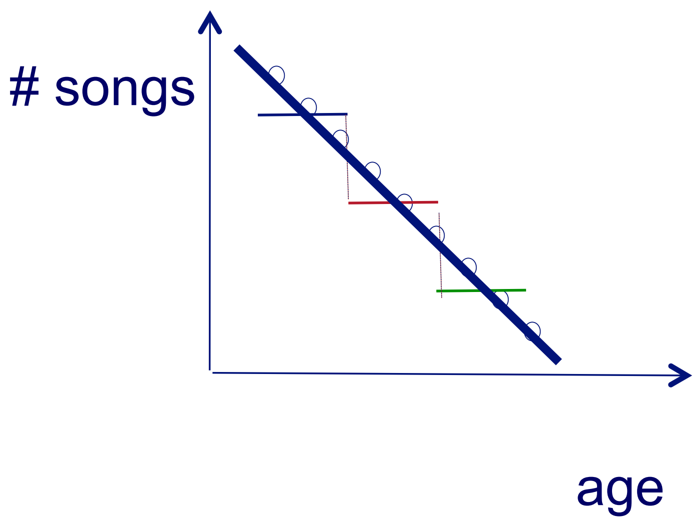


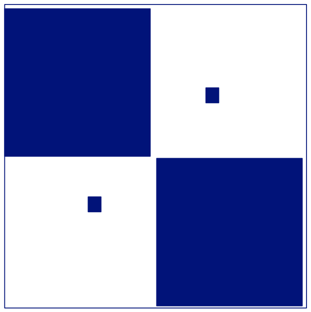
Piece-wise flat parts?











Wrong questions to ask!

Summary of Part#1

- *many* patterns in real graphs
  - Small & shrinking diameters
  - Power-laws everywhere
  - Gaussian trap
  - ‘no good cuts’
- Self-similarity (RMAT/Kronecker): good model

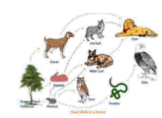
Summary of Part#1

- *many* patterns in real graphs
  - Small & shrinking diameters **90% Trust Intuition**
  - Power-laws everywhere **Take logarithms!**
  - Gaussian trap **Mode << Avg << Max**
  - 'no good cuts'
- Self-similarity (RMAT/Kronecker): good model

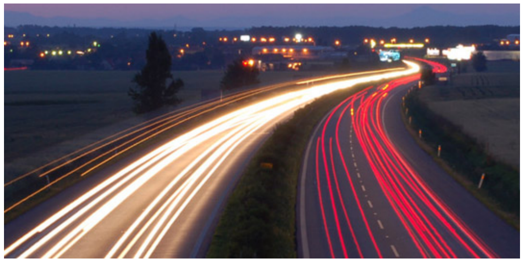
Part 2: Cascades & Immunization

Why do we care?

- Information Diffusion
- Viral Marketing
- Epidemiology and Public Health
- Cyber Security
- Human mobility
- Games and Virtual Worlds
- Ecology
- ....



Roadmap



- Introduction – Motivation
- Part#1: Patterns in graphs
- Part#2: Cascade analysis
  - (Fractional) Immunization
  - Epidemic thresholds
- Conclusions

Fractional Immunization of Networks

B. Aditya Prakash, Lada Adamic, Theodore

Iwashyna (M.D.), Hanghang Tong,
Christos Faloutsos

SDM 2013, Austin, TX

Whom to immunize?

- Dynamical Processes over networks

- Each circle is a hospital
- ~3,000 hospitals
- More than 30,000 patients transferred

[US-MEDICARE NETWORK 2005]

Problem: Given *k* units of disinfectant, whom to immunize?

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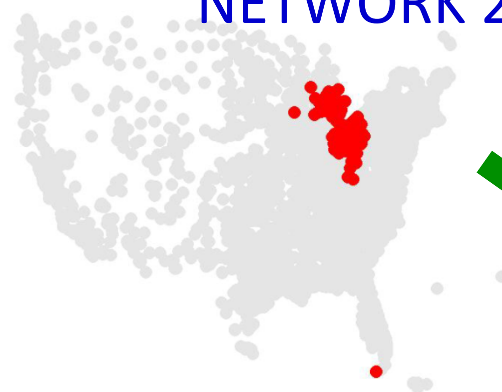
Whom to immunize?

~6x fewer!

[US-MEDICARE NETWORK 2005]

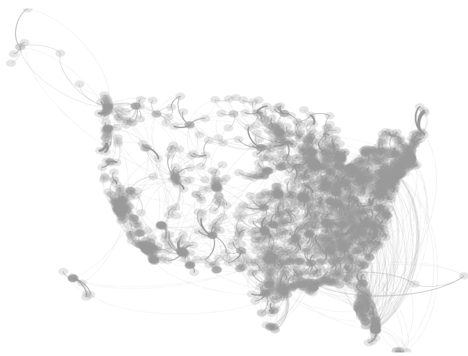


CURRENT PRACTICE

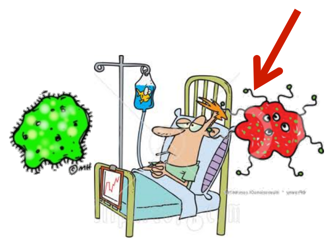
OUR METHOD

Hospital-acquired inf. : 99K+ lives, $5B+ per year

Fractional Asymmetric Immunization



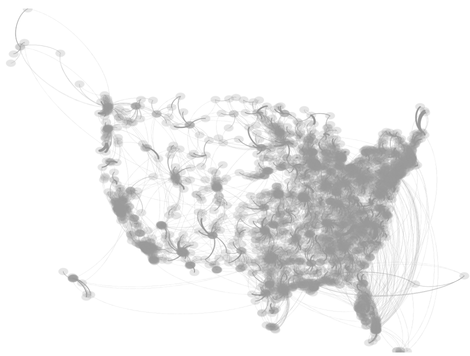
Drug-resistant Bacteria
(like XDR-TB)

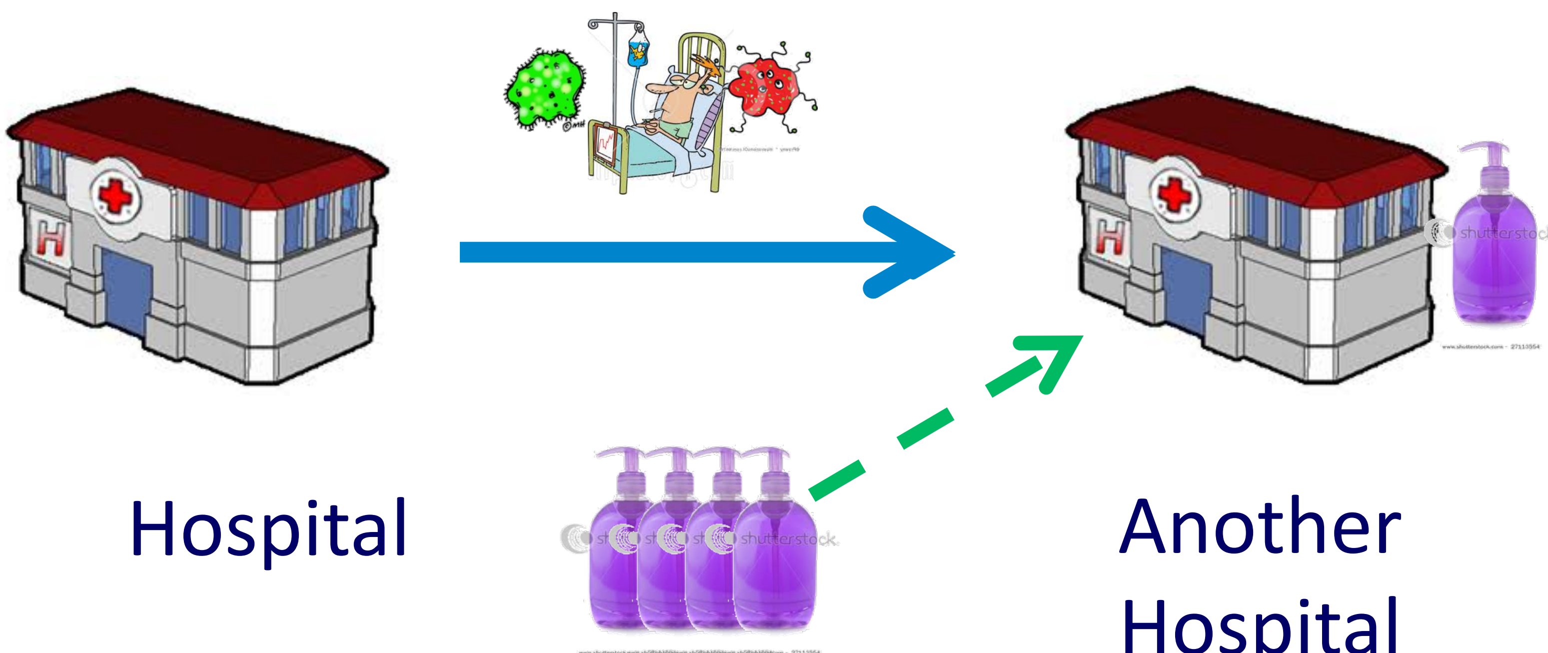


Hospital

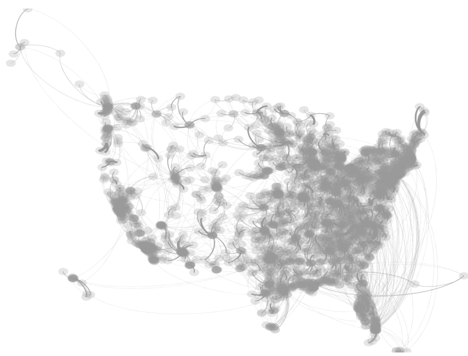
Another Hospital

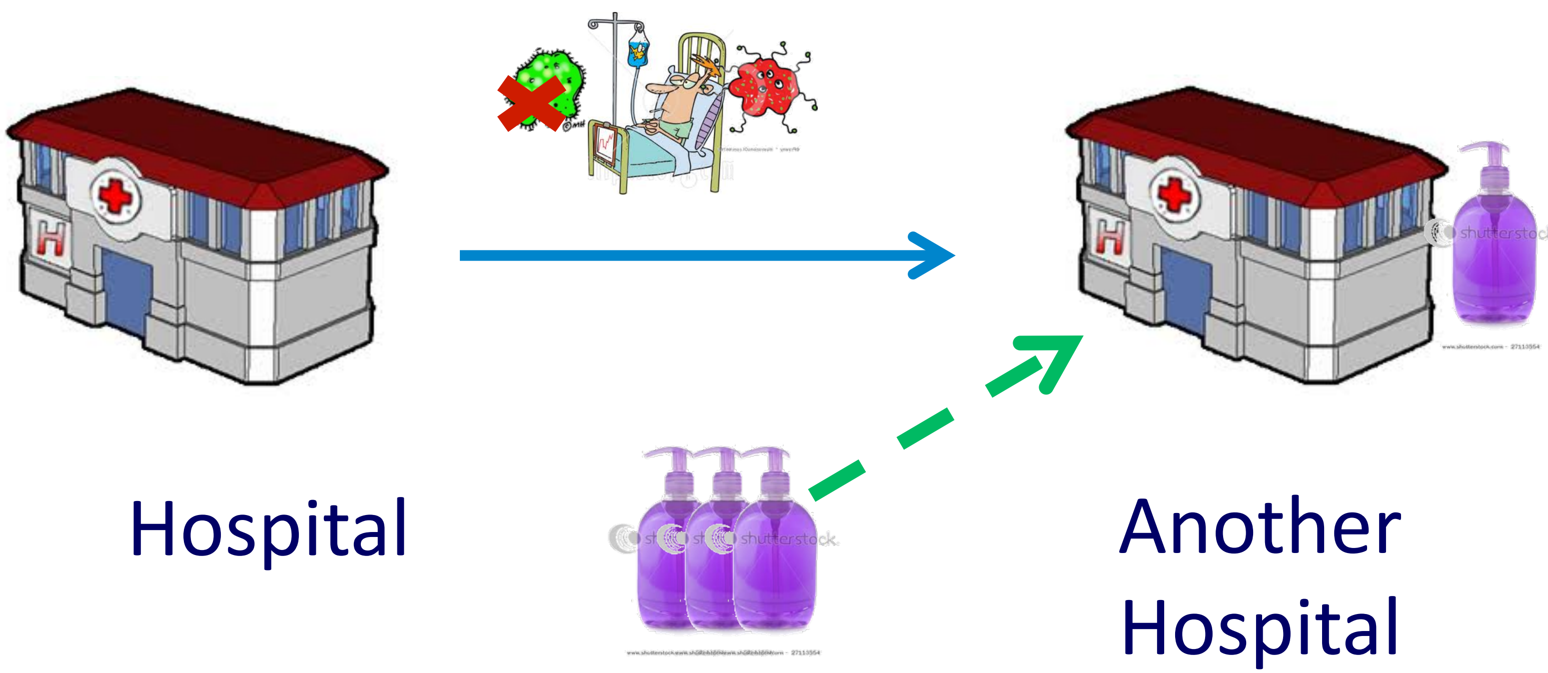
Fractional Asymmetric Immunization



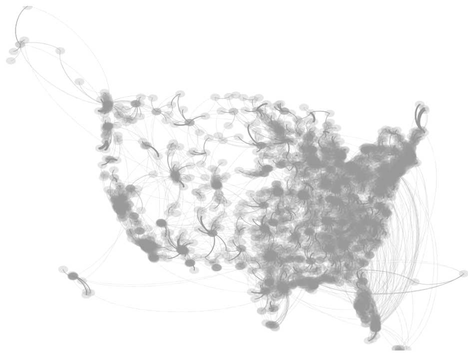


Fractional Asymmetric Immunization



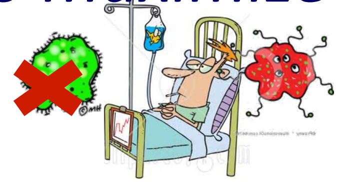


Fractional Asymmetric Immunization



***Problem**:*

Given k units of disinfectant, distribute them to maximize hospitals saved



Hospital

Another Hospital

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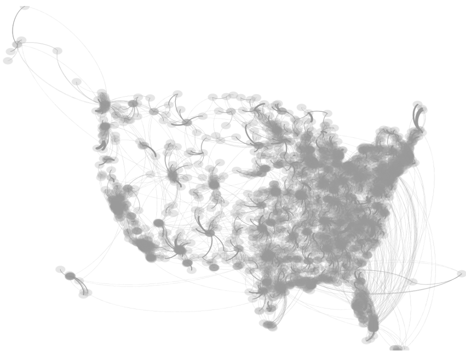
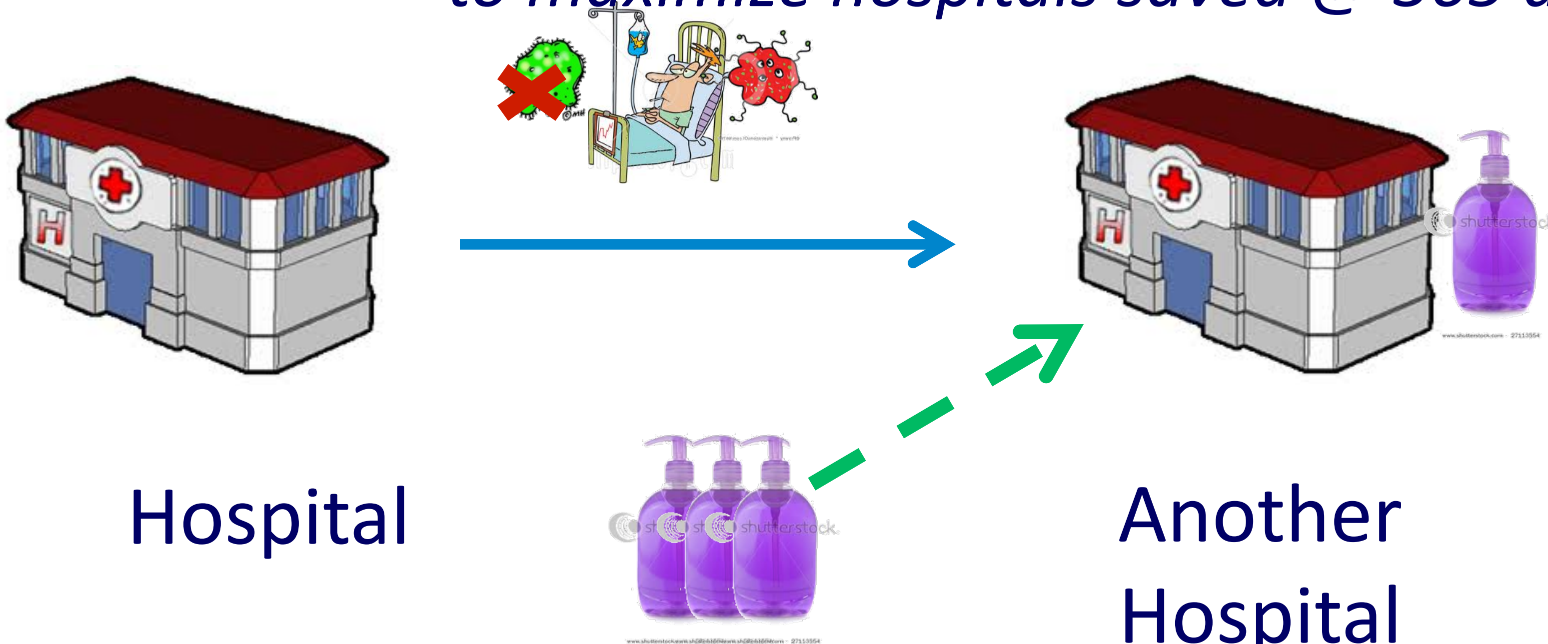
Fractional Asymmetric Immunization



***Problem**:*

Given k units of disinfectant,
distribute them
to maximize hospitals saved @ 365 days



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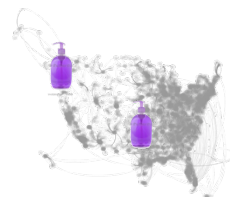
Straightforward solution:

Simulation:

1. Distribute resources
2. ‘infect’ a few nodes
3. Simulate evolution of spreading
   – (10x, take avg)
4. Tweak, and repeat step 1



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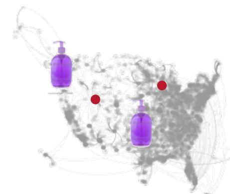

Carnegie Mellon

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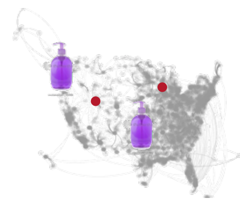

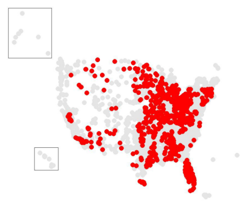

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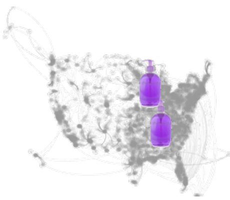
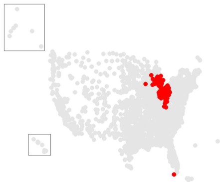
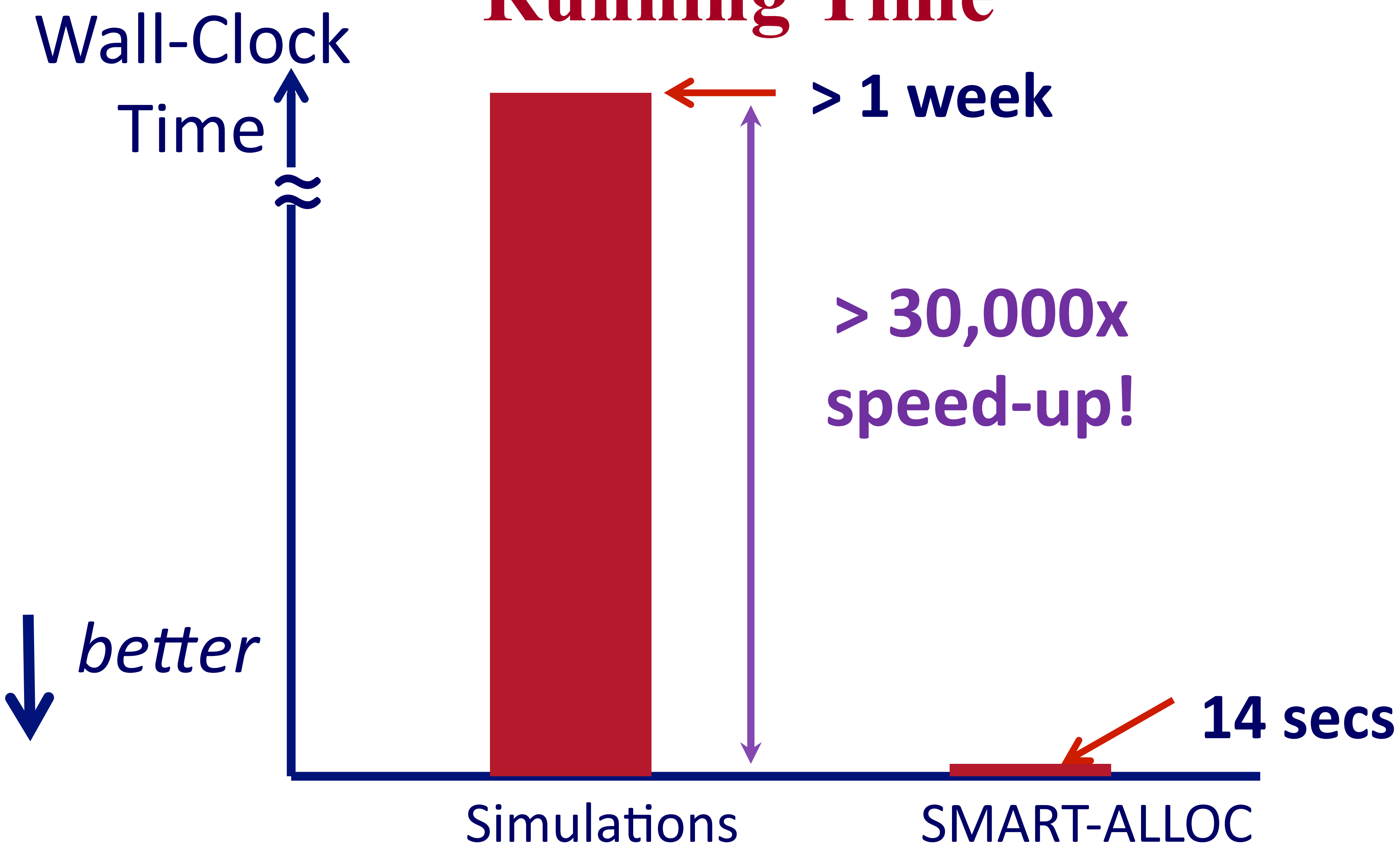
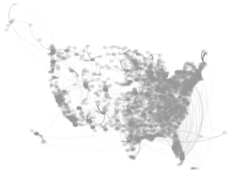




Running Time



Experiments

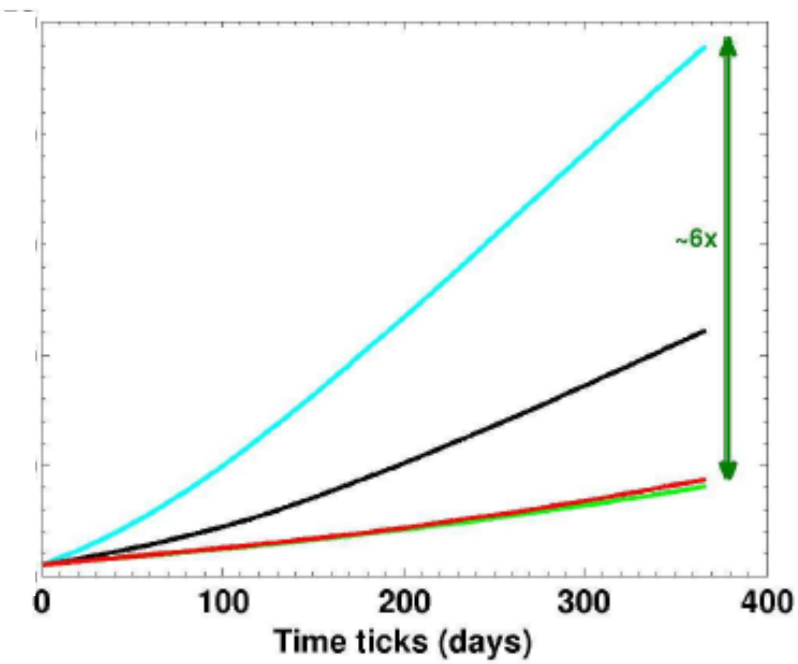


uniform

better

SMART-ALLOC

infected



epochs

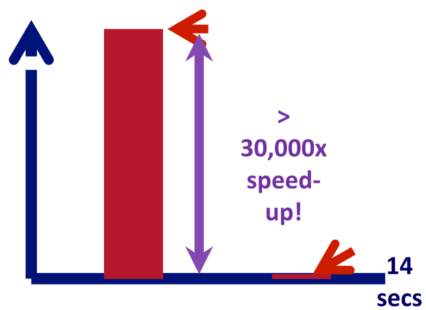
K = 120

What is the 'silver bullet'?

A: Try to decrease connectivity of graph

Q: how to measure connectivity?

- Avg degree? Max degree?
- Std degree / avg degree ?
- Diameter?
- Modularity?
- 'Conductance' (~min cut size)?
- Some combination of above?



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What is the 'silver bullet'?

A: Try to decrease connectivity of graph

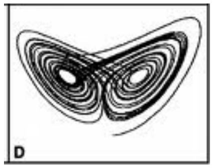
Q: how to measure connectivity?

A: first **eigenvalue** of adjacency matrix

Avg degree
Max degree
Diameter
Modularity
'Conductance'

Q1: why??

(Q2: dfn & intuition of eigenvalue ?)

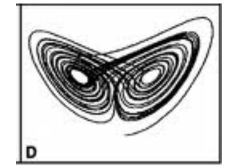


Why eigenvalue?

A1: ‘G2’ theorem and ‘**eigen-drop**’:

- For (almost) **any** type of virus
- For **any** network
- -> no epidemic, if small-enough first eigenvalue (λ_1) of *adjacency* matrix

Threshold Conditions for Arbitrary Cascade Models on Arbitrary Networks, B. Aditya Prakash, Deepayan Chakrabarti, Michalis Faloutsos, Nicholas Valler, Christos Faloutsos, ICDM 2011, Vancouver, Canada



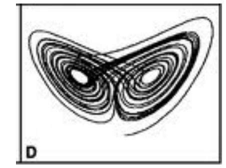
Why eigenvalue?

A1: ‘G2’ theorem and ‘**eigen-drop**’:

- For (almost) **any** type of virus
- For **any** network
- -> no epidemic, if small-enough first eigenvalue (λ_1) of *adjacency* matrix

- Heuristic: for immunization, try to min λ_1
- The smaller λ_1, the closer to extinction.

G2 theorem



Threshold Conditions for Arbitrary Cascade Models on Arbitrary Networks

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IEEE ICDM 2011, Vancouver

extended version, in arxiv

http://arxiv.org/abs/1004.0060

~10 pages proof

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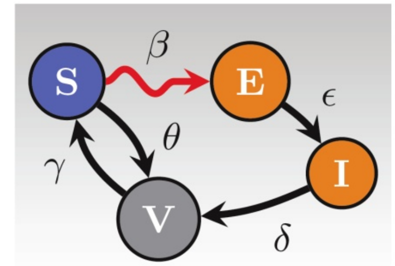
Our thresholds for some models

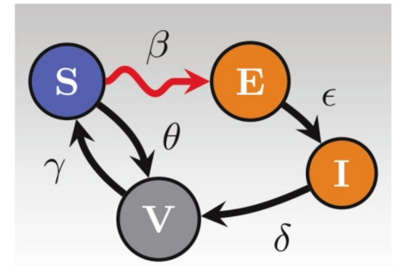


- *s = effective strength*
- *s < 1 : below threshold*

Models	Effective Strength (s)	Threshold (tipping point)
SIS, SIR, SIRS, SEIR	$s = \lambda \left(\frac{\beta}{\delta}\right)$	
SIV, SEIV	$s = \lambda \,.\, \left(\frac{\beta\gamma}{\delta(\gamma+\theta)}\right)$	$s = 1$
$SI_1I_2V_1V_2$ (**H.I.V.**)	$s = \lambda \,.\, \left(\frac{\beta_1 v_2 + \beta_2 \varepsilon}{v_2(\varepsilon + v_1)}\right)$	

Our thresholds for some models

- *s = effective strength*
- *s < 1 : below threshold*



No immunity

Temp. immunity

w/ incubation

	…e Strength (s)	Threshold (tipping point)
SIS, SIR, SIRS, SEIR	$s = \lambda \left(\frac{\beta}{\delta}\right)$	$s = 1$
SIV, SEIV	$s = \lambda \cdot \left(\frac{\beta\gamma}{\delta(\gamma+\theta)}\right)$	
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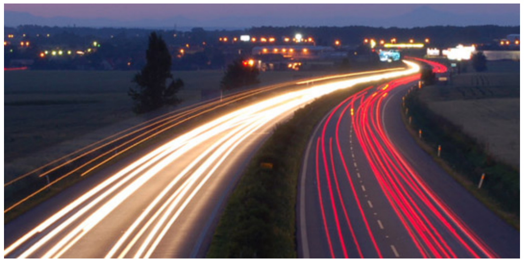
Roadmap



- Introduction – Motivation
- Part#1: Patterns in graphs
- Part#2: Cascade analysis
  - (Fractional) Immunization
  - ➡ intuition behind λ_1
- Conclusions

Intuition for λ

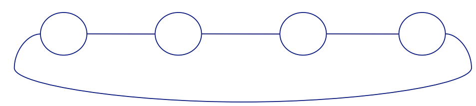
"Official" definitions:

- *Let **A** be the adjacency matrix. Then λ is the root with the largest magnitude of the characteristic polynomial of **A** [det(**A** – x**I**)].*
- Also: $\mathbf{A}\ \mathbf{x} = \lambda\ \mathbf{x}$

Neither gives much intuition!

"Un-official" Intuition

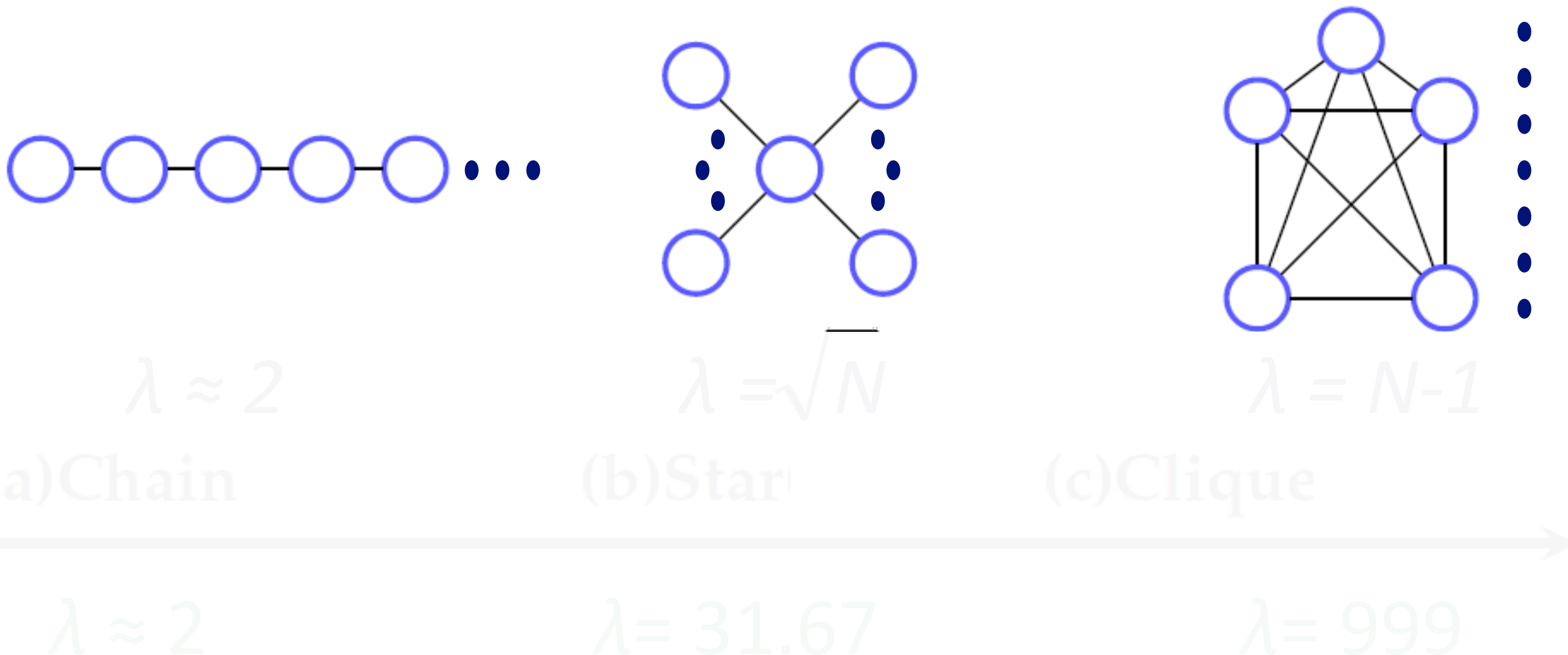
- For 'homogeneous' graphs, λ == *degree*



- λ ~ avg degree
  - done right, for skewed degree distributions

Largest Eigenvalue (λ)

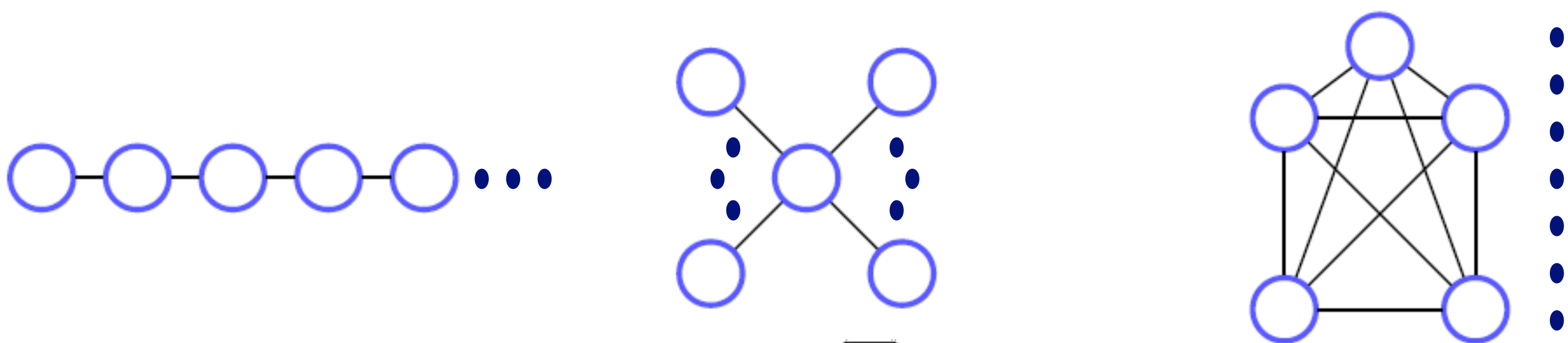
better connectivity ⟶ higher λ



N = 1000 nodes

Largest Eigenvalue (λ)

better connectivity ⟶ higher λ



$\lambda \approx 2$ | $\lambda = \sqrt{N}$ | $\lambda = N-1$

(a)Chain (b)Star (c)Clique

$\lambda \approx 2$ | $\lambda = 31.67$ | $\lambda = 999$

N = 1000 nodes

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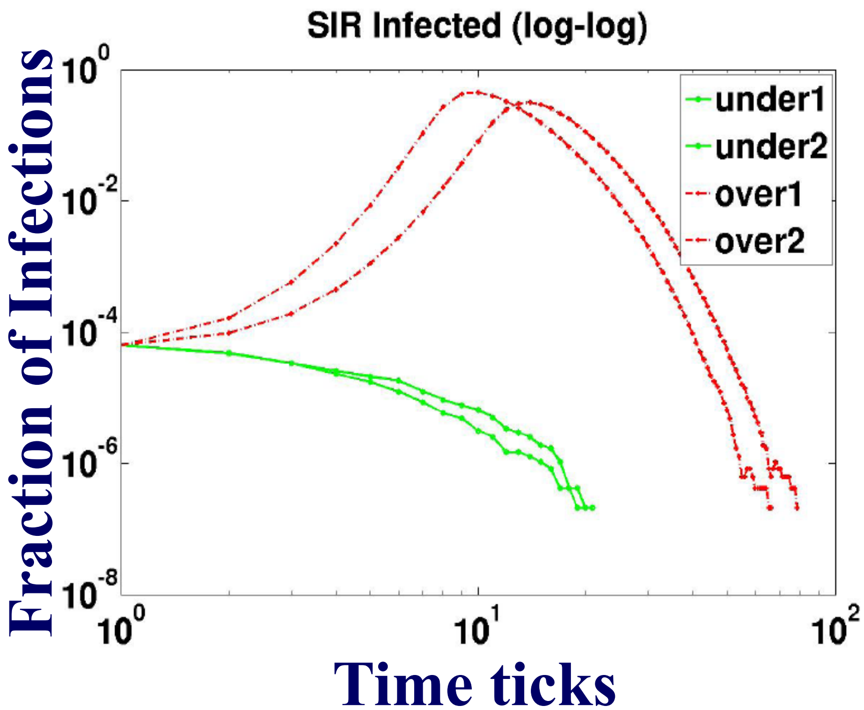
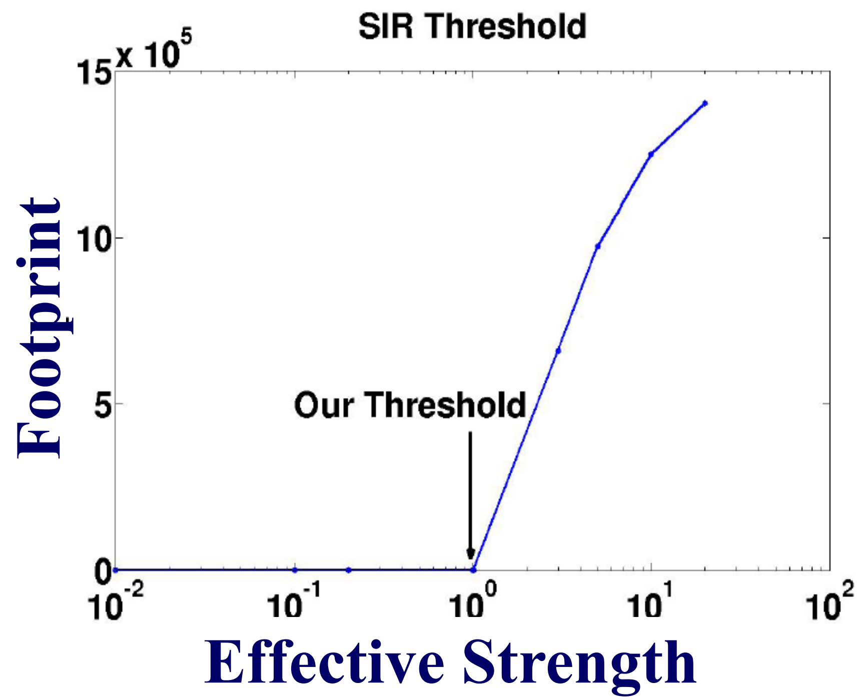
Examples: Simulations – SIR (mumps)





(a) Infection profile **(b) "Take-off" plot**

PORTLAND graph: *synthetic population,*

31 million links, 6 million nodes

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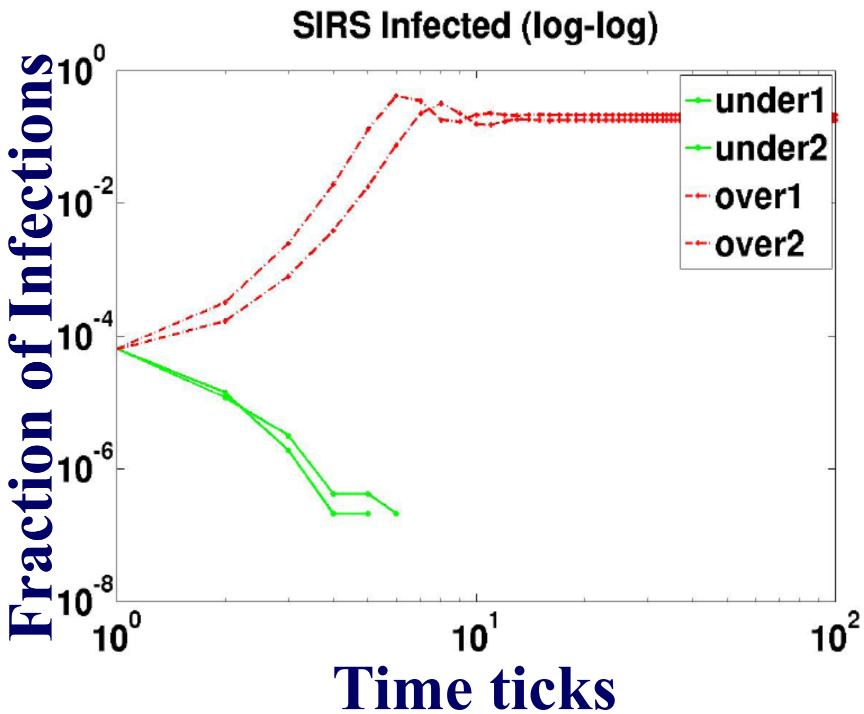
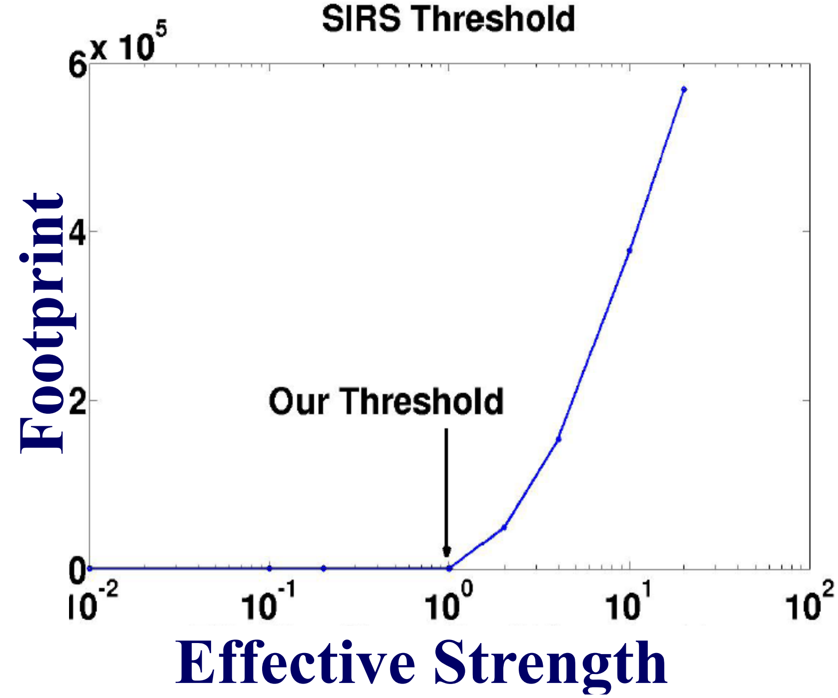
Examples: Simulations – SIRS (pertusis)





(a) Infection profile **(b) "Take-off" plot**

PORTLAND graph: *synthetic population,*

31 <u>million</u> links, 6 <u>million</u> nodes

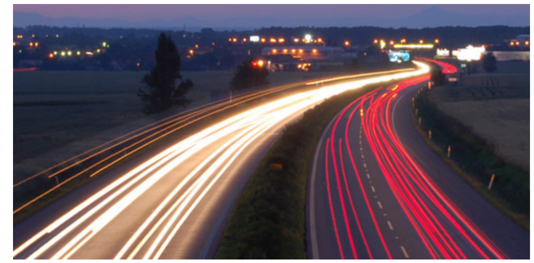
Immunization - conclusion

In (**almost any**) immunization setting,

- Allocate resources, such that to
- **Minimize** λ_1
- (*regardless* of virus specifics)

- Conversely, in a market penetration setting
  - Allocate resources to
  - Maximize λ_1

Roadmap



- Introduction – Motivation
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  - Epidemic thresholds
- ➡ Acks & Conclusions

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Thanks

Disclaimer: All opinions are mine; not necessarily reflecting the opinions of the funding agencies

Thanks to: NSF IIS-0705359, IIS-0534205, CTA-INARC; Yahoo (M45), LLNL, IBM, SPRINT, Google, INTEL, HP, iLab

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Project info: PEGASUS

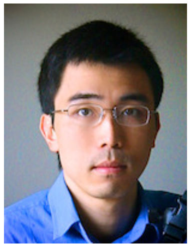
www.cs.cmu.edu/~pegasus

Results on large graphs: with Pegasus + hadoop + M45

Apache license

Code, papers, manual, video

Prof. U Kang



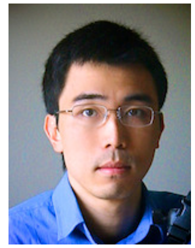
Prof. Polo Chau

Carnegie Mellon

Cast

Akoglu, Leman

Beutel, Alex



Chau, Polo

Kang, U

Koutra, Danai

McGlohon, Mary

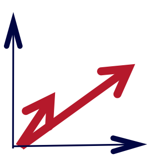
Prakash, Aditya

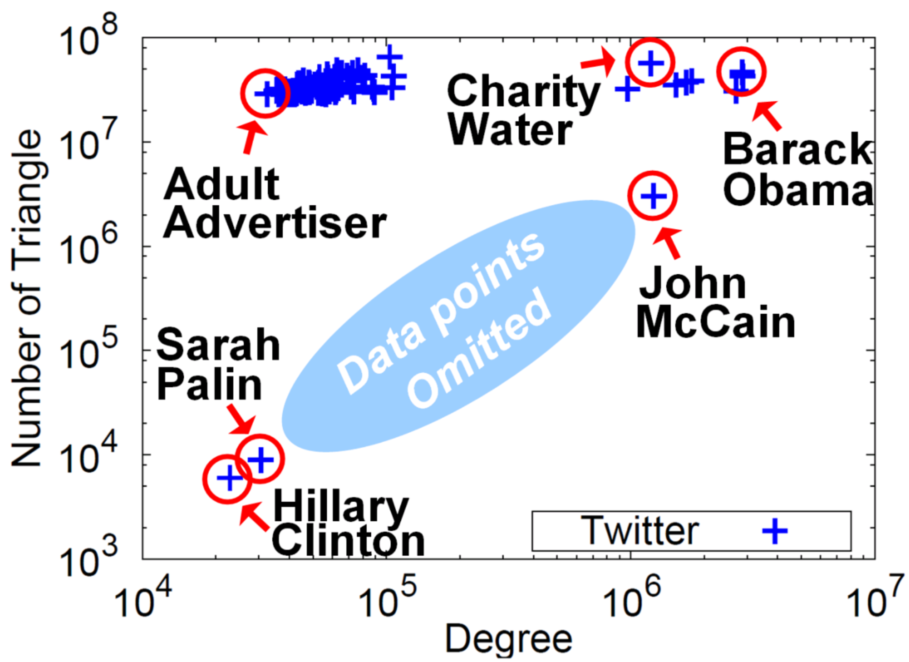
Papalexakis, Vagelis

Tong, Hanghang

CONCLUSION#1 – Big data

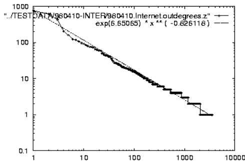
- **Large** datasets reveal patterns/outliers that are invisible otherwise

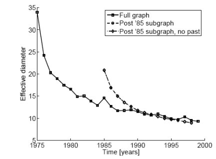


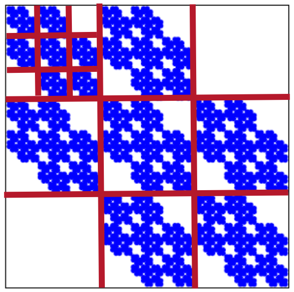


CONCLUSION#2 – self-similarity

- powerful tool / viewpoint
  - Power laws; shrinking diameters
  - **Gaussian trap** (eg., F.O.F.)
  - 'no good cuts'
  - RMAT – graph500 generator



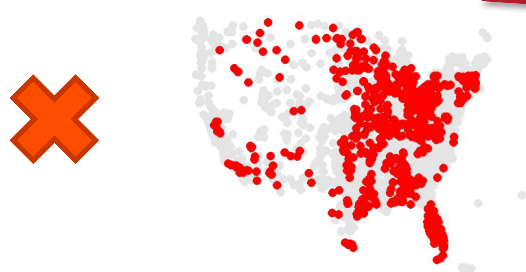




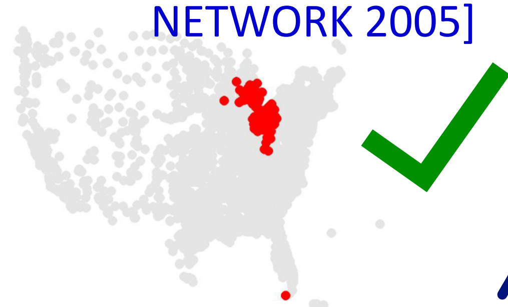
CONCLUSION#3 – eigen-drop

- Cascades & immunization: G2 theorem & **eigenvalue**

~6x fewer!

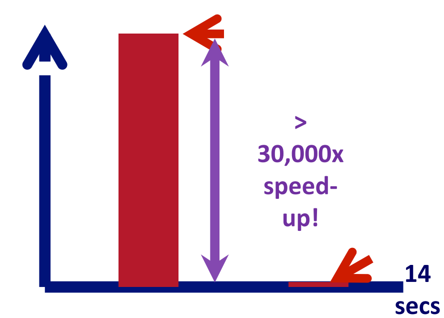


[US-MEDICARE NETWORK 2005]



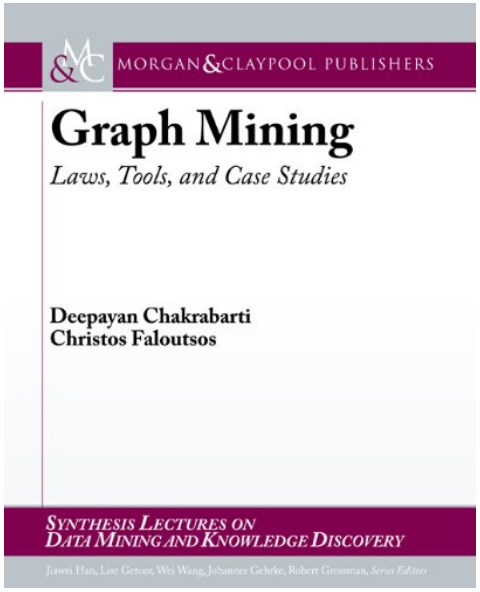
CURRENT PRACTICE

OUR METHOD



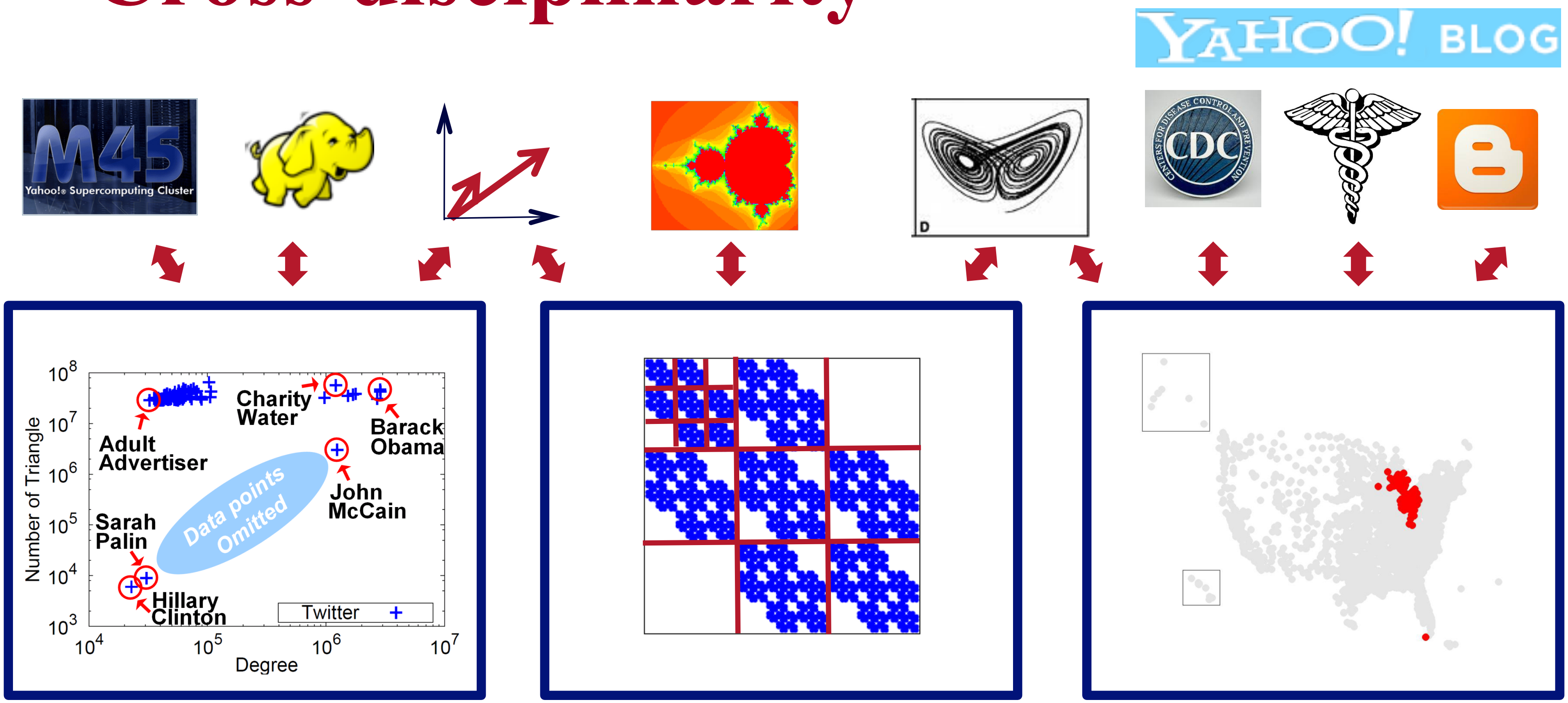
References

- D. Chakrabarti, C. Faloutsos: *Graph Mining – Laws, Tools and Case Studies*, Morgan Claypool 2012
- http://www.morganclaypool.com/doi/abs/10.2200/S00449ED1V01Y201209DMK006



TAKE HOME MESSAGE:

Cross-disciplinarity



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QUESTIONS?

Cross-disciplinarity